COS 217: Introduction to Programming Systems

Abstraction:

Data Structures

PRINCETON UNIVERSITY

Reminder — Midterm Exam!

This Wednesday – Oct 8, in class, at regular lecture time (10:40 am)

Info: https://www.cs.princeton.edu/courses/archive/fall25/cos217/exam1.php

Goals of this Lecture

1. Learn about Abstract Data Types

2. Learn (or refresh your memory) about:
   - Common data structures: linked lists and hash tables (if not taken 226, or 126 ...)

Why? Deep motivation:

- Common data structures serve as "high level building blocks" for programs
  - A mature programmer:
    - Rarely creates programs from scratch
    - Often creates programs using high level building blocks

"Every program depends on algorithms and data structures, but few programs depend on the invention of brand new ones." -- Kernighan & Pike

Why? Shallow motivation:

- Provide background pertinent to Assignment 3 (linked lists and hash tables)

Data Structures as Abstractions: Abstract Data Types

Data structures are abstractions, implemented using primitive types

- Linked lists or trees using pointers, ints, strings, ...
- Hash tables using arrays, pointers, ints, strings, ...

Or using lower-level data structures

- Symbol table, used by compiler, implemented using linked lists or hash tables
- Should client (user) know which data structure is used in the implementation?

Data Structures can follow the rules of good abstraction

- Separation of interface from implementation
- Assignment 3: Abstract Data Types

Symbol Table

The abstraction: a collection of key/value pairs

- Lookup *binding* by key, get value back
- For these slides, a key is a **string**; a value is an **int**
- Unknown number of key-value pairs

Examples

- (student name, class year)
  - ("Andrew Appel", 81), ("Jen Rexford", 91), ("JP Singh", 87)
- (baseball player, number)
  - ("Ruth", 3), ("Gehrig", 4), ("Mantle", 7)
- (variable name, value)
  - ("maxLength", 2000), ("i", 7), ("j", -10)

We will examine implementing this with linked lists and with hash tables

Should a Client Know Which Data Structure is Used?

- Dangerous w.r.t. separation of interface and implementation
  - Client should only be able to access symbol table through the functions it allows
- What if the client is given access to the underlying data structure (linked list or hash table)
  - Allowing client to modify the implementation makes their interface implementation-specific, and allows client deeper access (e.g. modification of the implementation)
- Symbol table ADT exposes a set of things you can with/to it
  - Find the value for a key
  - Insert or delete a key value pair
  - Whatever the symbol table ADT decides, nothing more

Agenda

Linked lists

Hash tables

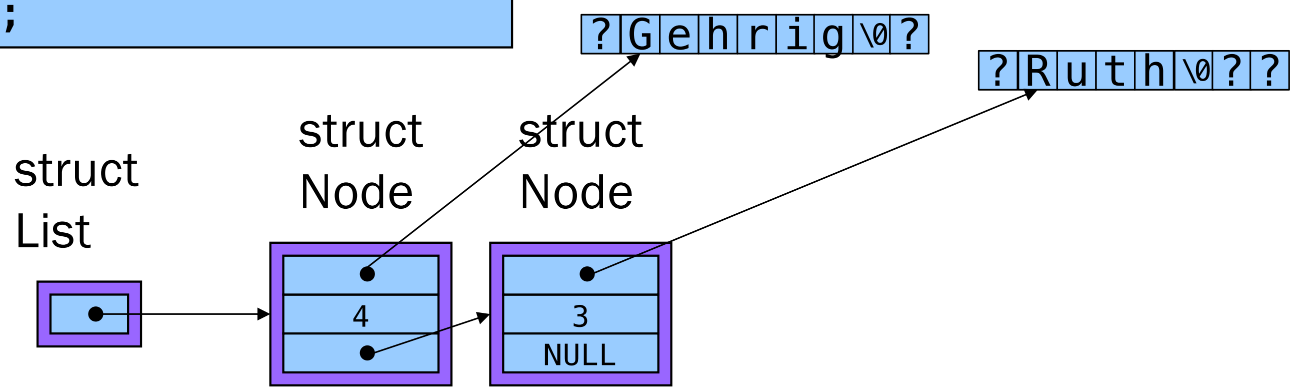
Hash table issues

Symbol table key ownership

Linked List Data Structure (for use by Symbol Table)

```
struct Node {
   const char *key;
   int value;
   struct Node *next;
};

struct List {
   struct Node *first;
};
```

