Sampling, Resampling, and Warping

COS 426, Fall 2022

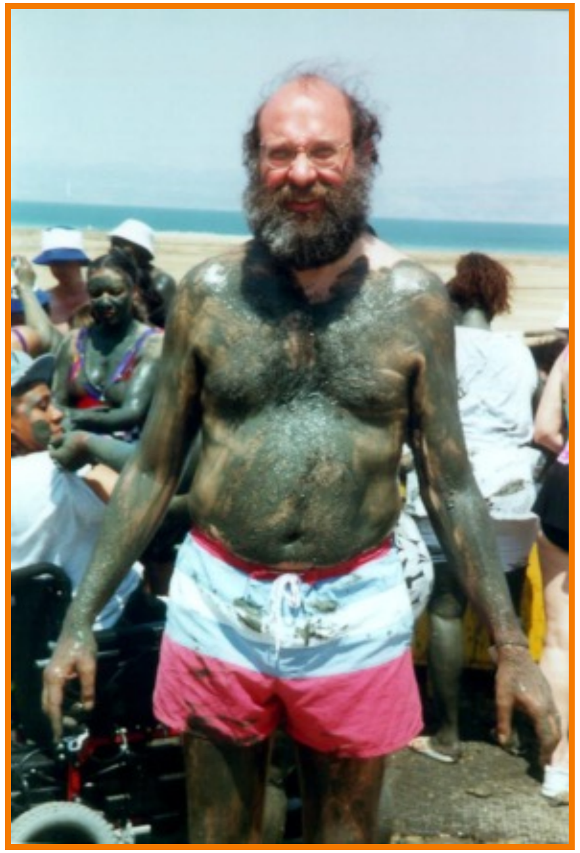
PRINCETON UNIVERSITY

Digital Image Processing

- Changing pixel values
  - Linear: scale, offset, etc.
  - Nonlinear: gamma, saturation, etc.
  - Histogram equalization
- Filtering over neighborhoods
  - Blur & sharpen
  - Detect edges
  - Median
  - Bilateral filter
- Moving image locations
  - Scale
  - Rotate
  - Warp
- Combining images
  - Composite
  - Morph
- Quantization
- Spatial / intensity tradeoff
  - Dithering

Image Warping

- Move pixels of an image



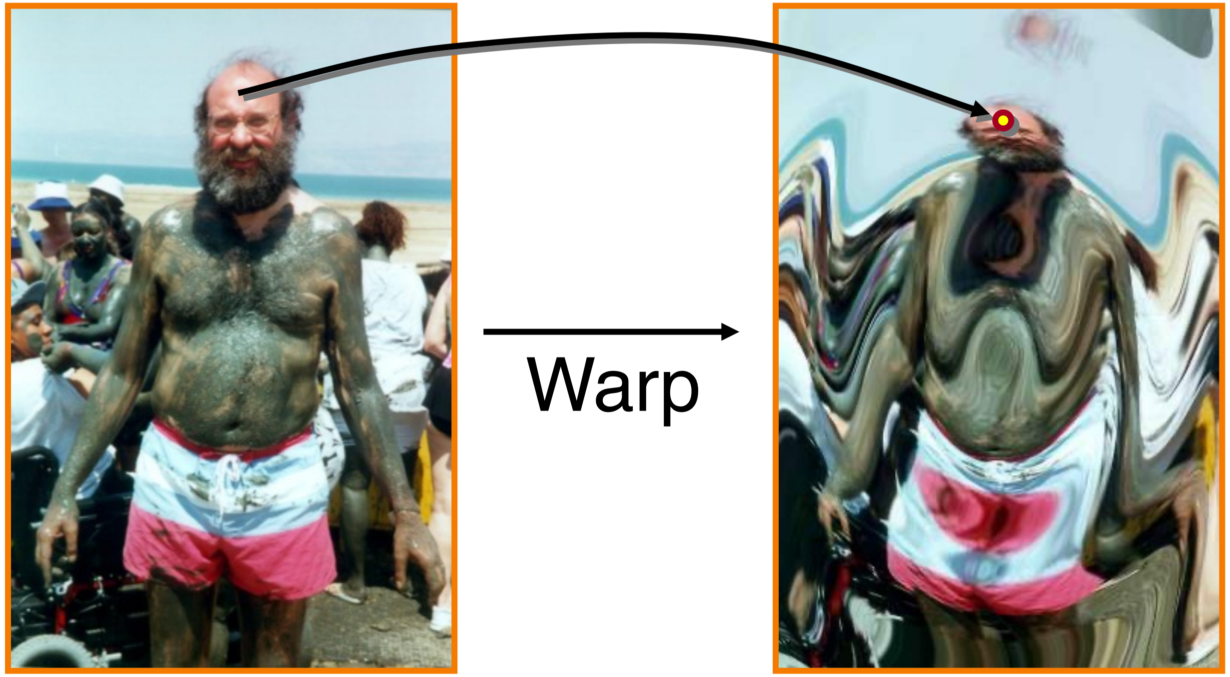
Source image

Warp

Destination image

Image Warping

- Issues:
  - Specifying where every pixel goes (**mapping**)

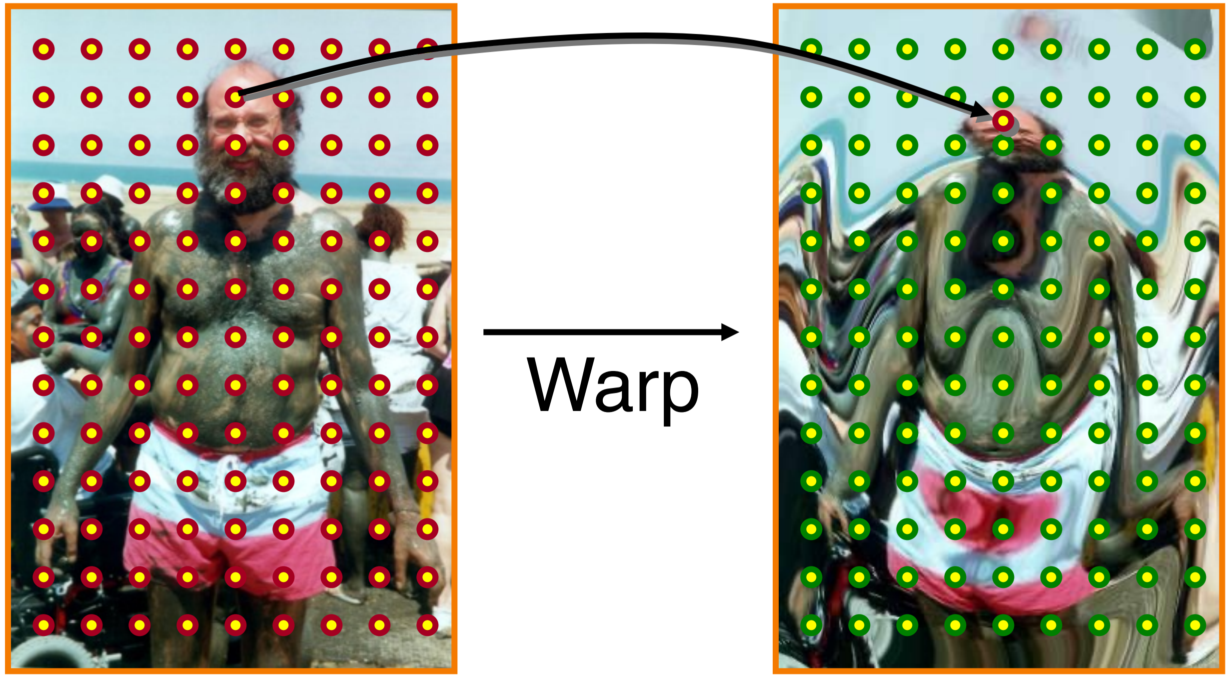


Source image

Destination image

Image Warping

- Issues:
  - Specifying where every pixel goes (mapping)
  - Computing colors at destination pixels (**resampling**)

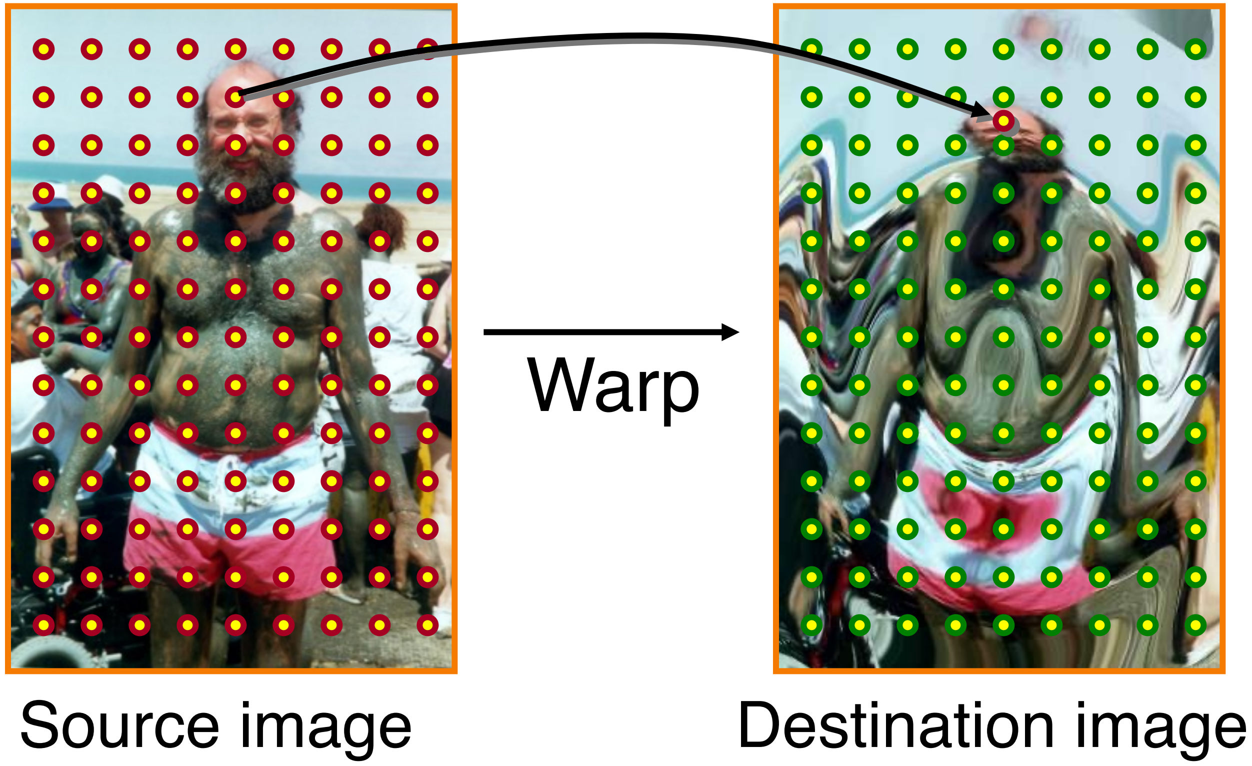


Source image

Destination image

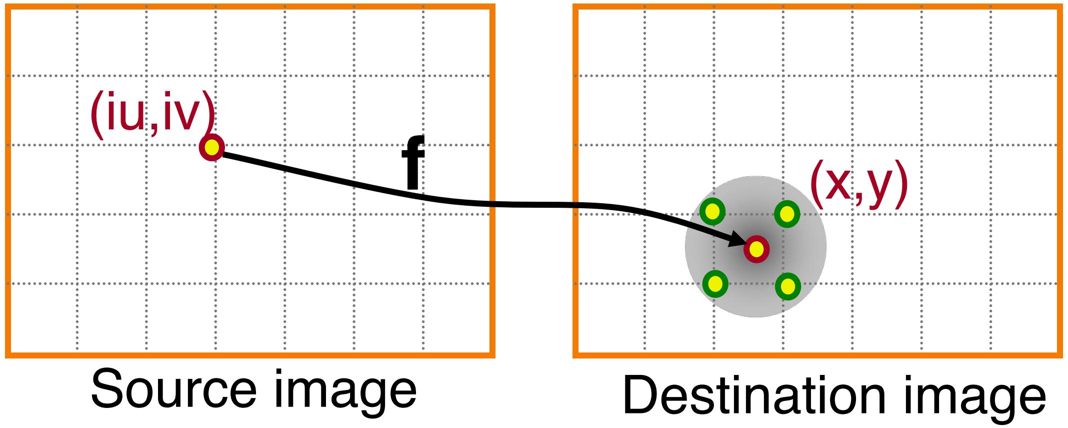
Image Warping

- Issues:
  - Specifying where every pixel goes (**mapping**)
  - Computing colors at destination pixels (resampling)

