COS 341, December 2, 1998
Due: December 9, 1998

Homework Set 8

Reading Assignments Read Chapter 9.

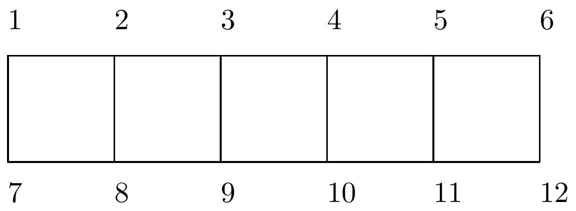
Written Assignments Do Exercises 5, 18, 19, 30 and 42 in Chapter 12.6.

Remark In your solution to Exercise 5, be sure to give a rigorous justification of your claim.

Special Problem 1 [Counted as 2 exercises] For any $n > 0$, let G_n be the ladder graph with n rungs, which has $2n$ vertices and $3n - 2$ edges. (The ladder graph G_6 is shown below.) Let t_n be the number of spanning trees of G_n. Clearly, $t_1 = 1, t_2 = 4$. Let $T(x) = \sum_{n \geq 1} t_n x^n$.

(a) Determine the value of t_3 and t_4.
(b) Derive a closed-form formula for $T(x)$.

Hint: Let s_n be the number of spanning trees of G_n that contain the edge $\{n, 2n\}$. Note $s_1 = 1$, $s_2 = 3$. Find a recurrence relation involving both s_n's and t_n's.



Special Problem 2 [Counted as 2 exercises] Let $G = (V, E)$ where $V = \{v_1, v_2, v_3, v_4, v_5, w_1, w_2, w_3\}$ and E consists of the edges $\{v_i, v_j\}$, $1 \leq i, j \leq 5$ and $\{v_i, w_j\}, 1 \leq i \leq 3$, $1 \leq j \leq 3$. Answer each of the following questions; justify your answers.

(a) What is $\omega(G)$, the size of the largest clique?
(b) What is $\alpha(G)$, the size of the largest independent set?
(c) What is $\chi(G)$, the chromatic number?
(d) Is G a planar graph?
(e) Is G an Eulerian graph?
(f) Does G contain a Hamiltonian cycle?