Thinking Inductively

COS 326
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Often, we either have a thing .... or we don’t:

Option types are used in this situation:  \( t \text{ option} \)

There’s \textit{one way} to build a pair, but \textit{two ways} to build an optional value:

- **None**  -- when we’ve got nothing
- **Some \( v \)**  -- when we’ve got a value \( v \) of type \( t \)
type point = float * float

let slope (p1:point) (p2:point) : float =

```
(x1, y1)

a

(x2, y2)
```
type point = float * float

let slope (p1:point) (p2:point) : float =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  deconstruct tuple
type point = float * float

let slope (p1:point) (p2:point) : float =
    let (x1,y1) = p1 in
    let (x2,y2) = p2 in
    let xd = x2 -. x1 in
    if xd != 0.0 then
        (y2 -. y1) /. xd
    else
        ???
Slope between two points

```ocaml
let slope (p1:point) (p2:point) : float option =
    let (x1,y1) = p1 in
    let (x2,y2) = p2 in
    let xd = x2 -. x1 in
    if xd != 0.0 then
        ???
    else
        ???
```

we need an option type as the result type

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type point = float * float

let slope (p1:point) (p2:point) : float option =
let (x1,y1) = p1 in
let (x2,y2) = p2 in
let xd = x2 -. x1 in
if xd != 0.0 then
    Some ((y2 -. y1) /. xd)
else
    None
Slope between two points

```
type point = float * float

let slope (p1:point) (p2:point) : float option =
  let (x1, y1) = p1 in
  let (x2, y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
    (y2 -. y1) /. xd
  else
    None
```

Has type `float`

Can have type `float option`
Slope between two points

type point = float * float

let slope (p1:point) (p2:point) : float option =
  let (x1, y1) = p1 in
  let (x2, y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
    (y2 -. y1) /. xd
  else
    None
type point = float * float

let slope (p1:point) (p2:point) : float option =
let (x1,y1) = p1 in
let (x2,y2) = p2 in
let xd = x2 -. x1 in
if xd != 0.0 then
  (y2 -. y1) /. xd
else
  None

Has type float
doubly WRONG: result does not match declared result
Remember the typing rule for if

```plaintext
if e1 : bool
and e2 : t and e3 : t (for some type t)
then if e1 then e2 else e3 : t
```

Returning an optional value from an if statement:

```plaintext
if ... then
  None : t option
else
  Some ( ... ) : t option
```
How do we use an option?

slope : point -> point -> float option

returns a float option
How do we use an option?

slope : point -> point -> float option

let print_slope (p1:point) (p2:point) : unit =
How do we use an option?

`slope : point -> point -> float option`

`let print_slope (p1:point) (p2:point) : unit =
  slope p1 p2`

returns a float option; to print we must discover if it is None or Some
How do we use an option?

```ocaml
slope : point -> point -> float option

let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
```
How do we use an option?

```
slope : point -> point -> float option

let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
    Some s ->
    | None ->

There are two possibilities

Vertical bar separates possibilities
```
How do we use an option?

`slope : point -> point -> float option`

```ocaml
let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
  Some s ->
  | None ->
```

The "Some s" pattern includes the variable s

The object between | and -> is called a pattern
How do we use an option?

```ocaml
slope : point -> point -> float option

let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
  | Some s ->
  | None ->

You can put a “|” on the first line if you want. It is generally considered better style to do so.
How do we use an option?

slope : point -> point -> float option

let print_slope (p1:point) (p2:point) : unit =
    match slope p1 p2 with
    Some s ->
        print_string ("Slope: " ^ string_of_float s)
    | None ->
        print_string "Vertical line.\n"
Writing Functions Over Typed Data

- Steps to writing functions over typed data:
  1. Write down the function and argument names
  2. Write down argument and result types
  3. Write down some examples (in a comment)
  4. **Deconstruct** input data structures
  5. **Build** new output values
  6. Clean up by identifying repeated patterns

- For option types:

  when the input has type `t option`,
  deconstruct with:

  ```
  match ... with
  | None  -> ...
  | Some s -> ...
  ```

  when the output has type `t option`,
  construct with:

  ```
  Some (...)
  None
  ```
MORE PATTERN MATCHING
Recall the Distance Function

```ocaml
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```
Recall the Distance Function

```ocaml
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

(x2, y2) is an example of a pattern – a pattern for tuples.

So let declarations can contain patterns just like match statements.

The difference is that a match allows you to consider multiple different data shapes.
Recall the Distance Function

type point = float * float

let distance (p1:point) (p2:point) : float =
    let square x = x *. x in
    match p1 with
    | (x1,y1) ->
      let (x2,y2) = p2 in
      sqrt (square (x2 -. x1) +. square (y2 -. y1))

There is only 1 possibility when matching a pair
Recall the Distance Function

type point = float * float

let distance (p1:point) (p2:point) : float =
    let square x = x *. x in
    match p1 with
    | (x1,y1) ->
        match p2 with
        | (x2,y2) ->
            sqrt (square (x2 -. x1) +. square (y2 -. y1))

We can nest one match expression inside another.
(We can nest any expression inside any other, if the expressions have the right types)
Better Style: Complex Patterns

```ocaml
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match (p1, p2) with
  | ((x1, y1), (x2, y2)) ->
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

we built a pair of pairs

Pattern for a pair of pairs:  
((variable, variable), (variable, variable)) 
All the variable names in the pattern must be different.
type point = float * float

let distance (p1:point) (p2:point) : float =
    let square x = x *. x in
    match (p1, p2) with
    | (p3, p4) ->
        let (x1, y1) = p3 in
        let (x2, y2) = p4 in
        sqrt (square (x2 -. x1) +. square (y2 -. y1))

A pattern must be **consistent with** the type of the expression in between `match ... with`
We use (p3, p4) here instead of ((x1, y1), (x2, y2))
Pattern-matching in function parameters

```ocaml
type point = float * float

let distance ((x1,y1):point) ((x2,y2):point) : float =
  let square x = x *. x in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

Function parameters are patterns too!
What's the best style?

```
let distance (p1:point) (p2:point) : float =
    let square x = x *. x in
    let (x1,y1) = p1 in
    let (x2,y2) = p2 in
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

```
let distance ((x1,y1):point) ((x2,y2):point) : float =
    let square x = x *. x in
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

Either of these is reasonably clear and compact.
Code with unnecessary nested matches/lets is particularly ugly to read.
You'll be judged on code style in this class.
What’s the best style?

This is how I'd do it ... the types for tuples + the tuple patterns are a little ugly/verbose ... but for now in class, use the explicit type annotations. We will loosen things up later in the semester.

let distance (x1, y1) (x2, y2) =
  let square x = x *. x in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
type point = float * float

(* returns a nearby point in the graph if one exists *)

nearby : graph -> point -> point option

let printer (g:graph) (p:point) : unit =
  match nearby g p with
  | None -> print_string "could not find one\n"
  | Some (x,y) ->
    print_float x;
    print_string ", ";
    print_float y;
    print_newline();
Other Patterns

Constant values can be used as patterns

```ocaml
let small_prime (n:int) : bool =
  match n with
  | 2 -> true
  | 3 -> true
  | 5 -> true
  | _ -> false

