
# Subdivision Surfaces

COS 426, Fall 2022

PRINCETON UNIVERSITY

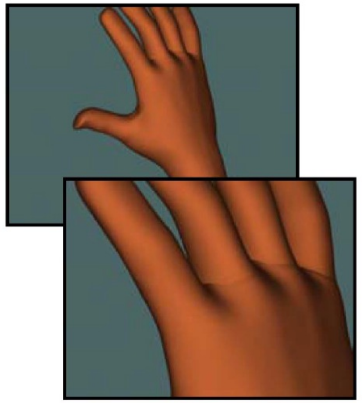
# 3D Object Representations

- Raw data
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Parametric
  - ➢ Subdivision
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific

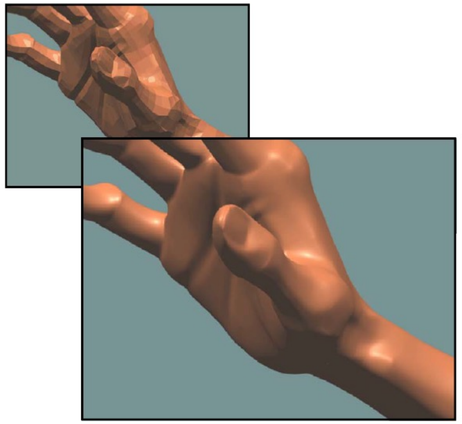
# Subdivision Surfaces

- Alternative to parametric surfaces, overcoming:
  - Many patches
  - Difficult to mark sharp features
  - Irregularities after deformation

Woody's hand (NURBS)

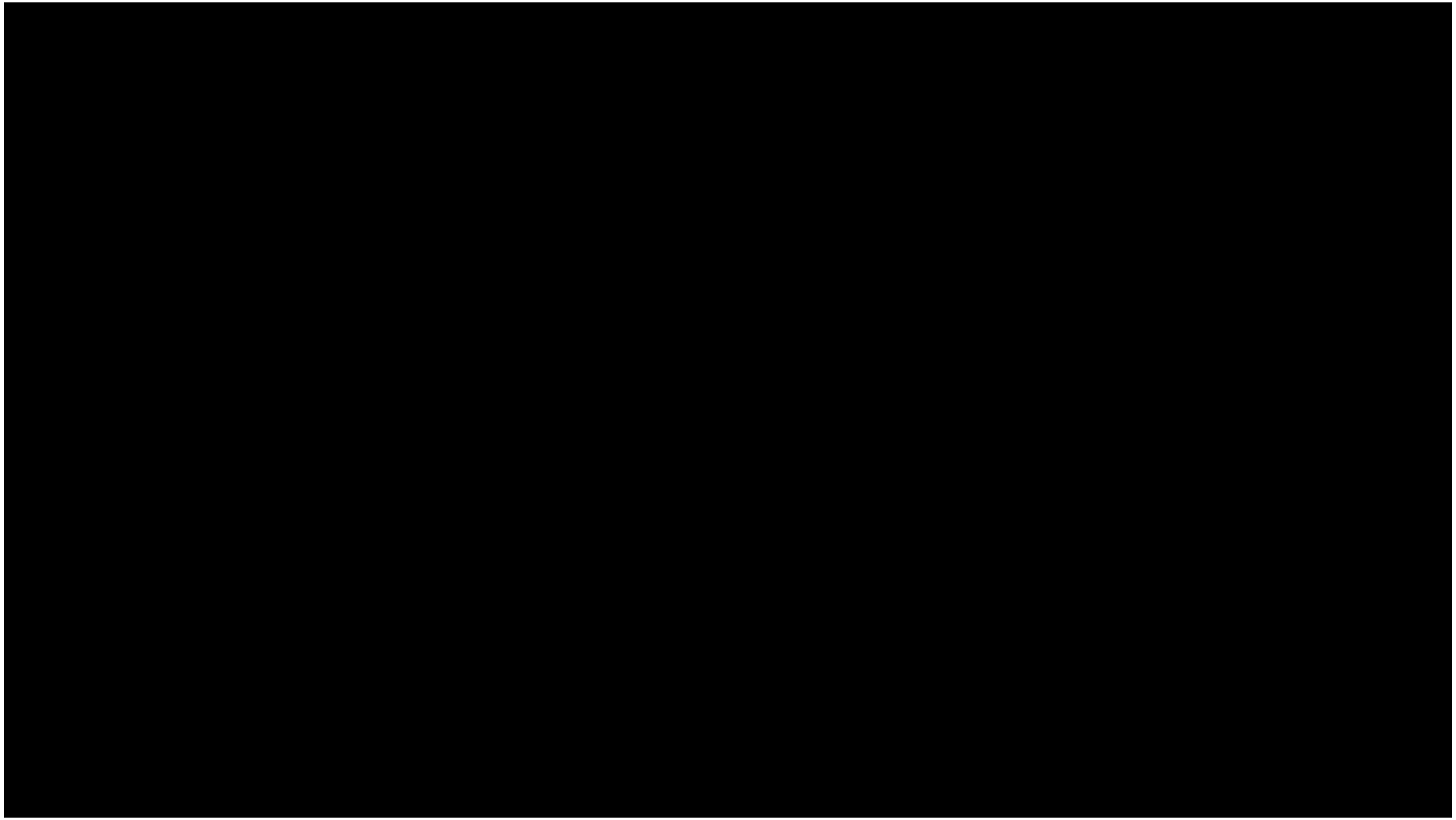


Geri's hand (subdivision)



Stanford Graphics course notes

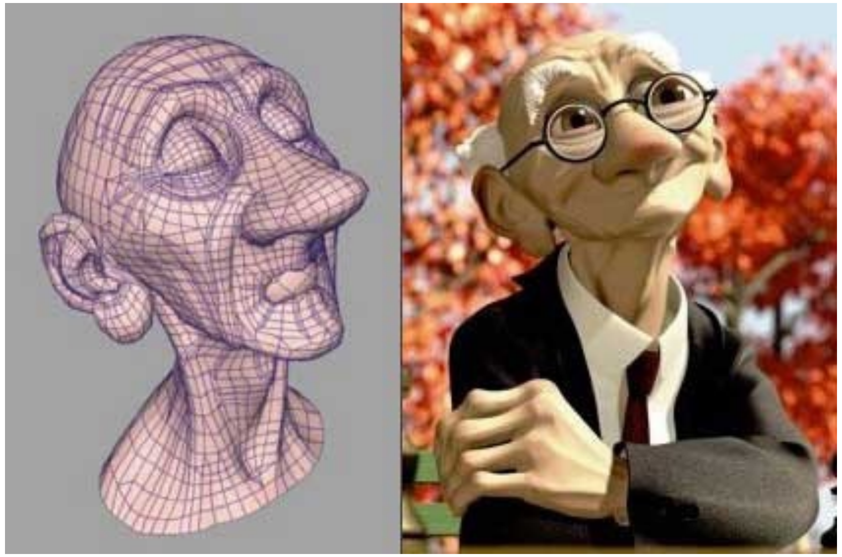
# Geri’s Game



Geri’s Game © Pixar Animation Studios

# Geri's Game

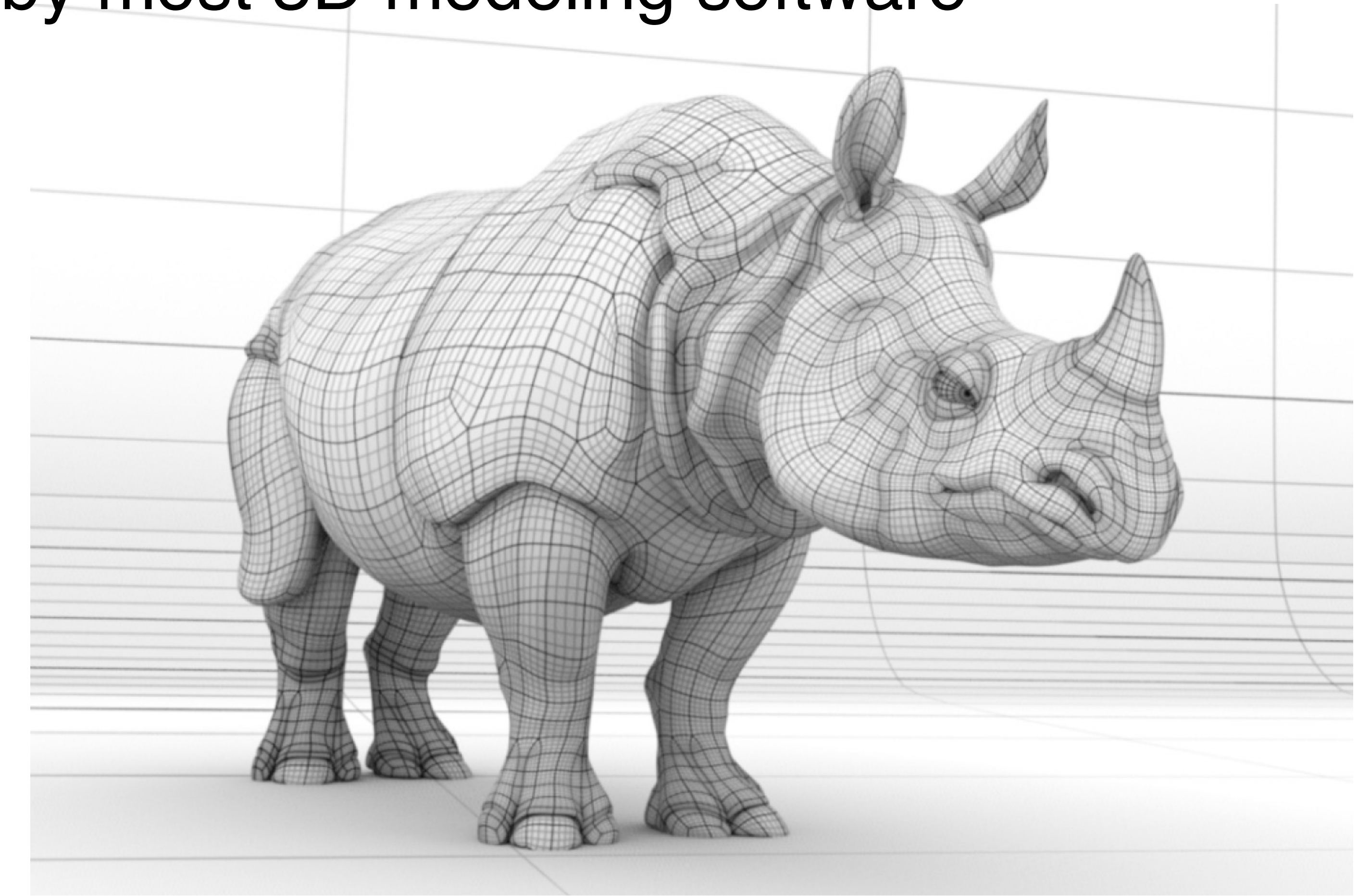
- "… served as a demonstration of a new animation tool called subdivision surfaces" (Wikipedia)
- Subdivision used for head, hands & clothing
- Academy Award winner

Geri's Game © Pixar Animation Studios

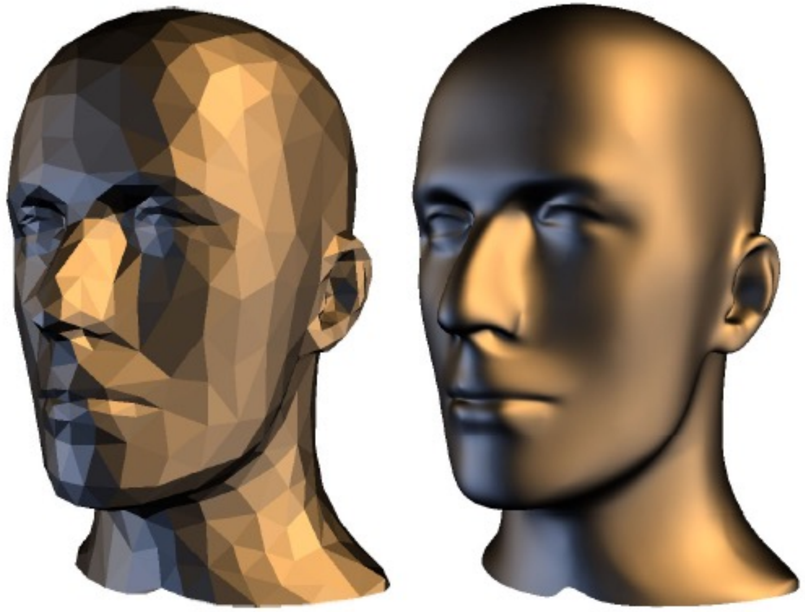
# Subdivision Surfaces

- Used in movie and game industries
- Supported by most 3D modeling software



# Subdivision Surfaces

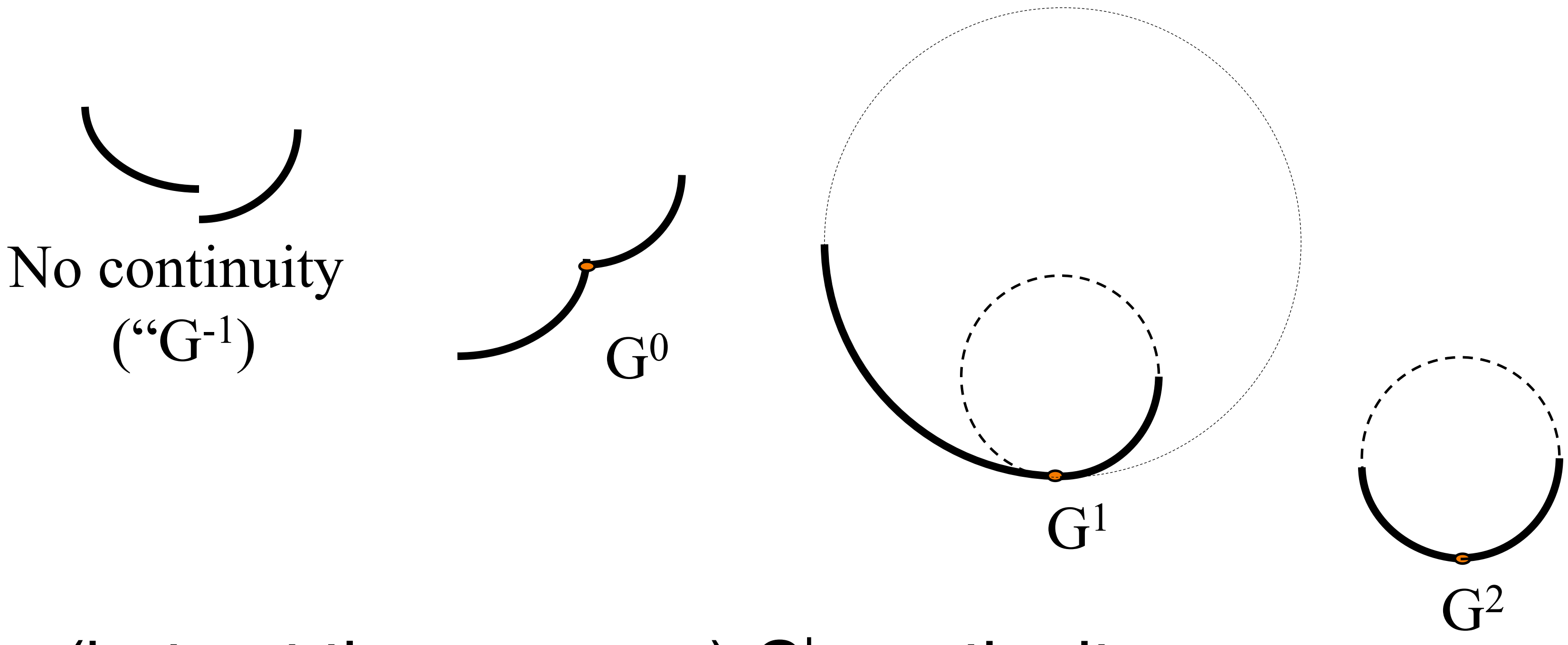
- What makes a good surface representation?
  - Accurate
  - Concise
  - Intuitive specification
  - Local support
  - Affine invariant
  - Arbitrary topology
  - ➢ Guaranteed continuity
  - Natural parameterization
  - Efficient display
  - Efficient intersections



Reif & Schroeder 2000

# Review on Continuity

- A curve / surface with $G^k$ continuity has a continuous k-th derivative, geometrically



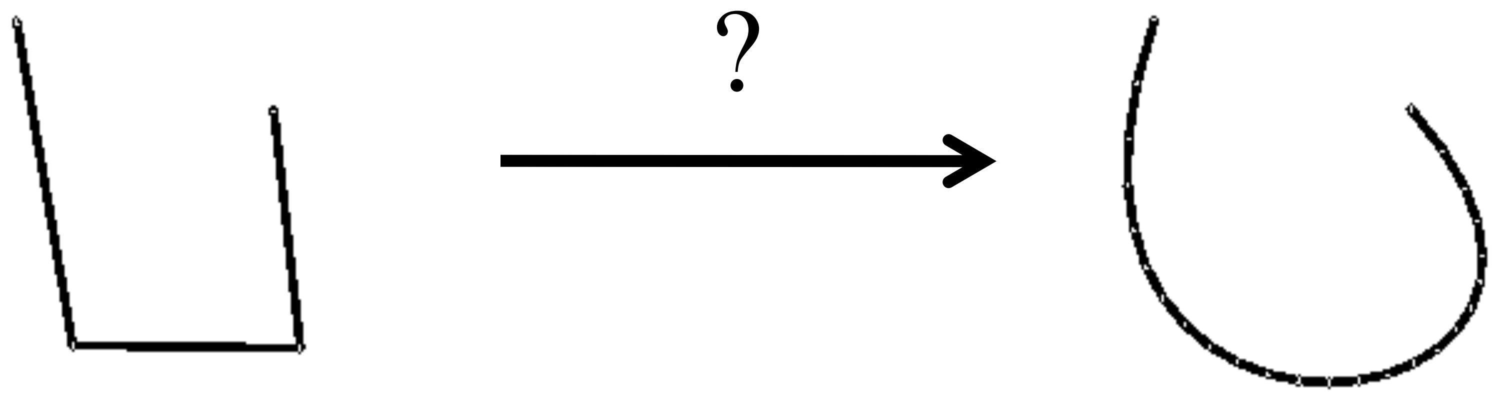

- Similar to (but not the same as) $C^k$ continuity, which refers to continuity with respect to parameter e.g.:

$$f_x(u) = r_x \cos(2\pi u)$$

*(but we're going to say $C^k$ from now on…)*

# Subdivision

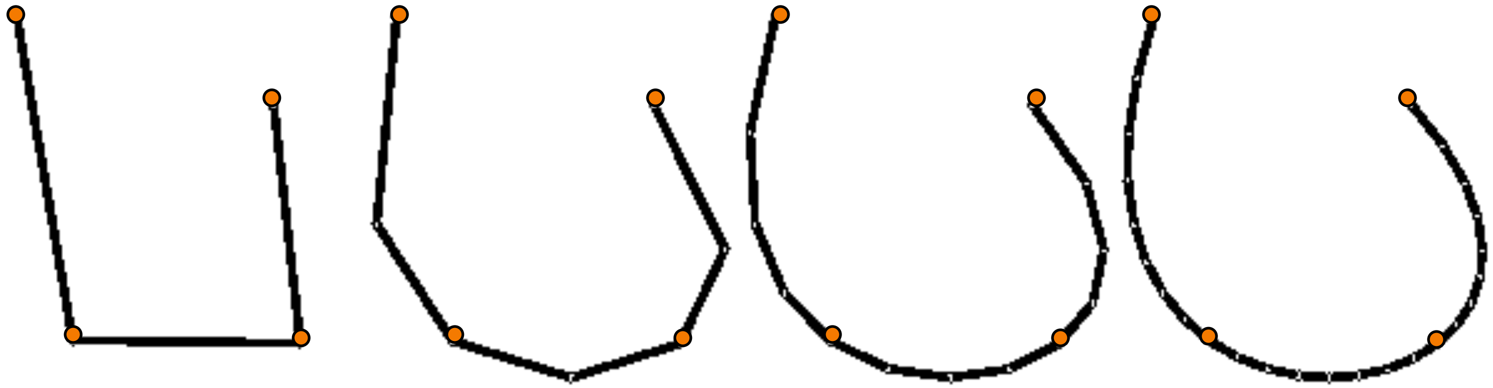
- How do you make a curve with guaranteed continuity?




