

Finishing Up Assignment 1: Image Processing

COS 426: Computer Graphics (Fall 2022)

Guðni Gunnarsson, Yuanqiao Lin, Yuting Yang

Picking up where we left off last week...

Luminance

- Brightness
- Contrast
- Gamma
- Vignette
- Histogram equalization

Color

- Grayscale
- Saturation
- White balance
- Histogram matching

Filter

- Gaussian
- Sharpen
- Edge detect
- Median
- Bilateral filter

Dithering

- Quantization
- Random dithering
- Floyd-Steinberg error diffusion
- Ordered dithering

Resampling

- Bilinear sampling
- Gaussian sampling
- Translate
- Scale
- Rotate
- Swirl

Composite

- Composite
- Morph

This week's precept
will focus specifically
on this topic

A Familiar Pattern



Notice anything familiar about the pattern?

Why Dither?

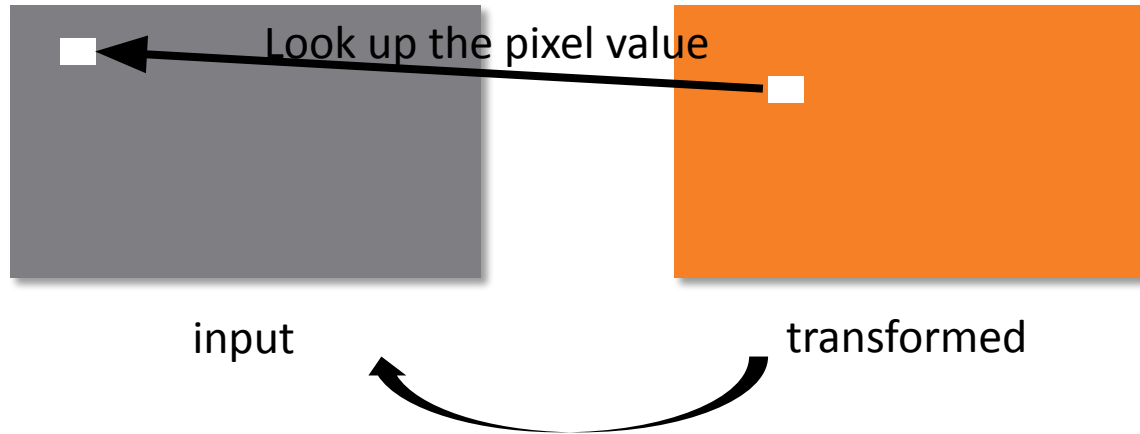


It's a Floyd-Steinberg dither over RGB channels (1 bit each)!

This filter was often used to compress web GIFs — look for the artifact in old-school animations!

Transformation (translate/scale/rotate/swirl)

- Inverse mapping



Inverse mapping guarantees that every pixel in the transformed image is filled!

Transformation (translate/scale/rotate/swirl)

- To fill in a pixel in the target image, apply the inverse transform to the pixel location and look it up in the input image (with resampling technique) for pixel value.
- i.e. For translation of $x' = x + tx$, $y' = y + ty$:

$$l'(x', y') = l(x' - tx, y' - ty)$$

- i.e. For scale of $x' = x * sx$, $y' = y * sy$:

$$l'(x', y') = l(x' / sx, y' / sy)$$

Composite

- $\text{output} = \alpha * \text{foreground} + (1 - \alpha) * \text{background}$
- α is the alpha channel foreground



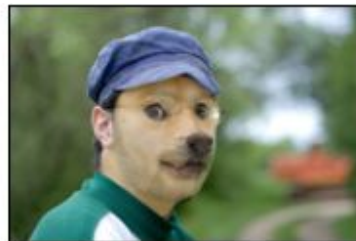
backgroundImg



foregroundImg



foregroundImg(alpha channel)

