Precept Topics
- Binary and Balanced Search Trees
- Midterm Review

Relevant Material
- Book chapters: 3.1, 3.2 and 3.3.

A. RECAP: Binary Search Trees and Balanced Search Trees

Your preceptor will briefly review key points of this week's lectures.

B. EXERCISE (Midterm Review): Subtree Counts (Spring '21 Midterm)

Design a *multiset* data type that supports adding integer keys and performing the following two types of queries:

- *Count*: Given an integer k, determine the number of integers in the multiset equal to k.
- *Rank*: Given an integer k, determine the number of integers in the multiset strictly less than k.

To do so, implement this API:

public class Multiset		
	`Multiset()`	*create an empty multiset*
`void`	`add(long k)`	*add the integer k to the multiset*
`int`	`count(long k)`	*number of integers in the multiset equal to k*
`int`	`rank(long k)`	*number of integers in the multiset strictly less than k*

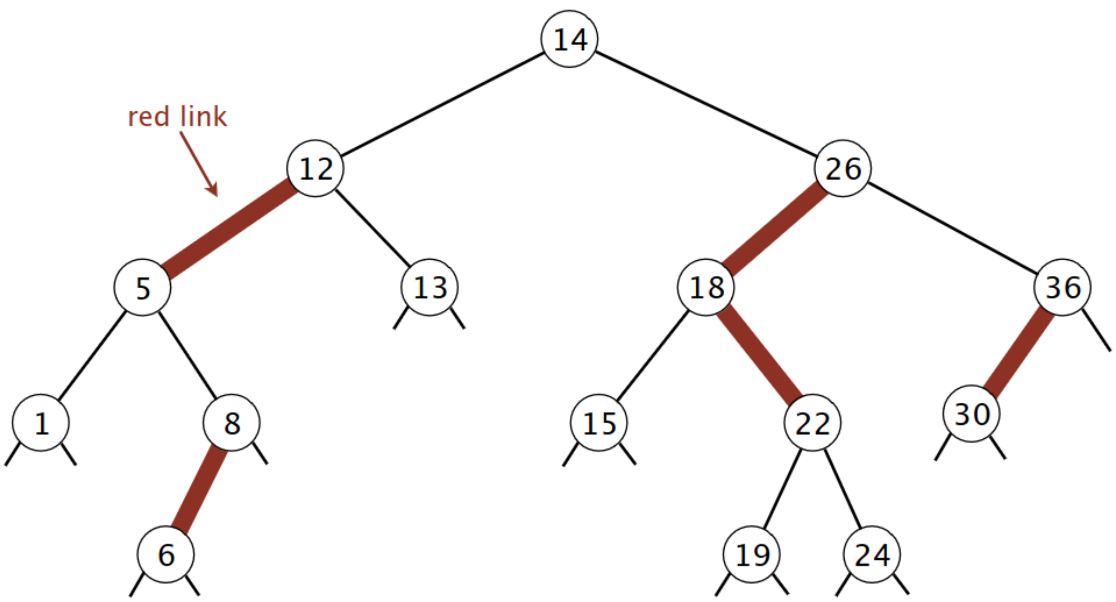
Note that, unlike in a symbol table with integer keys, an integer can appear in a multiset more than once. Here is an example:

```
Multiset multiset = new Multiset();    // [ ]
multiset.add(20);                      // [ 20 ]
multiset.add(30);                      // [ 20 30 ]
multiset.add(40);                      // [ 20 30 40 ]
multiset.add(30);                      // [ 20 30 30 40 ]
multiset.add(30);                      // [ 20 30 30 30 40 ]
multiset.add(20);                      // [ 20 20 30 30 30 40 ]
multiset.count(30);                    // 3
multiset.count(35);                    // 0
multiset.rank(30);                     // 2
multiset.rank(35);                     // 5
```

For full credit, each operation must take $O(\log n)$ time and use $O(n)$ extra space, where n is the number of integers added to the multiset. Half credit if `rank()` takes $\Theta(n)$ time.