Zorin & Schroeder
SIGGRAPH 99
Course Notes

# Subdivision

- How do you make a curve with guaranteed continuity?



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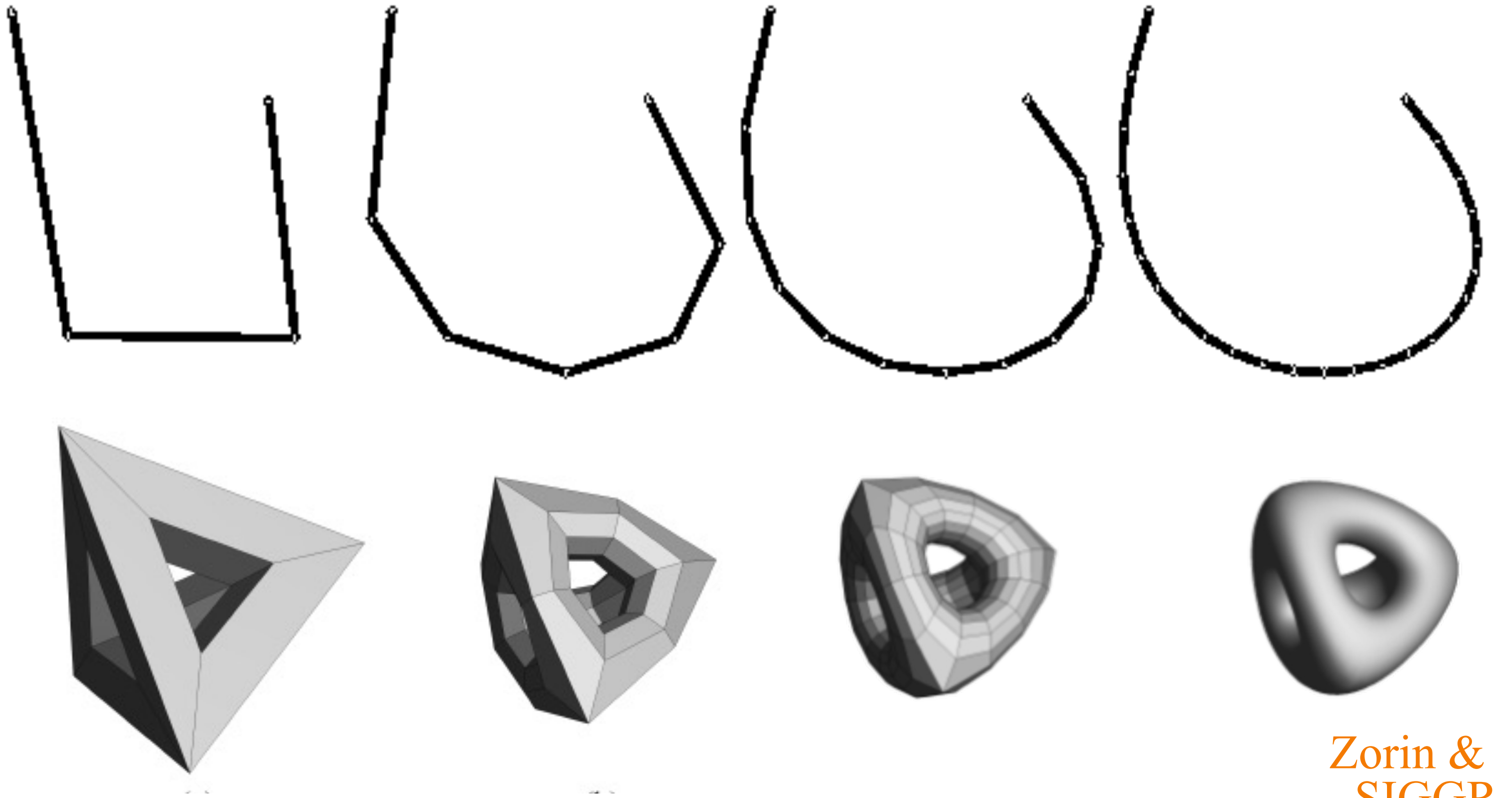
# Subdivision

- How do you make a *surface* with guaranteed continuity?



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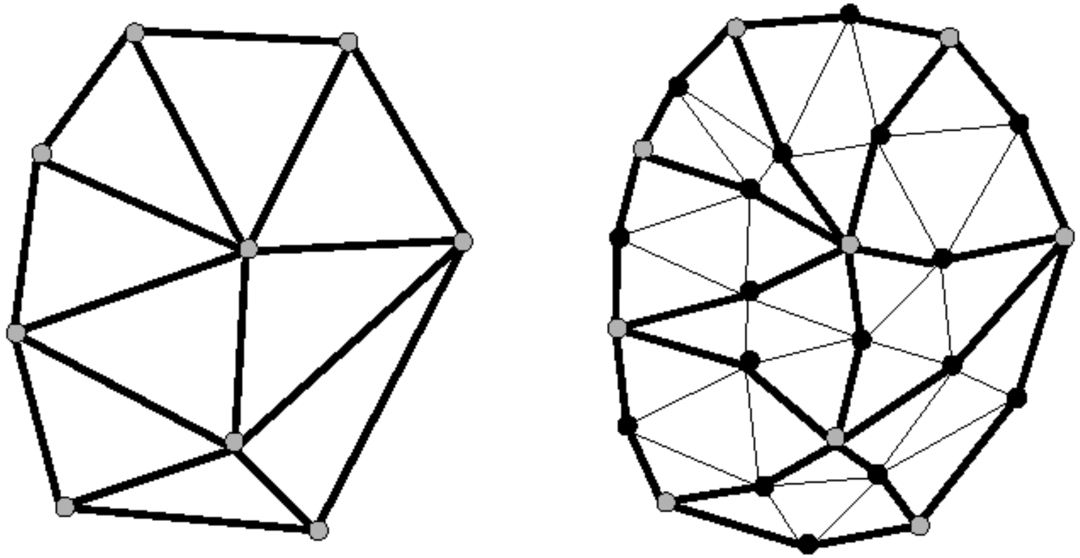
# Subdivision Surfaces

- Repeated application of
  - Topology refinement (splitting faces)
  - Geometry refinement (weighted averaging)



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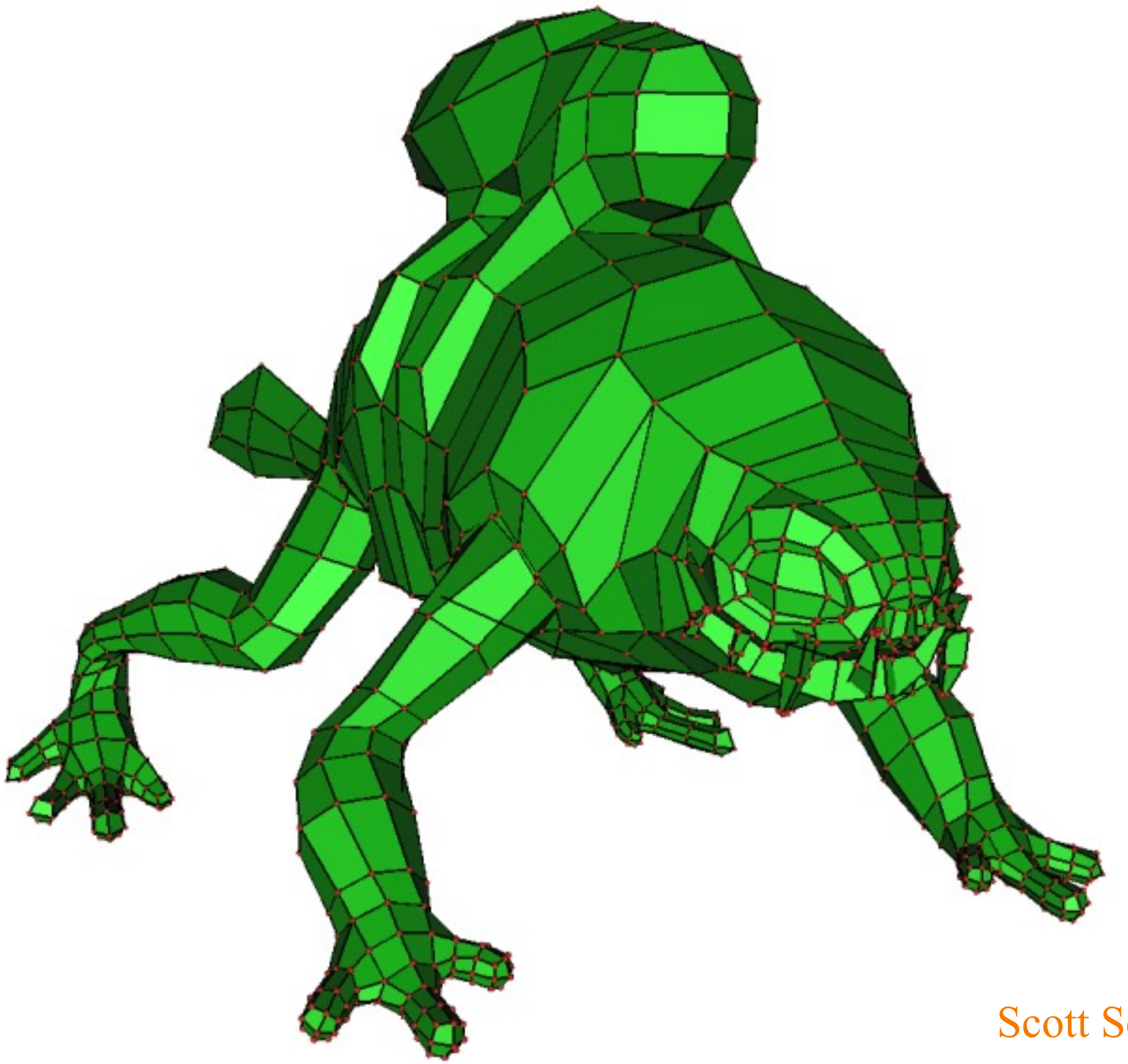
# Subdivision Surfaces – Examples

- Base mesh



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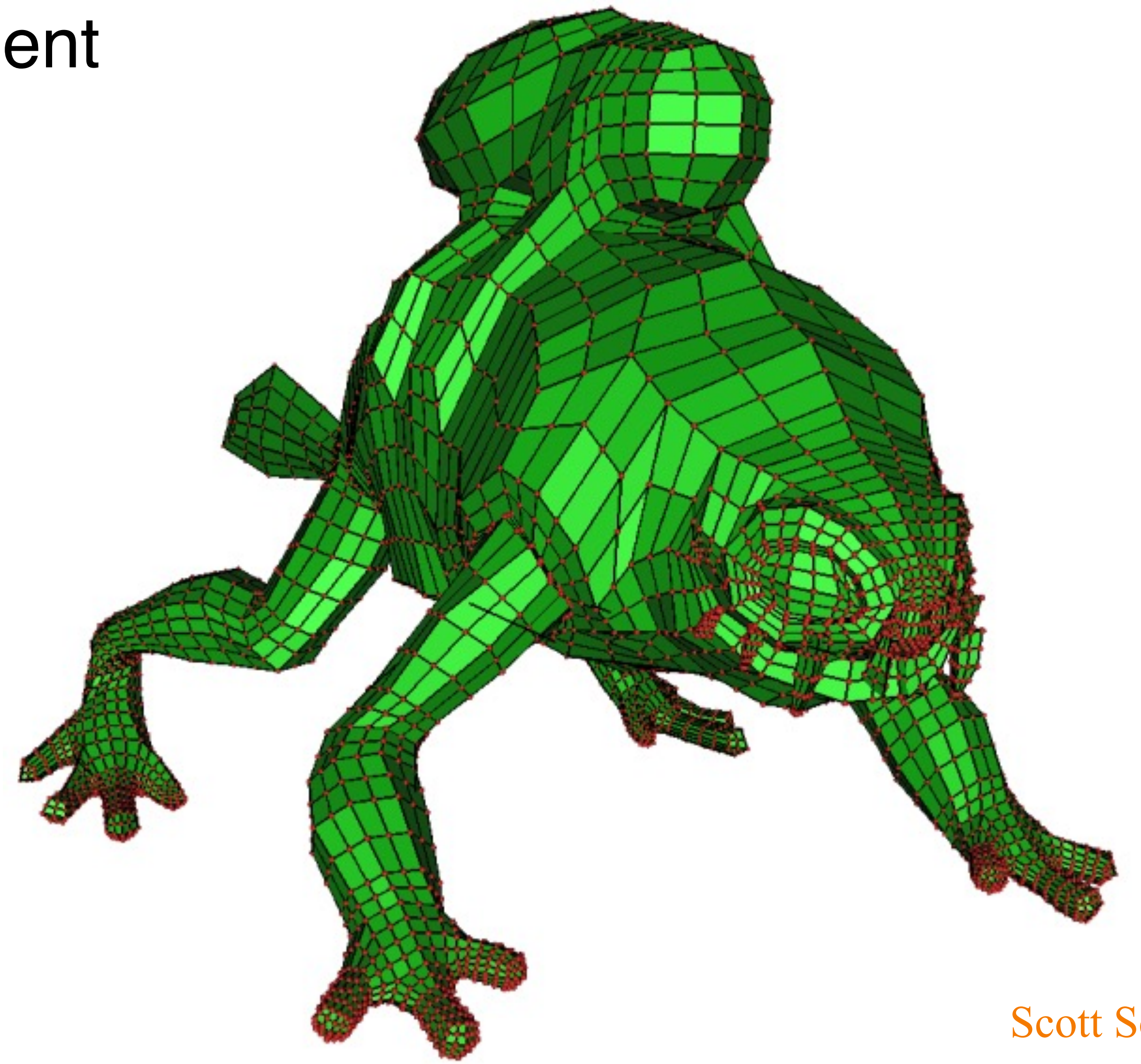
# Subdivision Surfaces – Examples

- Topology refinement



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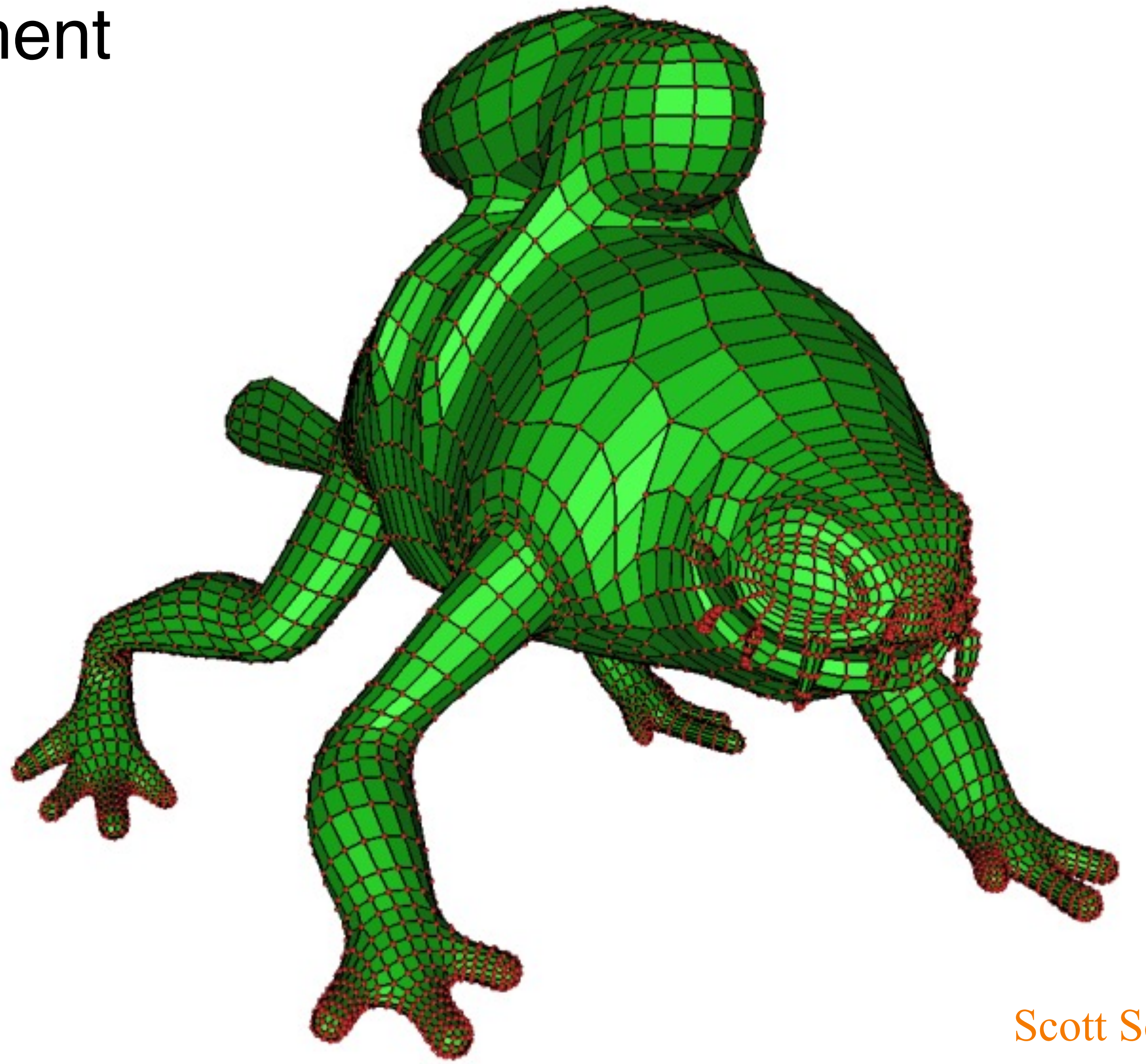
# Subdivision Surfaces – Examples

- Geometry refinement



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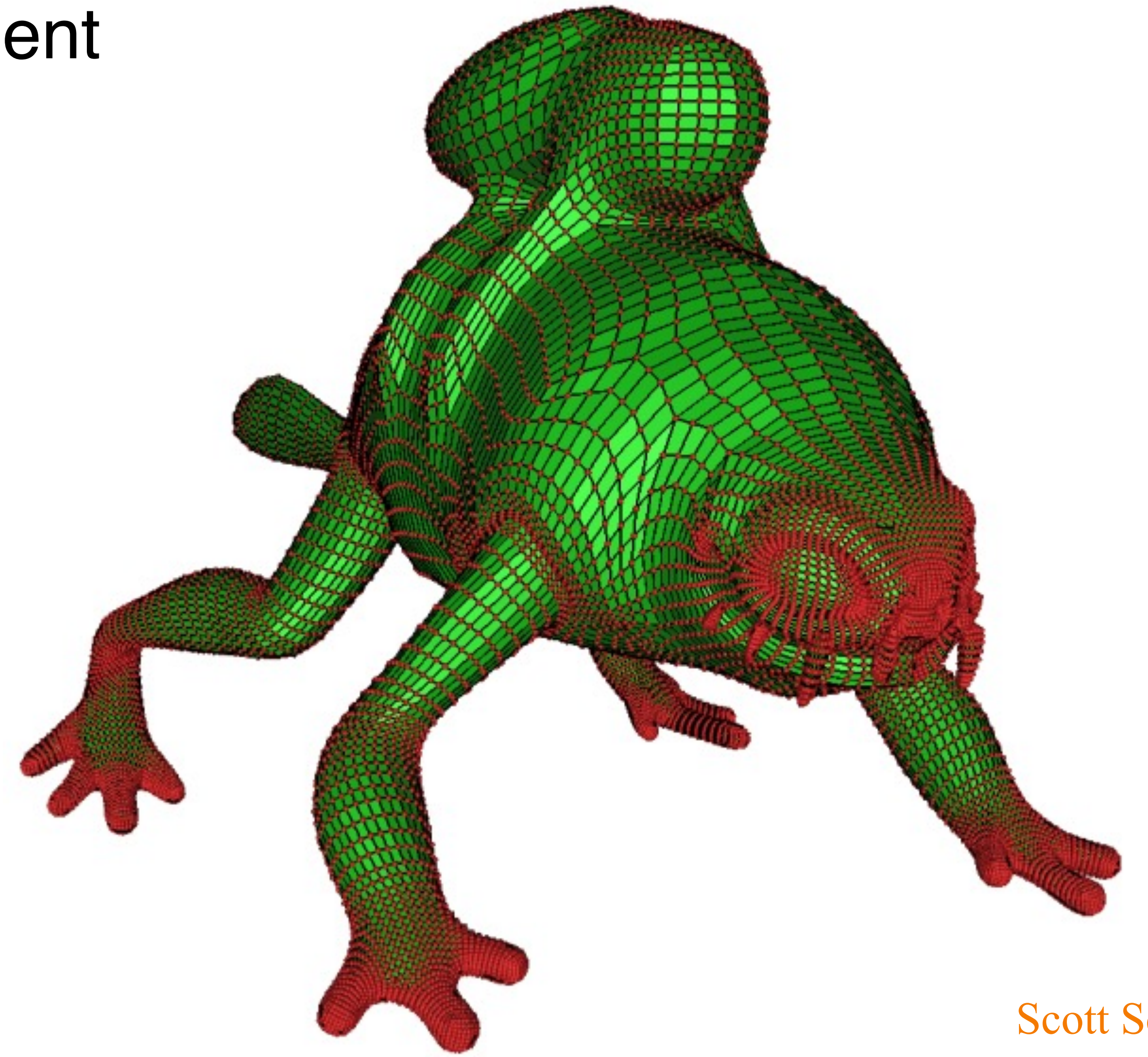
# Subdivision Surfaces – Examples

