COS 341, October 15, 1997
Handout Number 10

Homework Set 5

Reading Assignments Read Chapter 11.

Written Assignments Do exercises 1, 7 and 11 in Section 7.8.

Special Problem 1 (to be counted as 1 exercise) Let n be any positive integer. Evaluate

$$\sum_{0 \le k \le n} \sum_{0 \le m \le n} (k+m)^2 \binom{n}{k} \binom{n}{m}.$$

Your answer should be a closed-form expression of n.

Special Problem 2 (to be counted as 1 exercise) Let $n > 1$ be any integer. The road map of a certain town forms a $2 \times n$ grid (two East-West streets of length $(n-1)$ each, and n North-South streets of length 1 each). All the roads are two-way. If you want to go from the south-west corner point Q to the north-east corner point W, how many different routes can you take without traversing the same segment twice? Give your answer as a closed-form expression of n.

Remarks Let $g(n)$ be this number. Then $g(2) = 2, g(3) = 4$.

Special Problem 3 (counted as 1 exercise) Solve the following recurrence relation: $d_0 = d_1 = 1$, and for $n > 1$,

$$d_n = \frac{d_{n-1} d_{n-2}}{6 d_{n-1} + d_{n-2}}.$$