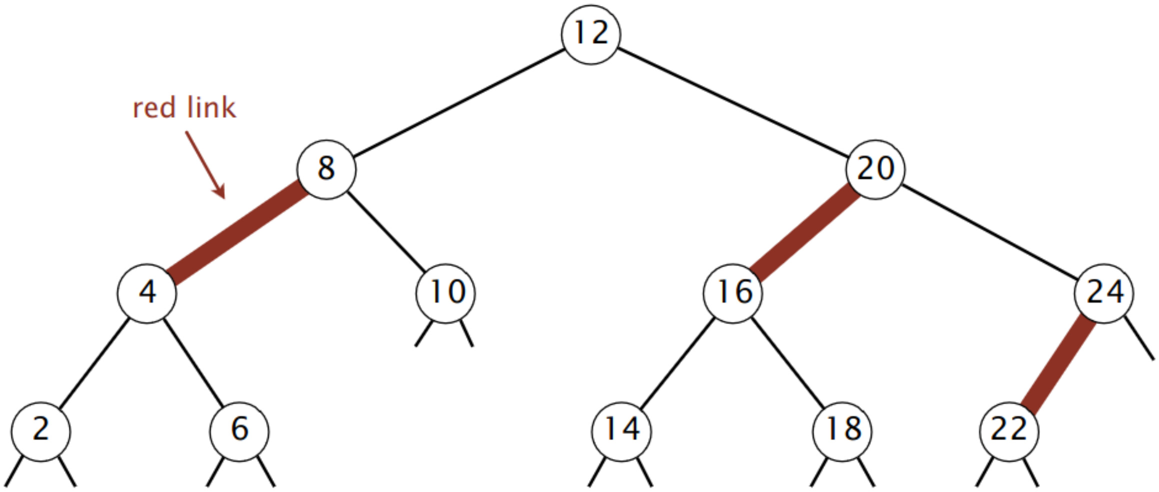
C. EXERCISES (Midterm Review): Red-Black Trees

1) (Spring '23 Midterm) The following BST satisfies perfect black balance, but violates color invariants:



Give a sequence of 4 elementary operations (*color flip*, *rotate left* or *rotate right*) that restore the color invariants.

2) (Fall '19 Midterm) Suppose that you insert the key 21 into the following left-leaning red–black BST:



Give the sequence of 4 elementary operations (*color flips* and *rotations*) that result.

D. EXERCISE (Midterm Review): Runtime Analysis (Fall '17 Midterm)

Consider an array that contains two successive copies of the integers 1 through n, in ascending order. For example, here is the array when $n = 8$:

1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8

Note that the length of the array is $2n$, not n.

(a) How many compares does *selection sort* make to sort the array as a function of n? Use tilde notation to simplify your answer.

$\sim$ [ ] compares

(b) How many compares does *insertion sort* make to sort the array as a function of n? Use tilde notation to simplify your answer.

$\sim$ [ ] compares

(c) How many compares does *mergesort* make to sort the array as a function of n? Assume n is a power of 2. Use tilde notation to simplify your answer.

$\sim$ [ ] compares

E. EXERCISE (Midterm Review): Memory Analysis (Fall '14 Midterm)

Suppose that you implement a left-leaning red-black BST using the following representation:

```
public class RedBlackBST<Key extends Comparable<Key>, Value> {
  private Node root;        // root of BST
  private int N;            // number of key-value pairs
  private class Node {
    private Key key;        // symbol table key
    private Value value;    // symbol table value
    private Node left;      // left child
    private Node right;     // right child
    private boolean color; // color of link from parent
    private int count;      // number of nodes in subtree rooted at this node
  }
  ...
}
```

Using the 64-bit memory cost model from lecture and the textbook, how much memory (in bytes) does a `RedBlackBST` object use as a function of the number of key-value pairs `N`? Use tilde notation to simplify your answer.

Include all memory except for the `Key` and `Value` objects themselves (because you do not know their types).

$\sim$ [ ] bytes