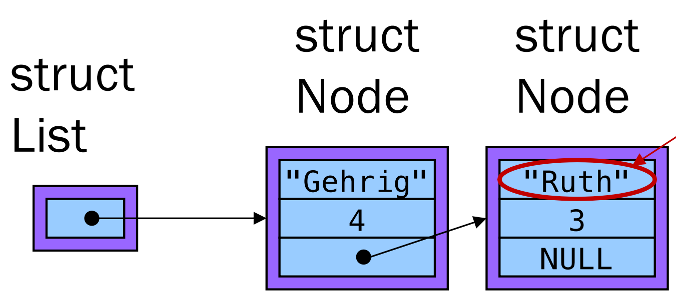
Your Assignment 3 data structures will be more general and perhaps more elaborate



Linked List Data Structure

```
struct Node {
   const char *key;
   int value;
   struct Node *next;
};

struct List {
   struct Node *first;
};
```

Your Assignment 3 data structures will be more general and perhaps more elaborate

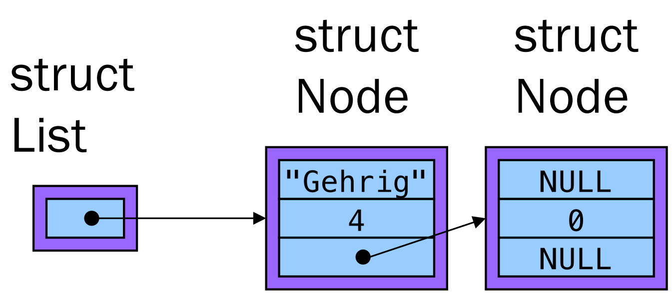


Really this is the address at which a string with contents “Ruth” resides

Accessing a Linked List

```
struct Node {
   const char *key;
   int value;
   struct Node *next;
};

struct List {
   struct Node *first;
};
```

```
struct List lineup;
struct Node g;
g.key = "Gehrig";
lineup.first = &g;
(*lineup.first).value = 4;
lineup.first->value = 4;
struct Node* r =
   calloc(1,sizeof(struct Node));
(*lineup.first).next = r;
lineup.first->next = r;
```



Preview of A3/Lecture+2: Encapsulation (wrong!)

Nothing stops a client from doing this!

```
p->first = NULL;
```

list.h

```
struct Node {const char* key; int value; struct Node *next;};
struct List {struct Node *first;};

struct List *new();
void insert(struct List *p, const char* key, int value);
void concat(struct List *p,
            struct List *q);
int nth_value(struct List *p, int n);
```

If you put the representation here, then it's not an **abstract** data type, it's just a data type.

client.c

```
#include "list.h"

int f(void) {
  struct List *p, *q;
  p = new();
  q = new();
  insert(p,"six",6);
  insert(p,"sept",7);
  insert(q,"cinq",5);
  concat(p,q);
  concat(q,p);
  return nth_value(q,1);
}
```

list_linked.c

```
#include "list.h"

struct List *new()
{
  struct List *p;
  p = calloc(1, sizeof(*p));
  if(p == NULL) { return NULL; }
  return p;
}

void insert(struct List *p, const char* key, int value) {...}

void concat(struct List *p, struct List *q) { ... }

int nth_value(struct List *p, int n) { ... }
```

Preview of A3/Lecture+2: Encapsulation (right!)

