COS 217: Introduction to Programming Systems

Numbers

(in C and otherwise)

PRINCETON UNIVERSITY

The Old Ice Breaker

Q: Why do computer programmers confuse Christmas and Halloween?

A: Because Dec 25 is Oct 31

Agenda

Number Systems: Decimal, Binary, Hex, Octal + conversions among them

Negative numbers in a computer

Integers in C : Representation and Operations

The Decimal Number System

From Latin *decem* ("ten")

Ten Symbols: e.g., **0 1 2 3 4 5 6 7 8 9**

- Different symbols in different languages
- The integers we use in C are decimal

Positional: **2945 ≠ 2495**

Base 10: $2945 = (2*10^3) + (9*10^2) + (4*10^1) + (5*10^0)$

The Unary Number System

From late Latin ūnus ("one")

One symbol: **1**

Effectively non-positional: $\mathtt{1111_U = 1111_U}$

Base 1: $\mathtt{1111 = (1*1^3) + (1*1^2) + (1*1^1) + (1*1^0)}$

Like tally marks (without crossing our fives)

- Integer 5 = 11111
- Integer 8 = 11111111
- Integer 25 = 1111111111111111111111111
- etc

Key Points

All the other base systems we'll consider seriously are positional

Counting, addition, subtraction, work the same way in them as in decimal, just with different bases (number of symbols available)

Some are good for humans (base 10) and some for computers (powers-of-two bases)

There are simple methods for conversion

So now we can go over them very quickly …

The Binary Number System

From late Latin *binarius* ("consisting of two"), from classical Latin *bis* ("twice"). Plus English suffix *-ary*

Two symbols: **`0 1`**

Positional: $\mathbf{1010_B \neq 1100_B}$

Base 2: $\mathbf{1010 = (1*2^3) + (0*2^2) + (1*2^1) + (0*2^0)}$

- "One zero one" in Base10 is integer value 101, and in Base 2 it is integer 5

Most (digital) computers use the binary number system

Why?

Terminology

- **Bit**: a single binary symbol ("binary digit")
- **Byte**: (typically) 8 bits
- **Nibble / Nybble**: 4 bits – we'll see a more common name for 4 bits soon.

The Hexadecimal Number System

From ancient Greek ἕξ (*hex,* “six”) + Latin-derived *decimal*

Sixteen symbols (“hexits”): 0 1 2 3 4 5 6 7 8 9 A B C D E F

Positional: $A13D_H \neq 3DA1_H$

Base 16: $A13D_H$: $A*16^3 + 1*16^2 + 3*16^1 + D*16^0 = 40960 + 256 + 48 + 13 =$

Computer programmers often use hexadecimal (“hex”)

- In C: 0x prefix (0xA13D, etc.)

Why?

That’s a zero, not a letter O

The Octal Number System

"octo" (Latin) ⇒ eight

Eight symbols: **0 1 2 3 4 5 6 7**

Positional: **1743 ≠ 7314**

Base 8: 7143_H: $7*8^3 + 1*8^2 + 4*8^1 + 3*8^0$

Computer programmers sometimes use octal (so does Mickey?)

- In C: 0 prefix (01743, etc.)
- Unix file permissions: chmod 755 myfile
  - Changes permissions to _rwxr_xr_x

Converting to Decimal: Expand using Base Template

Binary to Decimal:

$$
\begin{aligned}
100101_B &= (1*2^5)+(0*2^4)+(0*2^3)+(1*2^2)+(0*2^1)+(1*2^0) \\
&= 32 + 0 + 0 + 4 + 0 + 1 \\
&= 37
\end{aligned}
$$

MSB

LSB

Hex to Decimal:

$$
\begin{aligned}
25_H &= (2*16^1) + (5*16^0) \\
&= 32 + 5 \\
&= 37
\end{aligned}
$$

Similarly:

$$
\begin{aligned}
3B_H &= (3*16^1) + (11*16^0) \\
&= 48 + 11 \\
&= 59
\end{aligned}
$$