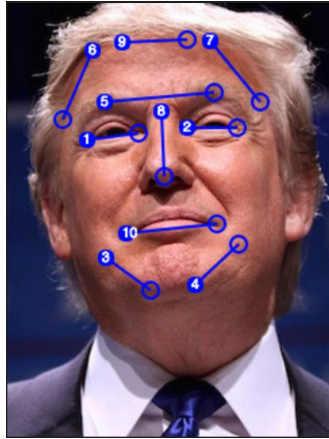


Result

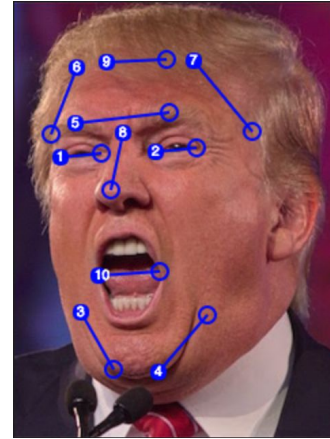
Morph

- Basic concepts
 - transform the background image to the foreground image
 - $\alpha = 0$: show background
 - $\alpha = 1$: show foreground
 - α is the blending factor / timestamp
- General approach
 - specify correspondences ([morphLines.html](#))
 - create an intermediate image with interpolated correspondences (α)
 - warp the background image to the intermediate correspondence
 - warp the foreground image to the intermediate correspondence
 - blend using α

Interpolate Morph Lines



Background Image



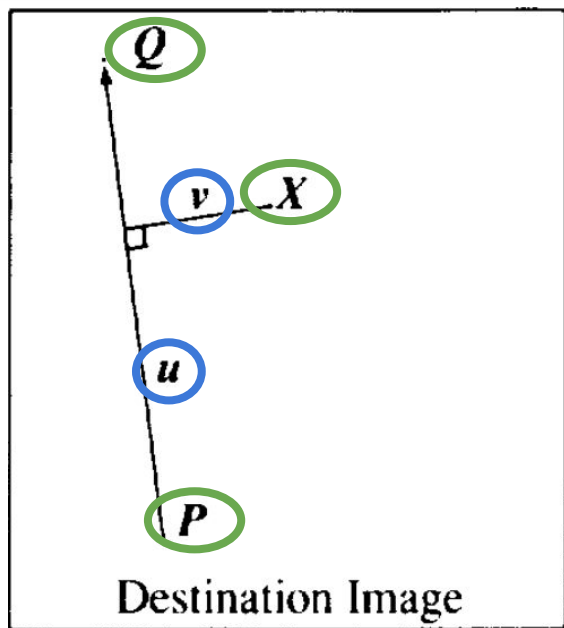
Foreground Image

$$\text{current_line}[i] = (1 - \alpha) * \text{background_lines}[i] + \alpha * \text{foreground_lines}[i]$$

Morph Algorithm Overview

1. Warp for a single line pair
2. Warp for many line pairs
3. For a fixed t , define the current line pairs as an interpolation between initial and final lines
4. Warp initial image I to **intermediate** I' and final image F to **intermediate** F' using current line pairs from Step 3
5. Alpha blend I' and F' using t
6. Vary t to get a morphing animation

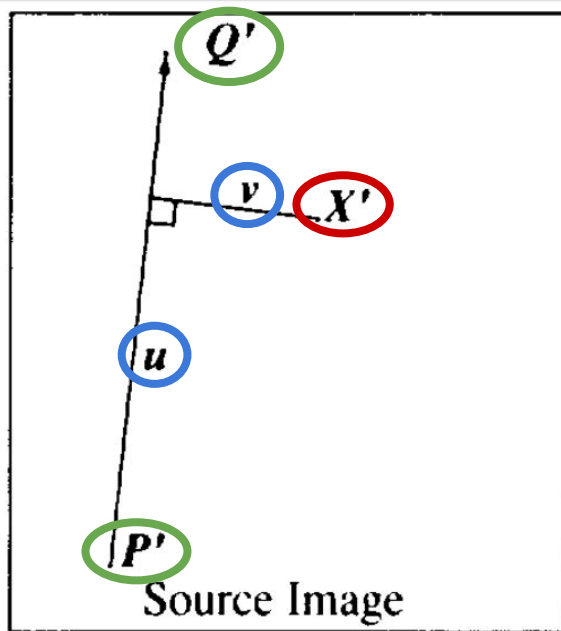
Warp Image (Single Line)



Destination Image






Warped background or foreground
(currently undefined)