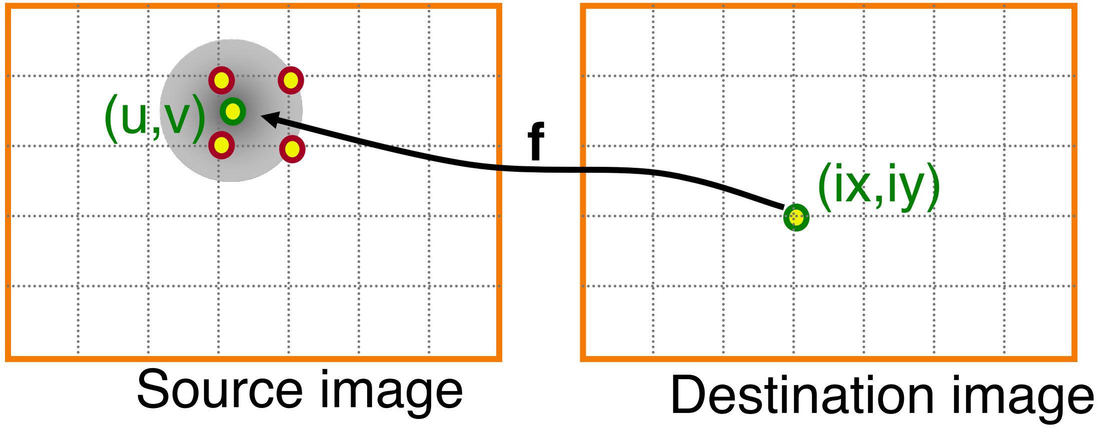


Two Options

- Forward mapping

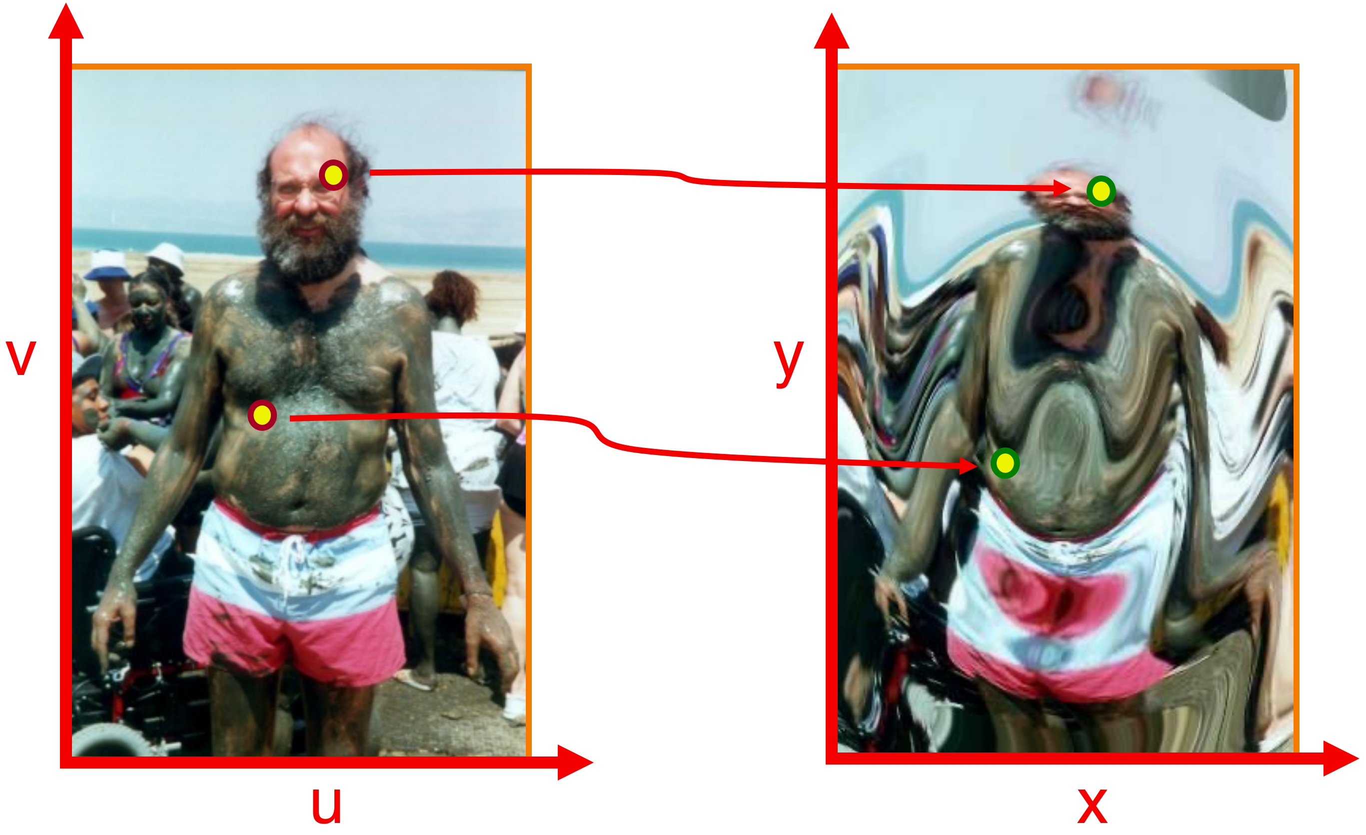


- Reverse mapping



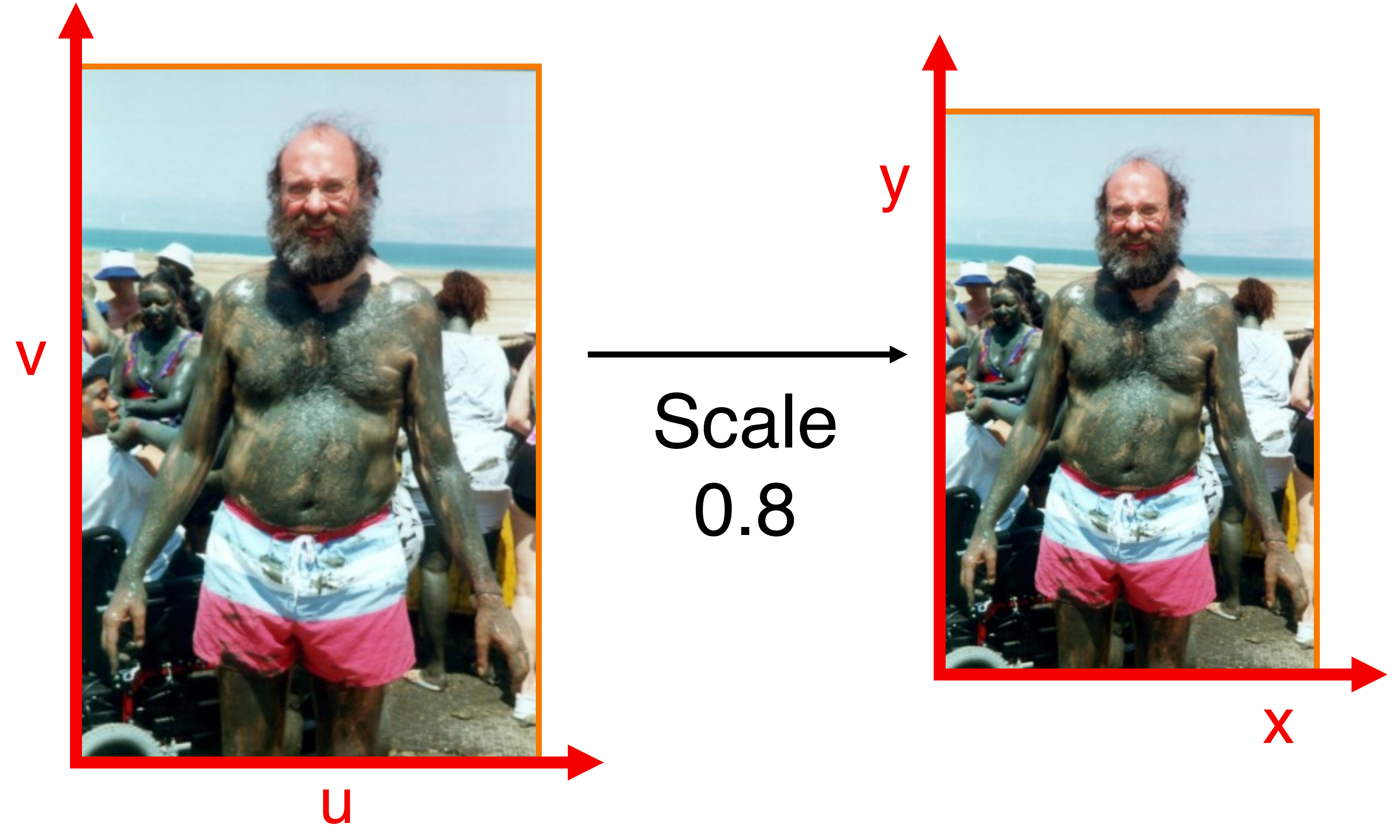
Mapping

- Define transformation
  - Describe the destination (x,y) for every source (u,v) (vice-versa, if reverse mapping)



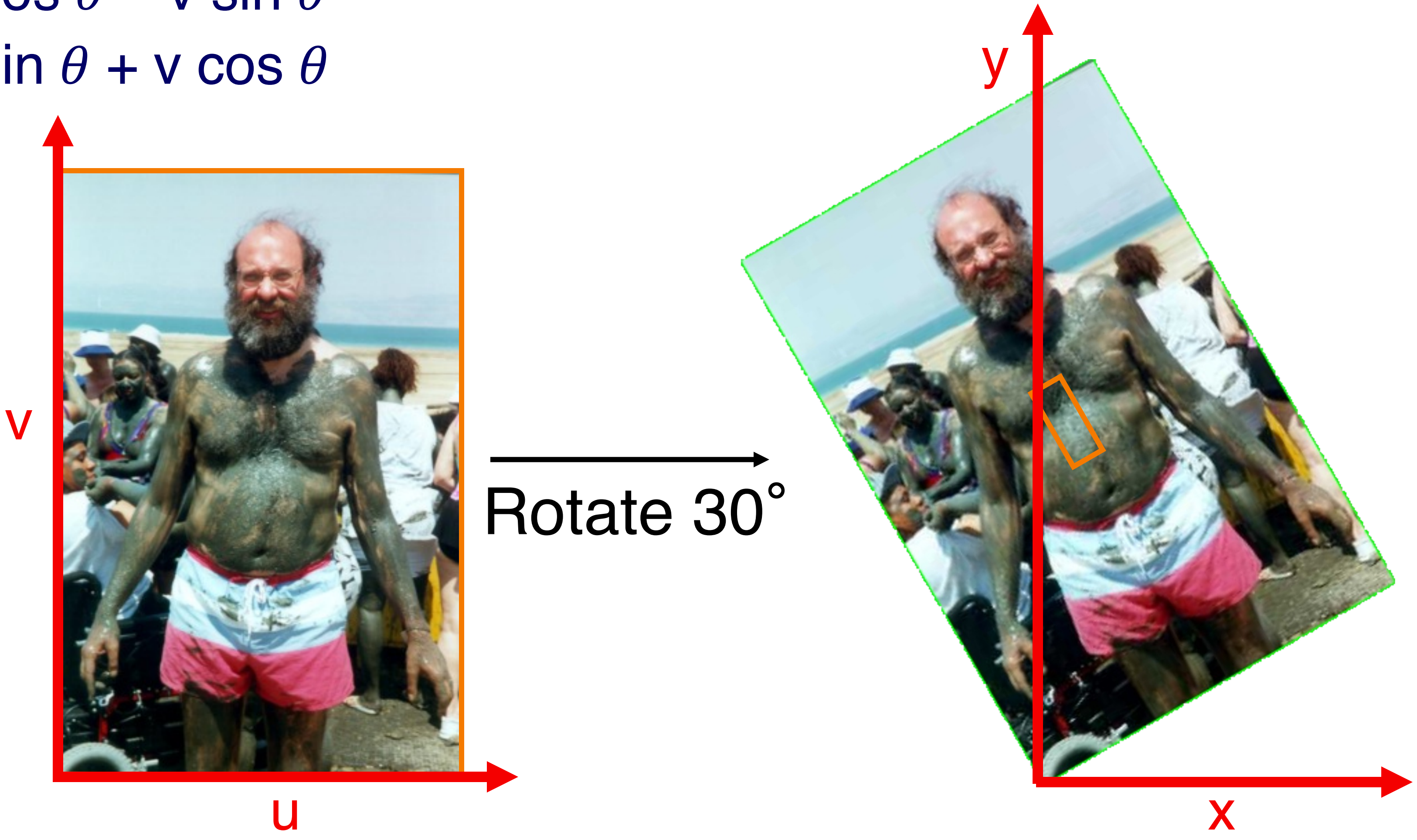
Parametric Mappings

- Scale by *factor*:
  - x = *factor* * u
  - y = *factor* * v



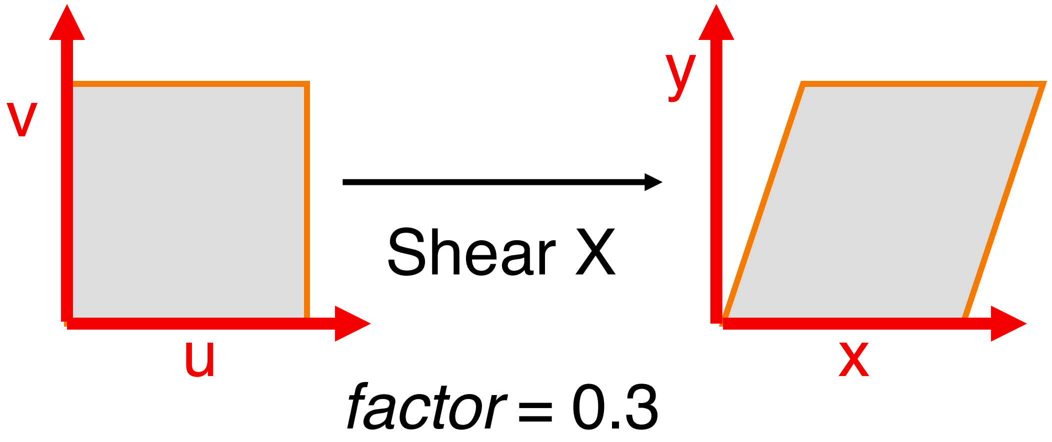
Parametric Mappings

- Rotate by θ degrees:
  - $x = u \cos\theta - v \sin\theta$
  - $y = u \sin\theta + v \cos\theta$