Octal to Decimal:

$$
\begin{aligned}
25_O &= (2*8^1) + (5*8^0) \\
&= 16 + 5 \\
&= 21
\end{aligned}
$$

Converting From Decimal: Subtraction Method

e.g., (Decimal) Integer to binary

- Determine largest power of 2 that's ≤ number. Then write template from that power down:

```
37 = (?*2^5)+(?*2^4)+(?*2^3)+(?*2^2)+(?*2^1)+(?*2^0)
```

- Then fill in template

```
 37 = (1*2^5)+(0*2^4)+(0*2^3)+(1*2^2)+(0*2^1)+(1*2^0)
-32
  5
 -4
  1                      100101B
 -1
  0
```

Converting from Decimal: Division Method

e.g. (Decimal) Integer to binary

- Repeatedly divide by 2, consider remainder

```
37 / 2 = 18 R 1
18 / 2 =  9 R 0
 9 / 2 =  4 R 1
 4 / 2 =  2 R 0
 2 / 2 =  1 R 0
 1 / 2 =  0 R 1
```

Read (L to R) from bottom to top: 100101_B

18 * 2^1 _ _ _ _ _ 1
9 * 2^2 _ _ _ _ 0 1
4 * 2^3 _ _ _ 1 0 1
2 * 2^4 _ _ 0 1 0 1
1 * 2^5 _ 0 0 1 0 1

Similarly, (Decimal) Integer to Hex:

```
37 / 16 = 2 R 5
 2 / 16 = 0 R 2
```

Read from bottom to top: 25_H

Conversion Among Power-of-2 Based Systems

Observations:

- For Hex, $16^1 = 2^4$, so every 1 hexit corresponds to a nybble (4 bits)
- Watch the left-padding with zeros

Binary to hexadecimal:

```
0010000100111101B
  2   1   3   DH
```

Number of bits in binary number not a multiple of 4? ⇒ pad with zeros on left

Hexadecimal to binary:

```
  2   1   3   DH
~~00~~10000100111101B
```

Discard leading zeros from binary number if appropriate

For octal, it's 3 bits

iClicker Question

Convert binary 101010 into decimal and hex

A. 21 decimal, A2 hex

B. 21 decimal, A8 hex

C. 18 decimal, 2A hex

D. 42 decimal, 2A hex

hint: might want to convert to hex first

Agenda

Number Systems: Decimal, Binary, Hex, Octal + conversions among them

Negative numbers in a computer

Integers in C: Representation and Operations

First, Unsigned (Non-Negative) Integers

Much easier than signed integers. Just binary numbers in the natural way we think of them. Addition, subtraction work same way. Only, can overflow due to fixed space

Mathematics

- Non-negative integers' range is 0 to ∞

Computers

- Range limited by computer's word size
- Word size is n bits $\Rightarrow$ range is 0 to $2^n - 1$ representing with an n bit binary number
- Exceed range $\Rightarrow$ overflow

Typical computers today

- n = 32 or 64, so range is 0 to $2^{32} - 1$ (~4 billion) or $2^{64} - 1$ (huge … ~1.8e19)

A pretend computer for upcoming slides

- Assume n = 4, so range is 0 to $2^4 - 1$ (15)
- (All points made generalize to larger word sizes like 32 and 64)

Representing Unsigned Integers

On 4-bit pretend computer

Unsigned Integer	Rep
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

Adding Unsigned Integers

Addition

```
                  1
   3           0011B
+ 10         + 1010B
  --           ----
  13           1101B
```

Start at right column
Proceed leftward
Carry 1 when necessary

```
                111
   7           0111B
+ 10         + 1010B
  --           ----
   1           0001B
```

Beware of overflow
All results are mod 24
17 mod 16 = 1

How would you detect overflow programmatically?

Subtracting Unsigned Integers

Subtraction

```
   10         1010B
–   7       – 0111B
   --         ----
    3         0011B
```

Start at right column
Proceed leftward
Borrow when necessary

```
    3         0011B
– 10        – 1010B
   --         ----
    9         1001B
```

Beware of overflow
All results are mod 24
-7 mod 16 = 9

How would you detect overflow programmatically?