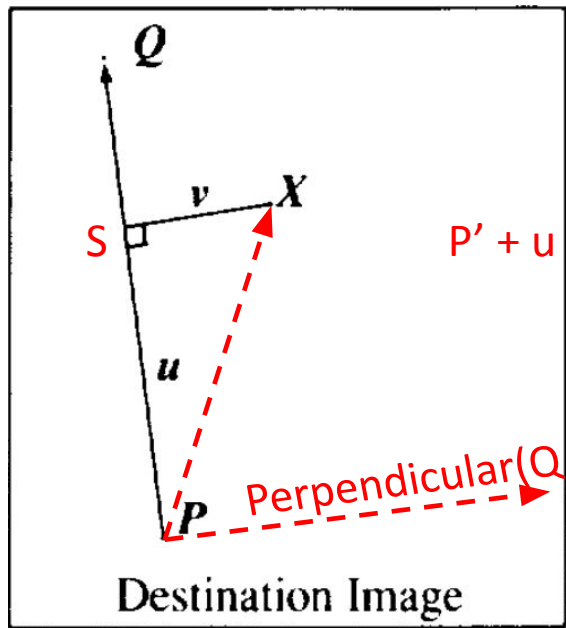
Source Image



Pixel source (background or foreground)

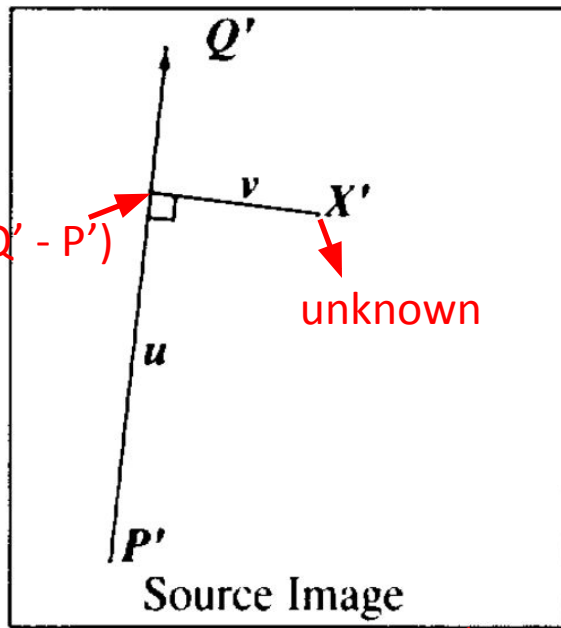
-  : known coordinates.
-  : unknown invariant.
-  : unknown coordinate.

Warp Image (Single Line)



$P' + u * (Q' - P')$

Perpendicular($Q - P$)



Let S be the projection point of X onto PQ

u = fraction of SP's signed length over PQ's absolute length

v = X's signed distance to PQ, or to say, signed length of SX

Warped background or foreground
(currently undefined)

Pixel source (background or foreground)

Warp Image (Single Line)

scalar

- $u = \frac{(X-P) \cdot (Q-P)}{\|Q-P\|^2} = \text{Projection of PX onto PQ}$

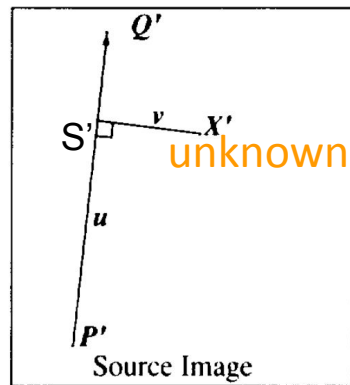
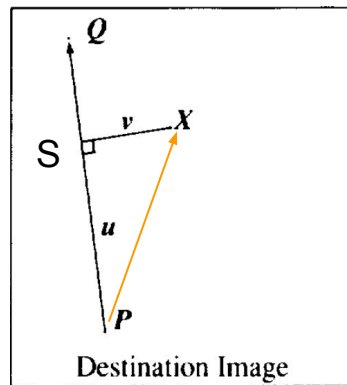
scalar