let iffy (b:bool) : int =
  match b with
  | true -> 0
  | false -> 1
```

the underscore pattern
matches anything
it is the "don't care" pattern
INDUCTIVE THINKING
An *inductive data type* $T$ is a data type defined by:

- base cases
  - don’t refer to $T$

- inductive cases
  - build new data of type $T$ from pre-existing data of type $T$
  - the pre-existing data is guaranteed to be *smaller* than the new values
An *inductive data type* $T$ is a data type defined by:

- **base cases**
  - don’t refer to $T$

- **inductive cases**
  - build new data of type $T$ from pre-existing data of type $T$
  - the pre-existing data is guaranteed to be *smaller* than the new values

**Example: a tree**

- **base case:**
  - the leaf of the tree

- **inductive case:**
  - the internal nodes of the tree
  - the left- and right- subtrees are the “smaller” data
To *program* a function over inductive data:

- think: what does my function need to do to be correct?
- solve the programming problem for the base cases
  - solve them one-by-one
- solve the programming problem for inductive cases:
  - solve them one-by-one
  - assume your function already works correctly on smaller data values
  - call your function, when necessary, on smaller data values
To \textit{prove} a function over inductive data is correct:

– think: what is the correctness theorem for this function?
– prove the function correct for the base cases
  • prove them one-by-one
– prove the function correct for the inductive cases:
  • prove them one-by-one
  • \textit{assume your function already works correctly on smaller data values}
  • \textit{use this assumption to reason about calls over smaller data values}
  • this assumption is called the \textit{induction hypothesis} of your proof
To prove a function over inductive data is correct:

– think: what is the correctness theorem for this function?
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  • prove them one-by-one
  • assume your function already works correctly on smaller data values
  • use this assumption to reason about calls over smaller data values
  • this assumption is called the induction hypothesis of your proof

To be a good programmer, you also need to be a good prover.
LISTS: AN INDUCTIVE DATA TYPE
Lists are Inductive Data

In OCaml, a list value is:

- `[ ]` (the empty list)
- `v :: vs` (a value `v` followed by a shorter list of values `vs`)
Lists are Inductive Data

In OCaml, a list value is:

- `[ ]` (the empty list)
- `v :: vs` (a value `v` followed by a shorter list of values `vs`)

An example:
- `2 :: 3 :: 5 :: [ ]` has type `int list`
- is the same as: `2 :: (3 :: (5 :: [ ]))`
- `"::"` is called "cons"

An alternative syntax ("syntactic sugar" for lists):

- `[2; 3; 5]`
- But this is just a shorthand for `2 :: 3 :: 5 :: []`. If you ever get confused fall back on the 2 basic `constructors`, `::` and `[]`
Typing Lists

Typing rules for lists:

(1) [ ] may have any list type, \( t \) list

(2) if \( e_1 : t \) and \( e_2 : t \) list
then \( (e_1 :: e_2) : t \) list
Typing Lists

Typing rules for lists:

(1) \([\ ]\) may have any list type \(t \text{ list}\)

(2) if \(e_1 : t\) and \(e_2 : t \text{ list}\) then \((e_1 :: e_2) : t \text{ list}\)

More examples:

(1 + 2) :: (3 + 4) :: [ ] : ??

(2 :: [ ]) :: (5 :: 6 :: [ ]) :: [ ] : ??

Typing rules for lists:

(1) [ ] may have any list type t list

(2) if e1 : t and e2 : t list
    then (e1 :: e2) : t list

More examples:

(1 + 2) :: (3 + 4) :: [ ] : int list

(2 :: [ ]) :: (5 :: 6 :: [ ]) :: [ ] : int list list

[ [2]; [5; 6] ] : int list list

(Remember that the 3\textsuperscript{rd} example is an abbreviation for the 2\textsuperscript{nd})
Another Example

What type does this have?

What type does this have?

```
# [2] :: [3];
Error: This expression has type int but an expression was expected of type int list
#
```
What type does this have?

\[
\]

int list

int list

Give me a simple fix that makes the expression type check?
Another Example

What type does this have?


int list

int list

Give me a simple fix that makes the expression type check?

Either: 2 :: [ 3 ] : int list

Analyzing Lists

Just like options, there are two possibilities when deconstructing lists. Hence we use a match with two branches

(* return Some v, if v is the first list element; return None, if the list is empty *)

let head (xs : int list) : int option =
Analyzing Lists

Just like options, there are two possibilities when deconstructing lists. Hence we use a match with two branches

\[
\text{let head (xs : int list) : int option =}
\]
\[
\text{match xs with}
\]
\[
\mid [] ->
\]
\[
\mid hd :: _ ->
\]

(* return Some v, if v is the first list element; return None, if the list is empty *)

we don't care about the contents of the tail of the list so we use the underscore
Analyzing Lists

Just like options, there are two possibilities when deconstructing lists. Hence we use a match with two branches

```ocaml
(* return Some v, if v is the first list element; return None, if the list is empty *)