Negative Numbers Attempt #1: Sign-Magnitude

Integer	Rep
-7	1111
-6	1110
-5	1101
-4	1100
-3	1011
-2	1010
-1	1001
-0	1000
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111

Definition

High-order bit indicates sign

`0 ⇒ positive, 1 ⇒ negative`

Remaining bits indicate magnitude

$0101_B = 101_B = 5$

$1101_B = -101_B = -5$

Pros and cons

+ easy to understand, easy to negate
+ symmetric
- two representations of zero
- different algorithms to add signed and unsigned

Not widely used for integers today

Negative Numbers: Two's Complement

-x is 0 – x.

```
                       1            11           111          1111
 00000000      00000000      00000000      00000000      00000000
- 01001011    - 01001011    - 01001011    - 01001011    - 01001011
----------    ----------    ----------    ----------    ----------
                       1            01           101          0101

    11111        111111       1111111      11111111
 00000000      00000000      00000000      00000000
- 01001011    - 01001011    - 01001011    - 01001011
----------    ----------    ----------    ----------
    10101        110101       0110101      10110101
```

How do you get 10110101 from 01001011? Flip the bits and add 1

Called two's complement because, in n bits, it's the same as $2^n - x$ (drop $n+1^{th}$ bit)

(just flipping bits gives us "one's complement." Also a way to represent –ve numbers)

Negative Numbers: Two's Complement

Integer	Rep
-8	1000
-7	1001
-6	1010
-5	1011
-4	1100
-3	1101
-2	1110
-1	1111
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111

Computing negative

neg(x) = flip all bits, and add 1. ~x + 1. onescomp(x) + 1

$neg(0101_B) = 1010_B + 1 = 1011_B$

$neg(1011_B) = 0100_B + 1 = 0101_B$

A definition: High-order bit has weight $-(2^{b-1})$

$1010_B = (1*-8)+(0*4)+(1*2)+(0*1) = -6$

$0010_B = (0*-8)+(0*4)+(1*2)+(0*1) = 2$

Pros and cons

- not symmetric ("extra" negative number; -(-8) = -8)

+ one representation of zero

+ same algorithms add/subtract signed and unsigned int

Adding Signed Integers in Two's Complement

pos + pos

```
             11
  3        0011B
+ 3      + 0011B
 --        ----
  6        0110B
```

pos + pos (overflow)

```
             111
  7        0111B
+ 1      + 0001B
 --        ----
 -8        1000B
```

pos + neg

```
          1111
  3        0011B
+ -1     + 1111B
 --        ----
  2        0010B
```

How would you detect overflow programmatically?

neg + neg

```
          11
 -3        1101B
+ -2     + 1110B
 --        ----
 -5        1011B
```

neg + neg (overflow)

```
         1  1
 -6        1010B
+ -5     + 1011B
 --        ----
  5        0101B
```

Subtracting Signed Integers in Two's Complement

How would you compute 3 – 4?

```
  3        0011B
- 4      - 0100B
 --        ----
  ?        ????B
```

Subtracting Signed Integers

Perform subtraction with borrows — or — Compute two's comp and add

```
  3        0011B
- 4      - 0100B
 --        ----
 -1        1111B
```

→

```
   3         0011B
+ -4       + 1100B
  --         ----
  -1         1111B
```

```
 -5        1011B
--2      - 1110B
 --        ----
 -3        1101B
```

→

```
             1
  -5         1011B
+  2       + 0010B
  --         ----
  -3         1101B
```

Why does Two's Complement Arithmetic Work

Answer: $[-b] \bmod 2^4 = [\text{twoscomp}(b)] \bmod 2^4$

$$
\begin{aligned}
&[-b] \bmod 2^4 \\
&= [2^4 - b] \bmod 2^4 \\
&= [2^4 - 1 - b + 1] \bmod 2^4 \\
&= [(2^4 - 1 - b) + 1] \bmod 2^4 \\
&= [\text{onescomp}(b) + 1] \bmod 2^4 \\
&= [\text{twoscomp}(b)] \bmod 2^4
\end{aligned}
$$