- $v = \frac{(X-P) \cdot \text{Perpendicular}(Q-P)}{\|Q-P\|}$ unit vector

- $X' = P' + u \cdot (Q' - P') + \frac{v \cdot \text{Perpendicular}(Q' - P')}{\|Q' - P'\|}$ unit vector

If $Q - P = (x, y)$,
 $\text{Perpendicular}(Q - P) = (y, -x)$

$$S' = P' + u \cdot (Q' - P')$$



Want to map X in destination image to unknown pixel X' in source image which contains current line

Warp Image (Single Line)

scalar

- $u = \frac{(X-P) \cdot (Q-P)}{\|Q-P\|^2}$ = Projection of PX onto PQ

scalar

- $v = \frac{(X-P) \cdot \text{Perpendicular}(Q-P)}{\|Q-P\|}$ unit vector

- $X' = P' + u \cdot (Q' - P') + \frac{v \cdot \text{Perpendicular}(Q' - P')}{\|Q' - P'\|}$ unit vector

- *dist* = shortest distance from X to PQ

- $0 \leq u \leq 1$: $\text{dist} = |v|$

- $u < 0$: $\text{dist} = \|X - P\|$

- $u > 1$: $\text{dist} = \|X - Q\|$

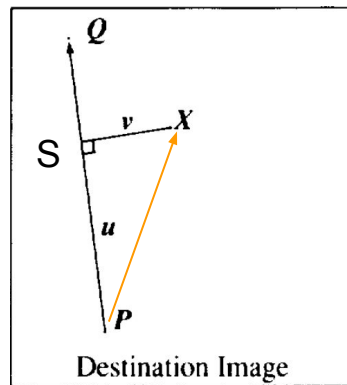
- $\text{weight} = \left(\frac{\text{length}^p}{a + \text{dist}}\right)^b$ Length of P'Q'

- we use $p = 0.5$, $a = 0.01$, $b = 2$

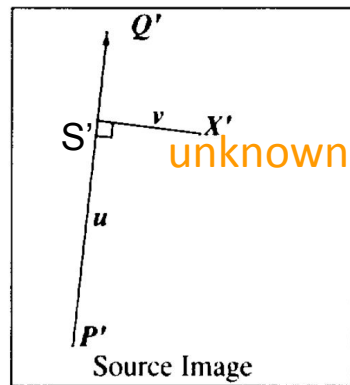
Contribution (weight) of line segment PQ to the warping of X's location

Each line segment contributes some weight

If $Q - P = (x, y)$,
 $\text{Perpendicular}(Q - P) = (y, -x)$



$$S' = P' + u \cdot (Q' - P')$$

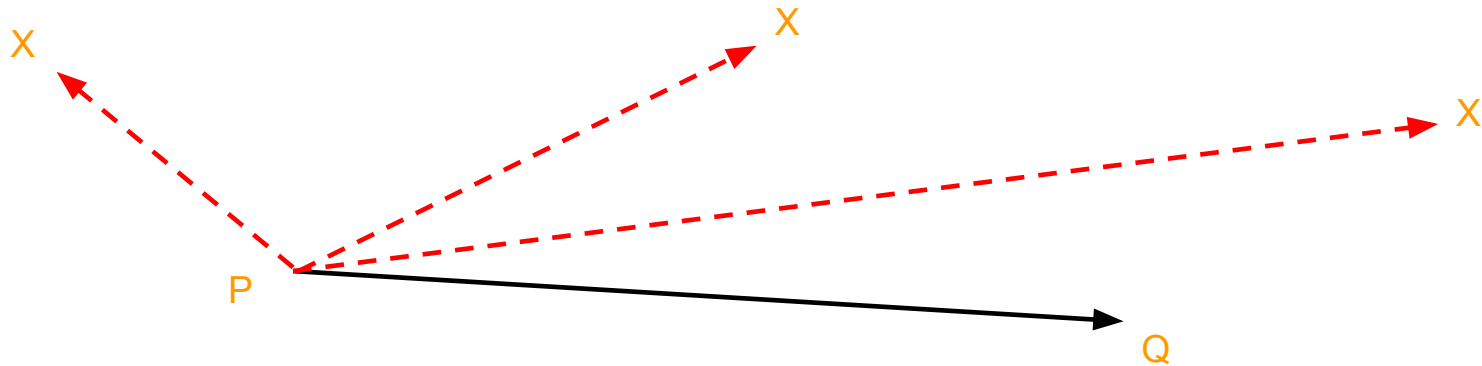


Want to map X in destination image to unknown pixel X' in source image which contains current line

