COS 487: Theory of Computation	Fall 1997
Assignment #3	
Due: Monday, November 3	*Sanjeev Arora*

Suggested reading (for lectures 7,8,9,10,11): Sipser Chapters 3, 4, 5, except for Section 5.2.

Problems (from lectures 7, 8, 9, 10,11):

1. (*Do not turn in this one*) A *planar TM* is like a TM, except the memory is an infinite 2-dimensional grid of memory cells. The machine consists of a DFA that moves from cell to cell, at each move taking a step in one of four directions: N, S, E or W. The DFA contains a finite number of rules of the form: "If in state q and reading the symbol a, then overwrite a with symbol b, move one cell North, and go into state q'." Argue that planar TMs can be simulated by 1-tape TMs.

2. Let the set of languages over the alphabet $\{0,1\}$ be divided into the following classes. *Class 1:* Regular. *Class 2:* Context-free but not Regular. *Class 3:* Decidable but not Context-free. *Class 4:* Recognizable but not Decidable. *Class 5:* Not Recognizable.

   If x is an integer, then let $[x]_2$ denote its binary representation. Classify each of the following languages and give a brief (1-2 lines) reason.

   (i) $\{[n]_2 : n \geq 0\}$.
   (ii) $\{[2^n]_2 : n \geq 0\}$.
   (iii) $\{[2^p]_2 : p \text{ is a prime number}\}$.
   (iv) $\{[2^{2n} + 2^n]_2 : n \geq 0\}$.
   (v) $\{[n]_2 :\ M \text{ accepts some string of length} \geq n\}$, where M is a Turing Machine.

3. This question concerns a deterministic push-down automaton with *two* stacks. It is allowed ϵ-transitions but is otherwise deterministic. (a) Give a formal definition of such an automaton, namely, how such an automaton is described and what it means for the automaton to decide a language. (b) Show that the class of languages decided by such automata is exactly the same as the class of languages decided by Turing Machines.

4. Is the following language decidable? Prove your answer.

   $$\{<G>: G \text{ is a CFG that generates some string of even length}\}$$

5. Give short proofs of the following facts about mapping reducibility ($\leq_m$). For any languages A, B, C: (a) $A \leq_m A$. (b) If $A \leq_m B$ and $B \leq_m C$ then $A \leq_m C$. (c) If $A \leq_m B$ then $\overline{A} \leq_m \overline{B}$. (d) If A is enumerable and $A \leq_m \overline{A}$ then A is decidable. (e) If A is decidable then $A \leq_m 0^*1^*$. (f) If A is enumerable then $A \leq_m K_{TM}$

6. Show that a language L is decidable iff the strings in L can be enumerated in an increasing order (i.e., in lexicographic order).

7. Show that every infinite recognizable language has an infinite subset that is decidable.

8. In this question we explore the notion of *oracle reducibility.* If A is a language, then a *Turing machine with oracle* A is a machine with a "magical" subroutine that can decide membership in A. In other words, the subroutine, when given a string w, tells the machine whether or not $w \in A$. Show that there is a Turing machine with oracle $HALT_{TM}$ that can solve the following problem with *two* questions to its oracle: Given any three (machine, input) pairs $(< M_1, x_1 >, < M_2, x_2 >, < M_3, x_3 >)$, decide for each pair whether or not it is in $HALT_{TM}$.