

Precept Outline

- Linear-time sorting
 - Radix sort
- Probability and Concentration

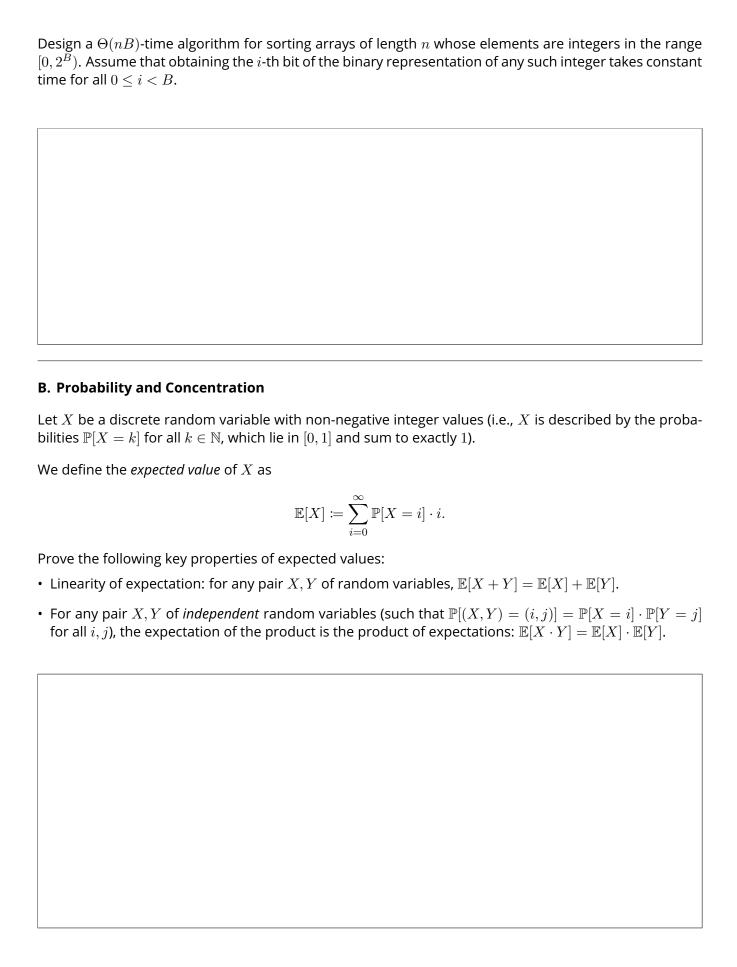
Relevant Book Sections

• Book chapters: 2.1, 2.2 and 2.5

A. Advanced Precept Problems

| Part 1: | Sorting | in Linear | Time |
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| Part 1: Sorting in Linear Time |
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| Design a $\Theta(n)$ -time algorithm for sorting arrays of length n whose elements are Java integers in $[0,n)$. |
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| Generalize the algorithm above to \emph{stably} sort objects with integer \emph{keys} between 0 and $n-1$. |
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| Part 2: Radix Sort |
| Design a $\Theta(n)$ -time algorithm for sorting arrays of length n whose elements are Java integers in $[0,n^2)$. Hint: consider running the algorithm above twice with different keys. |
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Prove *Markov's inequality*: for every t > 0,

$$\mathbb{P}[X \ge t] \le \frac{\mathbb{E}[X]}{t}.$$

The
$$\mathit{variance}$$
 of X is defined as
$$\mathrm{Var}[X] \coloneqq \mathbb{E}[(X - \mathbb{E}[X])^2].$$
 Prove $\mathit{Chebyshevs}$ inequality: for every $t > 0$,
$$\mathbb{P}[|X - \mathbb{E}[X]| \ge t] \le \frac{\mathrm{Var}[X]}{t^2}.$$
 In other words, prove that X concentrates around the mean with a quadratic tail bound. $\mathit{Hint: apply Markov's inequality to a well-chosen random variable.}$