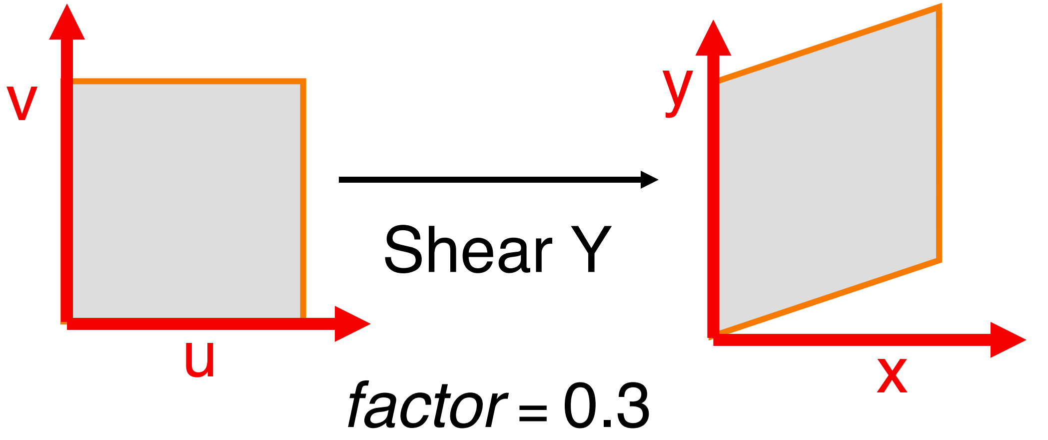


Parametric Mappings

- Shear in X by *factor:*
  - x = u + *factor* * v
  - y = v

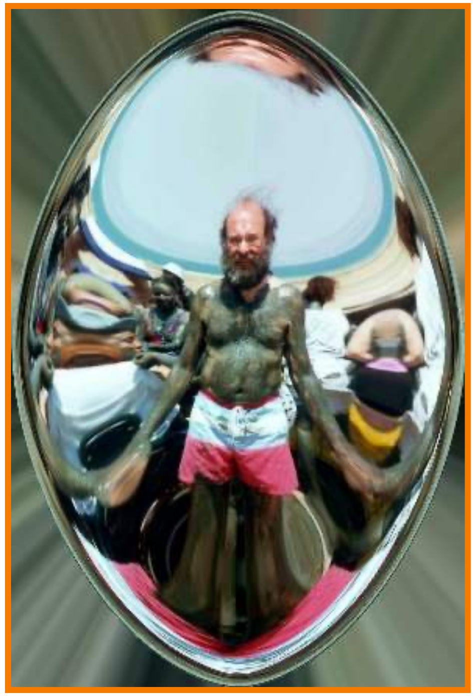


- Shear in Y by *factor:*
  - x = u
  - y = v + *factor* * u



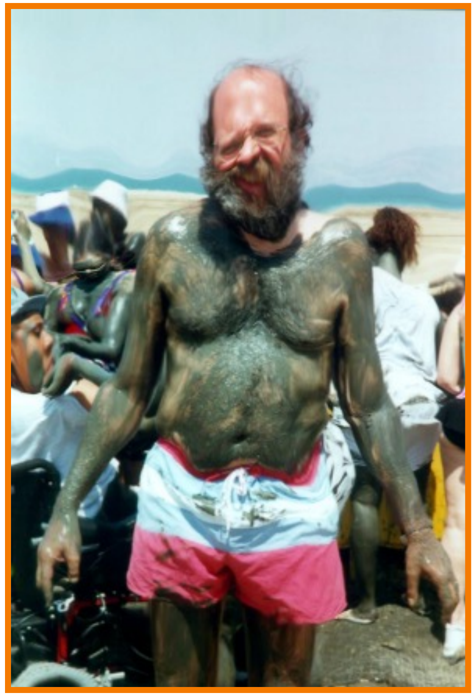
Other Parametric Mappings

- Any function of u and v:
  - $x = f_x(u,v)$
  - $y = f_y(u,v)$



Fish-eye

"Swirl"



"Rain"

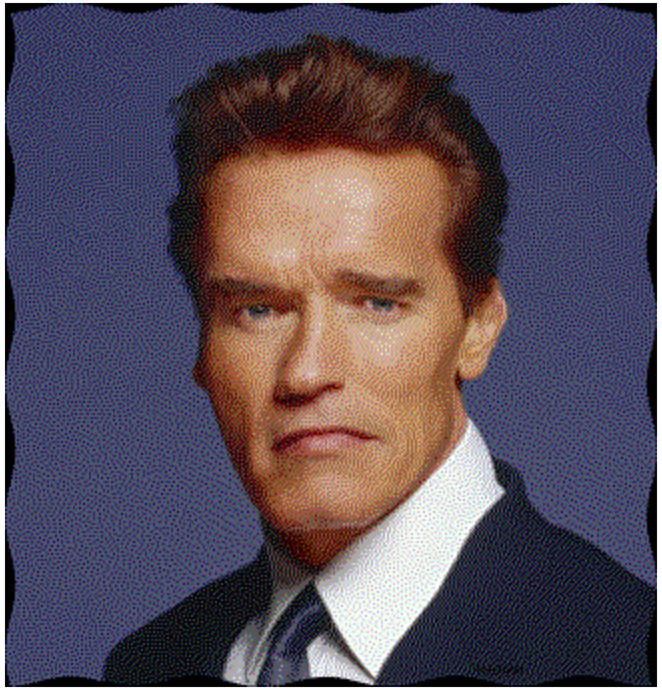
COS426 Examples

Aditya Bhaskara

Wei Xiang

More COS426 Examples

Sid Kapur

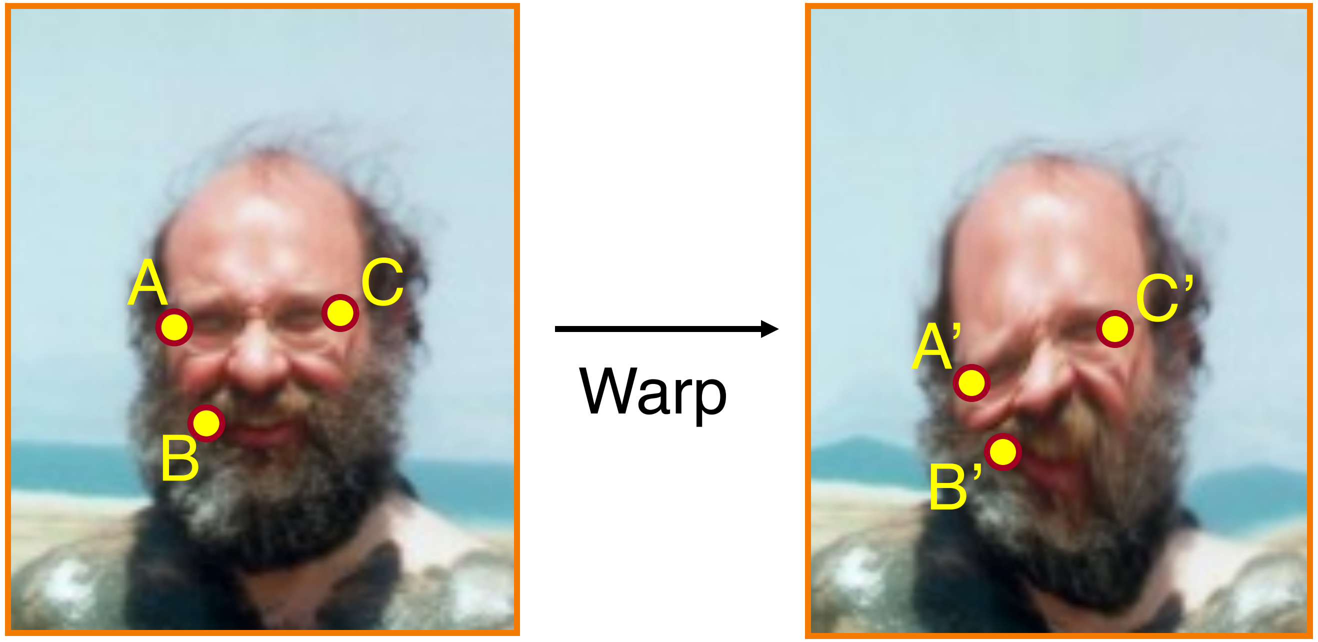


Eirik Bakke

Michael Oranato

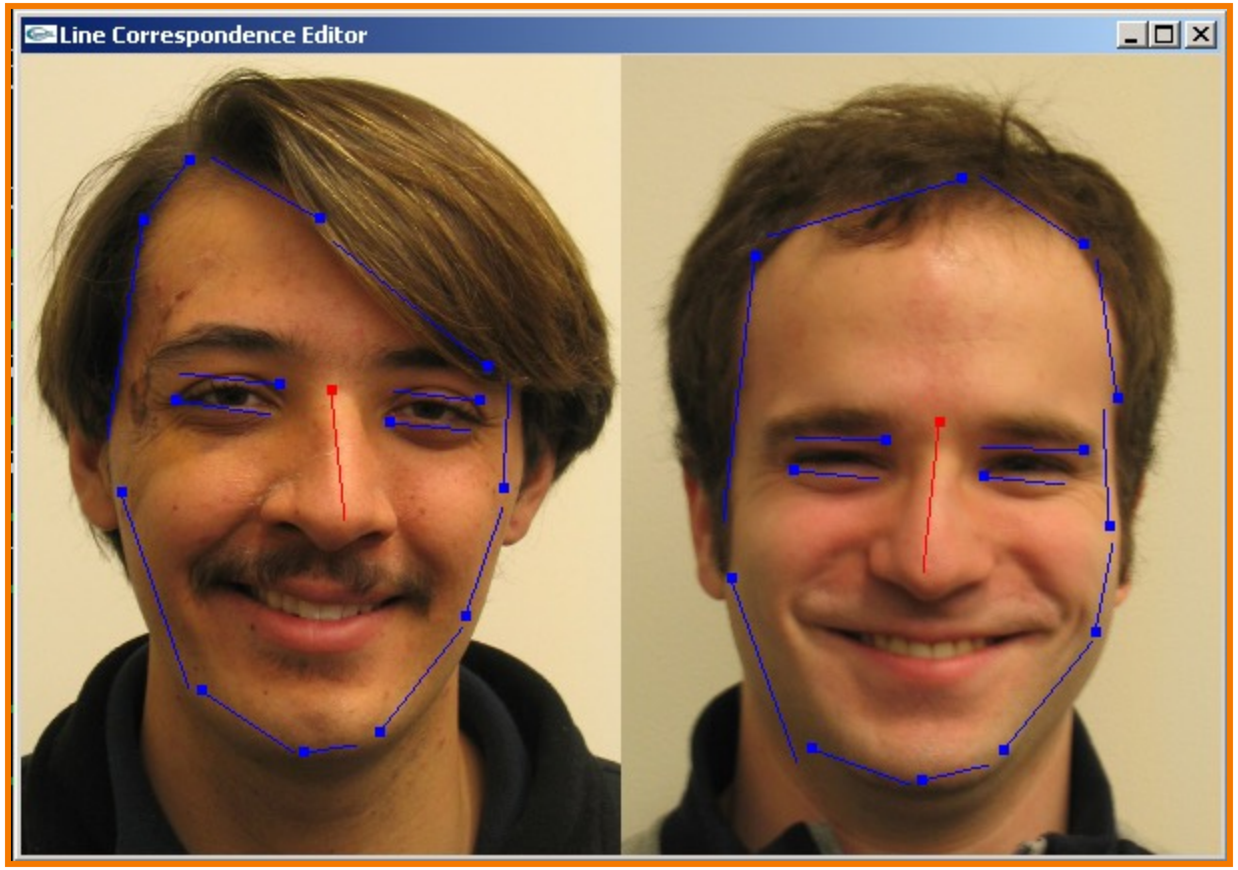
Point Correspondence Mappings

- Mappings implied by correspondences:
  - A ↔ A’
  - B ↔ B’
  - C ↔ C’



Line Correspondence Mappings

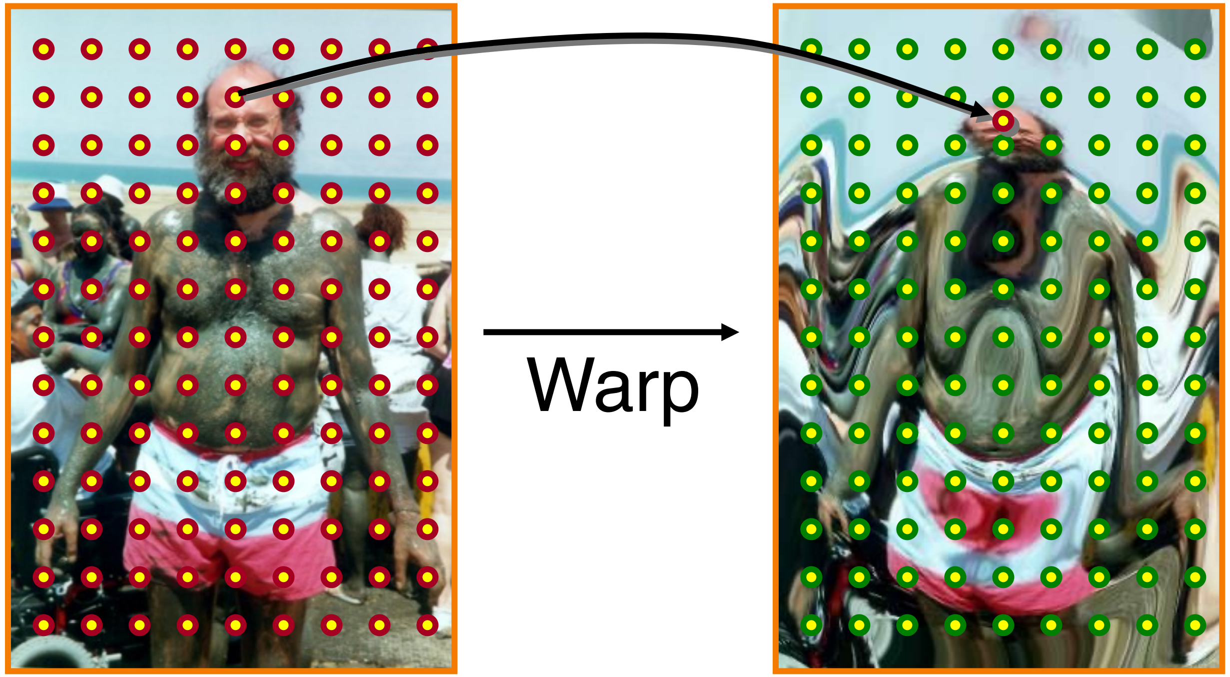
- Alternatively, Beier & Neeley [92] use pairs of *lines* to specify warp (more on this next time)



