COS 487: Theory of Computation	**Fall 1997**

Assignment #4

Due: Wednesday, November 19 — *Sanjeev Arora*

Suggested reading (for lectures 11–16): Sipser Chapter 6, 7. Handouts on Gödel's Theorem.

Problems (from lectures upto Nov. 12)

General Hint for all these questions: A short answer — i.e., one that fits in a paragraph — exists.

1. *(Do not turn in this one)* Describe an axiomatic system with finitely many axioms and derivation rules whose set of theorems is $\{0^n 1^n : n \geq 1\}$.

2. Peano Arithmetic is based upon a decidable axiomatic system. Suppose we consider any other axiomatic system whose set of axioms and derivation rules is enumerable. Show that the set THEOREMS, defined as

   $$\{<\varphi>: \ \varphi \text{ is well-formed and has a finite proof in the axiomatic system}\}$$

   is still enumerable.

3. Consider a new kind of finite automaton, an *All-Paths-PDA*. The automaton is defined just like a (nondeterministic) push-down-automaton, except an input x is said to be accepted iff *all* the states that the PDA is in at the end are accept-states. Note, in contrast, that an ordinary PDA is said to accept the string iff *at least one* of the states it is in at the end is an accept-state. Prove that the following set is not decidable. $\{<M>: \ M \text{ is an All-Paths-PDA that accepts no string}\}$.

4. A 2CNF formula is an AND of clauses, where each clause is an OR of at most two literals. Let 2SAT $= \{\varphi : \varphi$ is a satisfiable 2CNF formula$\}$. Show that 2SAT $\in$ P.

5. Point out the fallacy in the following proof that the language $\overline{CLIQUE} = \{<G, k>: $ every clique in graph G has size less than $k\}$ is in NP.

   In class we saw that $CLIQUE$ is in NP. Just take that NDTM and swap its accept and reject states.

6. Suppose P = NP. Show that then we can do the following in polynomial time (be very careful, since the problems listed below are not decision problems!). (i) Given a boolean formula, find a satisfying assignment for it (if one exists). (ii) Given a graph and a number k, find a clique of size k in the graph (if one exists).

7. We say that a language A is in co-NP iff $\overline{L} \in$ NP. We say that A is co-NP-complete if $A \in$ co-NP and every language $L \in$ co-NP is polynomial-time reducible to A. Show that the following language is co-NP-complete:

$$\{< G, k >: \text{ graph } G \text{ has no clique of size } \geq k\}.$$

8. For each of the following languages, state whether it is one or more of the following: in P, in NP, NP-complete, NP-hard. If you can't classify some language in any of these four categories, mention if you think that the exact classification is an open problem. Justify your answer in each case.
   (i) $K_{TM} = \{\langle M, w\rangle : M$ is a deterministic TM that accepts $w\}$.
   (ii) $\{\langle M, w, k\rangle : M$ is an NDTM that accepts w in time $k\}$.
   (iii) $\left\{\left\langle M, w, 1^k\right\rangle : M \text{ is an NDTM that accepts } w \text{ in time } k\right\}$.
9. (*Extra credit*) Describe an algorithm and some $c < 1$ such that the algorithm can decide satisfiability for 3CNF formulae on n variables in 2^{cn} time.