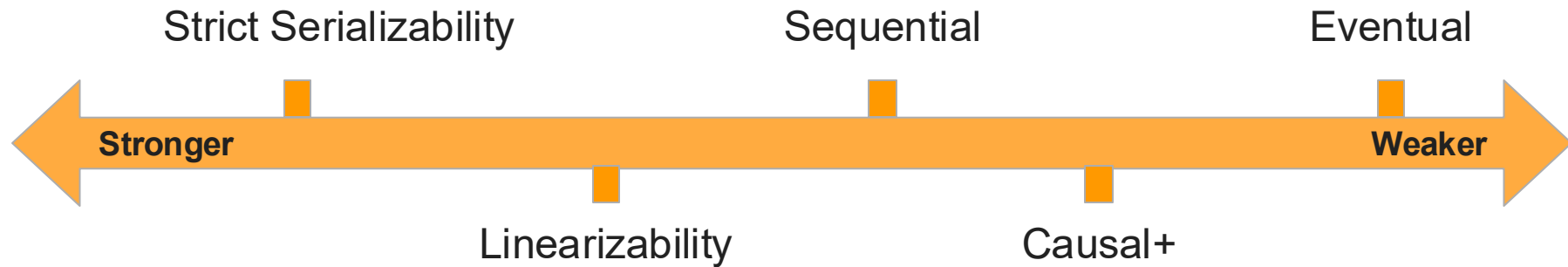


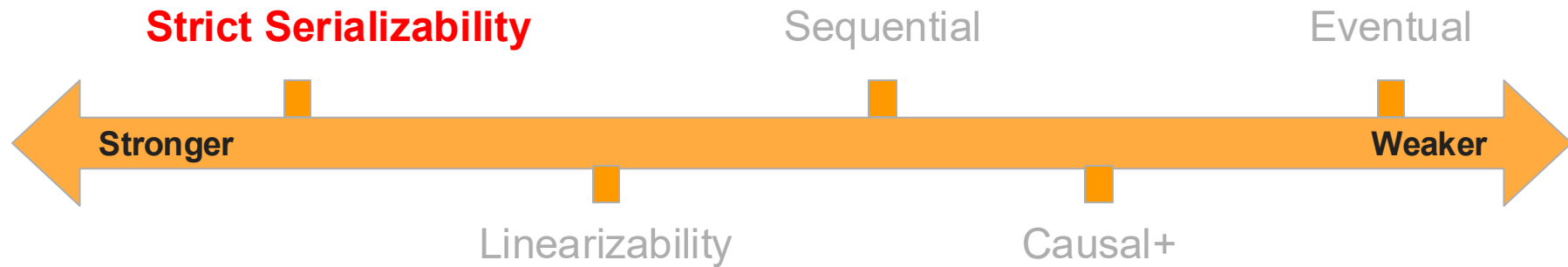
Consistency

November 2025

Consistency Models



Consistency Models



Strict Serializability

- **Transactions**: operations that span multiple objects (e.g., keys in KV store) *atomically* commit (or abort).
- **Total order**: There exists some legal total ordering of transactions.
 - Legal (intuitively defined for strict serializability): in the total ordering, read operations “see” the latest write operation.
- Preserves **real-time commit order**: if *txn A* commits before *txn B* begins, then *txn A* occurs before *txn B* in the total order.
 - Write ops in a committed txn are visible to all future txns’ read ops.
 - Intuition: once a read “sees” a txn and commits, all future reads must also “see” that txn.

Strict Serializability

Pros: applications can easily reason about correctness of transactions.

Cons: strict serializability imposes high read and write latencies on system.

Strict Serializability Example

Strictly Serializable? **Yes**

P1: $\{W(x)b, W(y)b\}$

P2: $\{W(x)a\}$

P3: $\{R(x)a\}$ $\{R(x)b\}$

P4: $\{R(x)b\}$ $\{R(y)b\}$

Strictly Serializable? **No**

P1: $\{W(x)b, W(y)b\}$

P2: $\{W(x)a\}$

P3: $\{R(y)b\}$ $\{R(x)a\}$

P4: $\{R(x)b\}$ $\{R(y)b\}$

Consistency Models



Linearizability

- **Total order**: There exists some legal total order of **operations (not txns)**.
- Difference from *strict serializability*?
 - Single-object operations! No transactions!
- **Preserves real-time ordering**: if an operation A completes before operation B begins, then op A occurs before op B in the total order.
 - A completed write op is visible to all future read ops.
 - Intuition: once a read “sees” a new write, all future reads must also “see” that write.

Pros: Easy to reason about correctness

Cons: High read and write latencies

Linearizability Example

Linearizable?

No

P1: W(x)a

P2: W(x)b

P3: R(x)b R(x)a

P4: R(x)b R(x)a

Linearizable?