list.h

```
struct List;
typedef struct List *List_T;

List_T new();
void insert(List_T p, const char* key, int value);
void concat(List_T p,
            List_T q);
int nth_value(List_T p, int n);
```

Including only the declaration in header file **enforces** the abstraction: it keeps clients from accessing fields of the struct, allowing implementation to change

Now this code won't compile!

```
p->first = NULL;
```

client.c

```
#include "list.h"

int f(void) {
  List_T p, q;
  p = new();
  q = new();
  insert(p,"six",6);
  insert(p,"sept",7);
  insert(q,"cinq",5);
  concat(p,q);
  concat(q,p);
  return nth_value(q,1);
}
```

list_linked.c

```
#include "list.h"

struct Node {const char *key; int value; struct Node *next;};
struct List {struct Node *first;};

struct List *new()
{
  struct List *p;
  p = calloc(1, sizeof(*p));
  if(p == NULL) {return NULL;}
  return p;
}

void insert(struct List *p, const char* key, int value) {...}
void concat(struct List *p, struct List *q) { ... }
int nth_value(struct List *p, int n) { ... }
```

Linked List Algorithms

Create

- Allocate `List` structure; set `first` to `NULL`
- Performance: O(1) ⇒ fast

Add (no check for duplicate key required)

- Insert new node containing key/value pair at front of list
- Performance: O(1) ⇒ fast

Add (check for duplicate key required)

- Traverse list to check for node with duplicate key
- Insert new node containing key/value pair into list
- Performance: O(n) ⇒ slow

Linked List Algorithms

Search

- Traverse the list, looking for given key
- Stop when key found, or reach end
- Performance: ???

Quick? Question

Q: How fast is searching for a key in a linked list?

A. Always fast – O(1)
B. Always slow – O(n)
C. On average, fast
D. On average, slow

Not well specified:

Depends on order of inserts, queries, etc.

Best answer is D.

Linked List Algorithms

Search

- Traverse the list, looking for given key
- Stop when key found, or reach end
- Performance: O(n) ⇒ slow

Free

- Free `Node` structures while traversing
- Free `List` structure
- Performance: O(n) ⇒ slow

Agenda

Linked lists

Hash tables

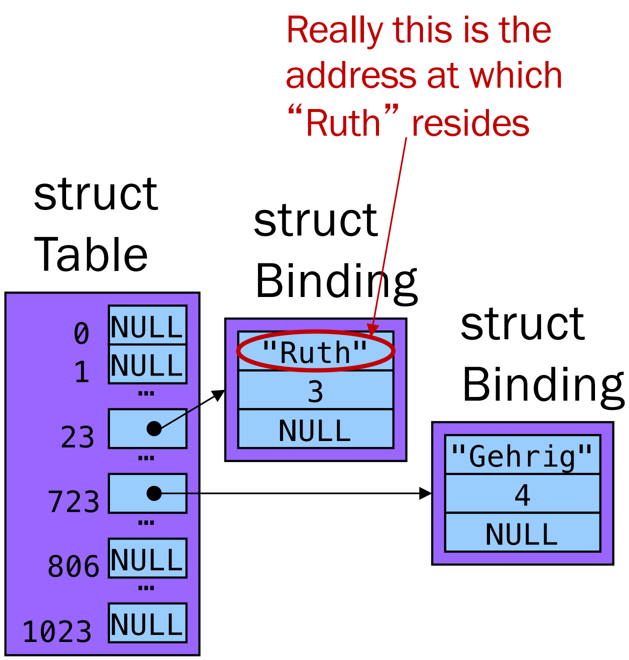
Hash table issues

Symbol table key ownership

Hash Table Data Structure (For COS 226 nerds – hashing with separate chaining)