let head (xs : int list) : int option =
  match xs with
  | [] -> None
  | hd :: _ -> Some hd
```

This function isn't recursive -- we only extracted a small, fixed amount of information from the list -- the first element
(* Given a list of pairs of integers, produce the list of products of the pairs

prods [(2,3); (4,7); (5,2)] == [6; 28; 10]
*)
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let rec prods (xs : (int * int) list) : int list =
A more interesting example

(* Given a list of pairs of integers, produce the list of products of the pairs

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let rec prods (xs : (int * int) list) : int list =
    match xs with
    | [] ->
    | (x,y) :: tl ->
A more interesting example

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A more interesting example

(* Given a list of pairs of integers, produce the list of products of the pairs

\[ \text{prods } [(2,3); (4,7); (5,2)] == [6; 28; 10] \]

*)

let rec prods (xs : (int * int) list) : int list =
match xs with
| [] -> []
| (x,y) :: tl -> ?? :: ??

the result type is int list, so we can speculate that we should create a list
(* Given a list of pairs of integers, produce the list of products of the pairs

prods [(2,3); (4,7); (5,2)] == [6; 28; 10]*)

let rec prods (xs : (int * int) list) : int list =
match xs with
| [] -> []
| (x,y) :: tl -> (x * y) :: ??

the first element is the product
A more interesting example

(* Given a list of pairs of integers, produce the list of products of the pairs

prods [(2,3); (4,7); (5,2)] == [6; 28; 10]
*)

let rec prods (xs : (int * int) list) : int list =
    match xs with
    | [] -> []
    | (x,y) :: tl -> (x * y) :: ??

to complete the job, we must compute the products for the rest of the list
(* Given a list of pairs of integers, produce the list of products of the pairs

prods [(2,3); (4,7); (5,2)] == [6; 28; 10] *)

let rec prods (xs : (int * int) list) : int list = match xs with |
| [] -> [] |
| (x,y) :: tl -> (x * y) :: prods tl
Three Parts to Constructing a Function

(1) Think about how to *break down* the input into cases:

```ml
let rec prods (xs : (int*int) list) : int list =
match xs with
  | [] -> ...
  | (x,y) :: tl -> ...
```

(2) *Assume* the recursive call on smaller data is correct.

(3) Use the result of the recursive call to *build* correct answer.

```ml
let rec prods (xs : (int*int) list) : int list =
  ...
  | (x,y) :: tl -> ...
  prods tl ...
```
Another example: zip

(* Given two lists of integers, return None if the lists are different lengths otherwise stitch the lists together to create Some of a list of pairs

zip [2; 3] [4; 5] == Some [(2,4); (3,5)]
zip [5; 3] [4] == None
zip [4; 5; 6] [8; 9; 10; 11; 12] == None
*)

(Give it a try.)
let rec zip (xs : int list) (ys : int list) : (int * int) list option =
let rec zip (xs : int list) (ys : int list) : (int * int) list option =

  match (xs, ys) with
let rec zip (xs : int list) (ys : int list) : (int * int) list option =

match (xs, ys) with
| ([], []) ->
| ([], y::ys') ->
| (x::xs', []) ->
| (x::xs', y::ys') ->
Another example: zip

```
let rec zip (xs : int list) (ys : int list)
    : (int * int) list option =

    match (xs, ys) with
    | ([], []) -> Some []
    | ([], ys') ->
    | (x::xs', []) ->
    | (x::xs', y::ys') ->
```
let rec zip (xs : int list) (ys : int list) : (int * int) list option =

match (xs, ys) with
| ([], []) -> Some []
| ([], y::ys') -> None
| (x::xs', []) -> None
| (x::xs', y::ys') ->
let rec zip (xs : int list) (ys : int list) : (int * int) list option =

  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y::ys') -> None
  | (x::xs', []) -> None
  | (x::xs', y::ys') -> (x, y) :: zip xs' ys'

is this ok?
Another example: zip

let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =

  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y::ys') -> None
  | (x::xs', []) -> None
  | (x::xs', y::ys') -> (x, y) :: zip xs' ys'

No! zip returns a list option, not a list!
We need to match it and decide if it is Some or None.
Another example: zip

```ocaml
let rec zip (xs : int list) (ys : int list) : (int * int) list option =
  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y::ys') -> None
  | (x::xs', []) -> None
  | (x::xs', y::ys') ->
    (match zip xs' ys' with
      None -> None
    | Some zs -> (x,y) :: zs)
```

Is this ok?
let rec zip (xs : int list) (ys : int list)
    : (int * int) list option =

  match (xs, ys) with
  | ([], []) -> Some []
  | ([], y::ys') -> None
  | (x::xs', []) -> None
  | (x::xs', y::ys') ->
      (match zip xs' ys' with
        None -> None
      | Some zs -> Some ((x,y) :: zs))
Another example: zip