- Topology refinement



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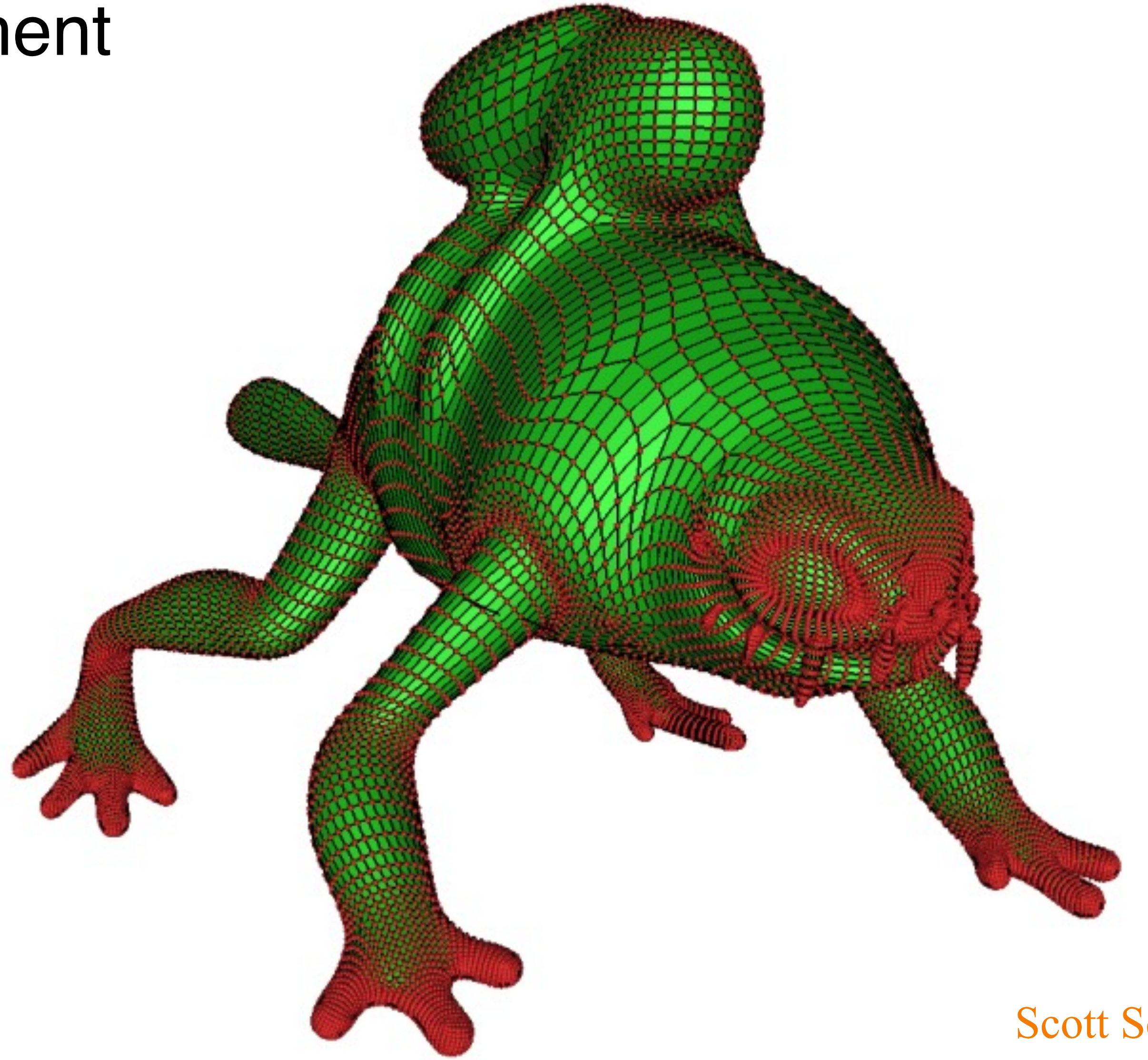
# Subdivision Surfaces – Examples

- Geometry refinement



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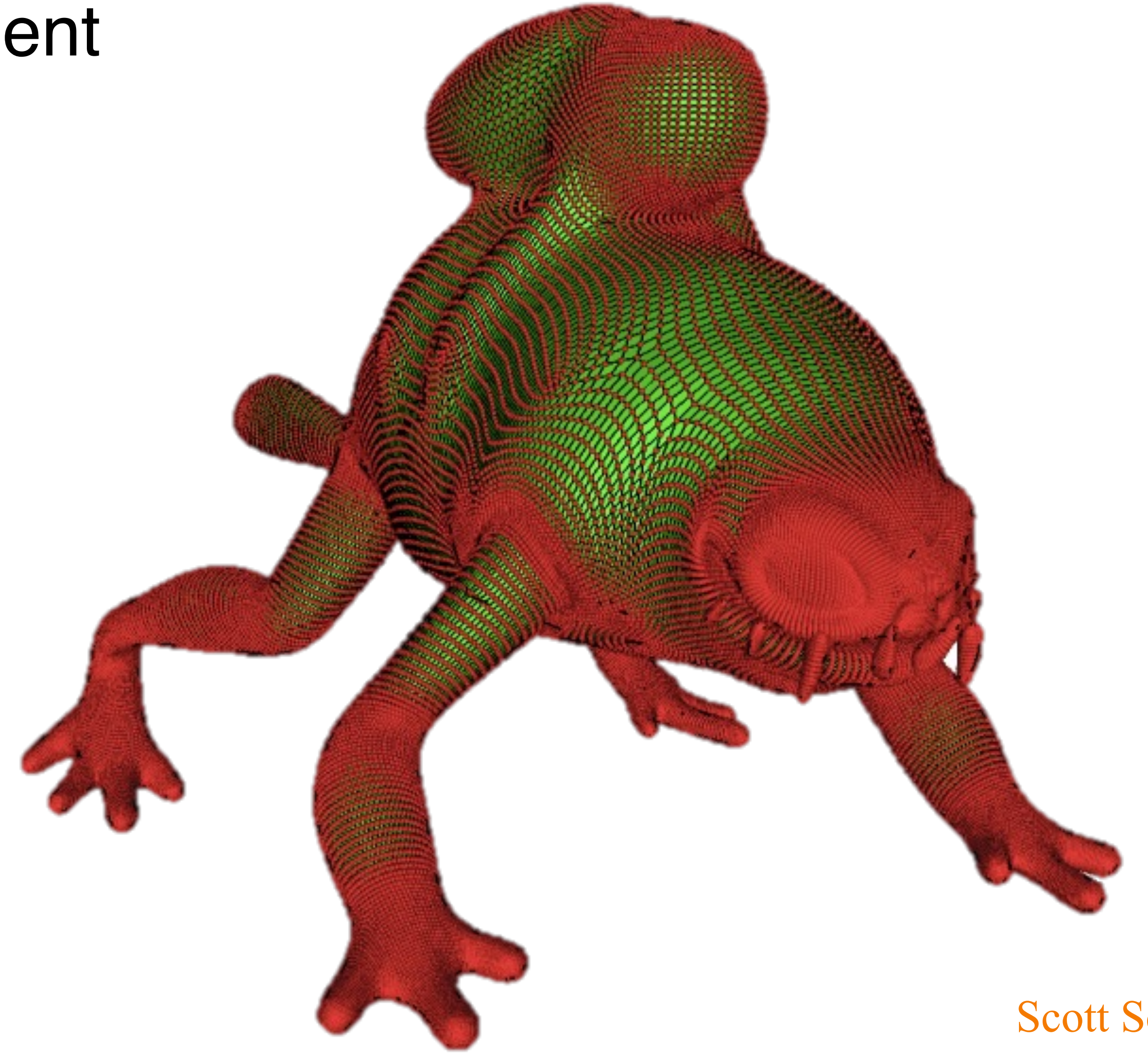
# Subdivision Surfaces – Examples

- Topology refinement



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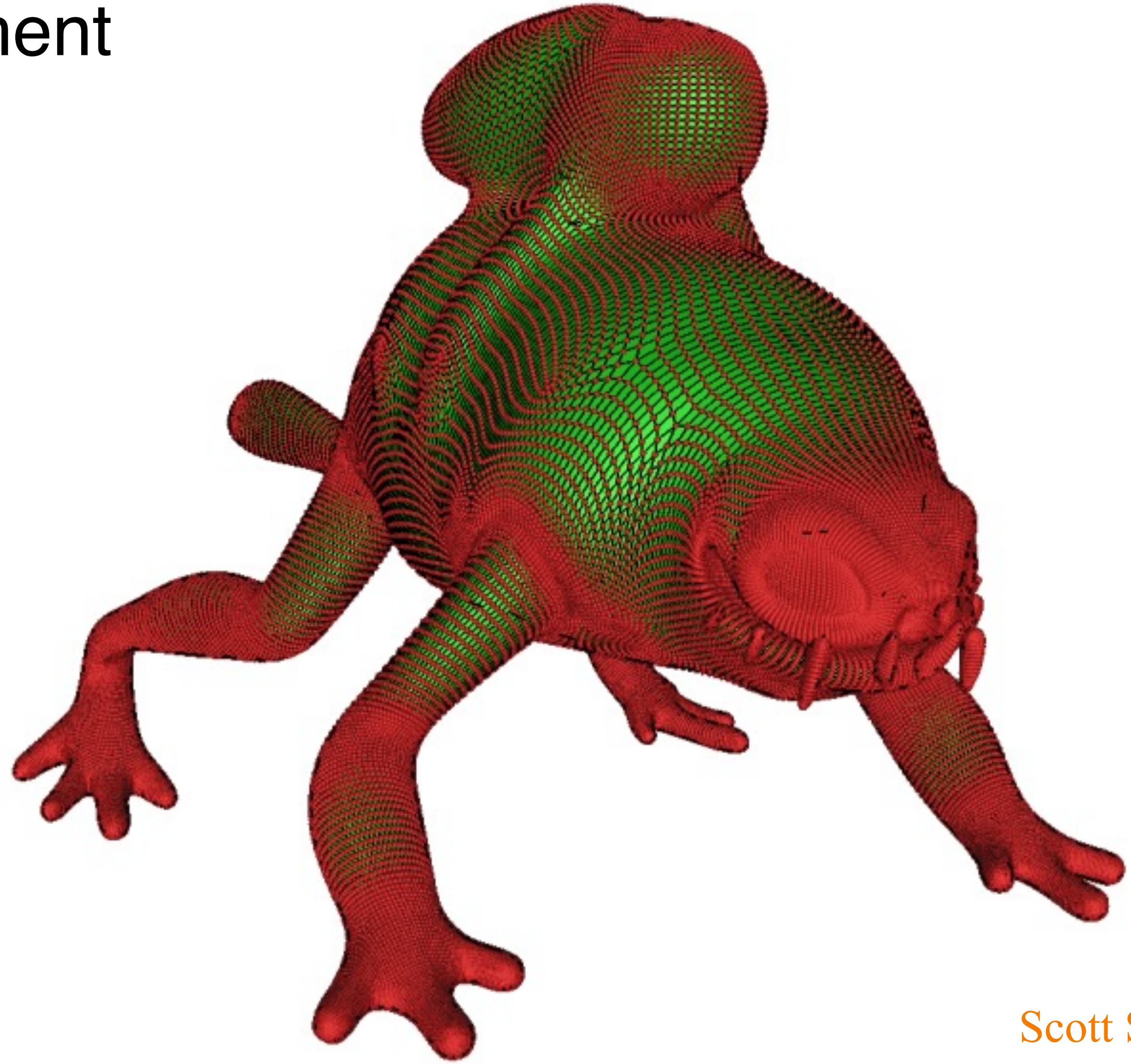
# Subdivision Surfaces – Examples

- Geometry refinement



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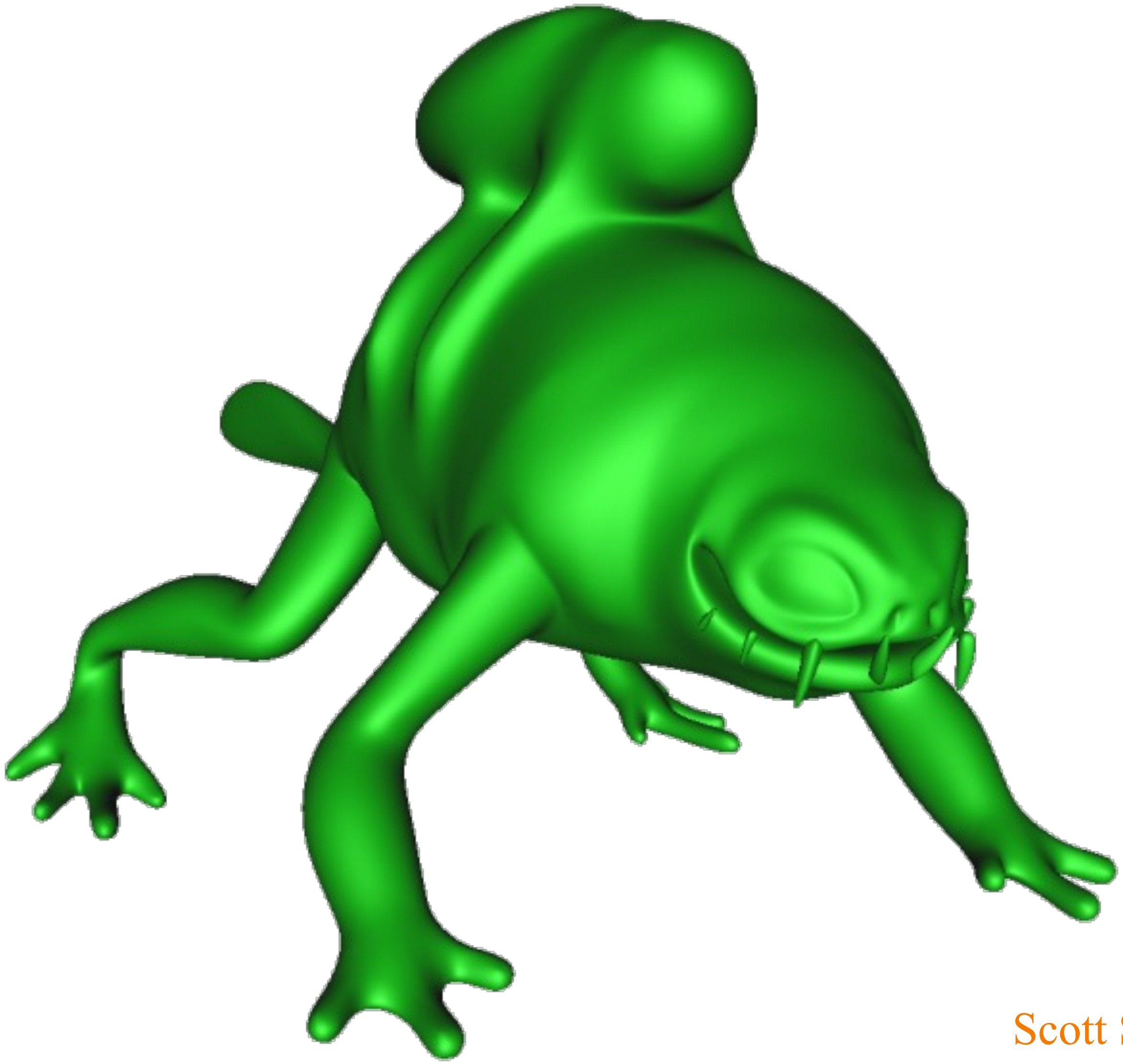
# Subdivision Surfaces – Examples

- Limit surface



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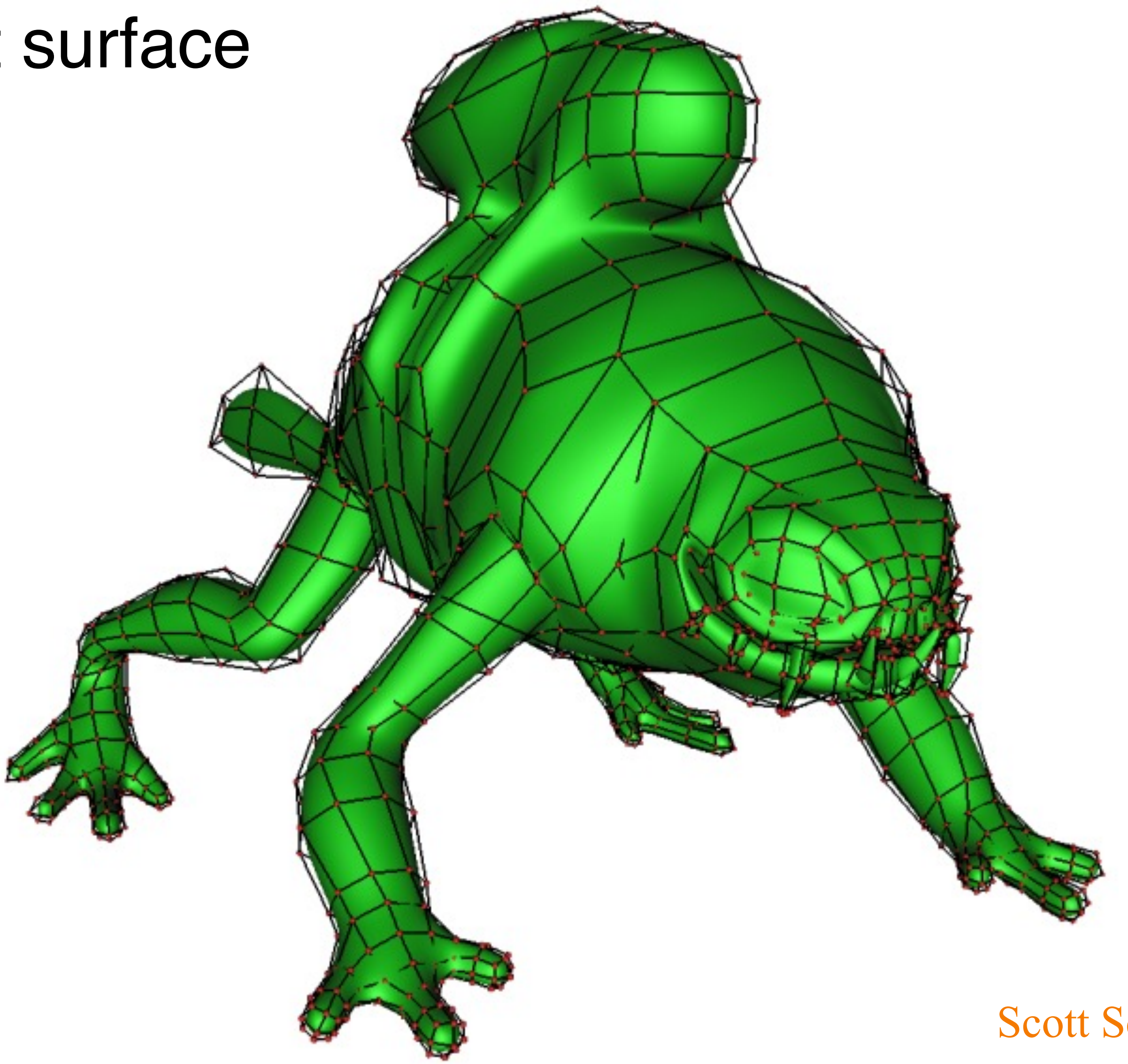
# Subdivision Surfaces – Examples

- Base mesh + limit surface



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# Design of Subdivision Rules

- What types of input?
  - Quad meshes, triangle meshes, etc.
- How to refine topology?
  - Simple implementations
- How to refine geometry?
  - Smoothness guarantees in limit surface
    - » Continuity (C0, C1, C2, …?)
  - Provable relationships between limit surface and original control mesh
    - » Interpolation of vertices?
    - » Surface within their convex hull?