Beier & Neeley
SIGGRAPH 92

Image Warping

- Issues:
  - Specifying where every pixel goes (mapping)
  - Computing colors at destination pixels (**resampling**)



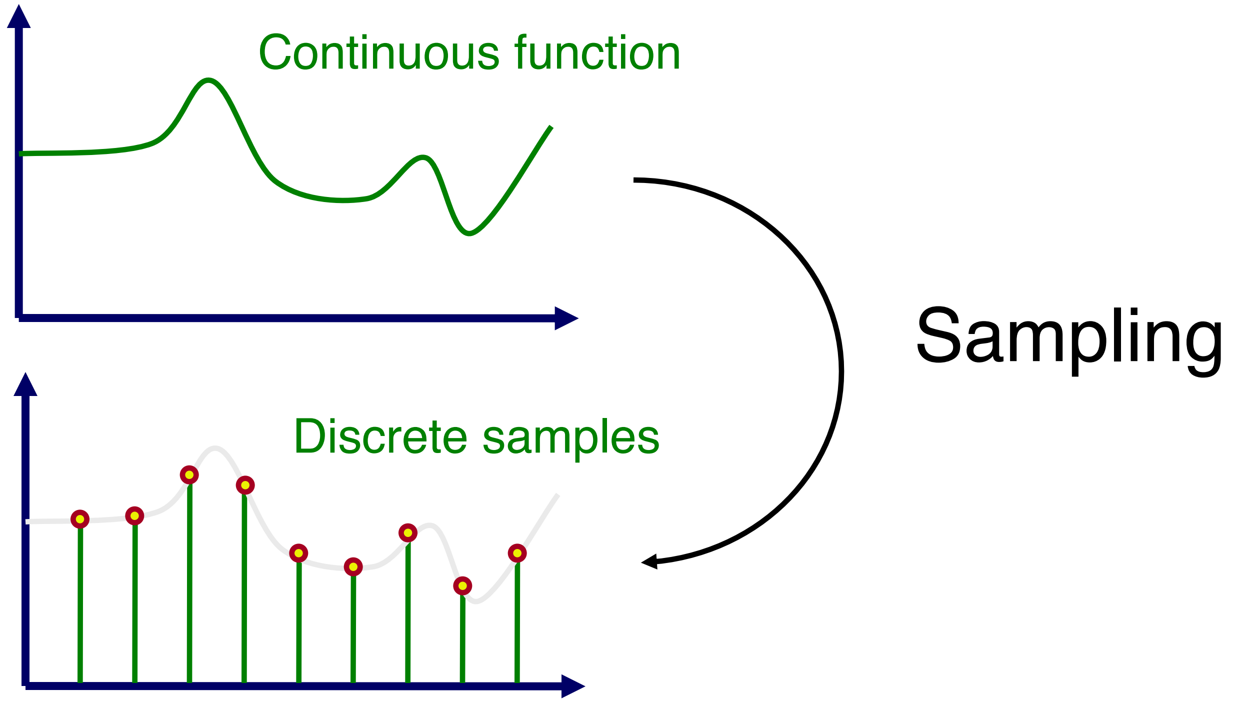
Source image

Destination image

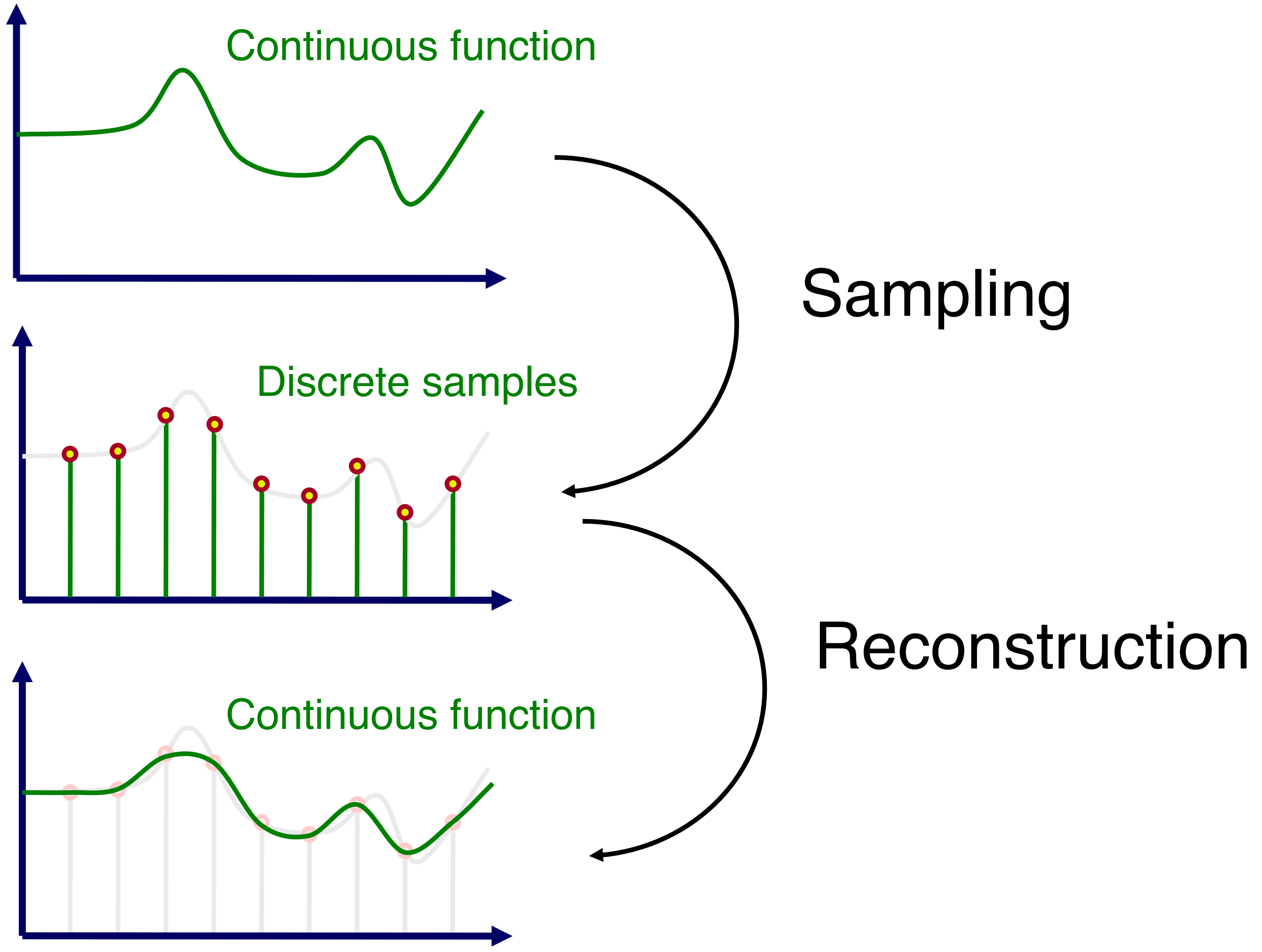
Digital Image Processing

When implementing operations that move pixels, must account for the fact that digital images are sampled versions of continuous ones

Sampling and Reconstruction



Sampling and Reconstruction



Sampling and Reconstruction

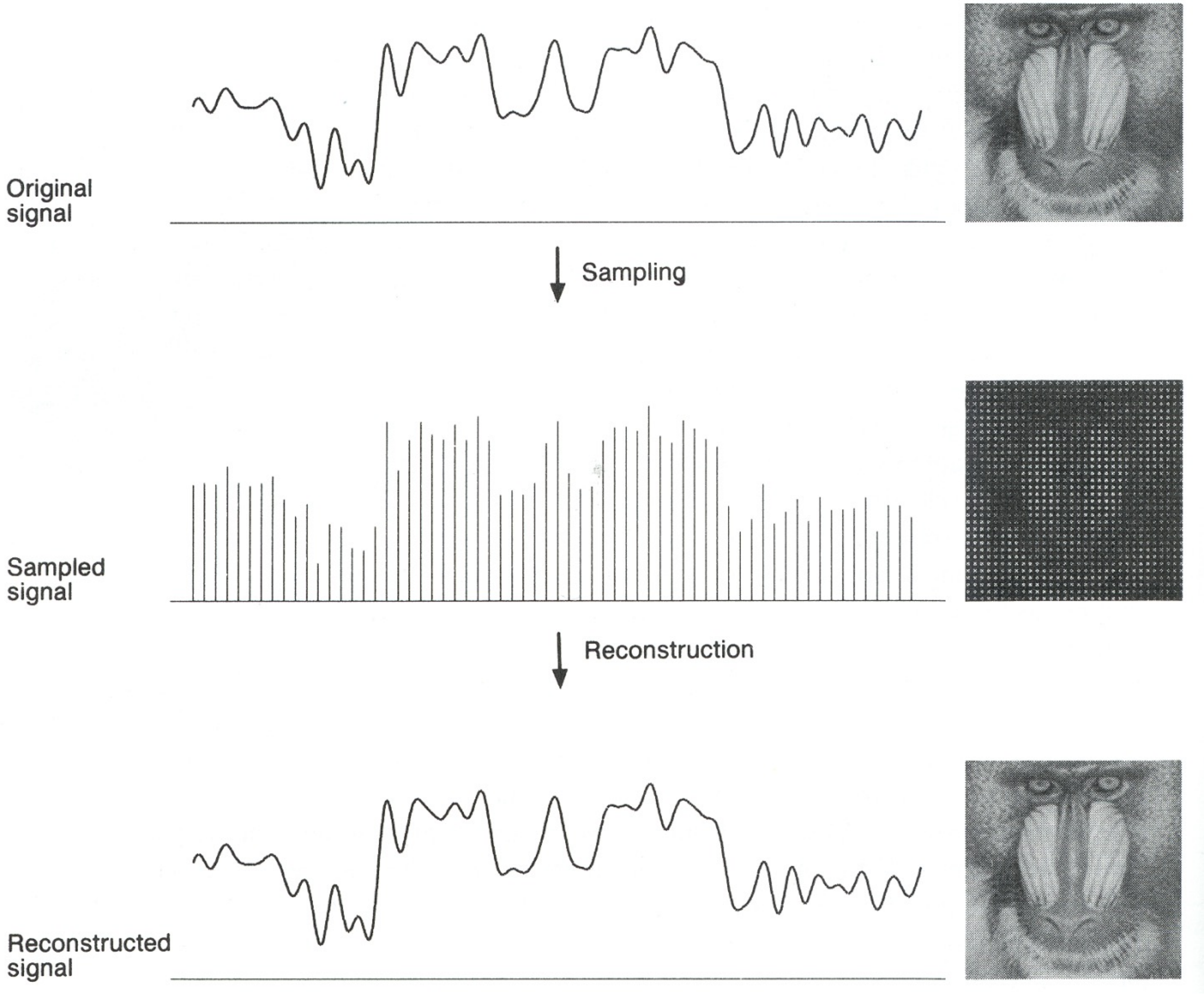
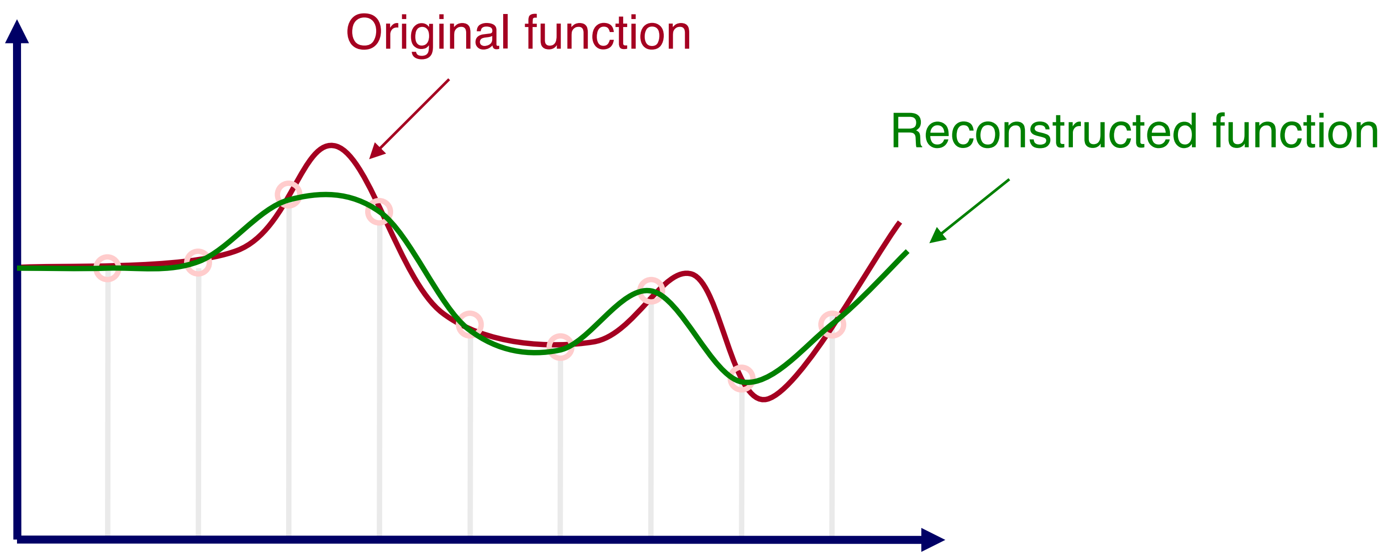


