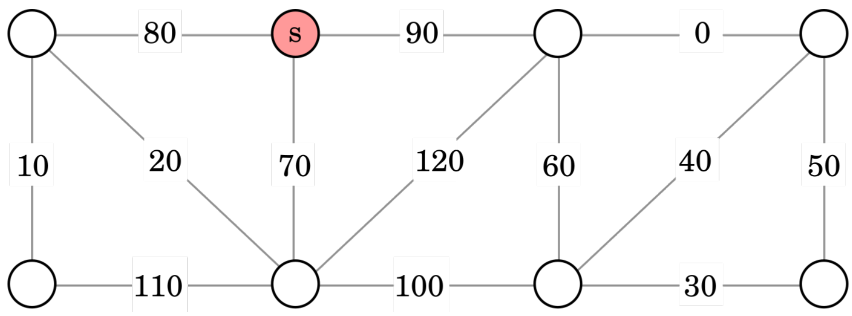
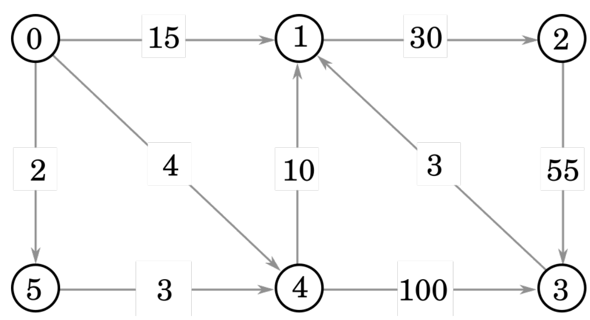
Precept Outline
- Review of Lectures 17 and 18:
  - Minimum Spanning Trees
  - Shortest Paths

Relevant Book Sections
- Book chapters: 4.3 and 4.4

A. Review: MSTs and Shortest Paths

Your preceptor will briefly review key points of this week's lectures. They may use the following graphs to trace examples:





B. Dorm Rooms and Routers

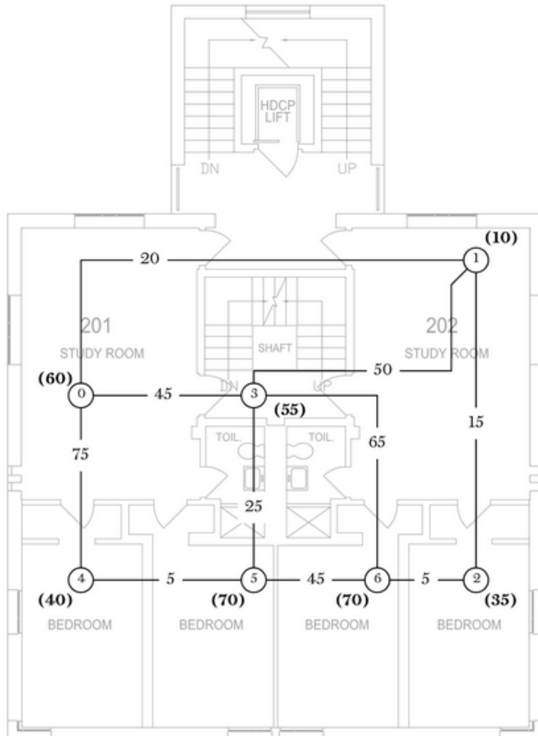
A college has just unveiled a brand-new dorm facility with n rooms. They need to make sure all of them have an internet connection (of course), and are looking for the most cost-effective way to do so. Room number i has internet access if either of the following is true:

- There is a router installed in room i.
- Room i is connected by a fiber path to a room j which has internet access.

Installing a router in room i costs $r_i > 0$, and putting down fiber between rooms i and j costs $f_{ij} > 0$.

The goal of this problem is to determine in which rooms to install a router, and in which pair of rooms to connect together with fiber, so as to minimize the total cost.

Formulate this as a *minimum spanning tree problem*: define a graph $G = (V, E)$ with vertices $V = \{1, 2, \ldots, n\}$ and edges/edge weights that depend on r_i and f_{ij}. You may use the example below to test your formulation.

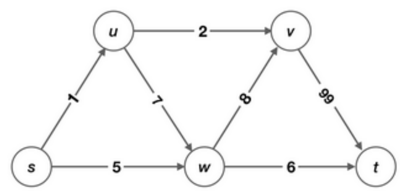


This instance contains 7 dorm rooms and 10 possible connections. The router installation costs are indicated in bold and parentheses; the fiber costs are given on the edges.

C. Shortest Teleport Path (Fall'14 Final)

Given an edge-weighted digraph G with non-negative edge weights, a source vertex s and a destination vertex t, find a shortest path from s to t where you are permitted to teleport across one edge for free. That is, the weight of a path is the sum of the weights of all but the largest edge weights in the path.

For example, in the edge-weighted digraph below, the shortest path from s to t is $s \to w \to t$ (with weight 11) but the shortest teleport path is $s \to u \to v \to t$ (with weight $1 + 2 + 0 = 3$).



A full solution should run in $O(E \log V)$ time and $O(V + E)$ extra space.