```ocaml
let rec zip (xs : int list) (ys : int list) : (int * int) list option =

  match (xs, ys) with
  | ([], []) -> Some []
  | (x::xs', y::ys') ->
    (match zip xs' ys' with
    None -> None
    | Some zs -> Some ((x,y) :: zs))
  | (_, _) -> None
```

Clean up.
Reorganize the cases.
Pattern matching proceeds in order.
let rec sum (xs : int list) : int =
  match xs with
  | hd::tl -> hd + sum tl
A bad list example

let rec sum (xs : int list) : int =
    match xs with
    | hd::tl -> hd + sum tl

Warning 8: this pattern-matching is not exhaustive.
Here is an example of a value that is not matched: []
val sum : int list -> int = <fun>
INSERTION SORT
Recall Insertion Sort

At any point during the insertion sort:

– some initial segment of the array will be sorted
– the rest of the array will be in the same (unsorted) order as it was originally

-5  -2  3  -4  10  6  7

sorted  unsorted
Recall Insertion Sort

At any point during the insertion sort:

– some initial segment of the array will be sorted
– the rest of the array will be in the same (unsorted) order as it was originally

At each step, take the next item in the array and insert it in order into the sorted portion of the list
The algorithm is similar, except instead of one array, we will maintain two lists, a sorted list and an unsorted list.

We'll factor the algorithm:
- a function to insert into a sorted list
- a sorting function that repeatedly inserts
let rec insert (x : int) (xs : int list) : int list =
(* insert x into sorted list xs *)

let rec insert (x : int) (xs : int list) : int list =
  match xs with
  | [] ->
  | hd :: tl ->

a familiar pattern: analyze the list by cases
(* insert x into sorted list xs *)

let rec insert (x : int) (xs : int list) : int list =
match xs with
| [] -> [x]
| hd :: tl ->

insert x into the empty list
let rec insert (x : int) (xs : int list) : int list =
match xs with
| []       -> [x]
| hd :: tl ->
  if hd < x then
    hd :: insert x tl
build a new list with:
• hd at the beginning
• the result of inserting x into the tail of the list afterwards
let rec insert (x : int) (xs : int list) : int list =
match xs with
| [] -> [x]
| hd :: tl ->
  if hd < x then
    hd :: insert x tl
  else
    x :: xs

(* insert x into sorted list xs *)

put x on the front of the list, the rest of the list follows
type il = int list

insert : int -> il -> il

(* insertion sort *)

let rec insert_sort(xs : il) : il =
type il = int list

insert : int -> il -> il

(* insertion sort *)

let rec insert_sort(xs : il) : il =

   let rec aux (sorted : il) (unsorted : il) : il =

      in
type il = int list

insert : int -> il -> il

(* insertion sort *)

let rec insert_sort(xs : il) : il =
  let rec aux (sorted : il) (unsorted : il) : il =
    in
  aux [] xs
type il = int list

insert : int -> il -> il

(* insertion sort *)

let rec insert_sort(xs : il) : il =

  let rec aux (sorted : il) (unsorted : il) : il =
      match unsorted with
      | [] ->
      | hd :: tl ->
    in
    aux [] xs
type il = int list

insert : int -> il -> il

(* insertion sort *)

let rec insert_sort(xs : il) : il =

  let rec aux (sorted : il) (unsorted : il) : il =
    match unsorted with
    | [] -> sorted
    | hd :: tl -> aux (insert hd sorted) tl
  in
  aux [] xs
Does Insertion Sort Terminate?

Recall that we said: inductive functions should call themselves recursively on *smaller data items*.

What about that loop in insertion sort?

```ocaml
let rec loop (sorted : il) (unsorted : il) : il =
  match unsorted with
  | []  -> sorted
  | hd :: tl -> loop (insert hd sorted) tl
```
Does Insertion Sort Terminate?

Recall that we said: inductive functions should call themselves recursively on *smaller data items*.

What about that loop in insertion sort?

```ocaml
let rec loop (sorted : il) (unsorted : il) : il =
  match unsorted with
  | [] -> sorted
  | hd :: tl -> loop (insert hd sorted) tl
```

growing!

shrinking!
Does Insertion Sort Terminate?

Recall that we said: inductive functions should call themselves recursively on *smaller data items*.

What about that loop in insertion sort?