Figure 19.9 FvDFH

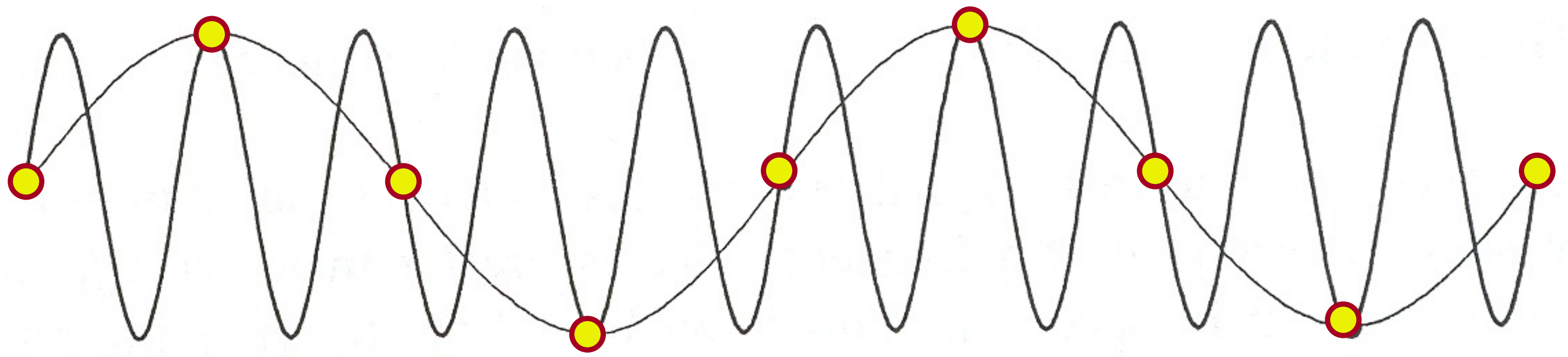
Sampling Theory

- How many samples are enough?
  - How many samples are required to represent a given signal without loss of information?
  - What signals can be reconstructed without loss for a given sampling rate?
- What happens when we use too few samples?



Sampling Theory

- What happens when we use too few samples?
  - **Aliasing:** high frequencies masquerade as low ones

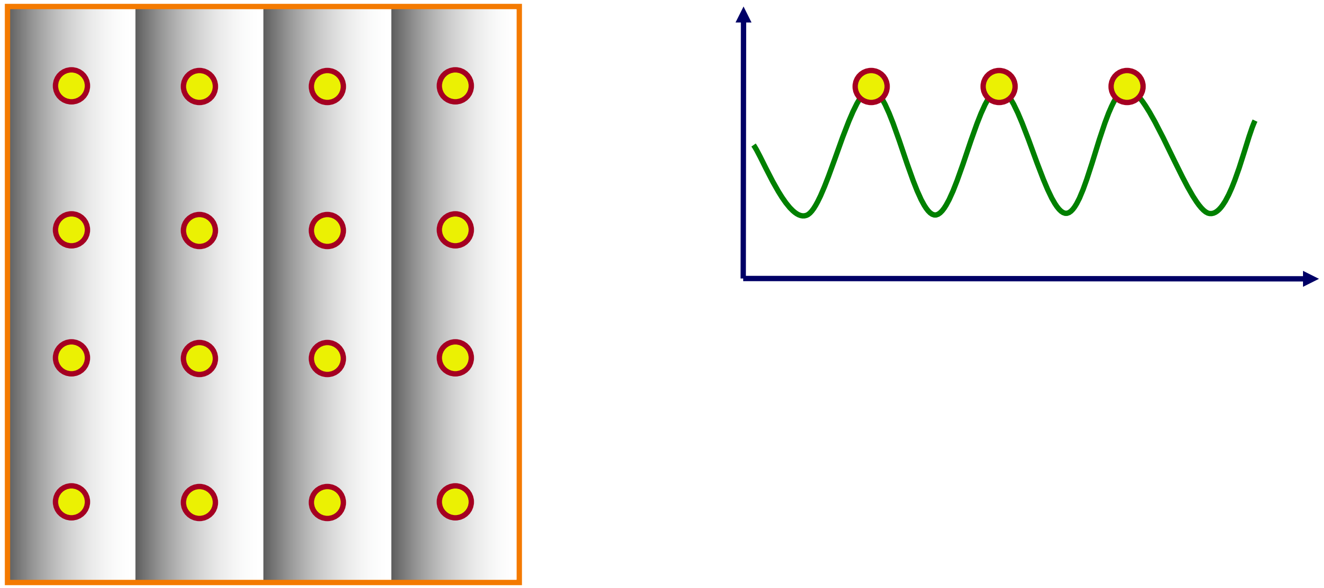


- Specifically, in graphics:
  - Spatial aliasing
  - Temporal aliasing

Figure 14.17 FvDFH

Spatial Aliasing

- Artifacts due to limited spatial resolution



Spatial Aliasing

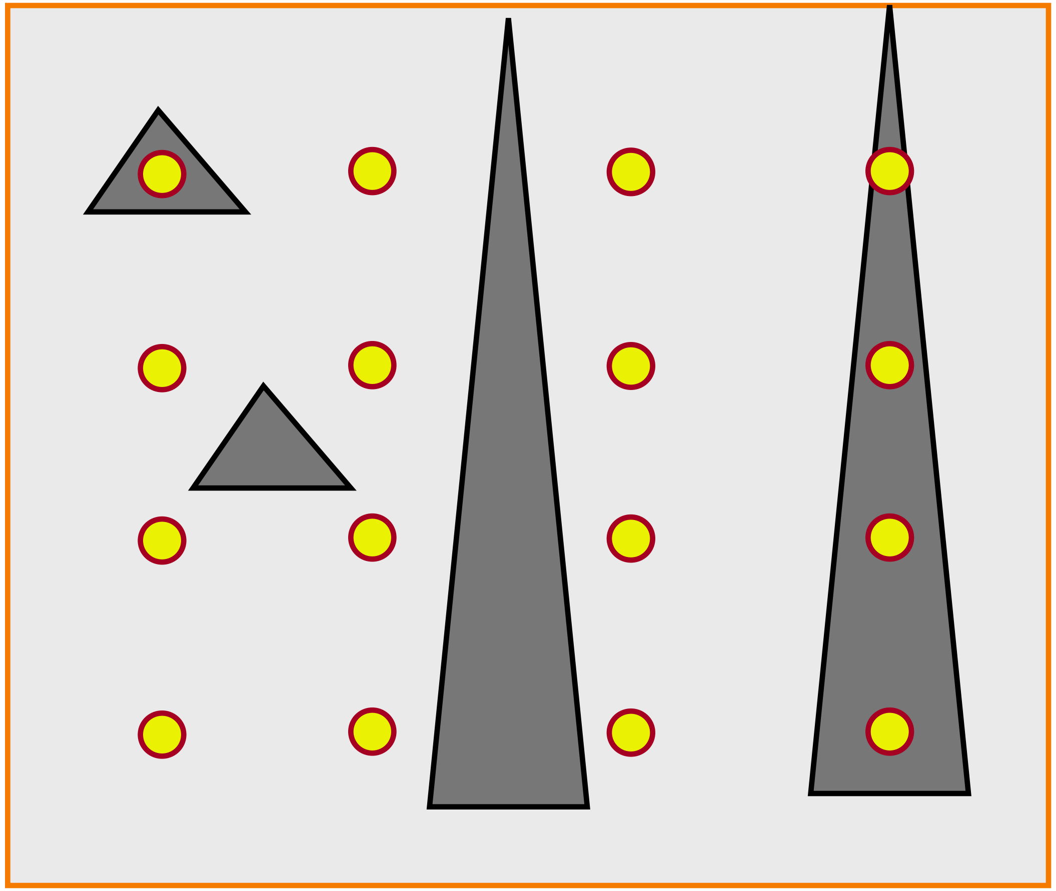
- Artifacts due to limited spatial resolution

(Barely) adequate sampling

Inadequate sampling

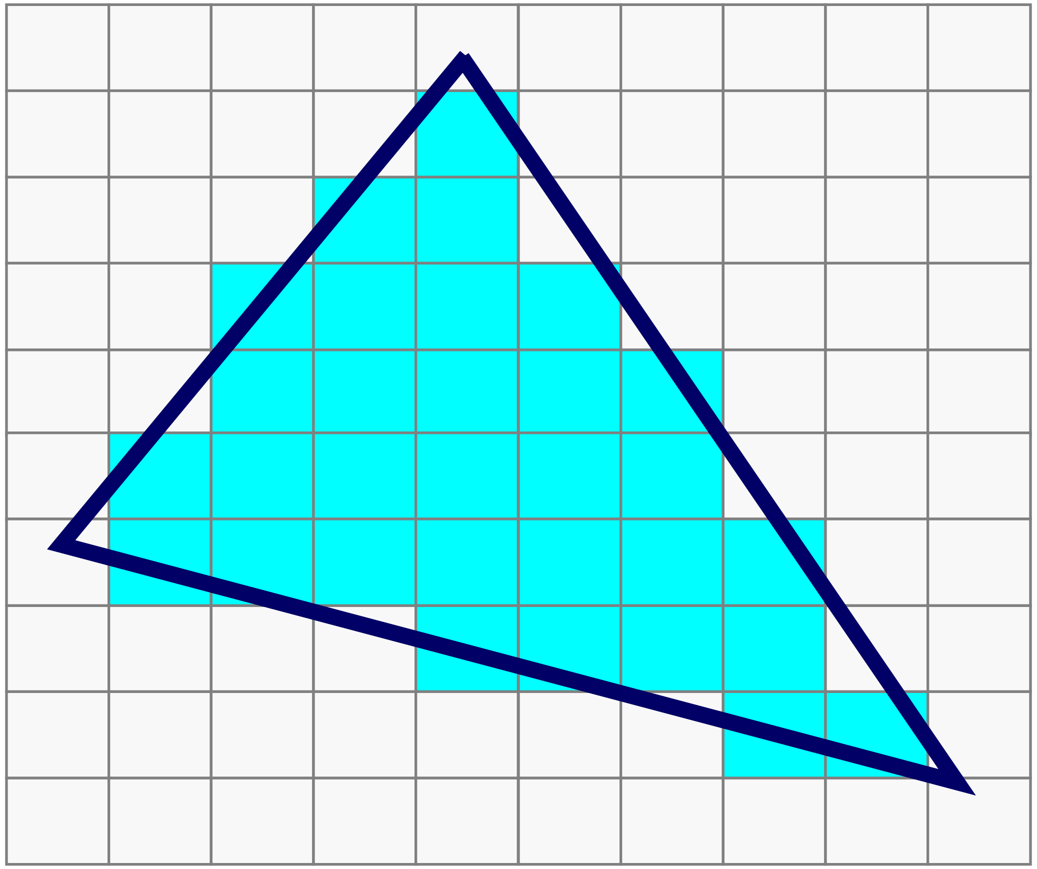
Spatial Aliasing

- Artifacts due to limited spatial resolution



Spatial Aliasing

- Artifacts due to limited spatial resolution



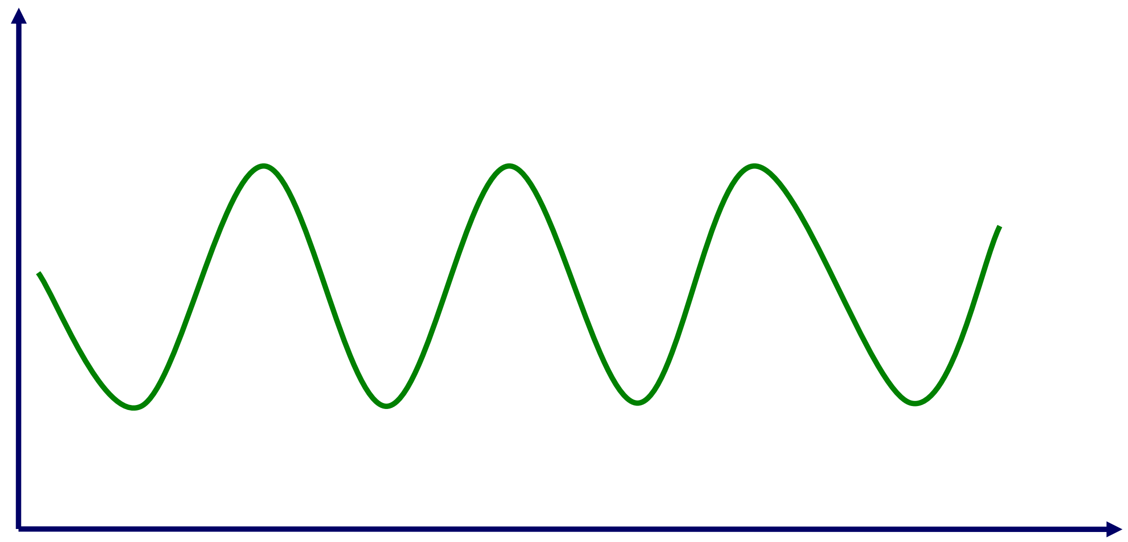
“Jaggies”

Temporal Aliasing

- Artifacts due to limited temporal resolution
  - Flickering
  - Strobing (“Backwards spinning wheel” effect)

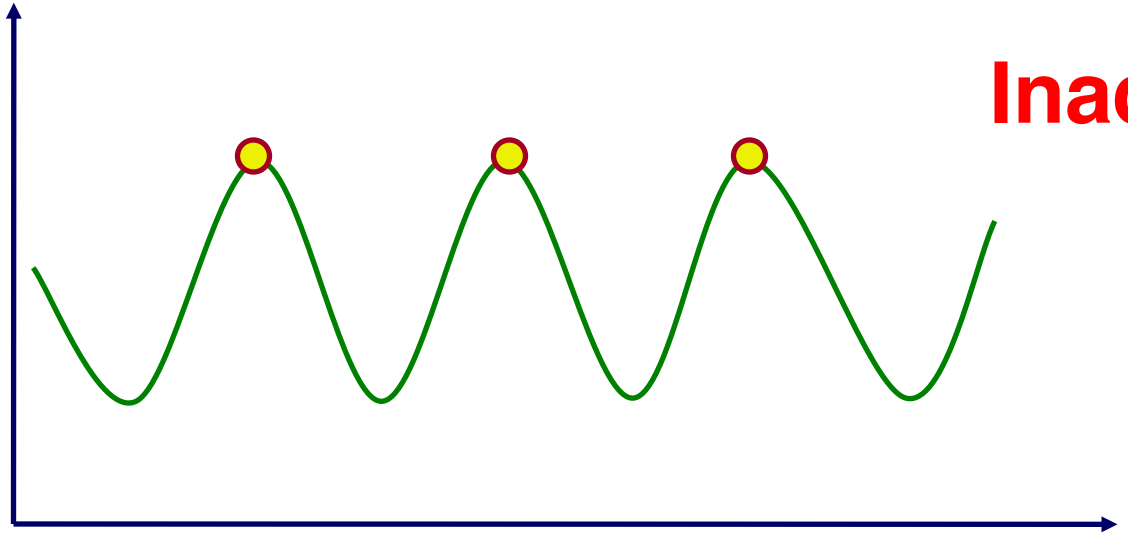
Sampling Theory

- How many samples are enough to avoid aliasing?
  - How many samples are required to represent a given signal without loss of information?
  - What signals can be reconstructed without loss for a given sampling rate?