So: $[a - b] \bmod 2^4 = [a + \text{twoscomp}(b)] \bmod 2^4$

$$
\begin{aligned}
&[a - b] \bmod 2^4 \\
&= [a + 2^4 - b] \bmod 2^4 \\
&= [a + 2^4 - 1 - b + 1] \bmod 2^4 \\
&= [a + (2^4 - 1 - b) + 1] \bmod 2^4 \\
&= [a + \text{onescomp}(b) + 1] \bmod 2^4 \\
&= [a + \text{twoscomp}(b)] \bmod 2^4
\end{aligned}
$$

Agenda

Number Systems: Decimal, Binary, Hex, Octal + conversions among them

Negative numbers in a computer

Integers in C: Representation and Operations

Integer Data Types in C

Integer types of various sizes: {`signed`, `unsigned`} {`char`, `short`, `int`, `long`}

- Shortcuts: `signed` assumed for `short`/`int`/`long`; `unsigned` means `unsigned int`
- `char` is 1 byte
  - Number of bits per byte is unspecified (but in the 21st century, safe to assume it's 8)
  - Signedness is system dependent, so for arithmetic use "signed char" or "unsigned char"
- Sizes of other integer types not fully specified but constrained:
  - `int` was intended to be "natural word size" of hardware, but isn't always
  - 2 ≤ `sizeof(short)` ≤ `sizeof(int)` ≤ `sizeof(long)`

On `armlab`:

- Natural word size: 8 bytes ("64-bit machine")
- `char`: 1 byte
- `short`: 2 bytes
- `int`: 4 bytes (compatibility with widespread 32-bit code)
- `long`: 8 bytes

What decisions did the designers of Java make?

Integer Types in Java vs. C

	Java	C

Unsigned types	`char // 16 bits`	`unsigned char`
		`unsigned short`
		`unsigned (int)`
		`unsigned long`

Signed types	`byte // 8 bits`
	`short // 16 bits`
	`int // 32 bits`
	`long // 64 bits`
		`signed char`
		`(signed) short`
		`(signed) int`
		`(signed) long`

1. Not guaranteed by C, but on **`armlab`**, **`short`** = 16 bits, **`int`** = 32 bits, **`long`** = 64 bits
2. Not guaranteed by C, but on **`armlab`**, **`char`** is unsigned

If Sizes aren't Fixed in C, How do I Tell Size?

`sizeof` operator returns the size of a type or a variable (or an expression's result)

- Applied at compile-time
- Operand can be a data type
- Operand can be an expression, from which the compiler infers a data type

Examples, on `armlab` using `gcc217`

- `sizeof(int)` evaluates to 4
- `sizeof(i)` evaluates to 4 if `i` is a variable of type `int`
- `sizeof(1+2)` evaluates to 4

Integer Literals in C

Prefixes, suffixes indicate bases, types (resp). Default base is decimal int: 123

- Prefixes to indicate a different base
  - Octal int: 0173 = 123
  - Hexadecimal int: 0x7B = 123
  - No prefix to indicate binary int literal
- Suffixes to indicate a different type
  - Use "L" suffix to indicate long literal
  - Use "U" suffix to indicate unsigned literal
  - No suffix to indicate char or short literals; instead, cast

char:	'{' (← really int, as seen last time), (char) 123, (char) 0173, (char) 0x7B
int:	123, 0173, 0x7B
long:	123L, 0173L, 0x7BL
short:	(short)123, (short)0173, (short)0x7B
unsigned int:	123U, 0173U, 0x7BU
unsigned long:	123UL, 0173UL, 0x7BUL
unsigned short:	(unsigned short)123, (unsigned short)0173, (unsigned short)0x7B

sizeof synthesis

Q: What is the value of the following sizeof expression on the armlab machines?

```
int i = 1;

sizeof(i + 2L)
```

A. 3

B. 4

C. 8

D. 12

E. error

Agenda

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Reading / Writing Numbers

Motivation

- Numbers come in as sequences of characters, but must be interpreted as numbers with types
- Could provide special read/write functions for each type: getchar(), putshort(), getint(), putfloat() ...
- Or parameterized functions: one function that takes a specification for the kind of data to expect