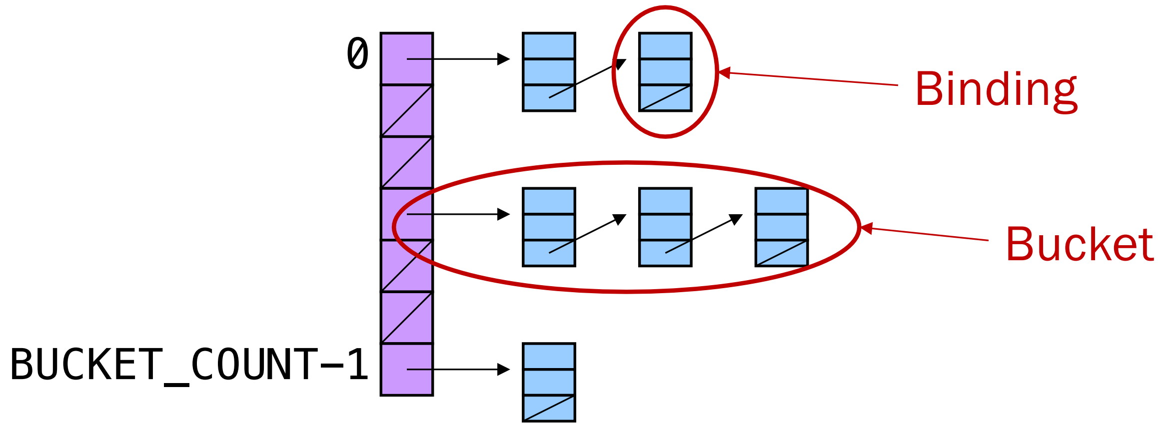
Array of linked lists

```
enum { BUCKET_COUNT = 1024 };

struct Binding {
   const char *key;
   int value;
   struct Binding *next;
};

struct Table {
   struct Binding *buckets[BUCKET_COUNT];
};
```



- You can handle hash tables just like you do linked lists
  - Just get to the right list first. How? By hashing the key

Hash Table Data Structure



Hash function maps given key to an integer

Mod integer by `BUCKET_COUNT` to determine proper bucket

Hash Table Example

Example: `BUCKET_COUNT` = 7

Add (if not already present) bindings with these keys:

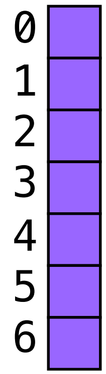
- the, cat, in, the, hat

Hash Table Example (cont.)

First key: "the"

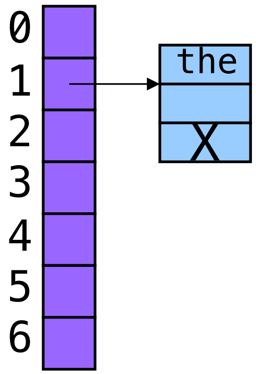
- hash("the") = 965156977; 965156977 % 7 = 1

Search `buckets[1]` for binding with key "the"; not found



Hash Table Example (cont.)

Add binding with key "the" and its value to `buckets[1]`

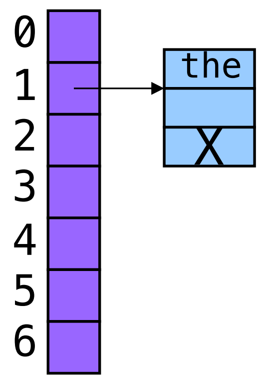


Hash Table Example (cont.)

Second key: "cat"

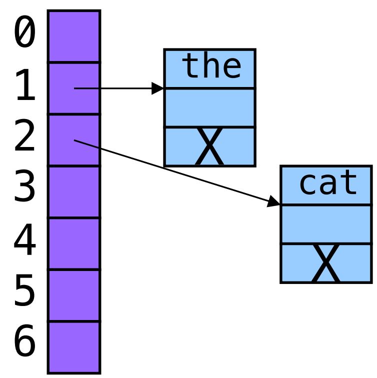
- hash("cat") = 3895848756; 3895848756 % 7 = 2

Search `buckets[2]` for binding with key "cat"; not found



Hash Table Example (cont.)