# Linear Subdivision

- Type of input
  - Quad mesh – four-sided polygons (quads)
- Topology refinement rule
  - Split every quad into four at midpoints
- Geometry refinement rule
  - Average vertex positions

Note: simple example to demonstrate how such schemes work, but not the best scheme…



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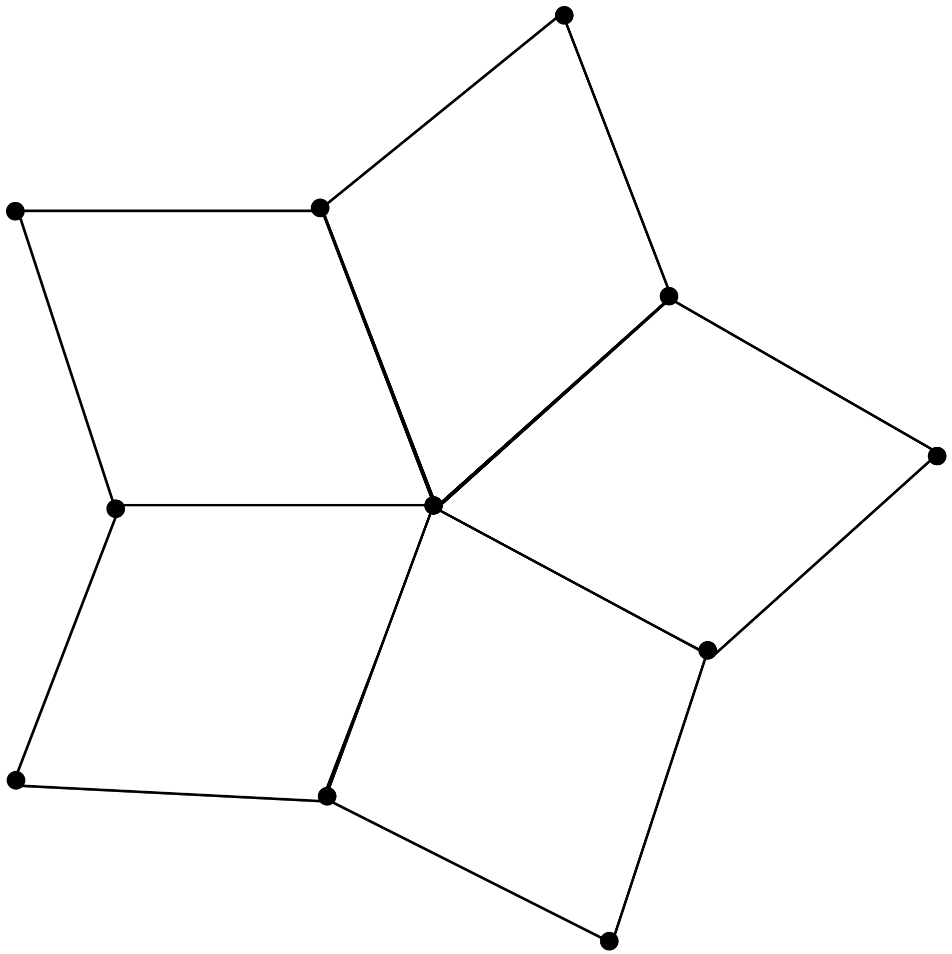
# Linear Subdivision

- Input





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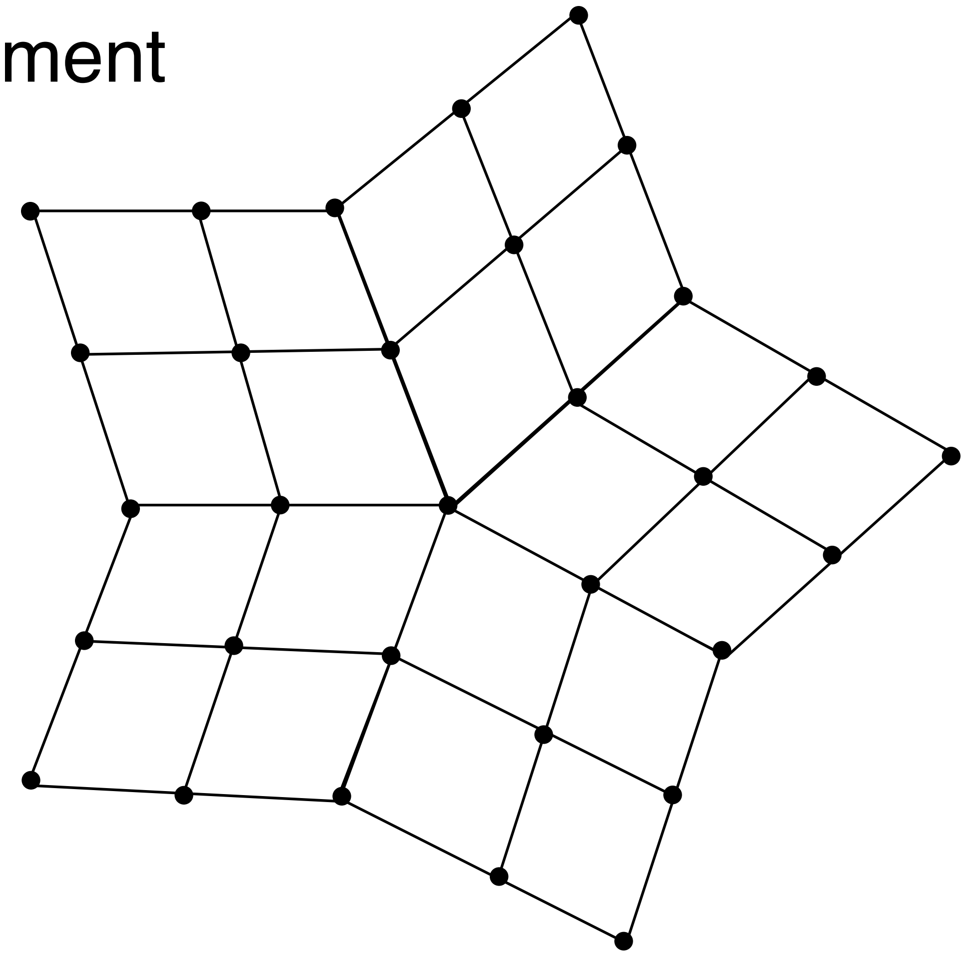
# Linear Subdivision

- Topology refinement





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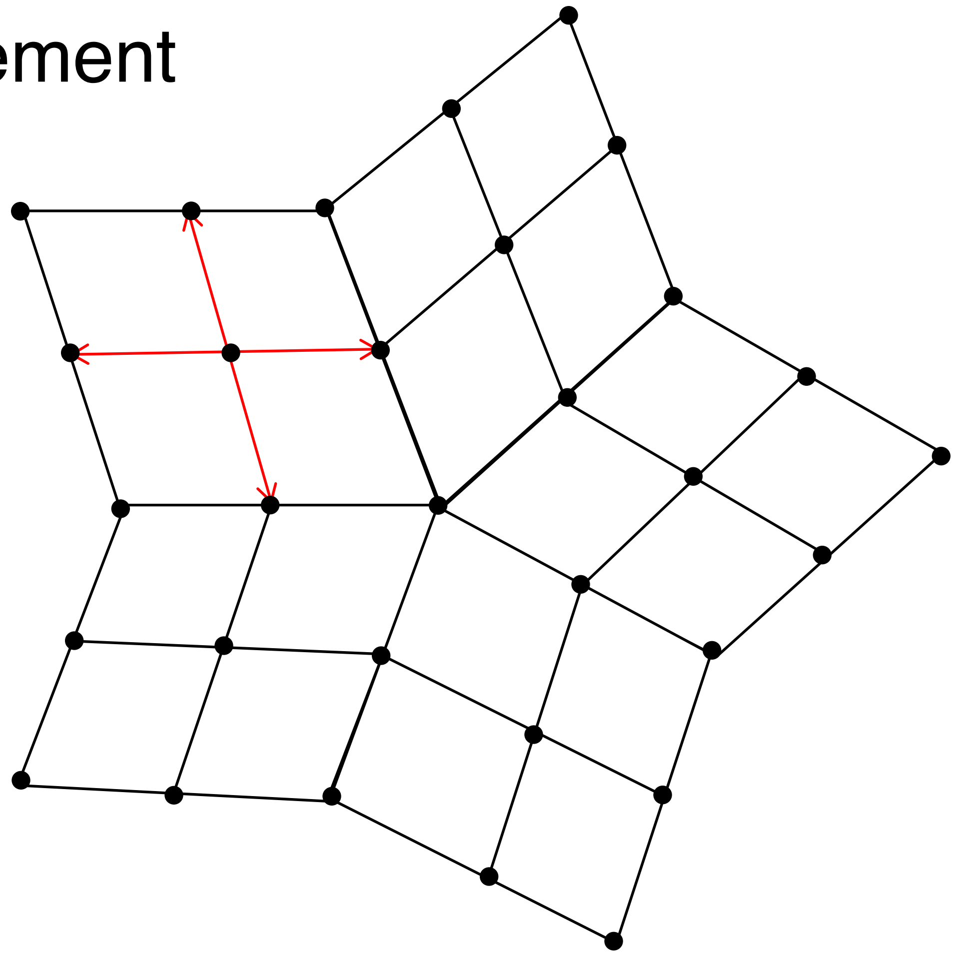
# Linear Subdivision

- Geometry refinement





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# Linear Subdivision

LinearSubivision ($F_0$, $V_0$, $k$ )
    for i = 1 …k levels
        ($F_i$, $V_i$) = RefineTopology($F_{i-1}$, $V_{i-1}$)
        RefineGeometry($F_i$, $V_i$)
    return ($F_k$, $V_k$)


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# Linear Subdivision

RefineTopology ( $F$, $V$ )

$newV = V$

$newF = \{\}$

for each face $F_i$

Insert new vertex $c$ at centroid of $F_i$ into $newV$

return ($newF$, $newV$)



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# Linear Subdivision

```
RefineTopology ( F, V )
    newV = V
    newF = {}
    for each face F_i
        Insert new vertex c at centroid of F_i into newV
        for j = 1 to 4
            Insert in newV new vertex e_j at
            centroid of each edge ( F_{i,j}, F_{i,j+1})



    return (newF, newV)
```


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# Linear Subdivision

RefineTopology ( $F$, $V$ )

$newV = V$

$newF = \{\}$

for each face $F_i$

Insert new vertex $c$ at centroid of $F_i$ into $newV$

for $j$ = 1 to 4

Insert in $newV$ new vertex $e_j$ at centroid of each edge ( $F_{i,j}$, $F_{i,j+1}$)

for $j$ = 1 to 4

Insert new face ($F_{i,j}$, $e_j$, $c$, $e_{j-1}$) into $newF$

return ($newF$, $newV$)



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# Linear Subdivision

```
RefineGeometry( F, V )
    newV = V
    newF = F
    for each vertex V_i in newV
        weight = 0;
        newV[i] = (0,0,0)




    return (newF, newV)
```



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# Linear Subdivision

```
RefineGeometry( F, V )
    newV = V
    newF = F
    for each vertex V_i in newV
        weight = 0;
        newV[i] = (0,0,0)
        for each face F_j connected to V_i
            newV[i] += centroid of F_j
            weight += 1.0;
        newV[i] /= weight
    return (newF, newV)
```



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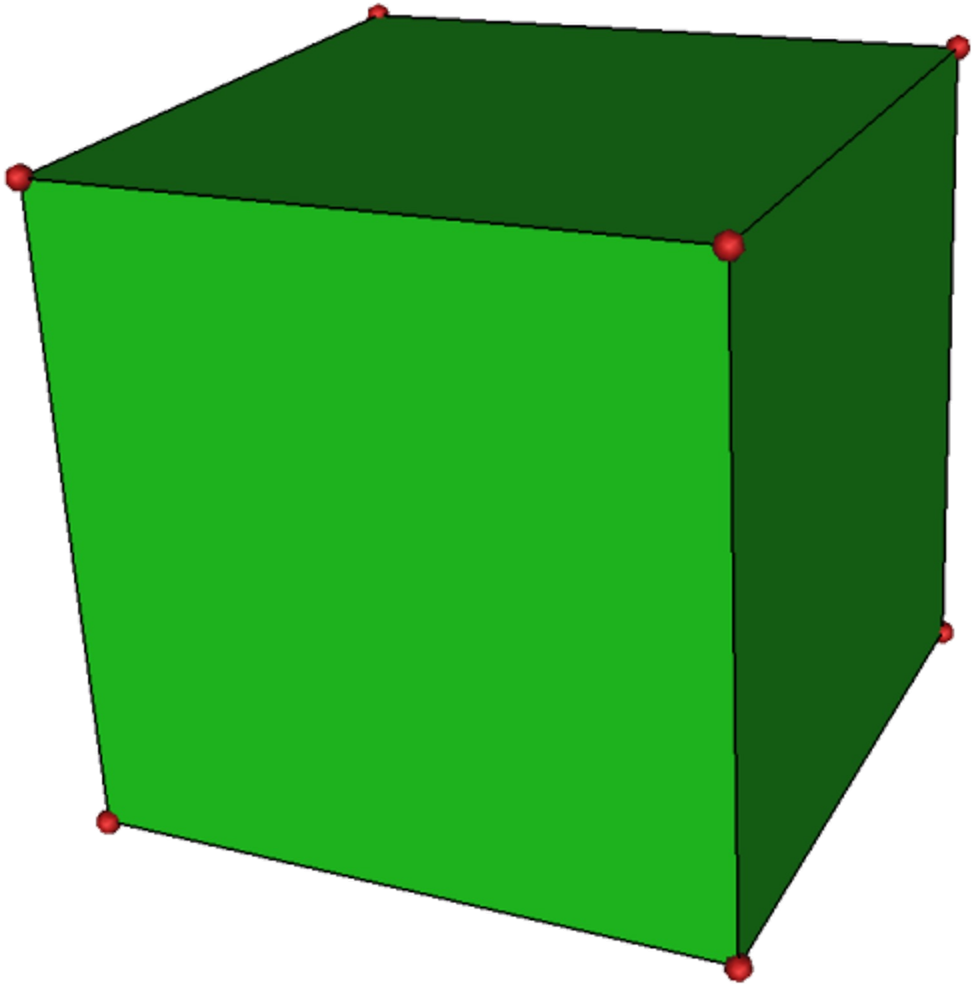
# Linear Subdivision

- Example



Input mesh

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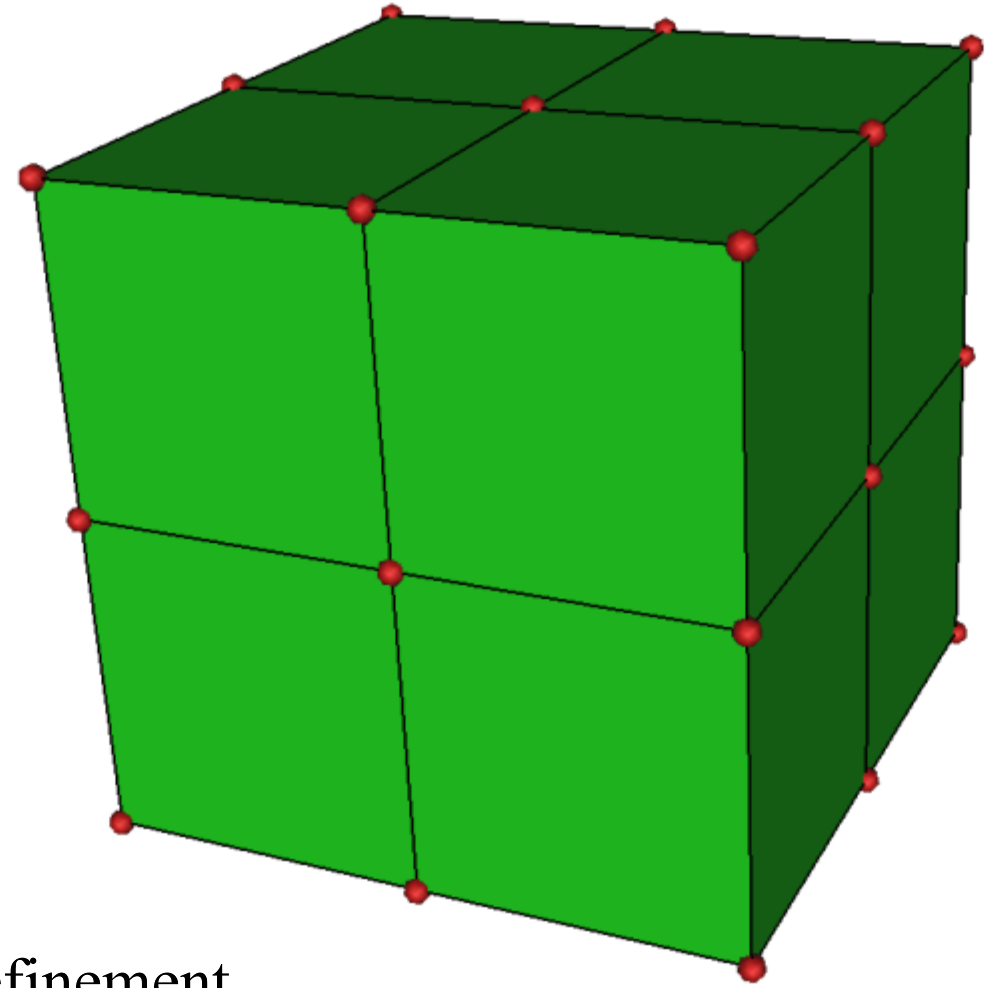
# Linear Subdivision

- Example



Topology refinement

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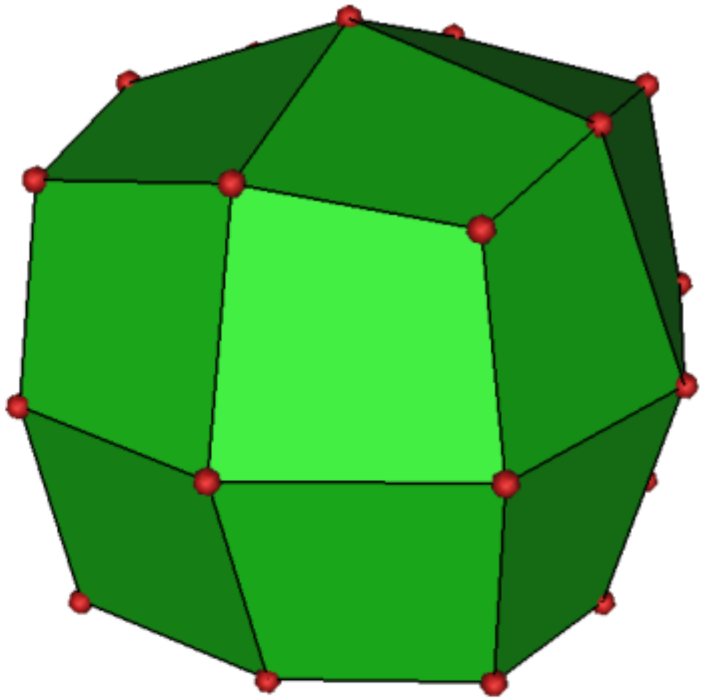
# Linear Subdivision