C library provides parameterized functions: scanf() and printf()

- Can read/write any primitive type of data
- First parameter is a format string containing conversion specs: size, base, field width
- Can read/write multiple variables with one call

See King book for details

Operators in C

- Typical arithmetic operators: + – * / %
- Typical relational operators: == != < <= > >=
  - Each evaluates to FALSE ⇒ 0, TRUE ⇒ 1
- Typical logical operators: ! && ||
  - Each interprets 0 ⇒ FALSE, non-0 ⇒ TRUE (remember, no Boolean type)
  - Each evaluates to FALSE ⇒ 0, TRUE ⇒ 1
- Cast operator: (type)
- Bitwise operators: ~ & | ^ >> <<
  - C designed to be close to the hardware
  - Bitwise operators enable fast calculations/arithmetic

Shifting Unsigned Integers

Bitwise right shift (>> in C): fill on left with zeros

```
10 >> 1 ⇒ 5
```
1010_B 0101_B

```
10 >> 2 ⇒ 2
```
1010_B 0010_B

What is the effect arithmetically?

Bitwise left shift (<< in C): fill on right with zeros

```
5 << 1 ⇒ 10
```
0101_B 1010_B

```
3 << 2 ⇒ 12
```
0011_B 1100_B

```
3 << 3 ⇒ 8
```
0011_B 1000_B ← Overflows. But Results are correct mod 2^4

What is the effect arithmetically?

Other Bitwise Operations on Unsigned Integers

Bitwise NOT (~ in C)

- Flip each bit (don't forget leading 0s)

```
~10 ⇒ 5
```
1010_B 0101_B

```
~5 ⇒ 10
```
0101_B 1010_B

Bitwise AND (& in C)

- AND (1=True, 0=False) corresponding bits

```
 10        1010B
& 7      & 0111B
 --        ----
  2        0010B
```

```
 10        1010B
& 2      & 0010B
 --        ----
  2        0010B
```

Useful for “masking” bits to 0. All bits ANDed with 0 are 0.

x & 0 is 0, x & 1 is x

Other Bitwise Operations on Unsigned Ints

Bitwise OR: (| in C)

- Logical OR corresponding bits

```
  10        1010B
|  1      | 0001B
  --        ----
  11        1011B
```

Useful for “masking” bits to 1. OR with 1 is 1.

x | 1 is 1, x | 0 is x

Bitwise exclusive OR (^ in C)

- Logical exclusive OR corresponding bits

```
  10        1010B
^ 10      ^ 1010B
  --        ----
   0        0000B
```

^ with 1 flips value of bit

x ^ x sets all bits to 0

Logical vs. Bitwise Ops

Logical AND (&&) vs. bitwise AND (&)

- `2 (TRUE) && 1 (TRUE) => 1 (TRUE)`

```
Decimal  Binary
      2  00000000 00000000 00000000 00000010
   && 1  00000000 00000000 00000000 00000001
   ----  -----------------------------------
      1  00000000 00000000 00000000 00000001
```

- `2 (TRUE)  & 1 (TRUE) => 0 (FALSE)`

```
Decimal  Binary
      2  00000000 00000000 00000000 00000010
    & 1  00000000 00000000 00000000 00000001
   ----  -----------------------------------
      0  00000000 00000000 00000000 00000000
```

Implication:

- Use **logical** AND to control flow of logic
- Use **bitwise** AND only when doing bit-level manipulation
- Same for OR and NOT

A *Bit* Complicated … challenge for the bored

How do you set bit k (where the least significant bit is bit 0) of unsigned variable u to zero (leaving everything else in u unchanged)?