Add binding with key "cat" and its value to `buckets[2]`

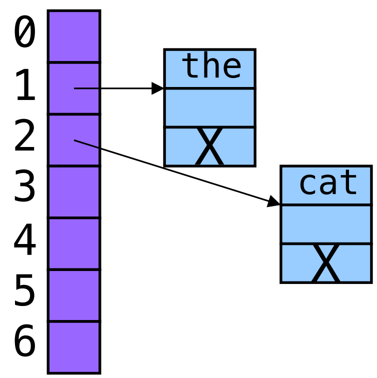


Hash Table Example (cont.)

Third key: "in"

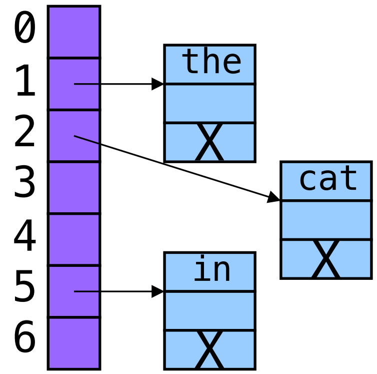
- hash("in") = 6888005; 6888005% 7 = 5

Search `buckets[5]` for binding with key "in"; not found



Hash Table Example (cont.)

Add binding with key "in" and its value to `buckets[5]`



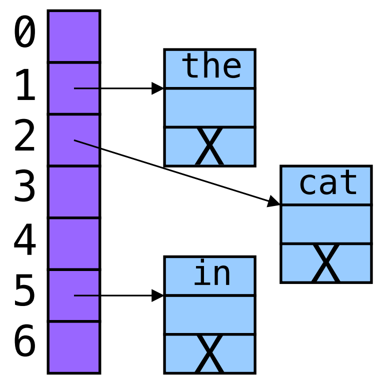
Hash Table Example (cont.)

Fourth word: "the"

- hash("the") = 965156977; 965156977 % 7 = 1

Search `buckets[1]` for binding with key "the"; found it!

- Don't change hash table

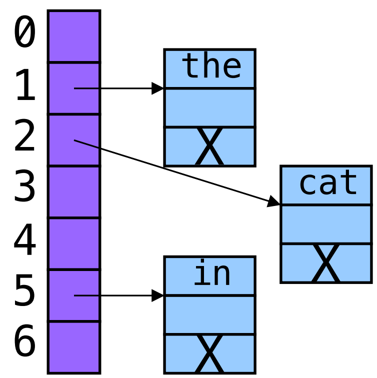


Hash Table Example (cont.)

Fifth key: "hat"

- hash("hat") = 865559739; 865559739 % 7 = 2

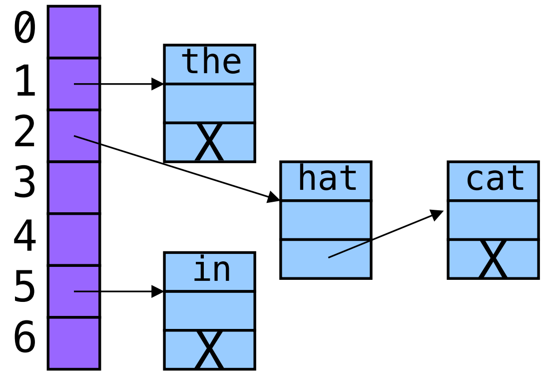
Search `buckets[2]` for binding with key "hat"; not found



Hash Table Example (cont.)

Add binding with key "hat" and its value to `buckets[2]`

- At front or back?



Hash Table Algorithms

Create

- Allocate `Table` structure; set each bucket to `NULL`
- Performance: O(1) ⇒ fast

Add

- Hash the given key
- Mod by `BUCKET_COUNT` to determine proper bucket
- Traverse proper bucket to make sure no duplicate key
- Insert new binding containing key/value pair into proper bucket
- Performance: ???

Now hash this one out …

Q: How fast is adding a key to a hash table?