- Example



Geometry refinement

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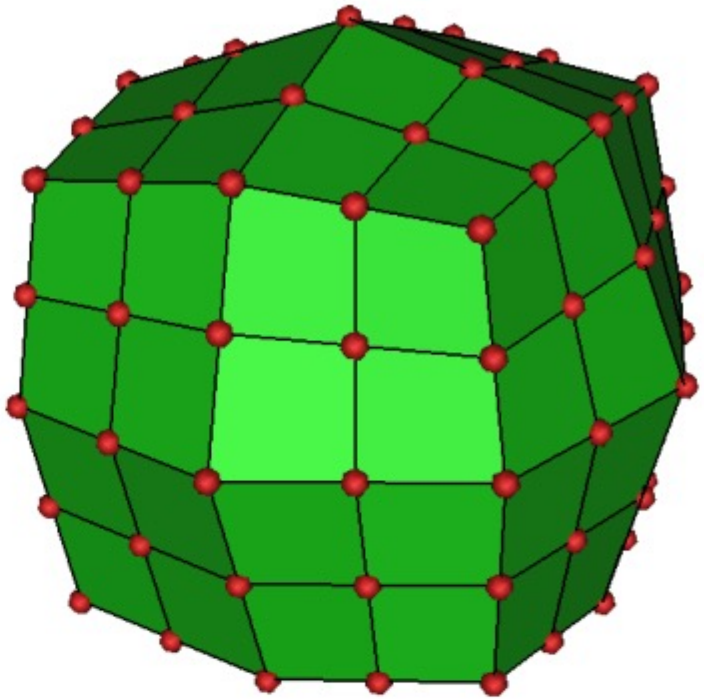
# Linear Subdivision

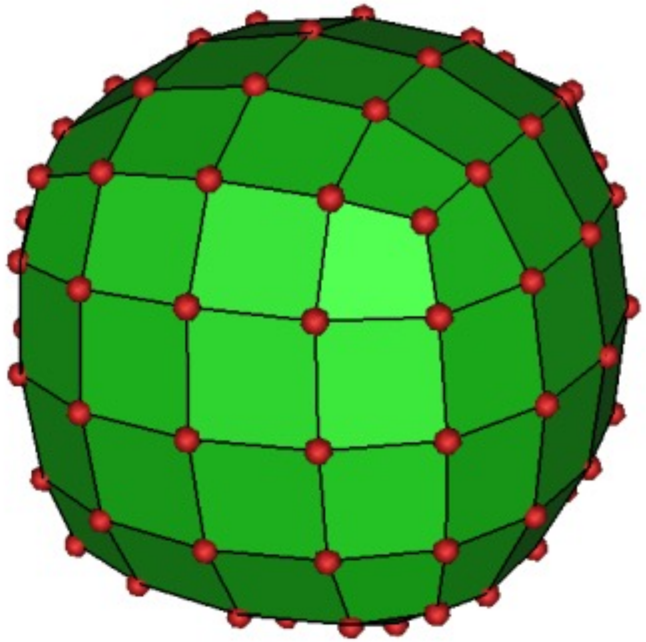
- Example



Topology refinement

# Linear Subdivision

- Example



Geometry refinement

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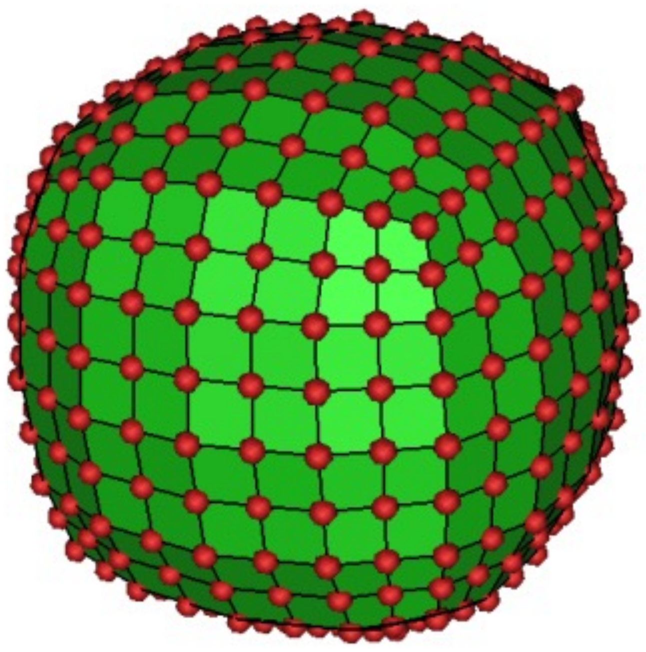
# Linear Subdivision

- Example



Topology refinement

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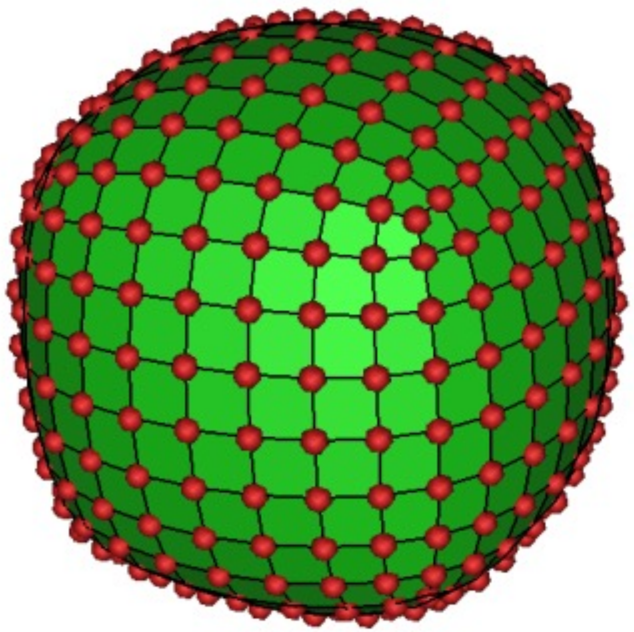
# Linear Subdivision

- Example



Geometry refinement

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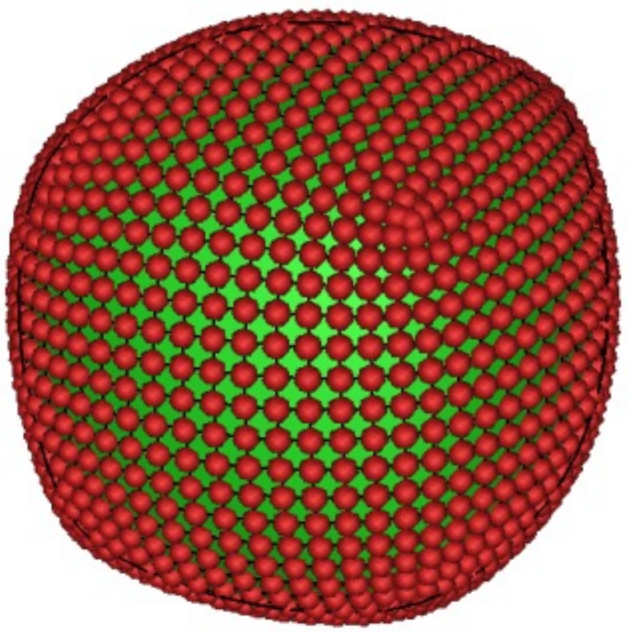
# Linear Subdivision

- Example



Topology refinement

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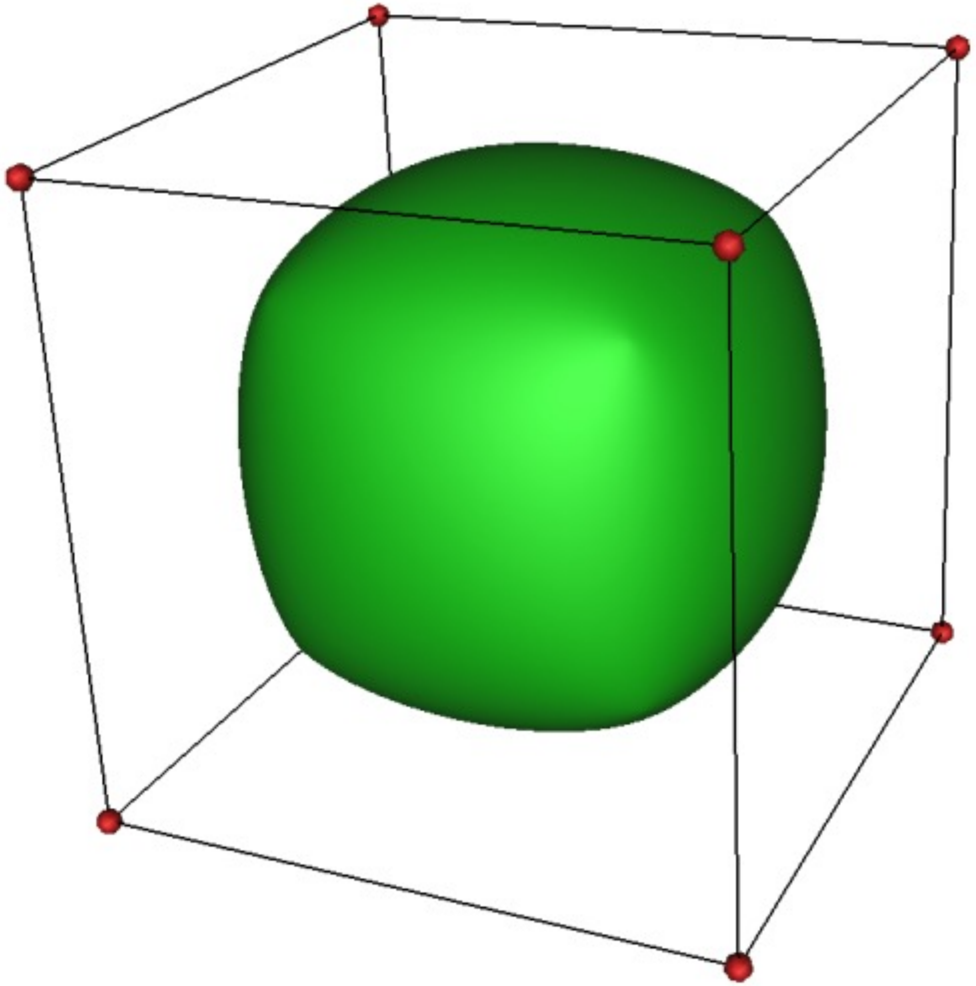
# Linear Subdivision

- Example



Final result

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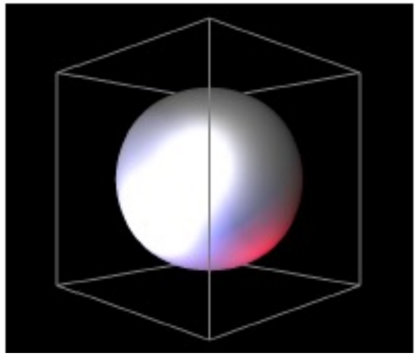
# Subdivision Demo

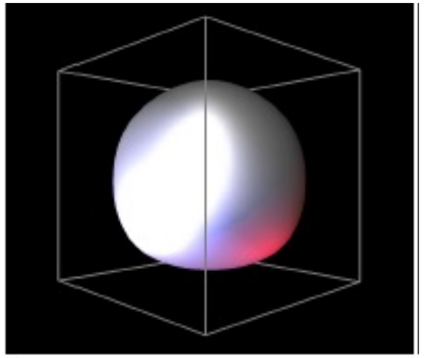
https://threejs.org/examples/webgl_modifier_subdivision.html

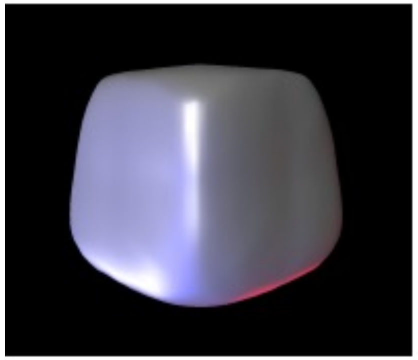
# Subdivision Schemes

- Common subdivision schemes
  - Catmull-Clark
  - Loop
  - Many others
- Differ in ...
  - Input topology
  - How refine topology
  - How refine geometry

… which makes differences in …
  - Provable properties

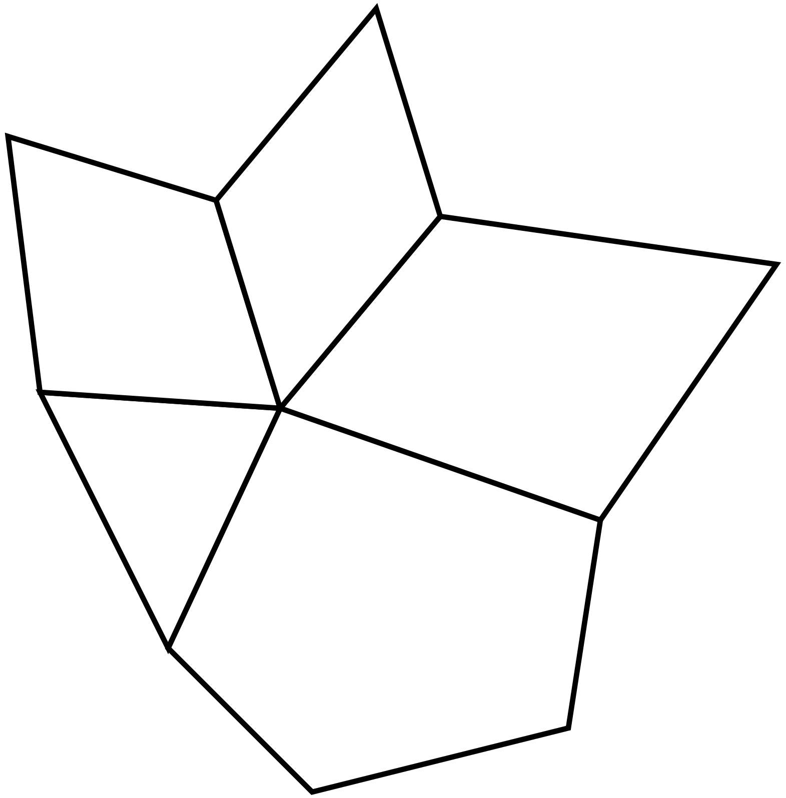






# Catmull-Clark Subdivision

- Input



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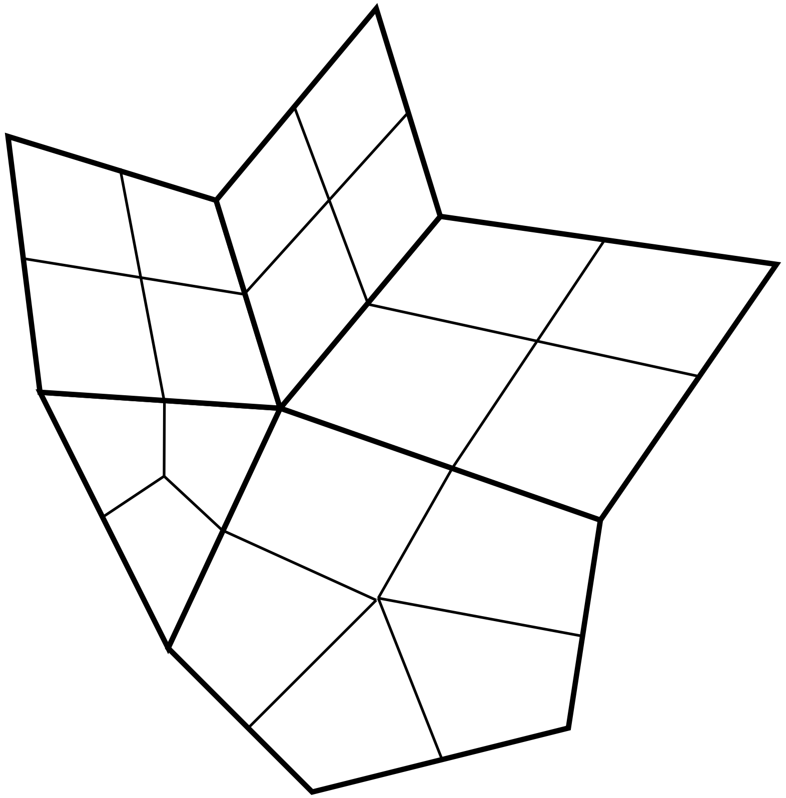
# Catmull-Clark Subdivision

- Topology refinement



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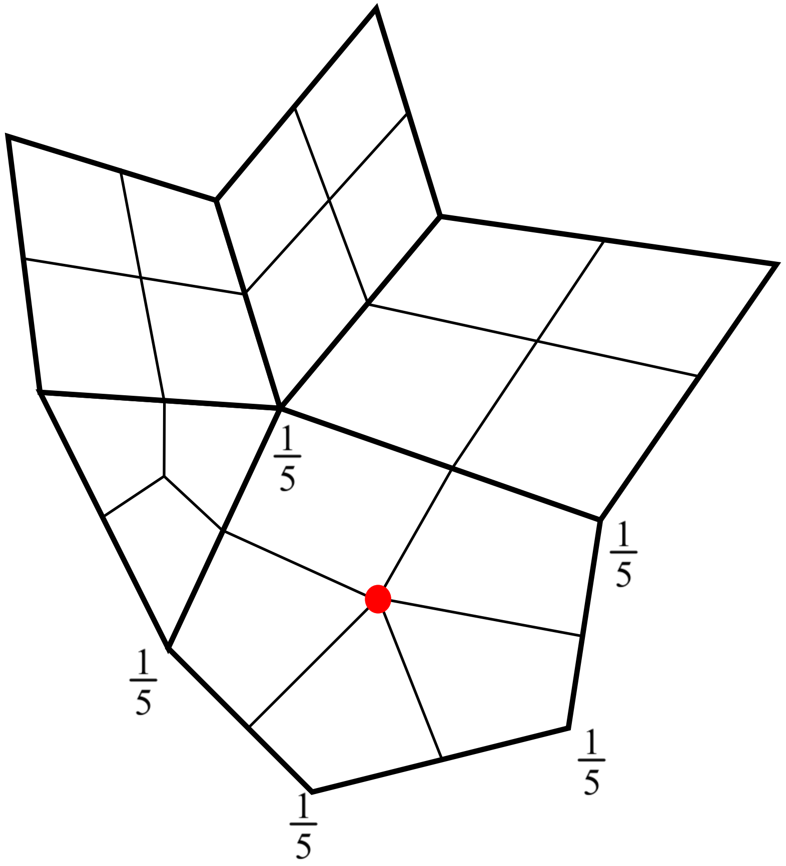
# Catmull-Clark Subdivision

- New “face points” at face centroids

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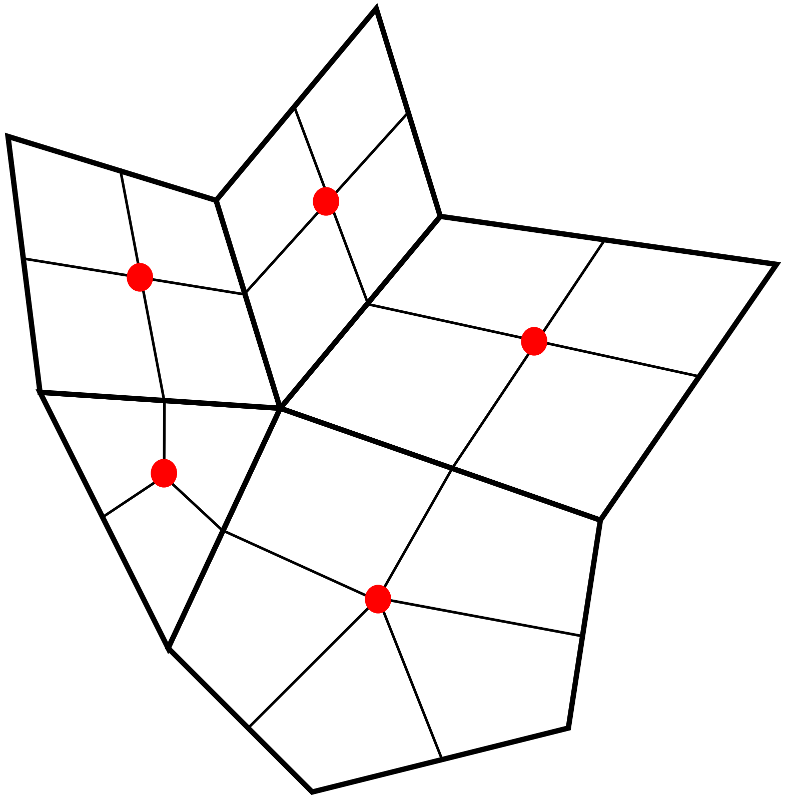
# Catmull-Clark Subdivision