```ocaml
let rec loop (sorted : il) (unsorted : il) : il =
  match unsorted with
  | [] -> sorted
  | hd :: tl -> loop (insert hd sorted) tl
```

Refined idea: Pick an argument up front. That argument must contain smaller data *on every recursive call*. 
Exercises

• Write a function to sum the elements of a list
  – sum [1; 2; 3] ==> 6

• Write a function to append two lists
  – append [1;2;3] [4;5;6] ==> [1;2;3;4;5;6]

• Write a function to reverse a list
  – rev [1;2;3] ==> [3;2;1]

• Write a function to turn a list of pairs into a pair of lists
  – split [(1,2); (3,4); (5,6)] ==> ([1;3;5], [2;4;6])

• Write a function that returns all prefixes of a list
  – prefixes [1;2;3] ==> [[]; [1]; [1;2]; [1;2;3]]

• suffixes...
A SHORT JAVA RANT
Definition and Use of Java Pairs

public class Pair {
    public int x;
    public int y;
    public Pair (int a, int b) {
        x = a;
        y = b;
    }
}

public class User {
    public Pair swap (Pair p1) {
        Pair p2 =
            new Pair(p1.y, p1.x);
        return p2;
    }
}

What could go wrong?
A Paucity of Types

The input \texttt{p1} to swap may be \texttt{null} and we forgot to check.

Java has no way to define a pair data structure that is \textit{just a pair}.

\textit{How many students in the class have seen an accidental null pointer exception thrown in their Java code?}
In OCaml, if a pair may be null it is a pair option:

```ocaml
type java_pair = (int * int) option
```
From Java Pairs to OCaml Pairs

In OCaml, if a pair may be null it is a pair option:

```ocaml
type java_pair = (int * int) option
```

And if you write code like this:

```ocaml
let swap_java_pair (p:java_pair) : java_pair =
  let (x,y) = p in
  (y,x)
```
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  let (x,y) = p in
  (y,x)
```

You get a *helpful* error message like this:

```ocaml
# ... Characters 91-92:
  let (x,y) = p in (y,x);;
    ^
Error: This expression has type java_pair = (int * int) option
     but an expression was expected of type 'a * 'b
```
type java_pair = (int * int) option

And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

let swap_java_pair (p:java_pair) : java_pair =
  match p with
  | Some (x,y) -> Some (y,x)
type java_pair = (int * int) option

And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

let swap_java_pair (p:java_pair) : java_pair =
  match p with
  | Some (x,y) -> Some (y,x)

OCaml to the rescue!

..match p with
  | Some (x,y) -> Some (y,x)
Warning 8: this pattern-matching is not exhaustive.
Here is an example of a value that is not matched:
None
From Java Pairs to OCaml Pairs

```ocaml
type java_pair = (int * int) option

let swap_java_pair (p:java_pair) : java_pair =
match p with
| Some (x,y) -> Some (y,x)
```

And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

```ocaml
let swap_java_pair (p:java_pair) : java_pair =
match p with
| None -> None
| Some (x,y) -> Some (y,x)
```

An easy fix!
Moreover, your pairs are probably almost never null!

Defensive programming & always checking for null is **AnNOying**
There just isn't always some "good thing" for a function to do when it receives a bad input, like a null pointer.

In OCaml, all these issues disappear when you use the proper type for a pair and that type contains no "extra junk".

```ocaml
type pair = int * int

let swap (p:pair) : pair =
  let (x,y) = p in (y,x)
```

Once you know OCaml, it is **hard** to write swap incorrectly. Your *bullet-proof* code is much simpler than in Java.
Java has a paucity of types
   – There is no type to describe just the pairs
   – There is no type to describe just the triples
   – There is no type to describe the pairs of pairs
   – There is no type ...

OCaml has many more types
   – use option when things may be null
   – do not use option when things are not null
   – OCaml types describe data structures more precisely
     • programmers have fewer cases to worry about
     • entire classes of errors just go away
     • type checking and pattern analysis help prevent programmers from ever forgetting about a case
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- Use Option when things may be null
- Do not use Option when things are not null
- OCaml types describe data structures more precisely
  - Programmers have fewer cases to worry about
  - Entire classes of errors just go away
  - Type checking and pattern analysis help prevent programmers from ever forgetting about a case

**SCORE: OCAML 1, JAVA 0**
Example problems to practice

• Write a function to sum the elements of a list
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  – prefixes [1;2;3] ==> [[]; [1]; [1;2]; [1;2;3]]

• suffixes...