COS 341, October 14, 1998 Due: October 21, 1998

Homework Set 4

Reading Assignments Read Chapter 6.

Written Assignments Do exercises 5, 10, 19, 21, 29 and 30 in Section 5.7.

Special Problem 1 (to be counted as 1 exercise) Let n > 0. Evaluate

$$f(n) = \sum_{0 \le k \le n-1} 2^k \frac{\binom{n-1}{k}}{\binom{n}{k}}.$$

Your answer should be a closed-form expression.

Special Problem 2 (to be counted as 1 exercise) Let $N = 10^4$. Roulettes in the Math Casino have the following rule. For each run, a random integer $n, 1 \le n \le N$ is uniformly generated. If $\lfloor n^{1/4} \rfloor$ divides n, you win \$5; otherwise, the \$1 bet you place is forfeited. Should you play the game? One way to make a rational decision is to calculate the probability p of winning a game, and play the game if and only if $p \ge 1/6$. Question: What is the value of p?

Remark For this last problem, you have to solve the problem with pencil and paper; you should not use a computer or calculator.