Warp Image (Single Line)

dist = shortest distance from X to PQ

- $0 \leq u \leq 1$: $\text{dist} = |v|$
- $u < 0$: $\text{dist} = ||X - P||$
- $u > 1$: $\text{dist} = ||X - Q||$



Warp Image (Many Lines)

For each pixel X in the destination

$DSUM = (0,0)$

$weightsum = 0$ Track total weight for later averaging

For each line $P_i Q_i$

calculate u, v based on $P_i Q_i$

calculate X'_i based on u, v and $P'_i Q'_i$

calculate displacement $D_i = X'_i - X_i$ for this line

$dist$ = shortest distance from X to $P_i Q_i$

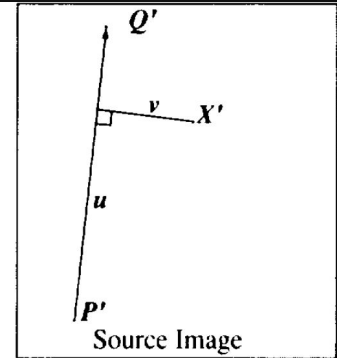
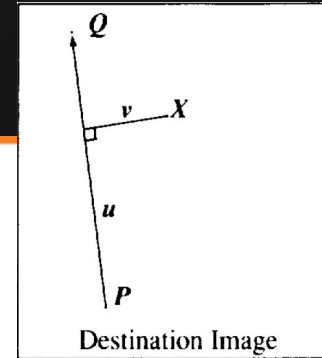
$weight = (length^P / (a + dist))^b$

$DSUM += D_i * weight$

$weightsum += weight$

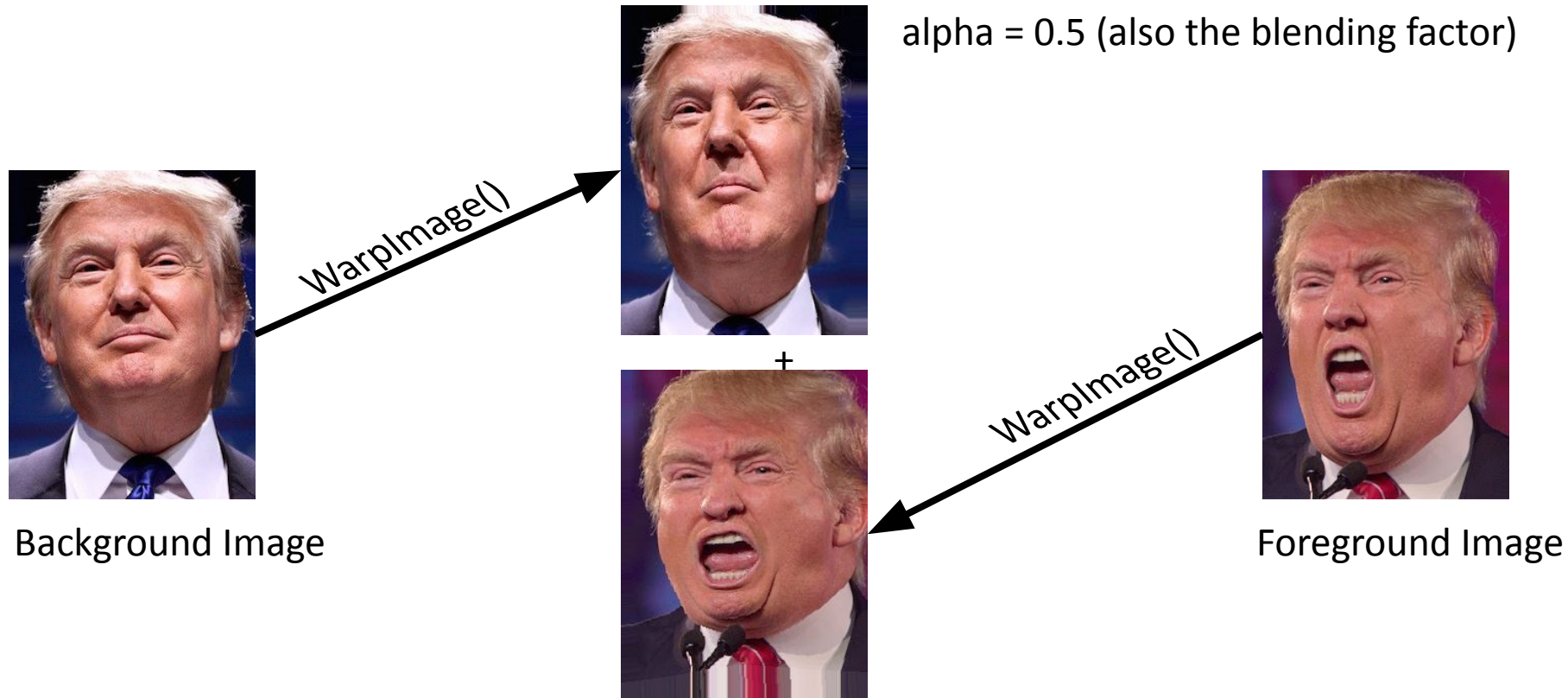
$X' = X + DSUM / weightsum$ Repeat for all lines and then average based on weight

