COS 487: Theory of Computation	**Fall 1997**

Mid-Term Exam

Due: November 7 — *Sanjeev Arora*

Instructions: Give yourself 4 hours to do the test. Write answers legibly in the space provided. If you need extra space for an answer, you may attach extra sheets, *which the instructor will read at his discretion.* You cannot consult another person in any way. You can however consult any notes/handouts from this class as well as the text. Feel free to quote, without proof, any results from class or the text.

DO NOT READ THE TEST BEFORE YOU ARE READY TO WORK ON IT.

Your Name:

Write and sign the honor code pledge[1] **here:**

[1] The pledge is "I pledge my honor that I have not violated the honor code during this exam."

Question 1 (15 points) Say whether the following statements are true or false, and give a brief (1-2 line) reason. No credit wil be given if the reason is incorrect.

(i) If L is any regular language, then every subset of L is regular.

(ii) If L is any regular language, then the following language is regular: (recall that w^R is just w written backwards)

$$L^R = \left\{ w^R : w \in L \right\}.$$

(iii) The following language is regular

$$\{w \in \{0,1\}^* : w \text{ has twice as many 1s as 0's}\}$$

Question 2 (20 points). Suppose L is context-free and R is regular. Answer the following questions and explain your reasoning.

(i) (10 points) Is $L - R$ context-free?

(ii) (10 points) Is $R - L$ context free?

Question 3 (25 points) Show that the following language is decidable.

$$E = \left\{ < M >:\ M \text{ is a DFA that accepts some string of the form } xx^R \right\}.$$

Question 4 (25 points) A *stringology* $S = (\Sigma, R, a)$ consists of a finite alphabet Σ, a *finish symbol* a, and a finite set R of *production rules* of the form $\alpha \to \beta$, where $\alpha, \beta \in \Sigma^*$. (Note that the string α could appear in more than one rule.) When *applied* to a string of the form $u\alpha v$, this rule *yields* $u\beta v$.

We say that a string w is *consumed* by the stringology S if there is a sequence of rules— the same rule could occur many times in this sequence— whose application to w yields the finish symbol a. Let $L(S)$ denote the language of strings consumed by S.

Show that the following language is undecidable:

$$\{< S, w >: S \text{ is a stringology that consumes } w\}.$$

THAT'S ALL FOLKS. GOOD LUCK. NEXT QUESTION IS OPTIONAL.

Question 5 (Optional) (25 points; you can work on this question after the alloted 4 hours) This question refers to the notion of oracle reducibility, which was introduced in Qs. 8 of Homework 3.

Suppose A is a language such that some Turing machine M needs just one query to oracle A to do the following: Given two strings x_1, x_2, decide for each whether or not it is in A.

Then show that A is decidable.