A. u &= (0 << k);

B. u |= (1 << k);

C. u |= ~(1 << k);

D. u &= ~(1 << k);

E. u = ~u ^ (1 << k);

Aside: Using Bitwise Ops for Arithmetic

Can use <<, >>, and & to do some arithmetic efficiently

x * 2^y == x << y

- 3*4 = 3*2^2 = 3<<2 ⇒ 12

Fast way to multiply by a power of 2

x / 2^y == x >> y

- 13/4 = 13/2^2 = 13>>2 ⇒ 3

Fast way to divide unsigned by power of 2

x % 2^y == x & (2^y-1)

- 13%4 = 13%2^2 = 13&(2^2-1) = 13&3 ⇒ 1

Fast way to mod by a power of 2

```
 13        1101B
& 3      & 0011B
 --        ----
  1        0001B
```

Many compilers will do these transformations automatically!

Shifting Signed Integers

Bitwise left shift (<< in C): fill on right with zeros

```
3 << 1 ⇒ 6
```
0011_B 0110_B

```
-3 << 1 ⇒ -6
```
1101_B 1010_B

```
-3 << 2 ⇒ 4
```
1101_B 0100_B

What is the effect arithmetically?

Results are mod 2^4

Bitwise right shift: fill on left with ???

Shifting Signed Integers (cont.)

Bitwise *logical* right shift: fill on left with zeros

```
6 >> 1 => 3
```

0110_B 0011_B

```
-6 >> 1 => 5
```

1010_B 0101_B

What is the effect arithmetically?

Bitwise *arithmetic* right shift: fill on left with sign bit

```
6 >> 1 ⇒ 3
```

0110_B 0011_B

```
-6 >> 1 ⇒ -3
```

1010_B 1101_B

What is the effect arithmetically?

In C, right shift (>>) could be logical (>>> in Java) or arithmetic (>> in Java)

- Not specified by standard (happens to be arithmetic on armlab)
- Best to avoid shifting signed integers

Other Operations on Signed Ints

Bitwise NOT (~ in C)

- Same as with unsigned ints

Bitwise AND (& in C)

- Same as with unsigned ints

Bitwise OR: (| in C)

- Same as with unsigned ints

Bitwise exclusive OR (^ in C)

- Same as with unsigned ints

Best to avoid using signed ints for bit-twiddling (you're not usually using the ints for their integer values when you do this, anyway)

Assignment Operator

Many high-level languages provide an assignment *statement*

C provides an assignment operator

- Operator takes operands as inputs and produces a result
- Performs assignment and then **evaluates to the assigned value. Why?**
  - Allows assignment to appear within larger expressions
  - Terseness of code
  - But be careful about precedence. Extra parentheses often needed. Can confuse reader.

Assignment Operator Examples

Examples

```
i = 0;
   /* Side effect: assign 0 to i.
      Evaluate to 0. */

j = i = 0; /* Assignment op has R to L associativity */
   /* Side effect: assign 0 to i.
      Evaluate to 0.
      Side effect: assign 0 to j.
      Evaluate to 0. */

while ((i = getchar()) != EOF) …
   /* Read a character or EOF value.
      Side effect: assign that value to i.
      Evaluate to that value.
      Compare that value to EOF.
      Evaluate to 0 (FALSE) or 1 (TRUE). */
```

Special Assignment Operators in C

Motivation

- The construct a = b + c is flexible
- The construct d = d + e is somewhat common
- The construct d = d + 1 is very common

Assignment in C

- Useful: Introduce += operator to do things like d += e
  - Extend to -= *= /= ~= &= |= ^= <<= >>=
  - All evaluate to whatever was assigned. i.e., if a was 1, a+=2 is a = a + 2 so evaluates to 3
- Pre-increment and pre-decrement: ++d --d. Evaluates to d+1 and d-1
- Post-increment and post-decrement (evaluate to *old* value): d++ d--. Evaluates to d

Plusplus Playfulness / Confusion Plusplus

Q: What are i and j set to in the following code?

```
i = 5;
j = i++;
j += ++i;
```

A. 5, 7

B. 7, 5

C. 7, 11

D. 7, 12

E. 7, 13

Incremental Iffiness

Q: What does the following code print?

```
int i = 1;
switch (i++) {
   case 1: printf("%d", ++i);
   case 2: printf("%d", i++);
}
```

A. 1

B. 2

C. 3

D. 22

E. 33

Sample Exam Question (Spring 2017, Exam 1)

1(b) (12 points/100) Suppose we have a 7-bit computer. Answer the following questions.