Yes

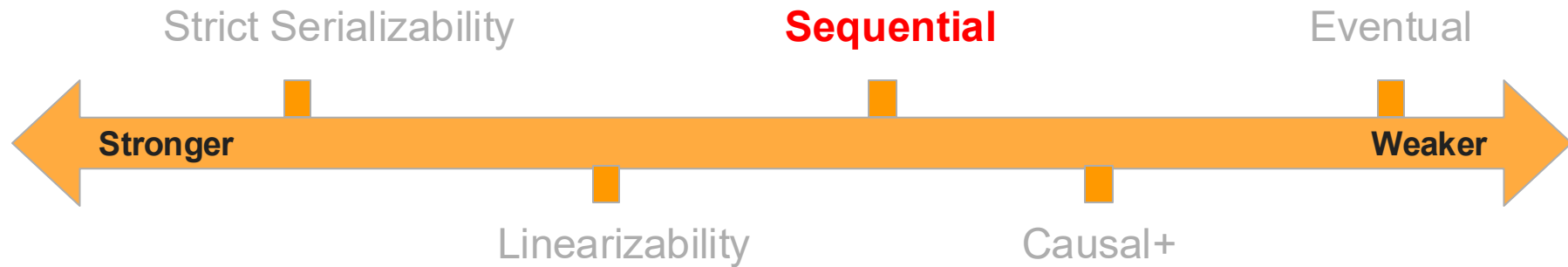
P1: W(x)a

P2: W(x)b

P3: R(x)a R(x)b

P4: R(x)a R(x)b

Consistency Models



Sequential Consistency

- **Total order**: there exists some legal total order of operations.
- **Preserves process ordering**: total order respects order of each process's operations.
- Difference from *linearizability*?
 - Order of ops across processes not determined by real-time

Pros: Can allow more orderings than linearizability → better performance

Cons: Many possible sequential executions → increased application complexity

Sequential Consistency Example

Sequentially Consistent? **Yes**

P1: W(x)a

P2: W(x)b

P3: R(x)b R(x)a

P4: R(x)b R(x)a

Sequentially Consistent? **No**

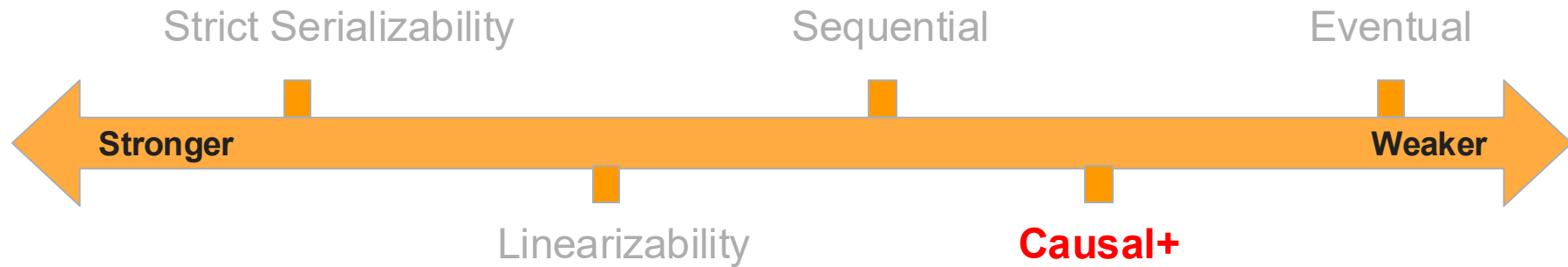
P1: W(x)a

P2: W(x)b

P3: R(x)b R(x)a

P4: R(x)a R(x)b

Consistency Models



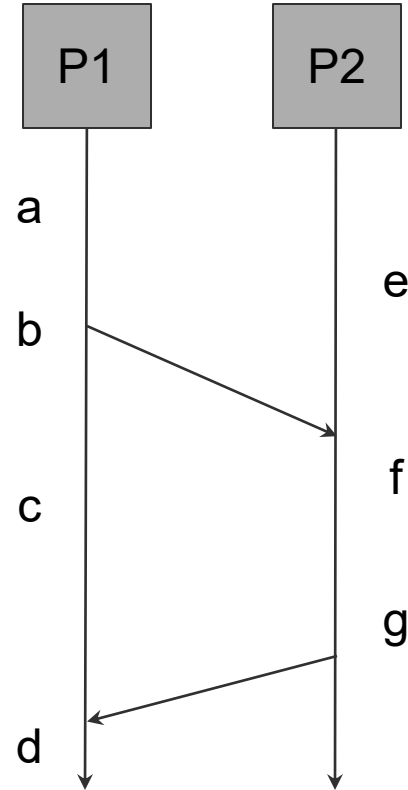
Causal+ Consistency

- **Partial order**: order causally related ops the same way across all processes
- **+**: replicas' total order eventually converges.
- Difference from *sequential consistency*?
 - Only causally related ops need to be ordered: **no guaranteed total order**.
 - Concurrent ops may be ordered differently across different processes.

Pros: preserves causality while improving efficiency.

Cons: harder to reason about concurrency.

