COS 341, October 1, 1997
Handout Number 6

Homework Set 3

Reading Assignments Read Chapter 6.

Written Assignments Do exercises 5, 10, 19, 27 and 29 in Section 5.7.

Special Problem 1 (to be counted as 1 exercise) Let $n > 0$. Evaluate

$$f(n) = \sum_{0 \le k \le n-1} 2^k \frac{\binom{n-1}{k}}{\binom{n}{k}}.$$

Your answer should be a closed-form expression.

Special Problem 2 (to be counted as 2 exercises) In the Math Casino Problem, suppose $N = s^6$ where $s > 0$ is an integer. The new rule is that a random $1 \le n \le N$ will be a winner for you if $\lfloor n^{1/6} \rfloor$ divides n. What is p, the probability that a random n is a winner? Give the answer as a closed-form expression involving s.

Special Problem 3 (to be counted as 2 exercises) In the National Basketball Association finals, the format is for the two teams to play a *best-of-seven* series. That is, the series stops whenever one team has accumulated 4 wins. Assume that the outcome of each game is like a random fair coin toss, i.e. each team has a 50% chance of winning that game, independent of what has happened so far in the series. Questions:
(a) What is the probability of a series going into the seventh game?
(b) Let n be any positive integer. What is the probability of a best-of-n series going into the n-th game? (We again assume that the outcome of each game is like a random fair coin toss.)