(i) (4 points) What is the largest unsigned number that can be represented in 7 bits?

In binary: In decimal:

(ii) (4 points) What is the smallest (i.e., most negative) signed number represented in 2's complement in 7 bits?

In binary: In decimal:

(iii) (2 points) Is there a number n, other than 0, for which n is equal to –n, when represented in 2's complement in 7 bits? If yes, show the number (in decimal). If no, briefly explain why not.

(iv) (2 points) When doing arithmetic addition using 2's complement representation in 7 bits, is it possible to have an overflow when the first number is positive and the second is negative? (Yes/No answer is sufficient, no need to explain.)

Sample Exam Question (Fall 2024, Exam 1)

1 (d) (1 point /32) If acc is of type int, write a statement that has the same effect as the line acc *= 10, without using multiplication and using no more than one addition operation.

(Hard!) Sample Exam Question (Fall 2020, Exam 1)

a. In the two ranges below, replace the "____" with the inclusive upper and lower bounds of decimal numbers that do not change value when moving from i-bit two's complement to (i+1)-bit two's complement (for example, when moving from four bits to represent integers to using five bits to do so). The two ranges consider two different possibilities for changing an i-bit value into an (i+1)-bit value:

If we make the change by prepending a 0 onto the front of the i-bit representation (e.g., 1001 -> 01001):

____ <= x <= ____

If we make the change by prepending a 1 onto the front of the i-bit representation (e.g., 1001 -> 11001):

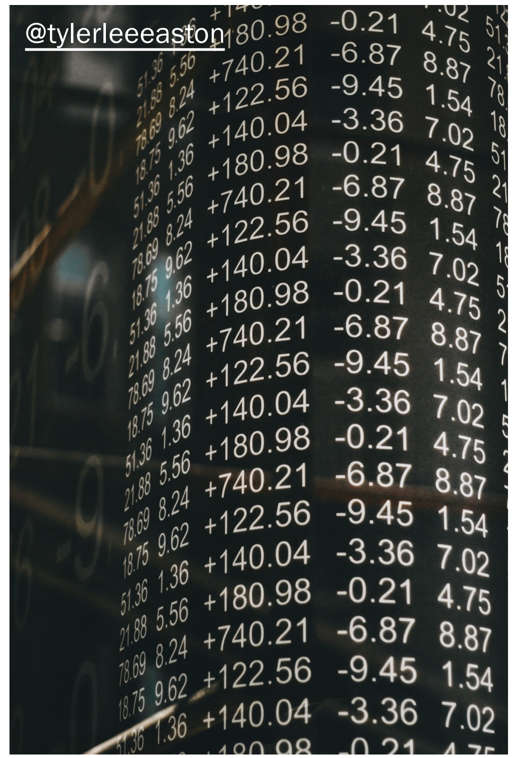
____ <= x <= ____

b. In the range below, replace the "____" with the inclusive upper and lower bounds of armlab C int literals for which the expression still compiles and does not change value when adding a 0 before the first character of the literal (for example, 217 -> 0217):

____ <= x <= ____

Hint 1: does a literal 09 compile?

Hint 2: the word ”expression” is intentional; note that the first character of a signed int is not necessarily a digit.



APPENDIX: FLOATING POINT

Rational Numbers

Mathematics

- A rational number is one that can be expressed as the ratio of two integers
- Unbounded range and precision

Computer science

- Finite range and precision
- Approximate using floating point number

Floating Point Numbers

Like scientific notation: e.g., c is

$$2.99792458 \times 10^{8} \text{ m/s}$$

This has the form

$$(\text{multiplier}) \times (\text{base})^{(\text{power})}$$

In the computer,

- Multiplier is called mantissa
- Base is almost always 2
- Power is called exponent

Floating-Point Data Types

C specifies:

- Three floating-point data types:
  float, double, and long double
- Sizes unspecified, but constrained:
- sizeof(float) ≤ sizeof(double) ≤ sizeof(long double)

On ArmLab (and on pretty much any 21st-century computer using the IEEE standard)

- float: 4 bytes
- double: 8 bytes

On ArmLab (but varying across architectures)

- long double: 16 bytes

Floating-Point Literals

How to write a floating-point number?