A. Always fast
B. Usually fast, but depends on how many keys are in the table
C. Usually fast, but depends on how many keys hash to the same bucket
D. Usually slow
E. Always slow

C

If bindings are spread across buckets, this is fast (though B is a concern).

Worst case: everything hashes to the same bucket – O(n)

Hash Table Algorithms

Search

- Hash the given key
- Mod by `BUCKET_COUNT` to determine proper bucket
- Traverse proper bucket, looking for binding with given key
- Stop when key found, or reach end
- Performance: Usually O(1) ⇒ fast

Free

- Traverse each bucket, freeing bindings
- Free `Table` structure
- Performance: O(n) ⇒ slow

Agenda

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Symbol table key ownership

How Many Buckets?

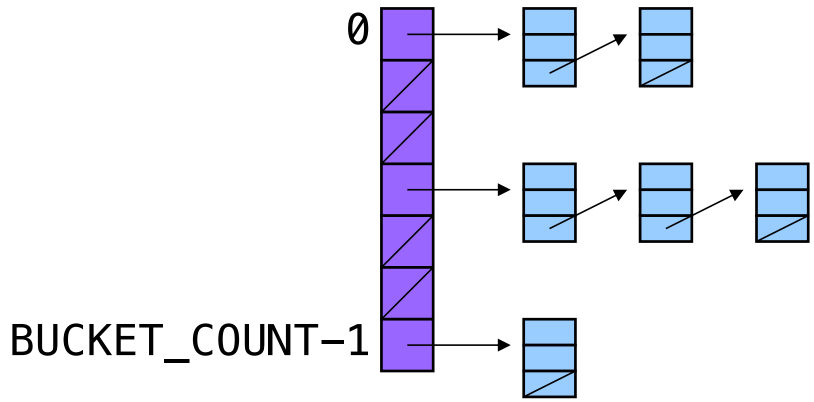
Many!

- Too few ⇒ large buckets ⇒ slow add, slow search

But not too many!

- Too many ⇒ memory is wasted

This is OK:

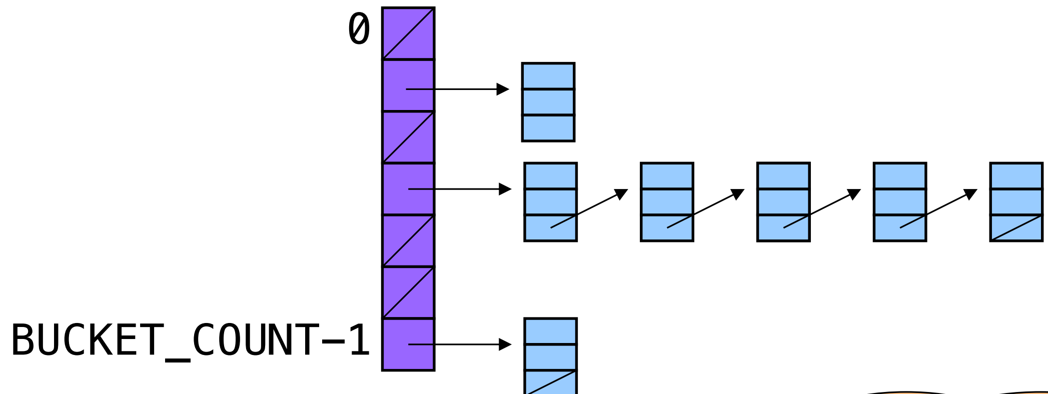


What Hash Function?

Should distribute bindings across the buckets well

- Distribute bindings over the range `0, 1, …, BUCKET_COUNT-1`
- Distribute bindings *evenly* to avoid very long buckets

This is not so good:



What would be the worst possible hash function?