- Works for polygons with any number of edges



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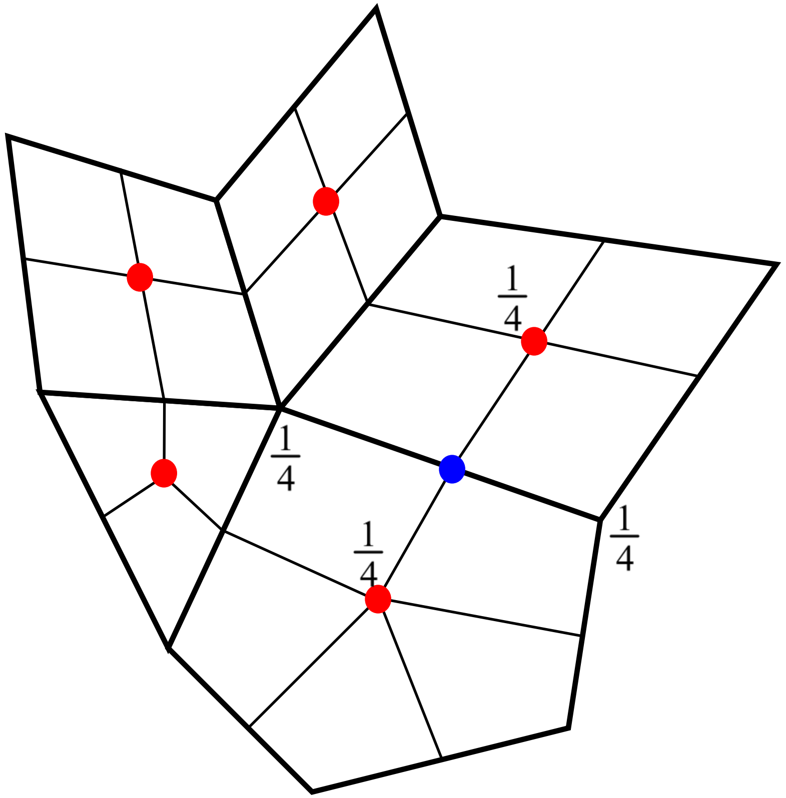
# Catmull-Clark Subdivision

- New “edge points” at average of edge vertices and face points

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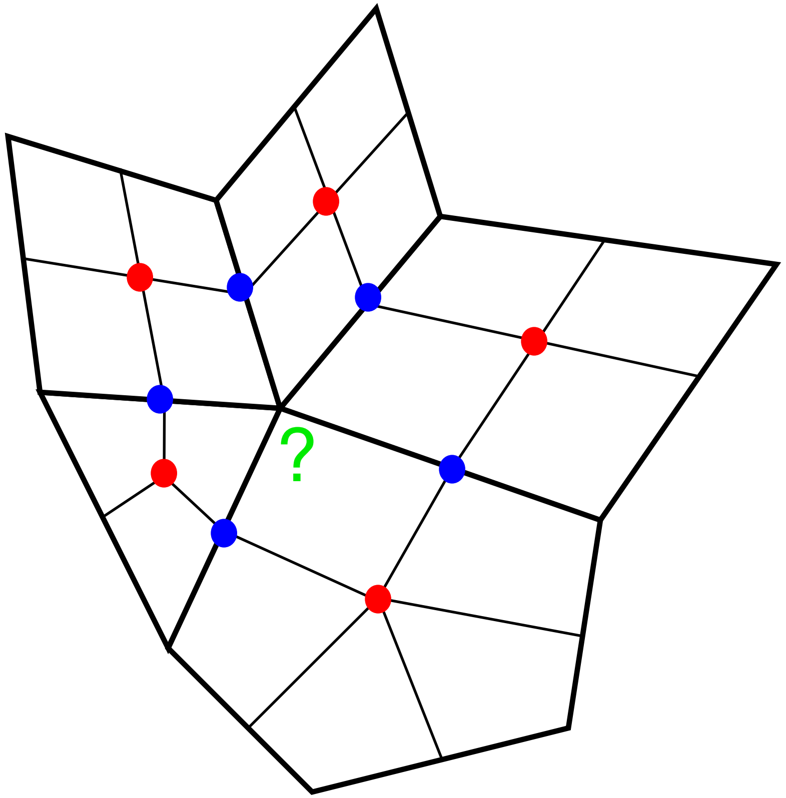
# Catmull-Clark Subdivision

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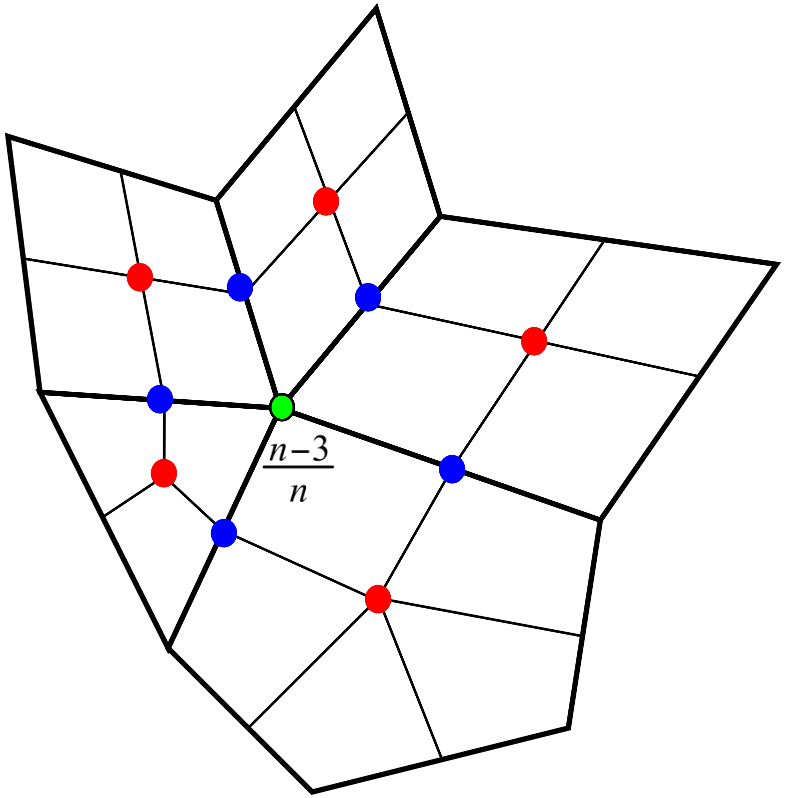
# Catmull-Clark Subdivision

- New (green point) = (4 * avg of (blue points) –1 * avg of (red points) + (n-3) * (green point) ) / n




n = #faces a point belongs to.

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# Catmull-Clark Subdivision

Linear Subdivision

Catmull-Clark Subdivision

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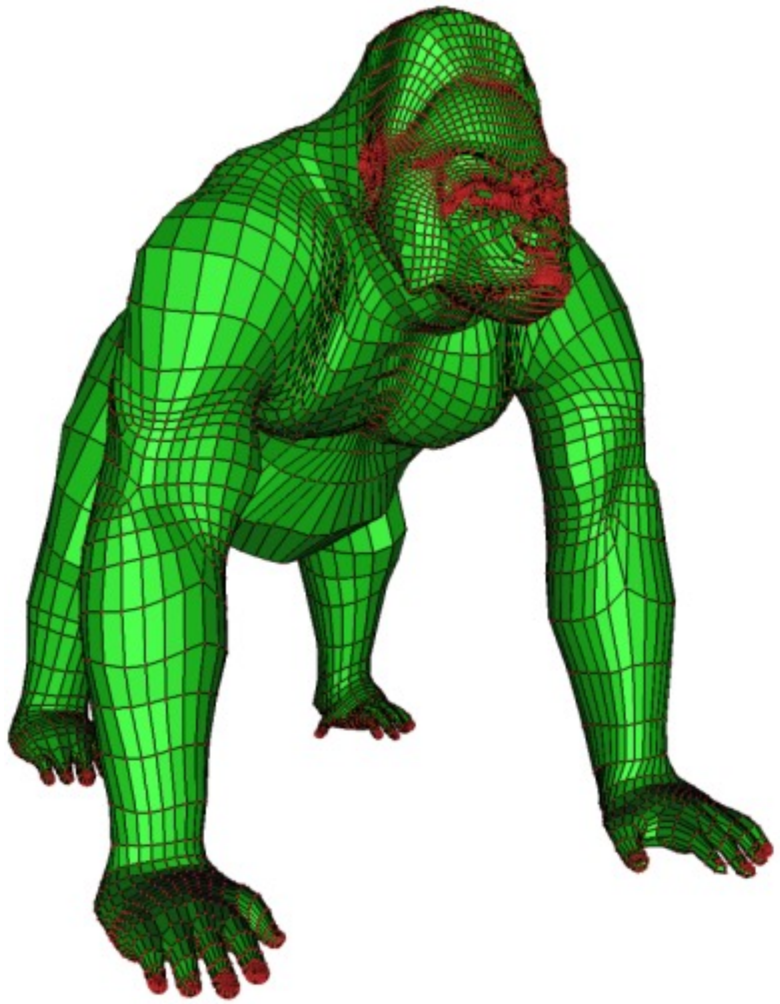
# Catmull-Clark Subdivision



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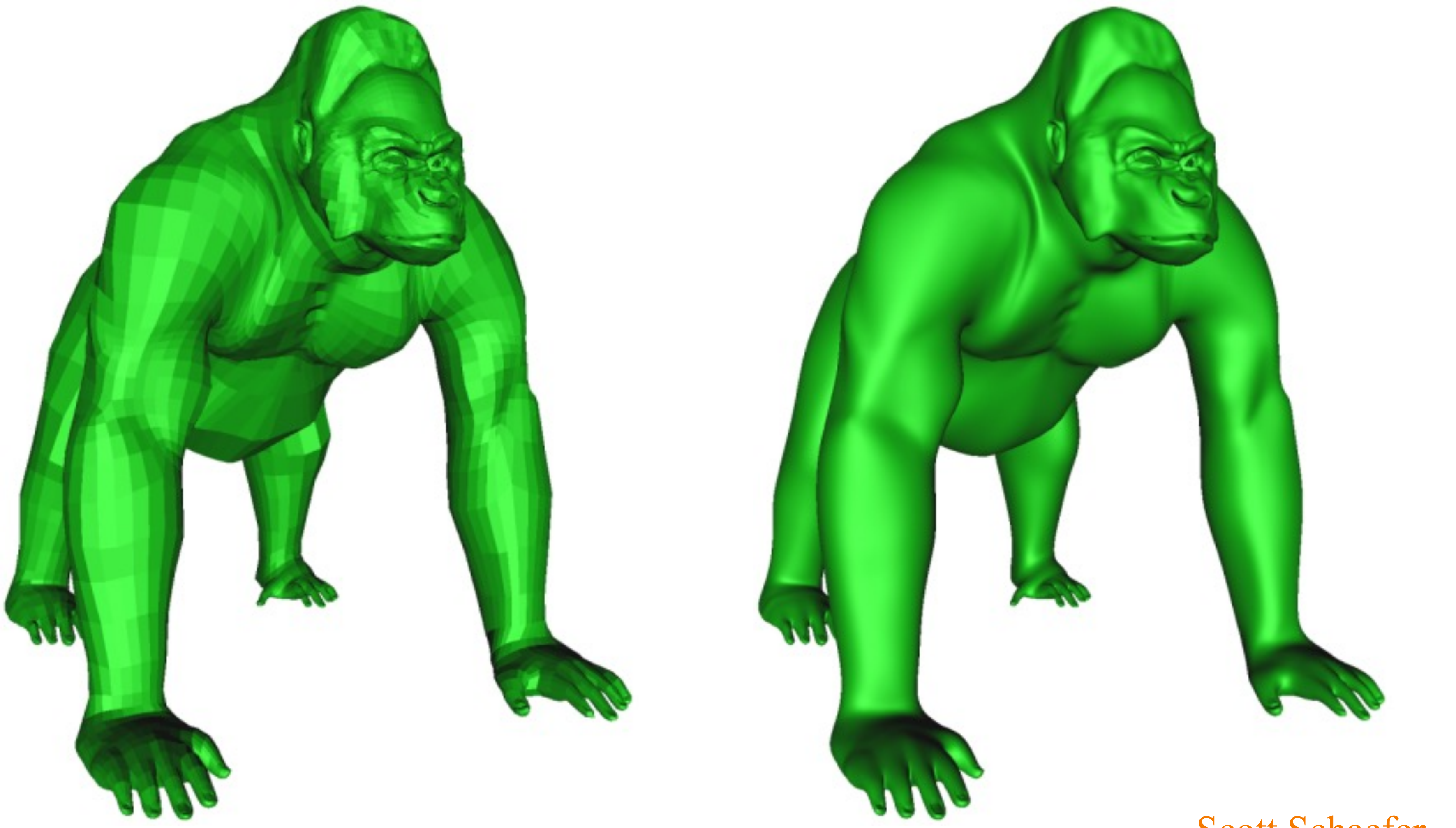
# Catmull-Clark Subdivision



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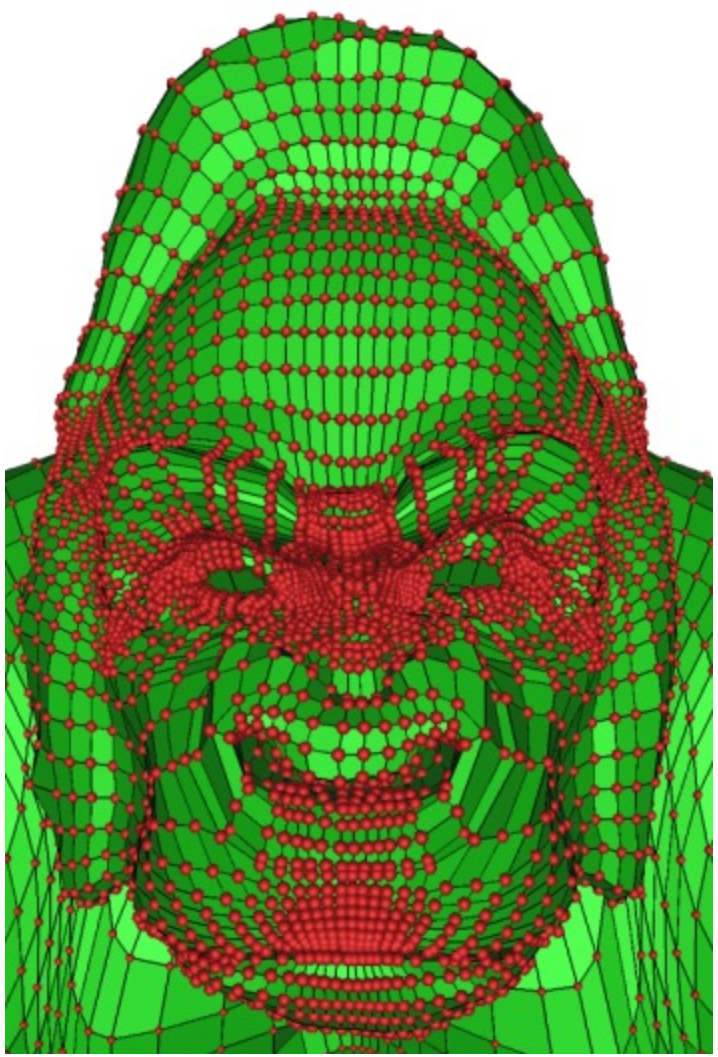
# Catmull-Clark Subdivision



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# Catmull-Clark Subdivision

# Catmull-Clark Subdivision

- One round of subdivision produces all quads
- Smoothness of limit surface
  - $C^2$ almost everywhere
  - $C^1$ at vertices with valence ≠ 4
- Relationship to control mesh
  - Does not interpolate input vertices
  - Within convex hull
- Most commonly used subdivision scheme in the movies…

Pixar

# Subdivision Schemes

- Common subdivision schemes
  - Catmull-Clark
  - ➢ Loop
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  - How refine topology
  - How refine geometry

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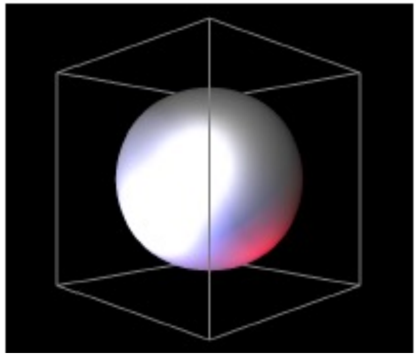
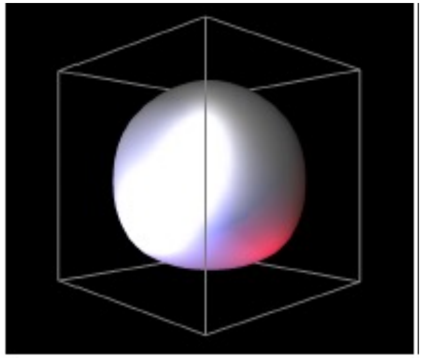
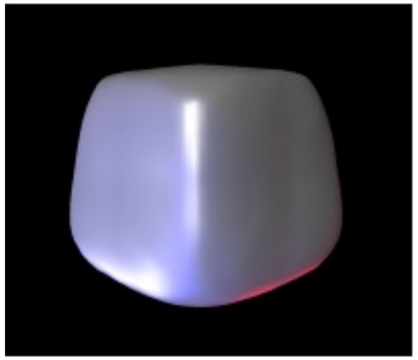
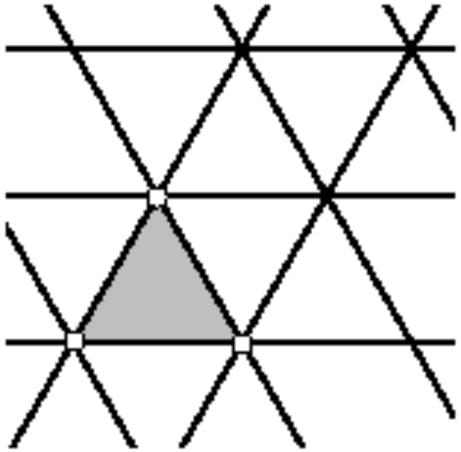






# Loop Subdivision

- Operates on pure triangle meshes
- Subdivision rules
  - Linear subdivision
  - Averaging rules for “even / odd” (white / black) vertices



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# Loop Subdivision

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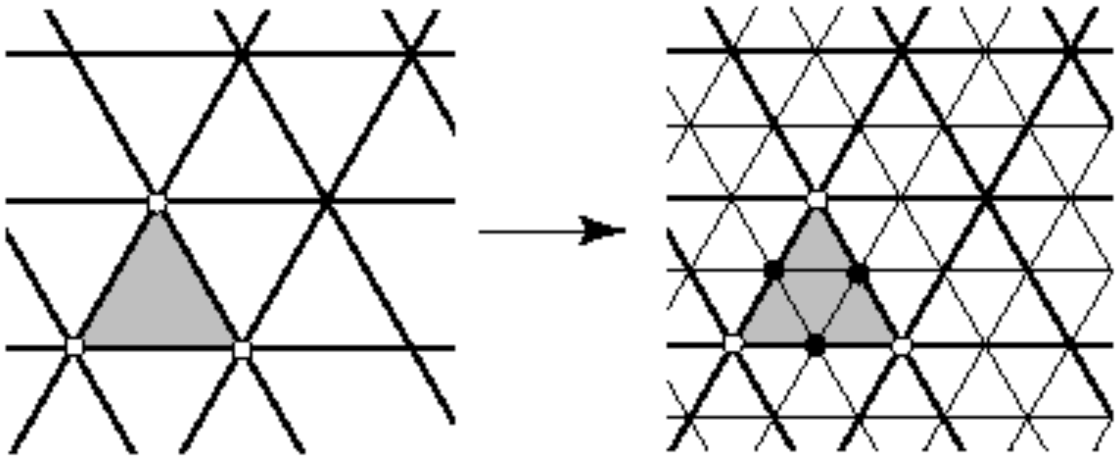