Sampling Theory

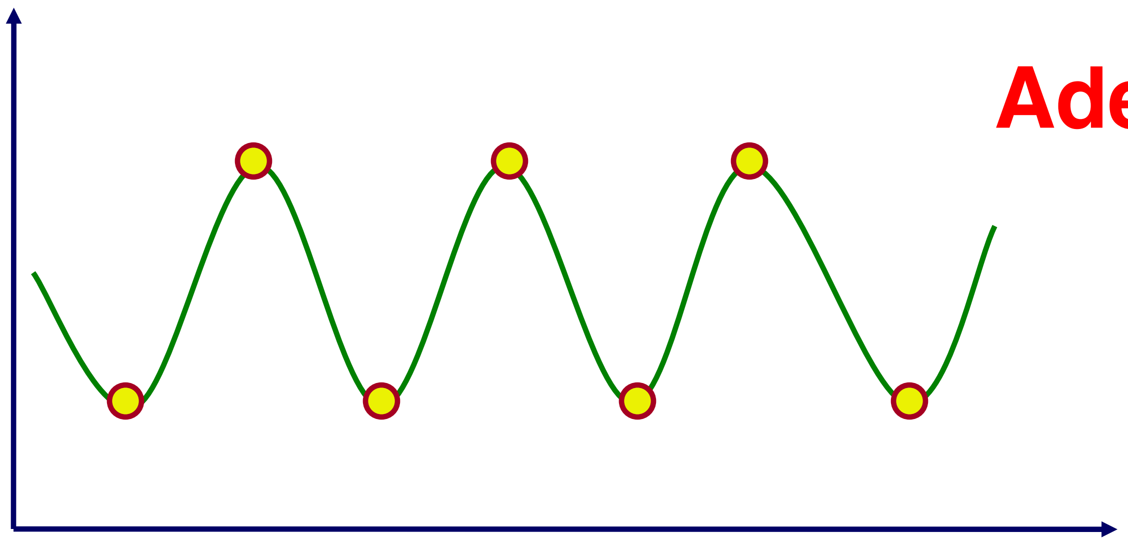
- How many samples are enough to avoid aliasing?
  - How many samples are required to represent a given signal without loss of information?
  - What signals can be reconstructed without loss for a given sampling rate?



Inadequate

Sampling Theory

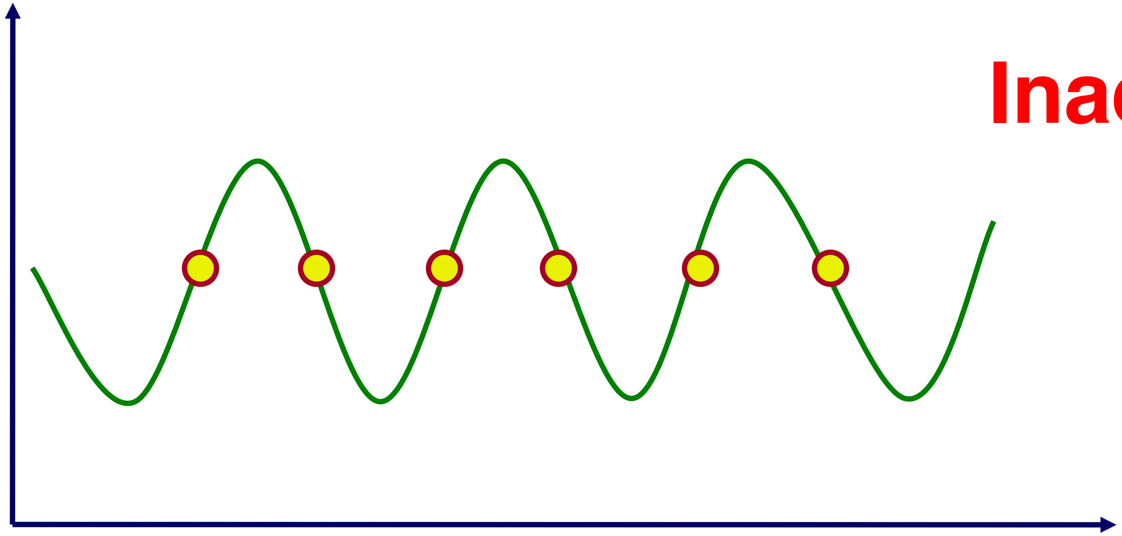
- How many samples are enough to avoid aliasing?
  - How many samples are required to represent a given signal without loss of information?
  - What signals can be reconstructed without loss for a given sampling rate?



Adequate?

Sampling Theory

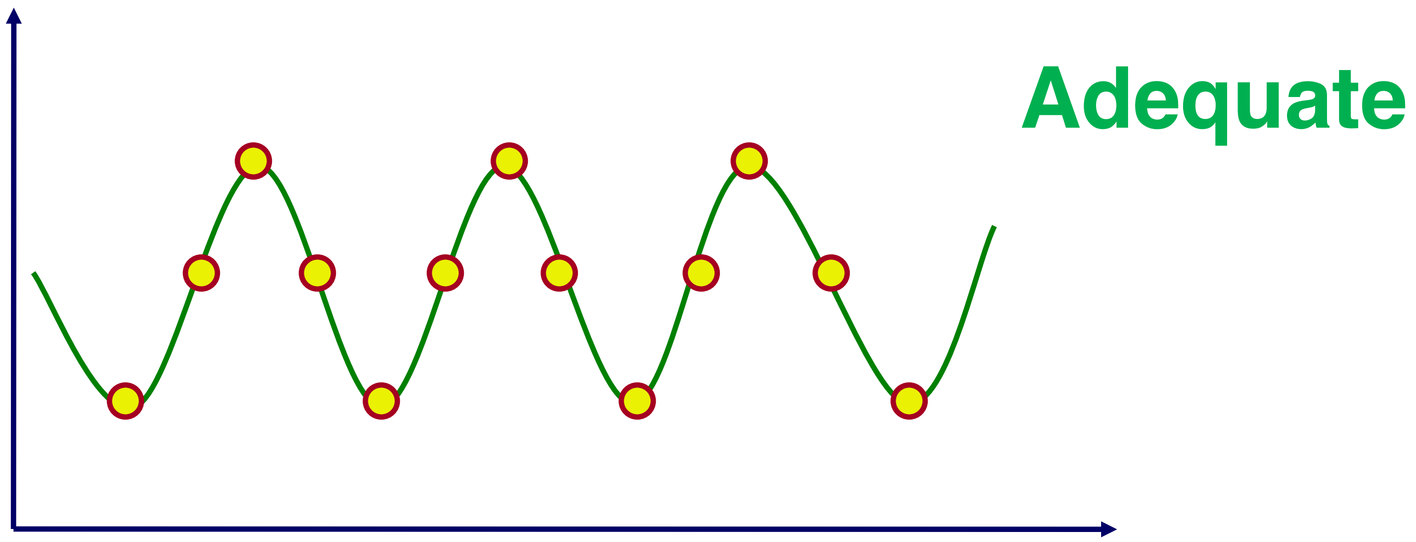
- How many samples are enough to avoid aliasing?
  - How many samples are required to represent a given signal without loss of information?
  - What signals can be reconstructed without loss for a given sampling rate?



Inadequate

Sampling Theory

- How many samples are enough to avoid aliasing?
  - How many samples are required to represent a given signal without loss of information?
  - What signals can be reconstructed without loss for a given sampling rate?



Sampling Theory

- How many samples are enough to avoid aliasing?
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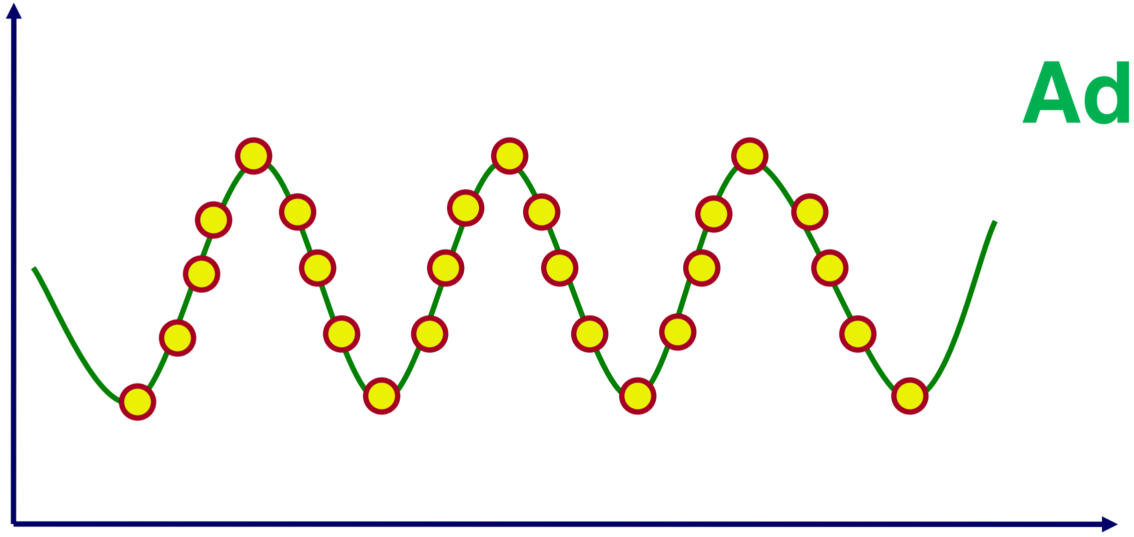
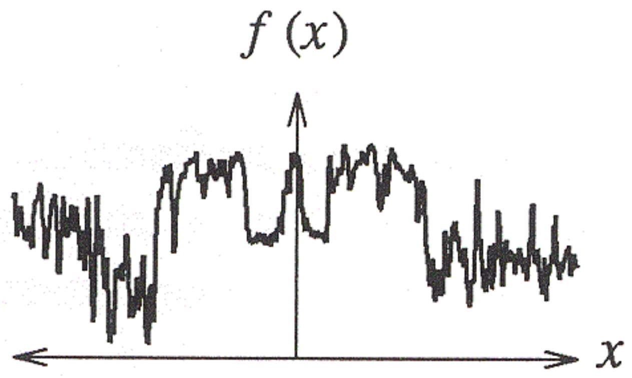
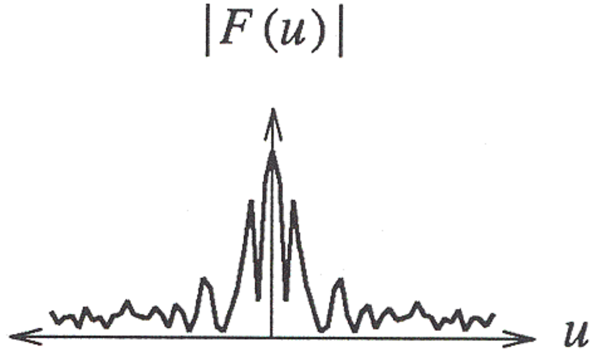


Adequate

Spectral Analysis

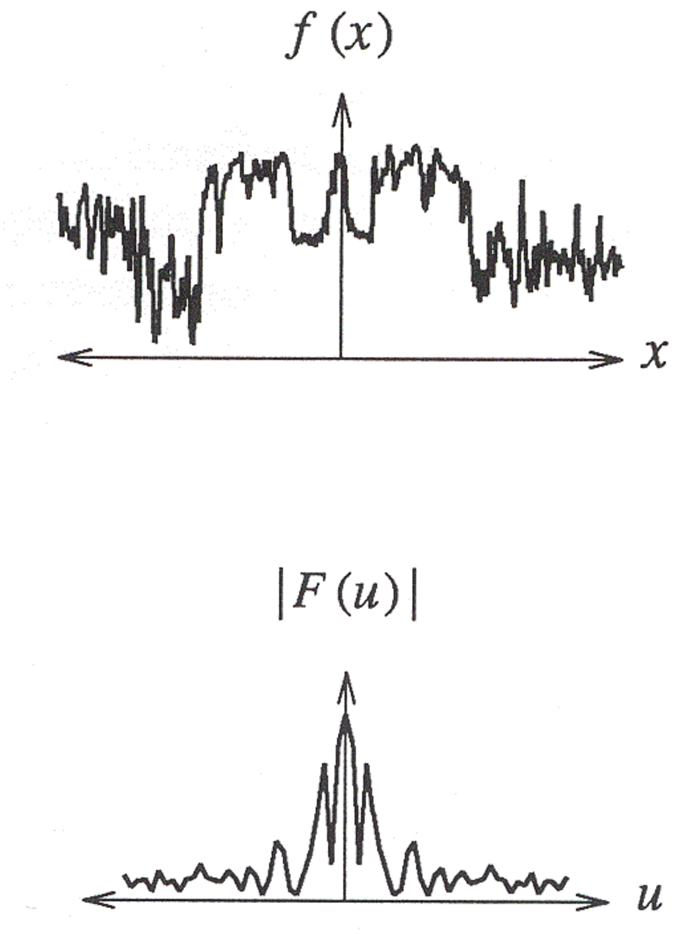
- Spatial domain:
  - Function: f(x)
  - Filtering: convolution
- Frequency domain
  - Function: F(u)
  - Filtering: multiplication





Any signal can be written as a sum of periodic functions.