In the spirit of A0, let's have a bash at this …

```
cmoretti@janet lec10 % for i in `cat names`; do echo ${#i} $i; done | sort -n | head -n 12 | column
3 Ark     3 Dev     3 Eve     3 Joy     3 Ray     3 Ryo
3 Ava     3 Era     3 Jie     3 Phu     3 Rin     3 Tom

cmoretti@janet lec10 % for i in `cat names`; do echo ${#i} $i; done | sort -nr | head -n 2 | column
13 Shreyassriram    13 Ourania-Maria

cmoretti@janet lec10 % for i in `cat names`; do echo ${#i}; done | sort -n | uniq -c
  12 3
  21 4
  42 5
  29 6
  25 7
  11 8
   6 9
   2 10
   2 13

cmoretti@janet lec10 % for i in `cat names`; do echo ${#i}; done | sort -n | uniq -c | wc -l
       9
```

How to Hash Strings?

Simple hash schemes don't distribute the keys evenly

- Number of characters, mod BUCKET_COUNT
- Sum the numeric codes of all characters, mod BUCKET_COUNT
- ...

A reasonably good hash function:

- Weighted sum of characters s_i in the string s
  - $(\Sigma\ a^i s_i)$ mod BUCKET_COUNT
- Best if a and BUCKET_COUNT are relatively prime (i.e., their GCD is 1)
  - e.g., a = 65599, BUCKET_COUNT = 1024

How to Hash Strings?

A bit of math, and translation to code, yields:

```
size_t hash(const char *s, size_t bucketCount)
{
   enum { HASH_MULT = 65599 };
   size_t i;
   size_t h = 0;
   for (i = 0; s[i] != '\0'; i++)
      h = h * HASH_MULT + (size_t)s[i];
   return h % bucketCount;
}
```

Agenda

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How to Protect Keys?

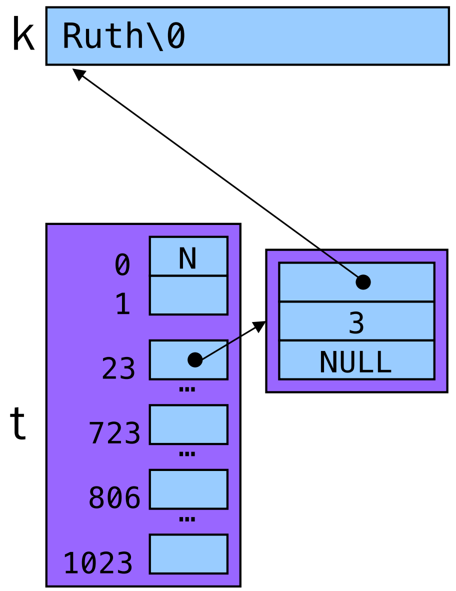
Suppose a hash table function `Table_add()` contains this code:

```
void Table_add(struct Table *t, const char *key, int value)
{   …
    struct Binding *p =
        (struct Binding*)malloc(sizeof(struct Binding));
    p->key = key;
    …
}
```

How to Protect Keys?

Problem: Consider this calling code:

```
struct Table *t;
char k[100] = "Ruth";
…
Table_add(t, k, 3);
```



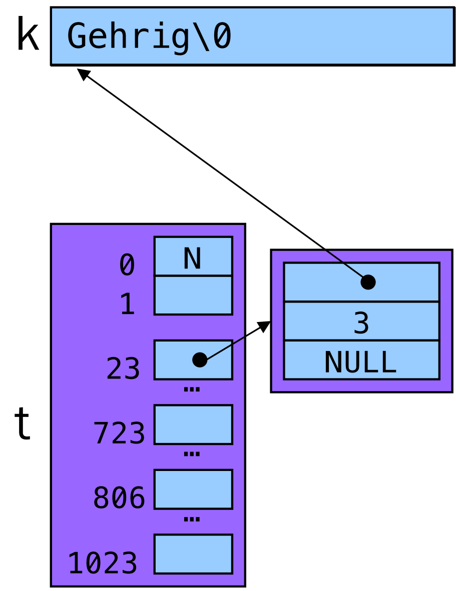
How to Protect Keys?

