

COS 330: Great Ideas in Theoretical Computer Science

Fall 2025

Precept 5

My Midterm Practice

Practice

Problem 1

Suppose that n_s students have to be assigned to n_p precepts. Each precept has a maximum capacity of c students, and each student has a list of precepts they are willing to attend. The goal is to assign each student to exactly one precept they are willing to attend, without exceeding the capacity of any precept.

(a) Let $n = n_s + n_p$. Describe an algorithm running in $O(n^3)$ time to determine the maximum number of students that can be assigned to precepts. Prove correctness and analyze the running time of your algorithm.

(b) Consider the same setting as above with common capacity c per precept. Suppose there is a precept P that appears on the lists of at most c (remaining) students. Prove that there exists an optimal assignment in which *all* those students are assigned to P . Conclude that you may greedily fix such a precept (assign those students to P and remove them and P from the instance) without reducing the maximum number of students that can be assigned overall.

(c) Suppose that we want to find the minimum capacity c such that all students can be assigned to precepts. Describe an algorithm to find this minimum capacity in $O(n^3 \log n)$ time. Prove correctness and analyze the running time of your algorithm.