Fourier Transform



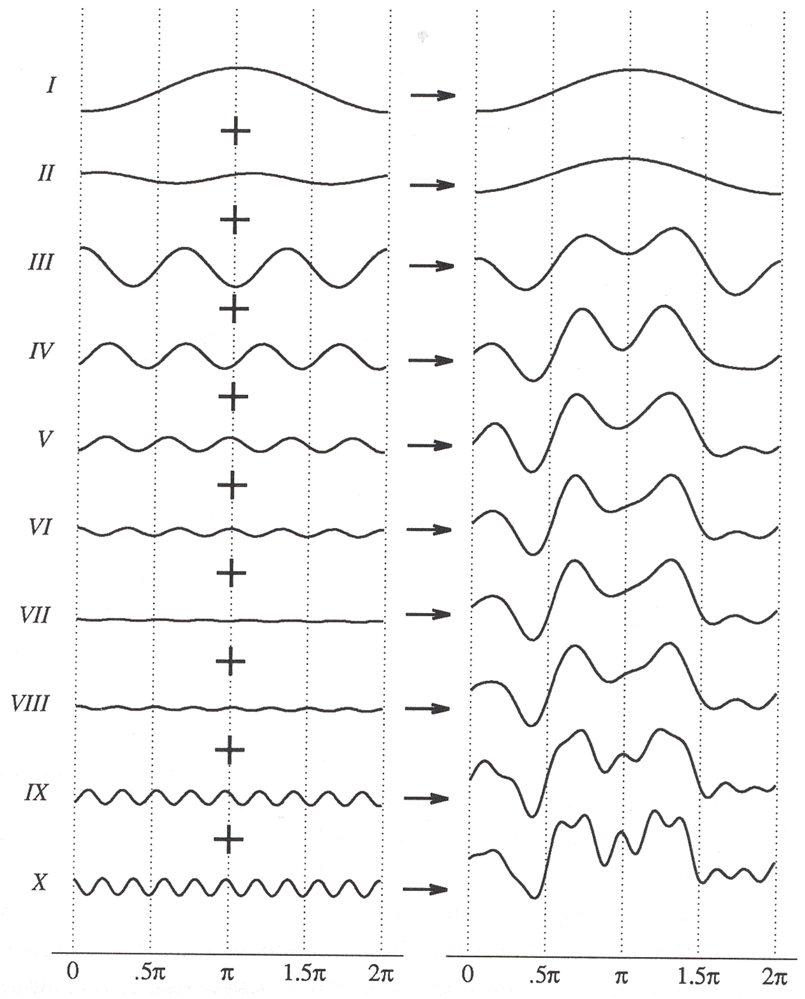


Figure 2.6 Wolberg

Fourier Transform

- Fourier transform:

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi xu} dx$$

- Inverse Fourier transform:

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{+i2\pi ux} du$$

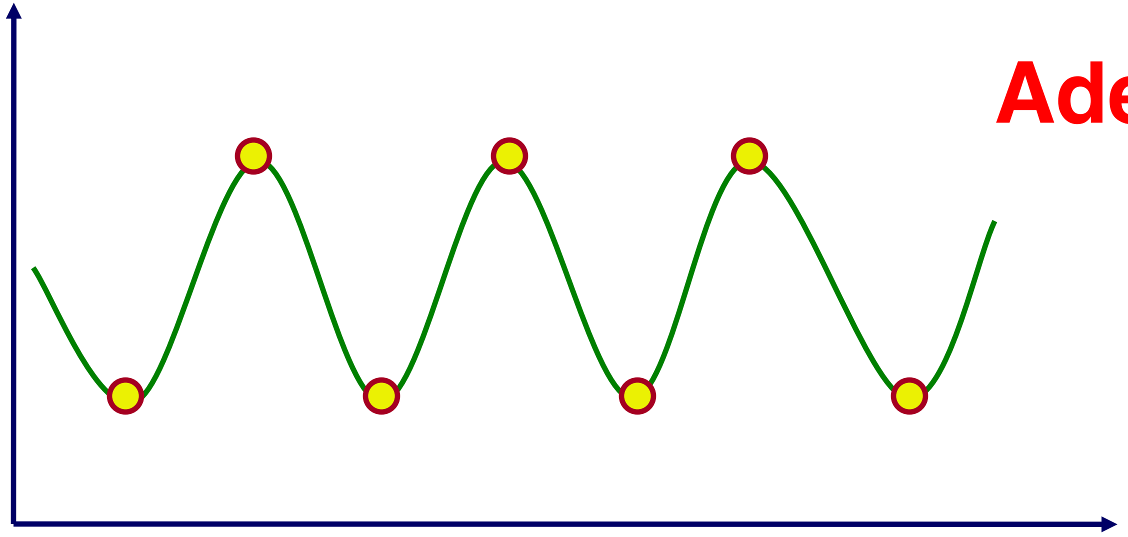
Sampling Theorem

- A signal can be reconstructed from its samples iff it has no content ≥ ½ the sampling frequency
  – Shannon
- The minimum sampling rate for a bandlimited function is called the "Nyquist rate"

A signal is *bandlimited* if its highest frequency is bounded. That frequency is called the bandwidth.

Why >?

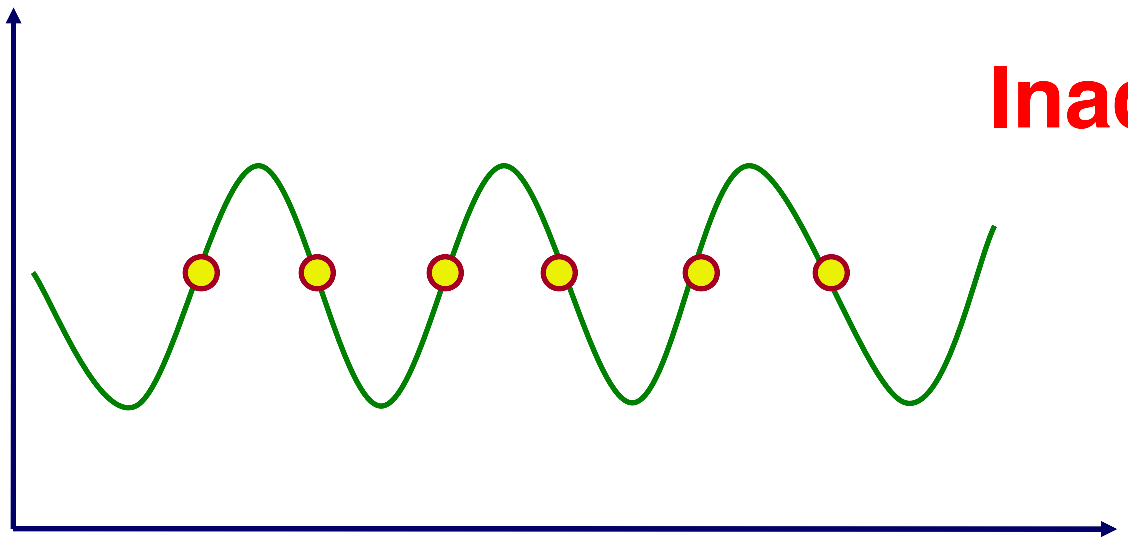
- Sampling rate must be > 2 bandwidth



Adequate?

Why >?

- Sampling rate must be > 2 bandwidth



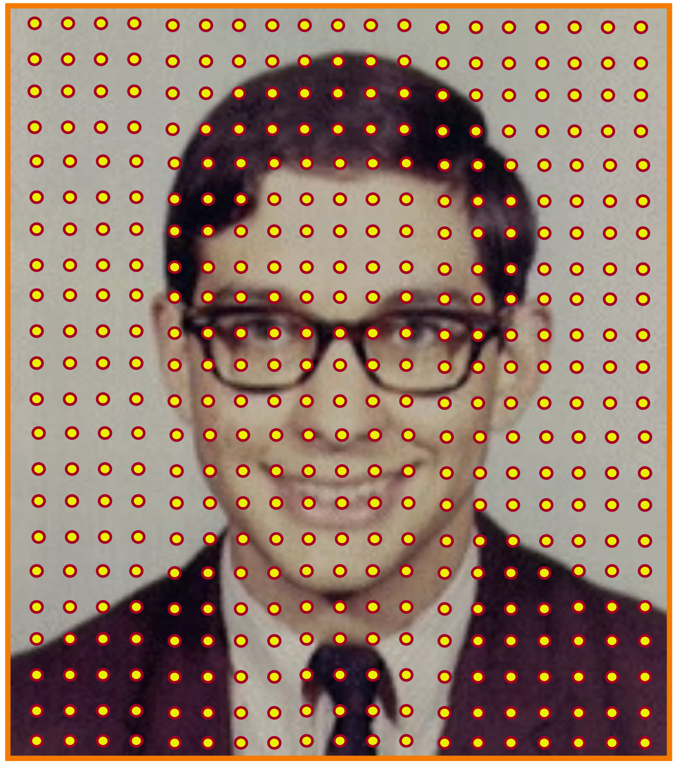
Inadequate

Antialiasing

- Sample at higher rate
  - Not always possible
  - Doesn't always solve the problem
- Pre-filter to form bandlimited signal
  - Use low-pass filter to limit signal to < 1/2 sampling rate
  - **Trades blurring for aliasing**

Image Processing

- Consider scaling the image (or, equivalently, reducing resolution)



Original image

1/4 resolution

Image Processing

↓ Real world

[Sample]

↓ Discrete samples (pixels)

[Reconstruct]

↓ Reconstructed function

[Transform]

↓ Transformed function

[Filter]

↓ Bandlimited function

[Sample]

↓ Discrete samples (pixels)

[Reconstruct]

↓ Display

Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

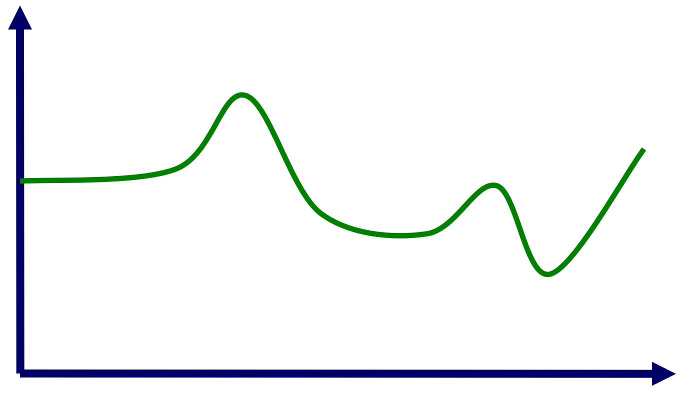
Bandlimited function

Sample

Discrete samples (pixels)

Reconstruct

Display



Continuous Function

Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

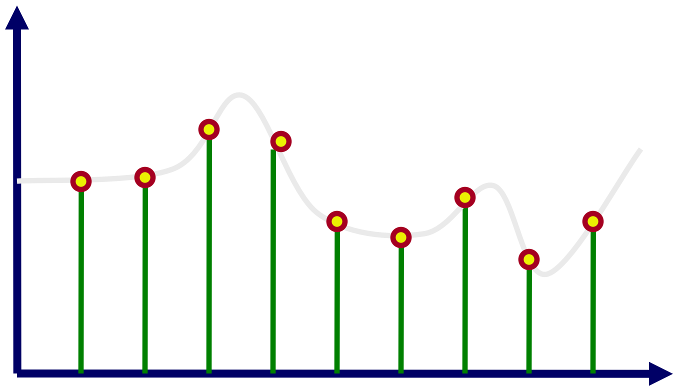
Bandlimited function

Sample

Discrete samples (pixels)

Reconstruct

Display



Discrete Samples

Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

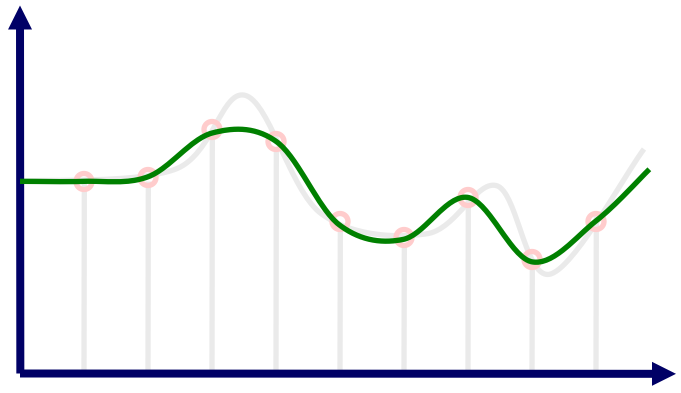
Bandlimited function

Sample

Discrete samples (pixels)