Problem: Consider this calling code:

```
struct Table *t;
char k[100] = "Ruth";
…
Table_add(t, k, 3);
strcpy(k, "Gehrig");
```

k is REALLY &k[0]!

What happens if the client searches t for "Ruth"? For "Gehrig"?



How to Protect Keys?

Solution: Table_add() saves a defensive copy of the given key

```
void Table_add(struct Table *t, const char *key, int value)
{   …
    struct Binding *p =
        (struct Binding*)malloc(sizeof(struct Binding));
    p->key = (const char*)malloc(strlen(key) + 1);
    strcpy((char*)p->key, key);
    …
}
```

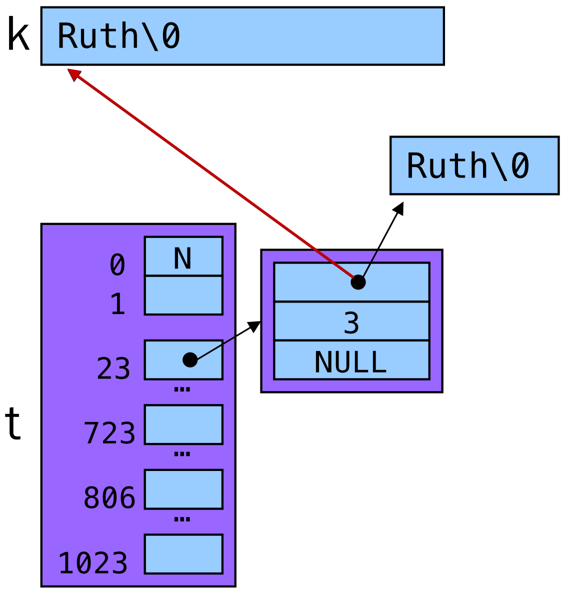
Why add 1?

What is missing from this code that you should have in yours?

How to Protect Keys?

Now consider same calling code:

```
struct Table *t;
char k[100] = "Ruth";
…
Table_add(t, k, 3);
```



How to Protect Keys?

Now consider same calling code:

```
struct Table *t;
char k[100] = "Ruth";
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Table_add(t, k, 3);
strcpy(k, "Gehrig");
```

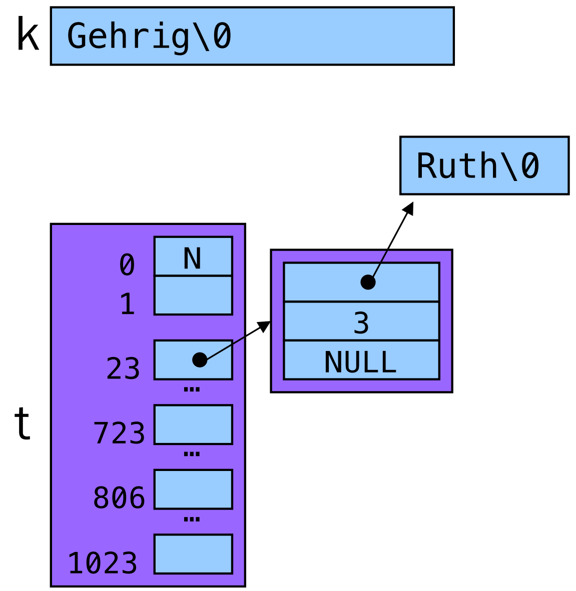

Hash table is not corrupted!

Who Owns the Keys?

Then the hash table **owns** its keys

- That is, the hash table allocated the memory in which its keys reside
- `Table_remove()` function must also free the memory in which the key resides, not just the binding containing the key

Summary

Common data structures and associated algorithms

- Linked list
  - (Maybe) fast add
  - Slow search
- Hash table
  - (Potentially) fast add
  - (Potentially) fast search
  - Very common

Hash table issues

- (Initial) Bucket array size
- Hashing algorithms

Symbol table concerns

- Key ownership