COS 341, October 14, 1998
Due: October 21, 1998

# Homework Set 4

**Reading Assignments** Read Chapter 6.

**Written Assignments** Do exercises 5, 10, 19, 21, 29 and 30 in Section 5.7.

*Special Problem 1* (to be counted as 1 exercise) Let $n > 0$. Evaluate

$$f(n) = \sum_{0 \le k \le n-1} 2^k \frac{\binom{n-1}{k}}{\binom{n}{k}}.$$

Your answer should be a closed-form expression.

*Special Problem 2* (to be counted as 1 exercise) Let $N = 10^4$. Roulettes in the *Math Casino* have the following rule. For each run, a random integer $n$, $1 \le n \le N$ is uniformly generated. If $\lfloor n^{1/4} \rfloor$ divides $n$, you win \$5; otherwise, the \$1 bet you place is forfeited. Should you play the game? One way to make a rational decision is to calculate the probability $p$ of winning a game, and play the game if and only if $p \ge 1/6$. *Question:* What is the value of $p$?

**Remark** For this last problem, you have to solve the problem with pencil and paper; you should not use a computer or calculator.