Ops	Concurrent
a,b	No
a,e	Yes
a,g	No
c,e	Yes
c,d	No
d,g	No
d,f	No
e,g	No
a,d	No



Causal+ Consistency Example

Causally+ Consistent? Yes

P1: W(x)a

P2: W(x)b

P3: R(x)b R(x)a

P4: R(x)a

Causally+ Consistent? No

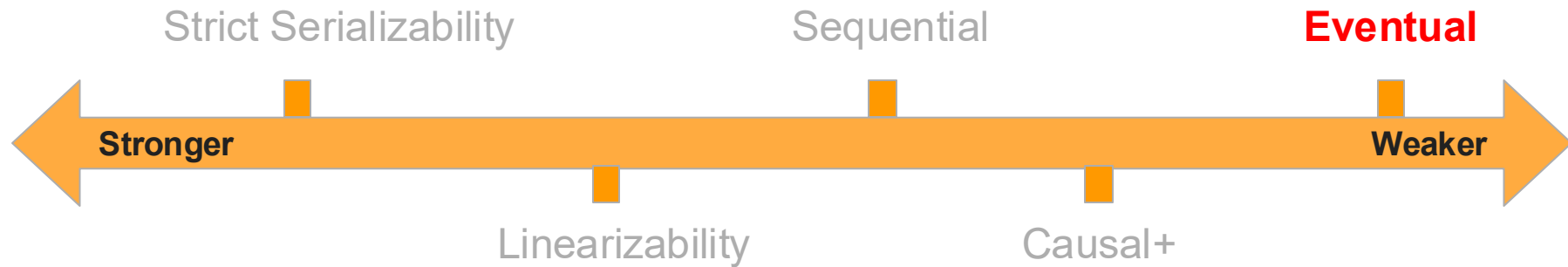
P1: W(x)a

P2: R(x)a W(x)b

P3: R(x)b R(x)a

P4: R(x)a

Consistency Models



Eventual Consistency

- **Eventual convergence**: If no more writes, all replicas *eventually* agree.
- Difference from *causal consistency*?
 - Does not preserve causal relationships
 - Is the “+” in causal+.
- Frequently used with application conflict resolution, anti-entropy

Pros: highly available; think Bayou.

Cons: no safety guarantees, need conflict resolution.

In a nutshell...

Strict Serializability: total order + real time guarantees over *transactions*

Linearizability: total order + real time guarantees over *operations*

Sequential Consistency: total order + process order

Causal+ Consistency: causally ordered + replicas eventually converge

Eventual Consistency: eventually, everyone should agree on state

Exercise 1:

P1: {W(x) 1, W(y) 2} {R(y) 4}

P2: {W(x) 1, R(y) 4}

P3: {W(x) 0, W(y) 4}

P4: {R(x) 0} {R(x) 1}

Consistency Model:

Strictly Serializable **Yes**

Linearizable **Yes**

Sequential **Yes**

Causal+ **Yes**

Eventual **Yes**

Exercise 2:

P1: W(x) 1 R(y) 4

P2: R(x) 1 R(y) 4

P3: R(x) 1 W(y) 4

P4: R(x) 1 R(y) 4

Consistency Model:

Linearizable **Yes**

Sequential **Yes**

Causal+ **Yes**

Eventual **Yes**

Exercise 3:

				Consistency Model:	
				Linearizable	No
P1:	W(x) 3		W(y) 7	Sequential	Yes
P2:	W(x) 1			Causal+	Yes
P3:		R(x) 1 R(x) 3	R(y) 7	Eventual	Yes
P4:		R(x) 1 R(x) 3	R(y) 7		
P5:		R(x) 1 R(x) 3	R(y) 7		

Exercise 4:

				Consistency Model:	
				Linearizable	No
P1:	W(x) 3		W(y) 7	Sequential	No
P2:	W(x) 1			Causal+	Yes
P3:		R(x) 1 R(x) 3	R(y) 7	Eventual	Yes
P4:		R(x) 3 R(x) 1	R(y) 7		
P5:		R(x) 1 R(x) 3	R(y) 7		

Exercise 5:

P1: W(x) 1

P2: W(x) 3

P3: W(x) 7

P4: R(x) 3 R(x) 7 R(x) 1

P5: R(x) 3 R(x) 1 R(x) 7

Consistency Model:

Linearizable **No**

Sequential **No**

Causal+ **Yes**

Eventual **Yes**

Exercise 6:

					Consistency Model:	
					Linearizable	No
P1:	W(x)	1			Sequential	No
P2:	W(x)	3			Causal+	Yes
P3:	R(x)	3	W(x)	7	Eventual	Yes
P4:		R(x)	3	R(x) 7	R(x) 1	
P5:		R(x)	3	R(x) 1	R(x) 7	

Exercise 7:

P1: $W(x) \rightarrow 1$

P2: $R(x) \mid W(x) \mid 3$

P3: $R(x) \leq 3$ $W(x) \leq 7$

P4: $R(x) \leq 3$ $R(x) \leq 7$ $R(x) \leq 1$

P5: $R(x) \leq 3$ $R(x) \leq 1$ $R(x) \leq 7$

Consistency Model:

Linearizable **No**

Sequential **No**

Causal+ No

Eventual **Yes**