- Either fixed-point or "scientific" notation
- Any literal that contains decimal point or "E" is floating-point
- The default floating-point type is double
- Append "F" to indicate float
- Append "L" to indicate long double

Examples

- double: 123.456, 1E-2, -1.23456E4
- float: 123.456F, 1E-2F, -1.23456E4F
- long double: 123.456L, 1E-2L, -1.23456E4L

IEEE Floating Point Representation

Common finite representation: IEEE floating point

- More precisely: ISO/IEEE 754 standard

Using 32 bits (type **`float`** in C):

- 1 bit: sign (0⇒positive, 1⇒negative)
- 8 bits: exponent + 127
- 23 bits: binary fraction of the form 1.bbbbbbbbbbbbbbbbbbbbbbb

Using 64 bits (type **`double`** in C):

- 1 bit: sign (0⇒positive, 1⇒negative)
- 11 bits: exponent + 1023
- 52 bits: binary fraction of the form 1.bbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbbb

When was floating-point invented?

mantissa (noun): decimal part of a logarithm, 1865, ← Answer: long before computers!
from Latin mantisa "a worthless addition, makeweight"

COMMON LOGARITHMS $\log_{10} x$

x	0	1	2	3	4	5	6	7	8	9	Δ_m +	1	2	3
50	·6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	9	1	2	3
51	·7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	8	1	2	2
52	·7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	8	1	2	2
53	·7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	8	1	2	2
54	·7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	8	1	2	2
55	·7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	8	1	2	2

Floating Point Example

`11000001110110110000000000000000`

32-bit representation

Sign (1 bit):

- 1 ⇒ negative

Exponent (8 bits):

- $10000011_B = 131$
- $131 - 127 = 4$

Mantissa (23 bits):

- $1.10110110000000000000000_B$
- $1 + (1*2^{-1})+(0*2^{-2})+(1*2^{-3})+(1*2^{-4})+(0*2^{-5})+ (1*2^{-6})+(1*2^{-7}) +(0*2^{-\cdots})= 1.7109375$

Number:

- $-1.7109375 * 2^4 = -27.375$

Floating Point Consequences

"Machine epsilon": smallest positive number you can add to 1.0 and get something other than 1.0

For float: $\varepsilon \approx 10^{-7}$

- No such number as 1.000000001
- Rule of thumb: "almost 7 digits of precision"

For double: $\varepsilon \approx 2 \times 10^{-16}$

- Rule of thumb: "not quite 16 digits of precision"

These are all relative numbers

Floating Point Consequences, cont

Just as decimal number system can represent only some rational numbers with finite digit count...

- Example: 1/3 cannot be represented

Decimal Approx	Rational Value
.3	3/10
.33	33/100
.333	333/1000
...	

Binary number system can represent only some rational numbers with finite digit count

- Example: 1/5 cannot be represented

Binary Approx	Rational Value
0.0	0/2
0.01	1/4
0.010	2/8
0.0011	3/16
0.00110	6/32
0.001101	13/64
0.0011010	26/128
0.00110011	51/256
...	

Beware of round-off error

- Error resulting from inexact representation
- Can accumulate
- Be careful when comparing two floating-point numbers for equality

Floating away …

What does the following code print?

```
double sum = 0.0;
double i;
for (i = 0.0; i != 10.0; i++)
   sum += 0.1;
if (sum == 1.0)
   printf("All good!\n");
else
   printf("Yikes!\n");
```

A. All good!

B. Yikes!

C. (Infinite loop)

D. (Compilation error)

B: Yikes!

… loop terminates, because we can represent 10.0 exactly by adding 1.0 at a time.

… but sum isn't 1.0 because we can't represent 1.0 exactly by adding 0.1 at a time.