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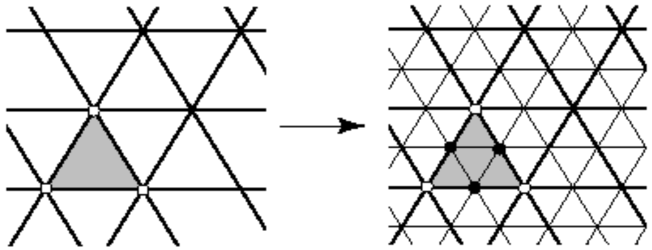
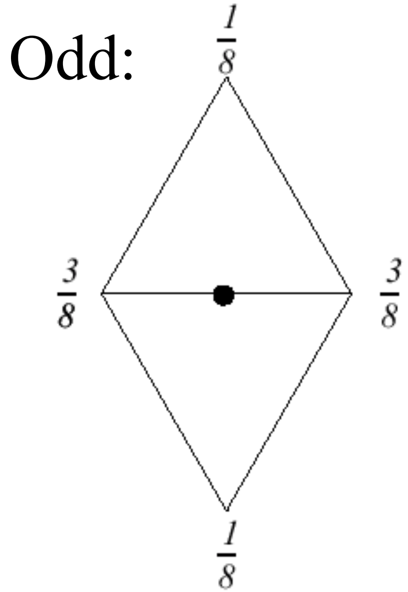
# Loop Subdivision

- Averaging rules
  - Weights for “odd” and “even” vertices



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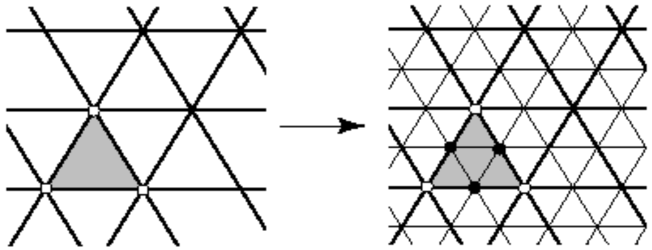
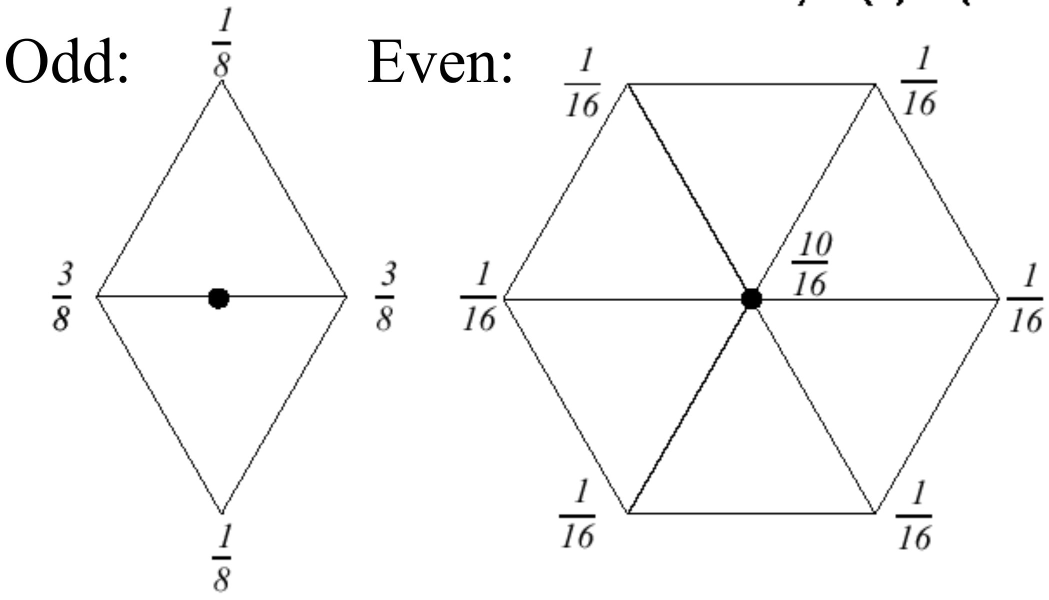
# Loop Subdivision

- Averaging rules
  - Weights for “odd” and “even” vertices






… but what about vertices with valence ≠ 6 ?

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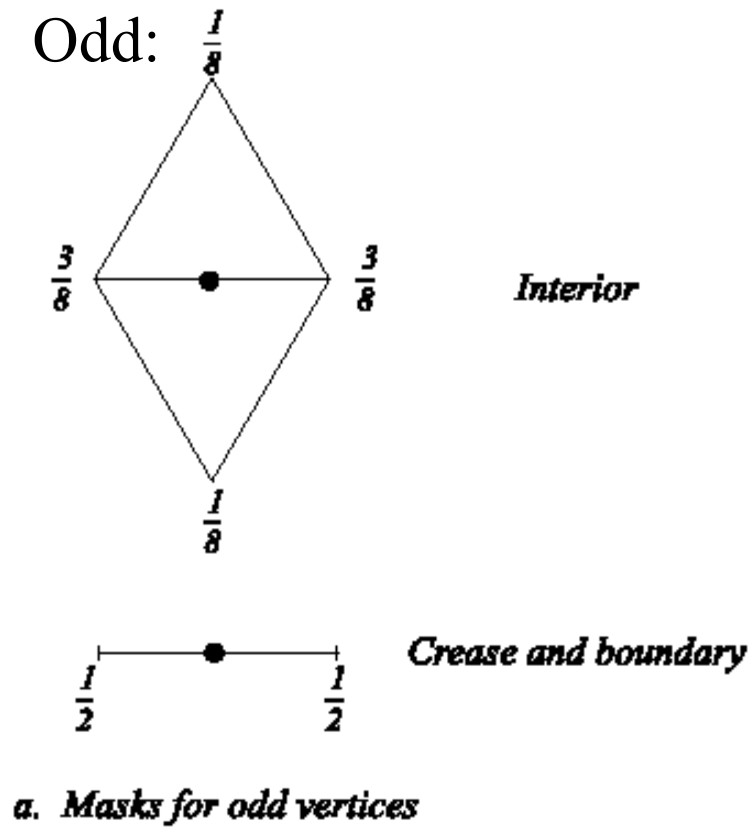
# Loop Subdivision

- Rules for *extraordinary vertices* and *boundaries*:

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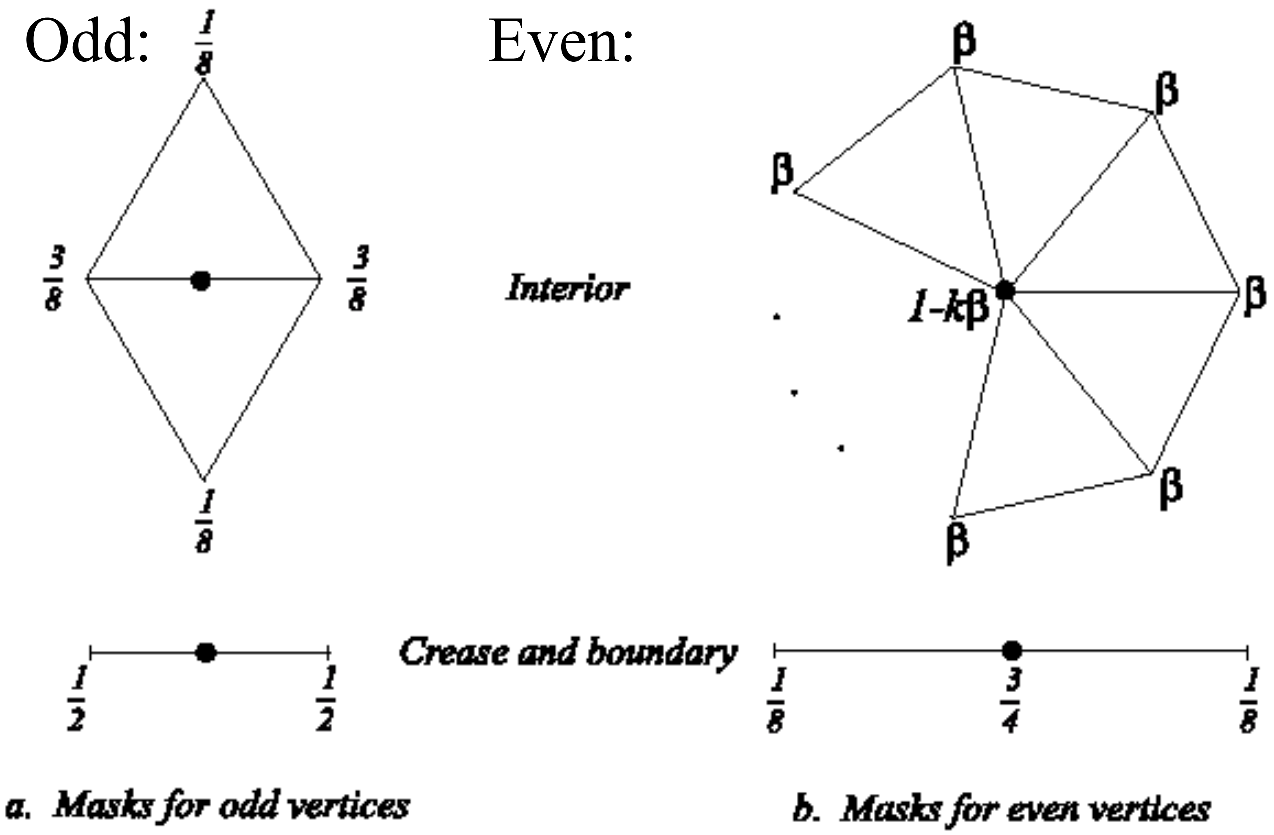
# Loop Subdivision

- Rules for *extraordinary vertices* and *boundaries*:

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# Loop Subdivision

- How to choose $\beta$?
  - Analyze properties of limit surface
  - Interested in continuity of surface and smoothness

» Original Loop

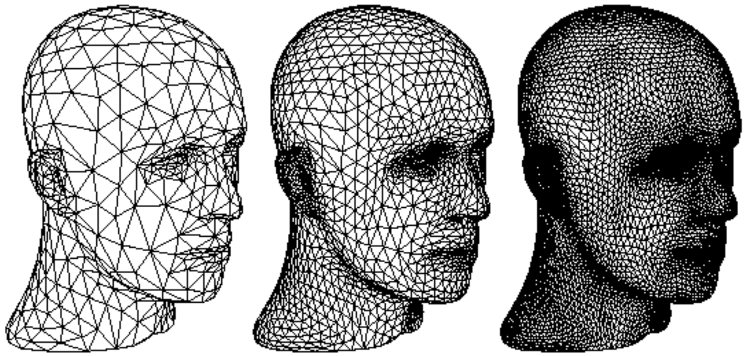
$$\beta = \frac{1}{n}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2\right)$$

» Warren

$$\beta = \begin{cases} \frac{3}{8n} & n > 3 \\ \frac{3}{16} & n = 3 \end{cases}$$

# Loop Subdivision

- Operates only on triangle meshes
- Smoothness of limit surface
  - $C^2$ almost everywhere
  - $C^1$ at vertices with valence ≠ 6
- Relationship to control mesh
  - Does not interpolate input vertices
  - Within convex hull



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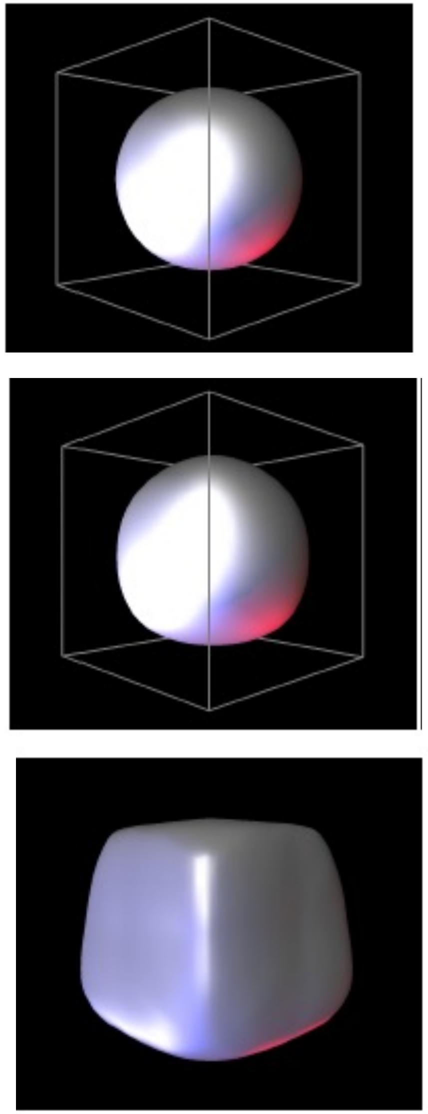
# Subdivision Schemes

- Common subdivision schemes
  - Catmull-Clark
  - Loop
  - Many others
- Differ in ...
  - Input topology
  - How refine topology
  - How refine geometry

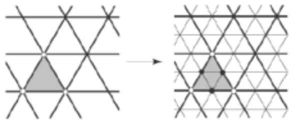
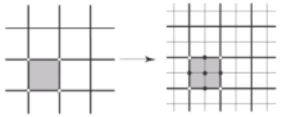
… which makes differences in …

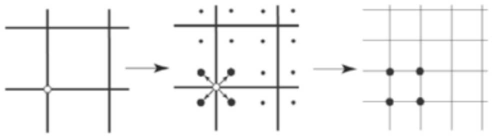
  - Provable properties



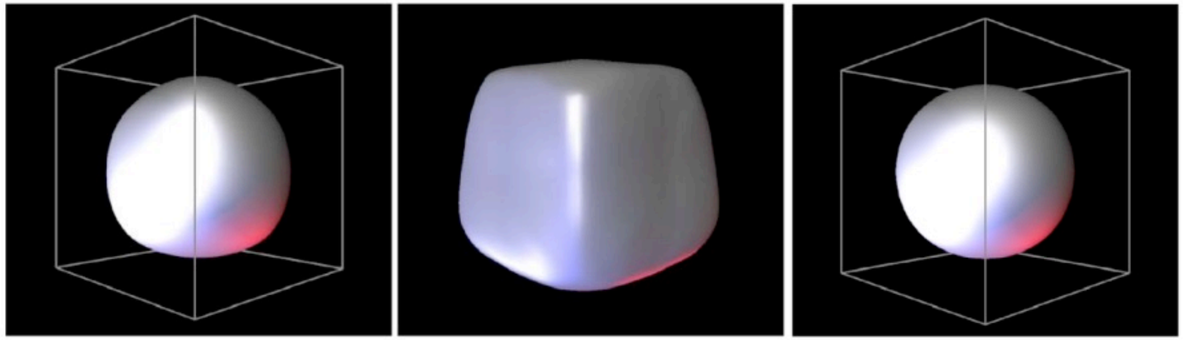
# Subdivision Zoo

- Other subdivision schemes

| Primal (face split) | | |
|---|---|---|
| |  *Triangular meshes* |  *Quad Meshes* |
| *Approximating* | Loop($C^2$) | Catmull-Clark($C^2$) |
| *Interpolating* | Mod. Butterfly ($C^1$) | Kobbelt ($C^1$) |



| **Dual (vertex split)** |
|---|
| Doo-Sabin, Midedge($C^1$) |
| Biquartic ($C^2$) |



*Loop* *Butterfly* *Catmull-Clark*

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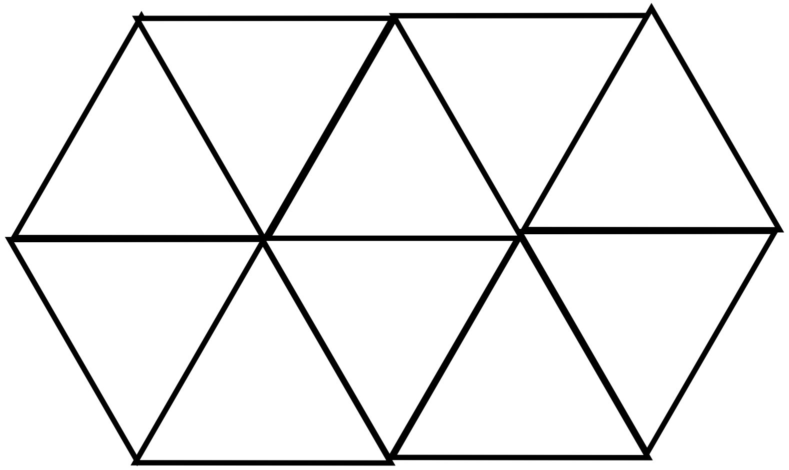
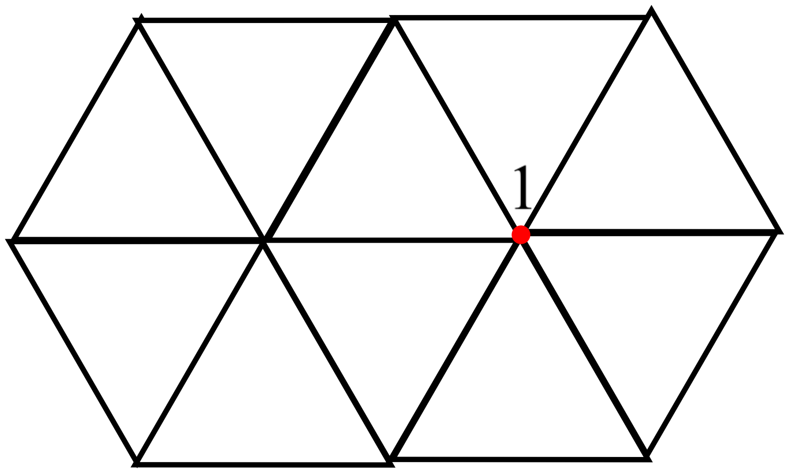
# Other Subdivision Schemes