$destinationImage(X) = sourceImage(X')$



Algorithm described
before for a single line

Blending



Blending

Vary this alpha to get an animation



$\alpha = 0.5$ (also the blending factor)



Background Image



Foreground Image

Morph Algorithm Sketch

```
GenerateAnimation(Image0, L0[...], Image1, L1[...])
begin
  foreach intermediate frame time t do
    for i = 0 to number of line pairs do
      L[i] = line t-th of the way from L0[i] to L1[i]
    end
    Warp0 = WarpImage(Image0, L0, L)
    Warp1 = WarpImage(Image1, L1, L)
    foreach pixel p in FinalImage do
      Result(p) = (1-t) Warp0 + t Warp1
    end
  end
end
```

Q&A

Course Logistics Update

- New course website incoming!
 - Preview at <https://reillybova.github.io/COS426-Website/>
 - Should have everything, but may be slightly buggy as we work out kinks
 - If you notice any problems, please make a public Piazza post under the “website” folder
- Web Framework specs (for those interested):
 - [ReactJS](#) for state-based logic and modularity
 - [MaterialUI](#) to build a [Material Design](#) compliant interface
 - [GatsbyJS](#) to compile the React App to static server files (allows us to host site as a normal webpage, and makes it blazing fast)
 - Content generate from Markdown

Fill out the Assignment 0 Feedback Form

Do this **now** — it takes less than a minute:

- <https://forms.gle/o2ea1iJ978zY6Kd78>

Ordered dithering

Pseudo code for n-bit case:

```
i = x mod m
j = y mod m
err = I(x, y) - floor_quantize(I(x, y))
threshold = (D(i, j) + 1) / (m^2 + 1)
if err > threshold
    P(x, y) = ceil_quantize(I(x, y))
else
    P(x, y) = floor_quantize(I(x, y))
```

- $\text{floor_quantize}(p)$
 $= \text{floor}(p * (2^{n-1})) / (2^{n-1})$
- $\text{ceil_quantize}(p)$
 $= \text{ceil}(p * (2^{n-1})) / (2^{n-1})$

$$m = 4, D = \begin{bmatrix} 15 & 7 & 13 & 5 \\ 3 & 11 & 1 & 9 \\ 12 & 4 & 14 & 6 \\ 0 & 8 & 2 & 10 \end{bmatrix}$$



n=1 example

An Update on the Bilateral Filter

- Compute color distance in RGB space, scaled to [0, 255].

$$w(i, j, k, l) = e^{\left(-\frac{(i-k)^2 + (j-l)^2}{2\sigma_d^2} - \frac{\|I(i, j) - I(k, l)\|^2}{2\sigma_r^2} \right)}$$

Spatial distance component Color distance component

