Introducing Assignment 4: Rasterizer
Agenda

- Overview of Rasterizer
- Rasterization Pipeline
  - Transformation Pipeline
  - Triangle Pipeline
  - Pixel Shading (Coloring)
  - Texture Mapping
What is Rasterization?

- Renders 3D primitives to a 2D image using projection
Rasterization vs Ray Tracing

● **Pros:**
  ○ Less computationally expensive
    ■ “Shoot rays from screen to objects” vs “Project objects to screen”
  ○ Takes advantage of spatial coherence of 3D objects
    ■ “Since this pixel is determined by a point on this triangle, then the neighboring pixels are likely determined by the same tri.”

● **Con:** Less realistic light behavior than ray tracing

● **Useful for live rendering**
  ○ Video games, Assignment 2(!)
Rasterization Pipeline

Transform into 3D world coordinates
Illuminate according to lighting and model reflectance
Transform into 3D camera coordinate system
Transform into 2D camera coordinate system
Clip primitives outside camera’s view
Transform into image coordinate system
Draw pixels
Transformation Pipeline

In A4:
All meshes are made of triangular faces

Your function transforms a triangle with 3D world coordinates to a projected triangle with 2D image coordinates
Viewing Transformation

- Going from world coordinates to camera coordinates
- The “pose” of a camera is written as \([R|t]\), a 4x4 matrix
  - \(R\) is a 3x3 rotation matrix, \(t\) is a 3x1 translation term
  - The last row exists because we are using homogeneous coords.
- \([R|t]\) transforms a point represented in camera coordinates to world coordinates
- So to do the opposite, we apply the inverse of \([R|t]\)
Homogeneous Coordinates

- It has a fourth dimension, but think of it as another representation of 3D coordinates.
- To transform a 4D homogeneous coord to 3D coord:
  - \((x, y, z, w) \rightarrow (x/w, y/w, z/w)\)
- A 3D coordinate \((x,y,z)\) is equivalent to \((x, y, z, 1)\) in 4D homogeneous coordinates.
- Important because the projection matrices we provide are in 4D homogeneous coordinates
Perspective Projection Transformation

- Going from 3D camera coordinates to 2D screen coordinates
  - More specifically, we want to convert to Normalized Device Coordinates (NDC)
View Volumes

- Camera Coordinates
  - A truncated pyramid frustum view, bounded by \([l, r]\) in \(x\), \([b, t]\) in \(y\), and \([-n, -f]\) in \(z\)
  - Positive \(z\) axis is going towards camera
View Volumes

- Normalized Device Coordinates
  - The “canonical” view volume bounded by a cube
  - Maps \([l, r]\) \(\rightarrow\) \([-1, 1]\) in \(x\),
  - \([b, t]\) \(\rightarrow\) \([-1, 1]\) in \(y\),
  - \([-n, -f]\) \(\rightarrow\) \([-1, 1]\) in \(z\)
Intuition of Transform to Canonical View

- Think about one dimension at a time
  - How can we scale it so the edges are going to be bound by [-1,1]?
Perspective Projection Matrix

- The matrix that transforms from frustum view to canonical view
  
  projMat = \[
  \begin{pmatrix}
  \frac{2n}{r-l} & 0 & \frac{r+l}{r-l} & 0 \\
  0 & \frac{2n}{t-b} & \frac{r-l}{t-b} & 0 \\
  0 & 0 & \frac{-f+n}{f-n} & \frac{2fn}{f-n} \\
  0 & 0 & -1 & 0 \
  \end{pmatrix}
  \]

- Remember to divide your result by w to get the 3D equivalent
- If your resulting z is not within the bounds of the canonical view, skip the triangle because it shouldn’t be seen
Viewport Transformation

- Going from Normalized Device Coordinates to image coordinates
  - X: [-1,1] -> [0, image width]
  - Y: [-1,1] -> [0, image height]

- Should we save Z?
  - Yes, need it for Z-buffering
    - Determining which object is closer to camera if they take up the same pixel, the closer thing gets rendered
Implementation Hints for Transform

- The world to camera transformation and perspective projection matrices are already precomputed for you!
  - $\text{viewMat} = \text{projMat} \times [R|t]^{-1}$
- You have to apply this matrix to the triangle in 3D space to project it to Normalized Device Coordinates, then scale it to image coordinates
Pipeline of Rendering a Triangle

- Now we know how to transform, how do we render it?
- **Transform** a 3D triangle in world space to 2D triangle in image space
- Compute bounding box of the triangle
- For each pixel \((x, y)\) in the bounding box:
  - Check if it’s in the triangle w/ barycentric coordinates. If not, skip this pixel
  - Use barycentric coords to interpolate the z value for this pixel
  - If this z value is bigger than the value in z buffer for this pixel, skip this pixel
  - **Render the pixel**, and save this z value to z buffer for this pixel
A point in a triangle can be represented as a convex combination of the three vertices.

- If any of the weights \( t_i \) are negative, then the point is not in the triangle.

Determine \( t_1 \) as the mass at \( A_1 \) that will balance a mass \( t_2 + t_3 \) at \( Q \).

Efficient 2D algorithm on slides 30-33 at

https://www.cs.drexel.edu/~david/Classes/CS430/Lectures/L-10_NURBSDrawing.pdf
Pipeline of Rendering a Pixel

- Now we know which pixel to render, how do we color it?
- For a pixel to render,
  - Compute the normal and position of this pixel in \textit{world coords} **
    - Use barycentric coords to interpolate
  - Find view position of the camera in \textit{world coords}
  - Find light source position(s) in \textit{world coords}
  - Get the material of this pixel (getPhongMaterial) **
  - Apply the given shader (either flat, Gourand, or Phong) using the above variables to color the pixel
    - Very similar to A3!
Overview of Shaders

- **Flat**
  - Color of pixel is determined by face normal and centroid
    - Calculate color once per triangle

- **Gouraud**
  - Color of pixel is an interpolation of the colors at the vertices
    - Calculate color x3 per triangle

- **Phong**
  - Color of pixel determined by its normal found by interpolation
    - Calculate color once **per pixel** in triangle
Texture Mapping

● If a mesh has a texture map, you have to define the uv coordinate for getPhongMaterial

● UV Coordinates
  ○ A vertex of the triangle with uv coordinate (u,v) in the texture map will have that color of texture map at (u,v)
  ○ Make sure uvs[] is defined for the triangle because not all meshes have texture maps
Texture Mapping

- Normal Mapping
  - Adds additional detail to texture map
  - Stores normal vector information in an image I
  - The image uses same UV coordinates
  - For a vertex with UV coordinate (u,v)
    - Get RGB at I(u,v)
    - Compute normal \( \text{XYZ} = \text{normalize}(2*\text{RGB} - 1) \)
  - Use this new normal when calculating color with Phong Reflection Model