- Butterfly subdivision



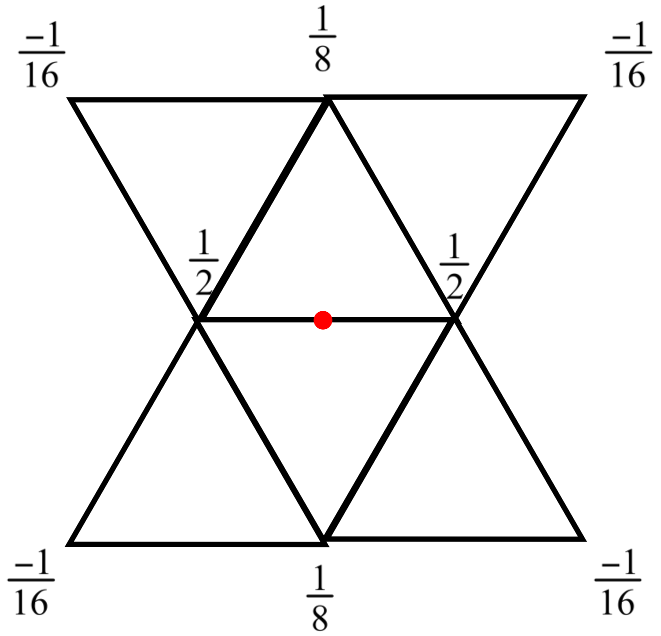
# Other Subdivision Schemes

- Butterfly subdivision



# Other Subdivision Schemes

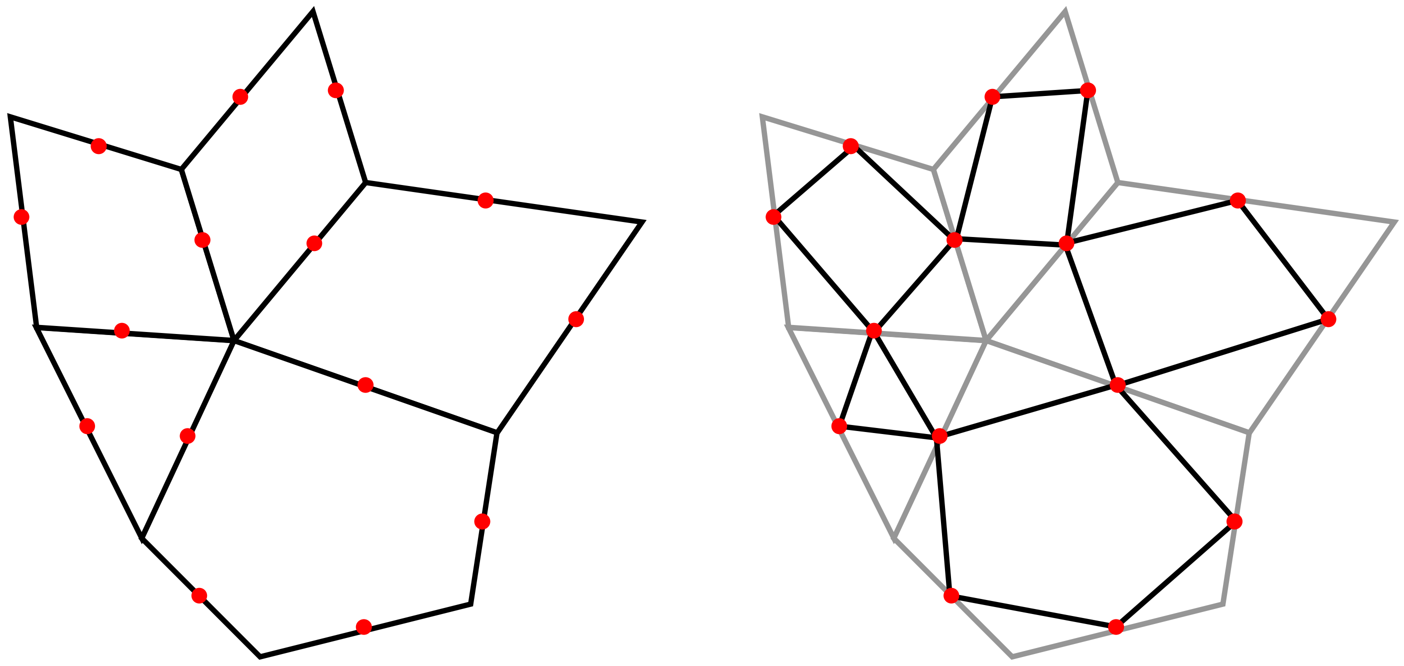
- Butterfly subdivision




Scott Schaefer

# Other Subdivision Schemes

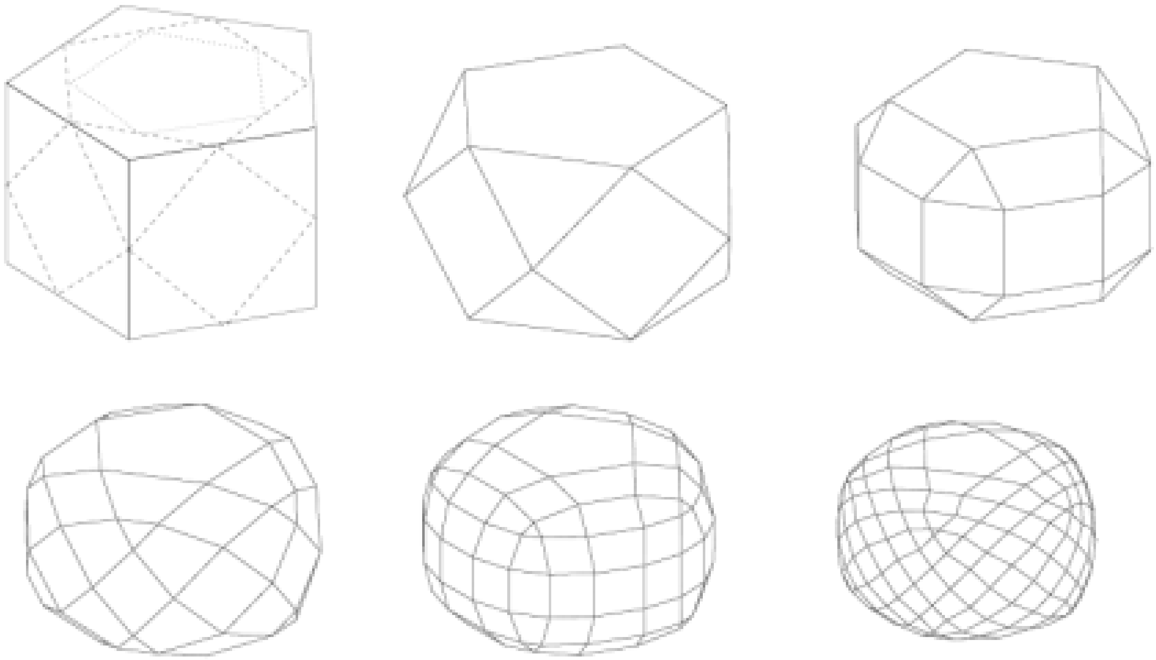
- Vertex-split subdivision
  (Doo-Sabin, Midedge, Biquartic)



One step of Midegde subdivision

# Other Subdivision Schemes

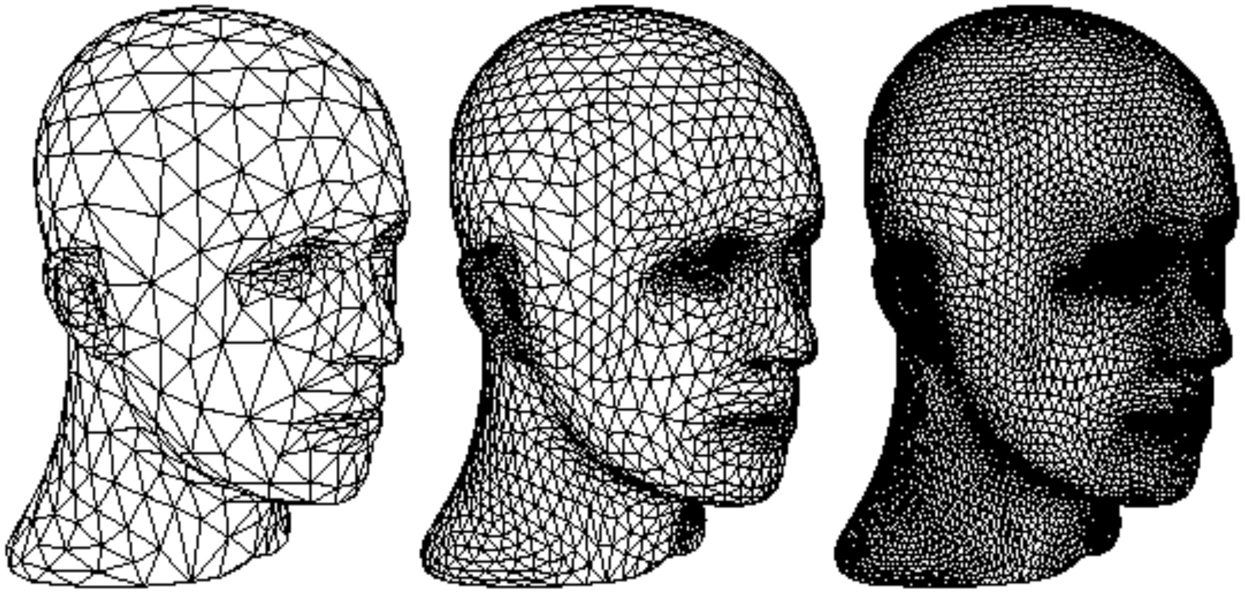
- Vertex-split subdivision
  (Doo-Sabin, Midedge, Biquartic)



Multiple steps of Midedge subdivision

# Drawing Subdivision Surfaces

- Goal:
  - Draw best approximation of smooth limit surface
  - **With limited triangle budget**



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# Drawing Subdivision Surfaces

- Goal:
  - Draw best approximation of smooth limit surface
  - With limited triangle budget
- Solution:
  - Stop subdivision at different levels across the surface
  - Stop-criterion depending on quality measure
- Quality of approximation can be defined by
  - Projected (screen) area of final triangles
  - Local surface curvature



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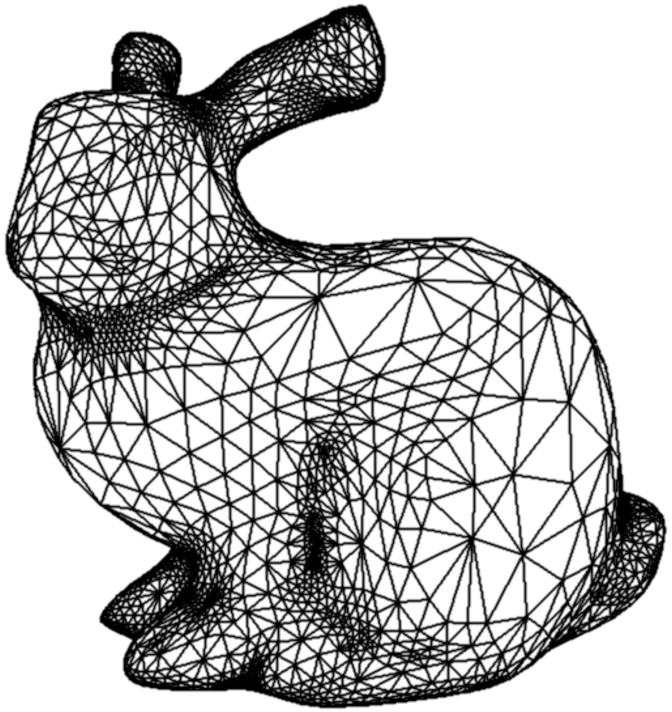
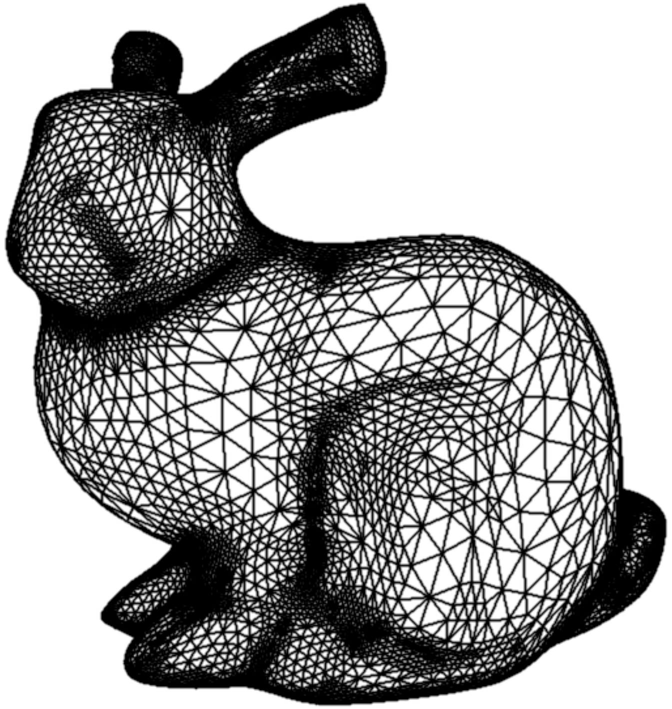
# Adaptive Subdivision



10072 Triangles

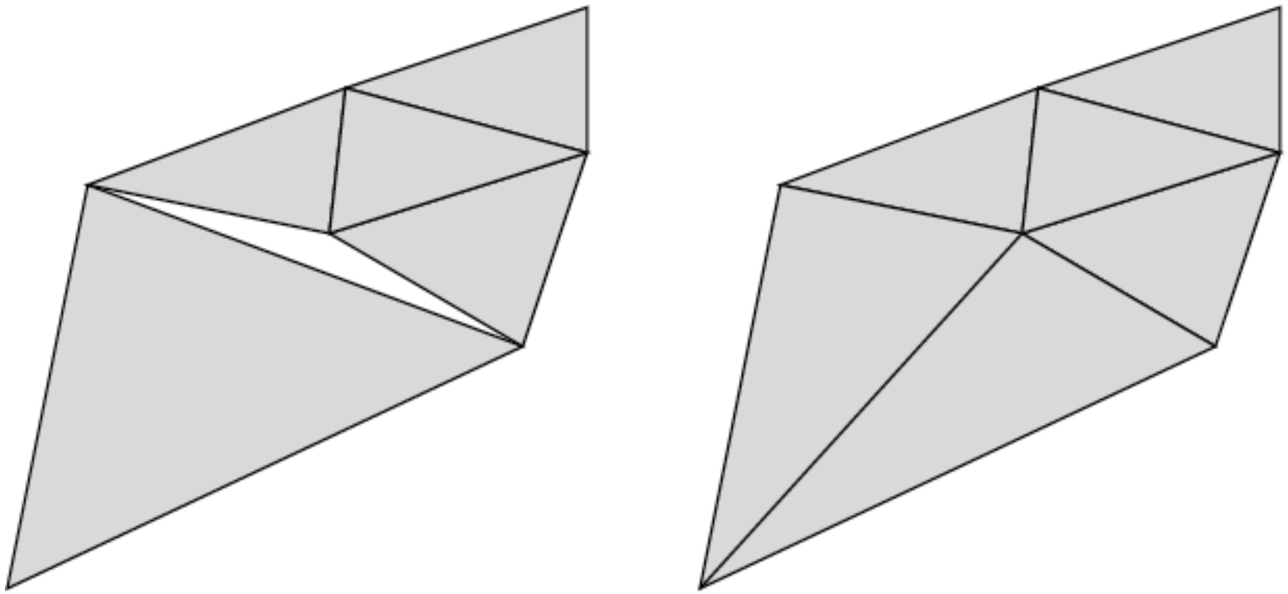


228654 Triangles

[Kobbelt 2000]

# Adaptive Subdivision

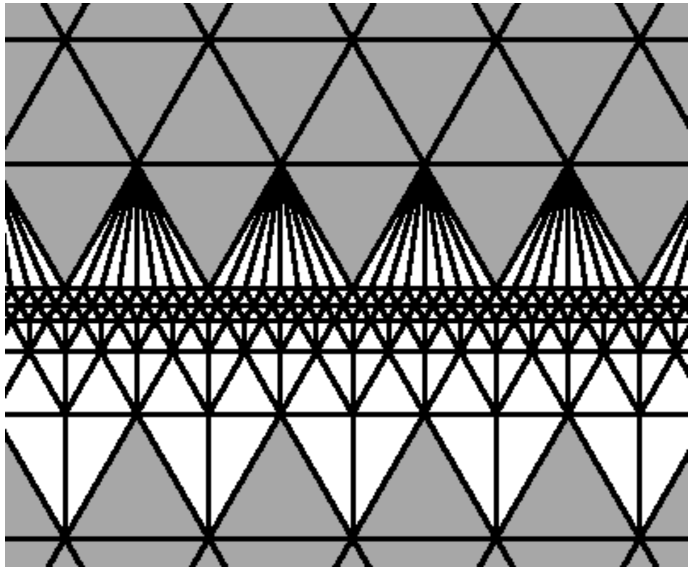
- Problem:
  - Different levels of subdivision may lead to gaps in the surface



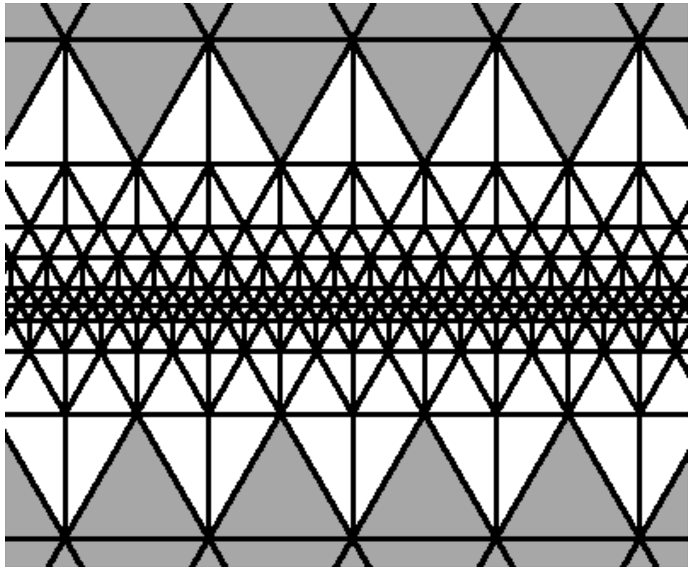
[Kobbelt 2000]

# Adaptive Subdivision

- Solution:
  - Replacing incompatible coarse triangles by *triangle fan*
  - Balanced subdivision: neighboring subdivision levels must not differ by more than one



Unbalanced

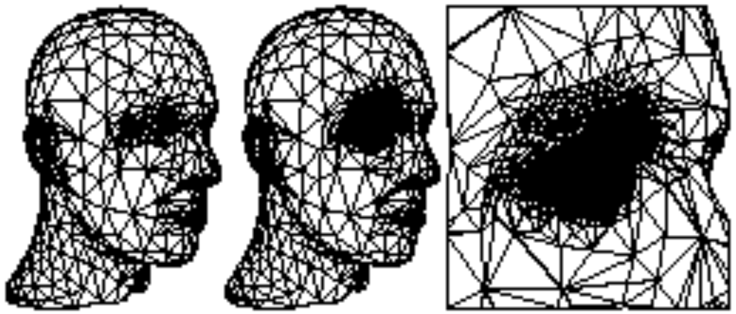


Balanced

[Kobbelt 2000]

# Subdivision Surface Summary

- Advantages:
  - Simple method for describing complex surfaces
  - Relatively easy to implement
  - Arbitrary topology
  - Intuitive specification
  - Local support
  - Guaranteed continuity
  - Multiresolution

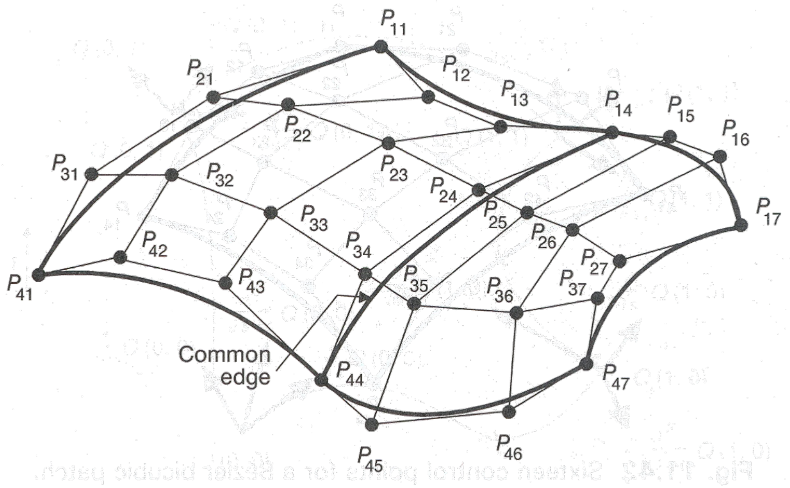


- Difficulties:
  - Parameterization
  - Intersections

# Comparison

Parametric surfaces

- Provide parameterization
- More restriction on topology of control mesh
- Some require careful placement of control mesh vertices to guarantee continuity (e.g., Bézier)




Subdivision surfaces

- No parameterization
- Subdivision rules can be defined for arbitrary topologies
- Provable continuity for all placements of control mesh vertices

# Comparison

| Feature | Polygonal Mesh | Parametric Surface | Subdivision Surface |
|---|---|---|---|
| Accurate | No | Yes | Yes |
| Concise | No | Yes | Yes |
| Intuitive specification | No | Yes | Yes |
| Local support | Yes | Yes | Yes |
| Affine invariant | Yes | Yes | Yes |
| Arbitrary topology | Yes | No | Yes |
| Guaranteed continuity | No | Yes | Yes |
| Natural parameterization | No | Yes | No |
| Efficient display | Yes | Yes | Yes |
| Efficient intersections | No | No | No |