Reconstruct

Display



Reconstructed Function

Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

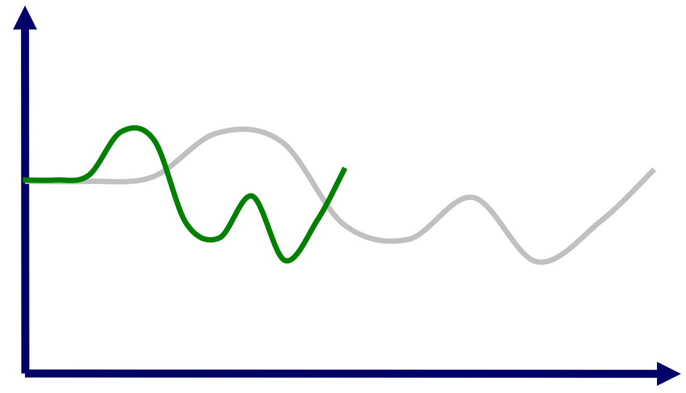
Bandlimited function

Sample

Discrete samples (pixels)

Reconstruct

Display



Transformed Function

Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

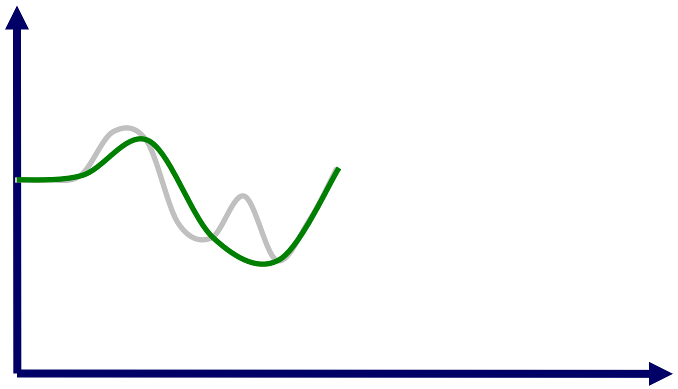
Bandlimited function

Sample

Discrete samples (pixels)

Reconstruct

Display



Bandlimited Function

Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

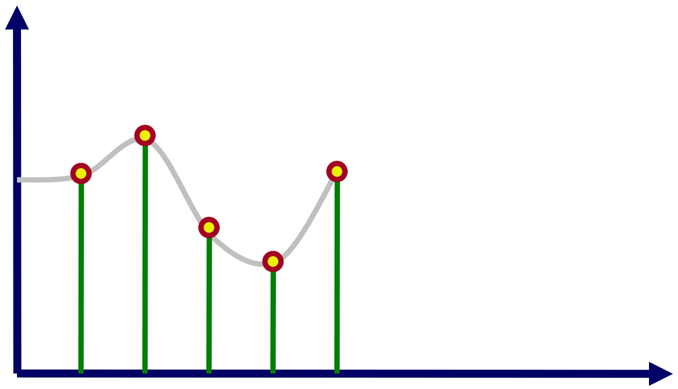
Bandlimited function

Sample

Discrete samples (pixels)

Reconstruct

Display



Discrete samples

Image Processing

Real world

Sample

Discrete samples (pixels)

Reconstruct

Reconstructed function

Transform

Transformed function

Filter

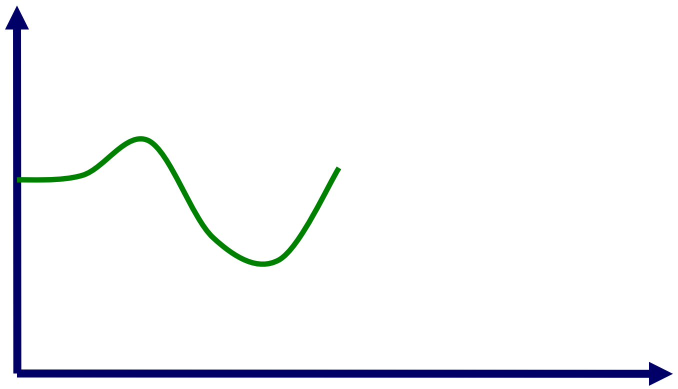
Bandlimited function

Sample

Discrete samples (pixels)

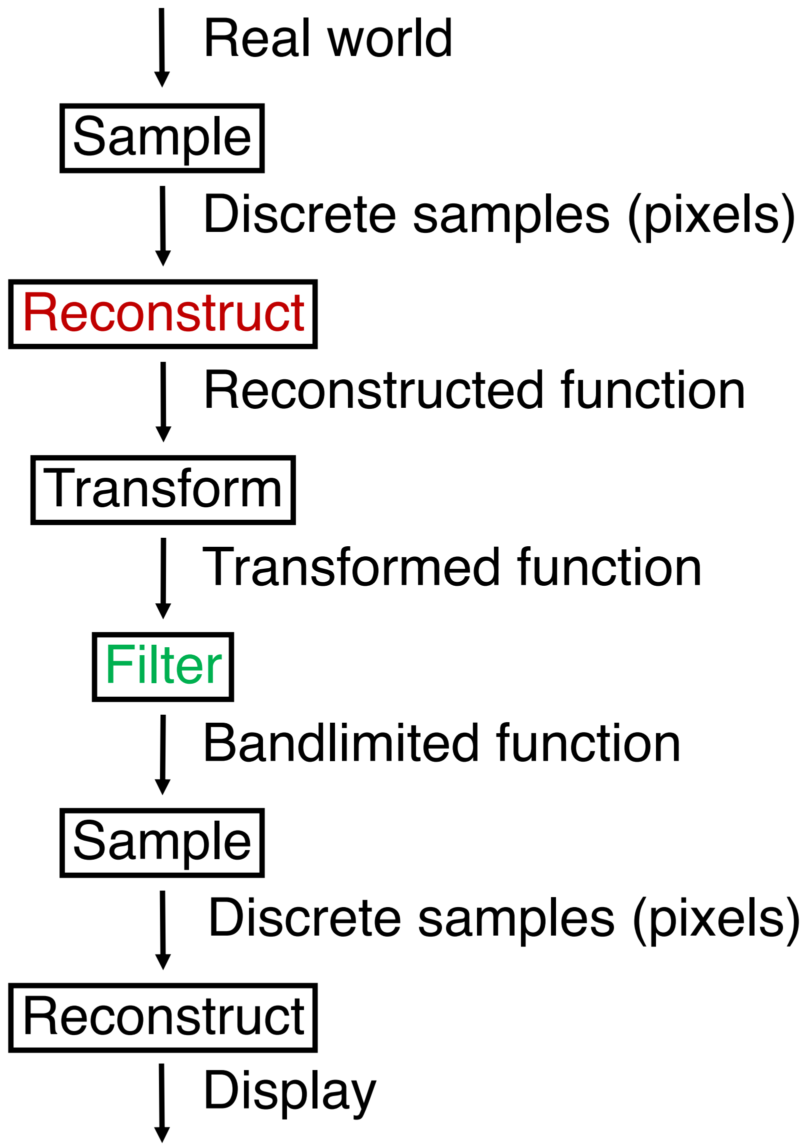
Reconstruct

Display



Display

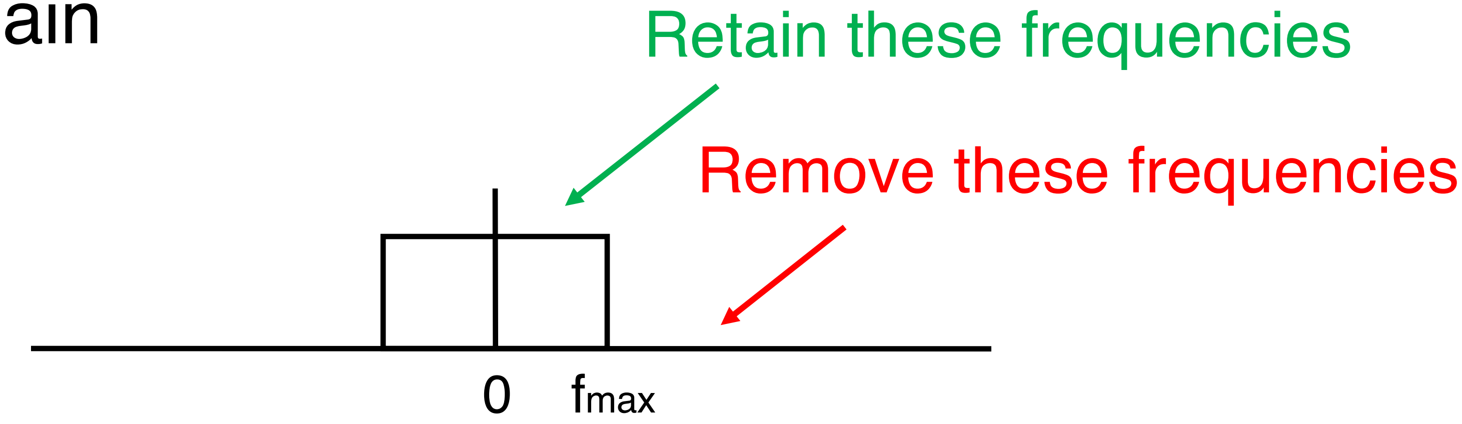
Image Processing



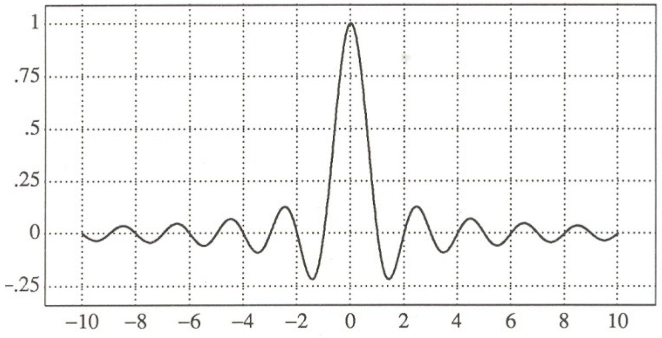
- Ideal resampling requires correct filtering to avoid artifacts
- Reconstruction filter especially important when magnifying
- Bandlimiting filter especially important when minifying

Ideal Image Processing Filter

- Frequency domain (multiplication)



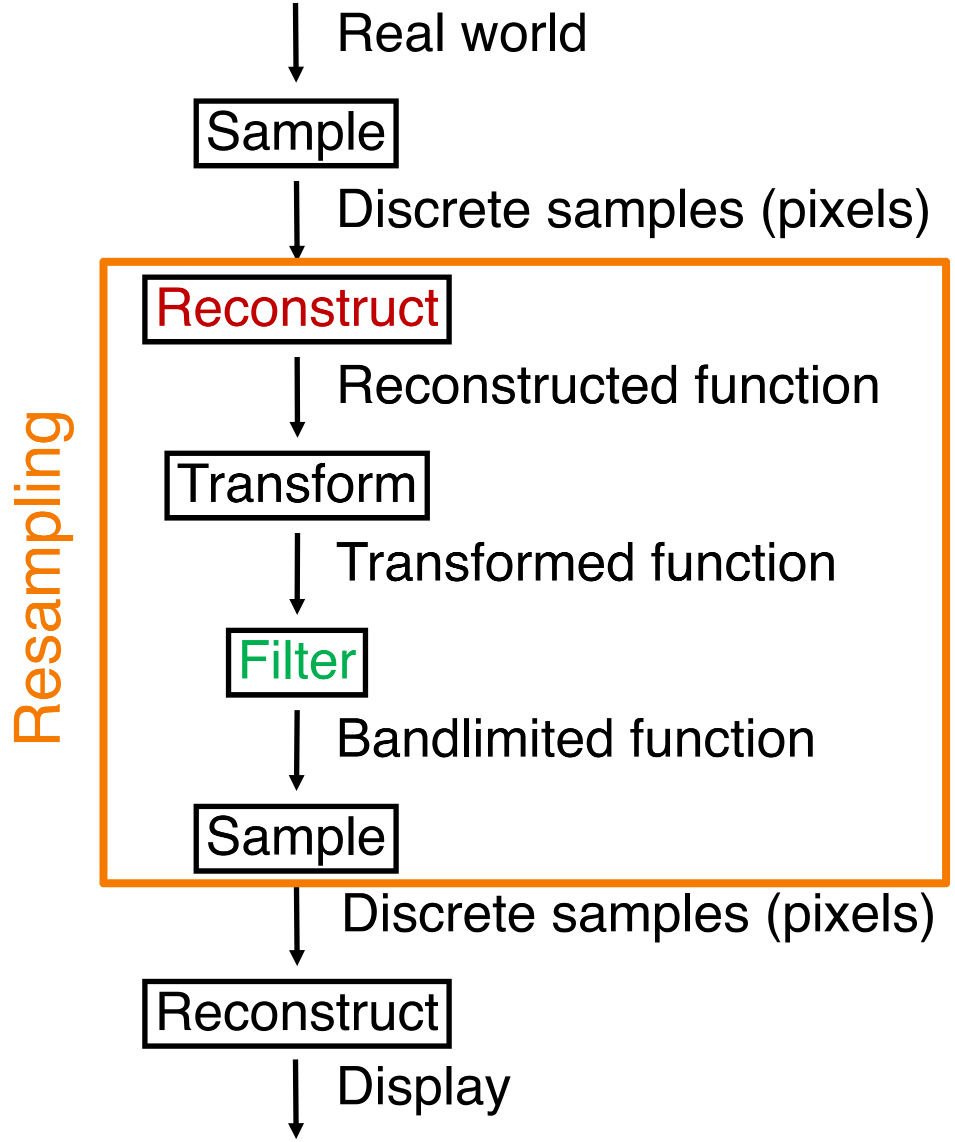
- Spatial domain (convolution)



$$Sinc(x) = \frac{\sin \pi x}{\pi x}$$

Figure 4.5 Wolberg

Practical Image Processing

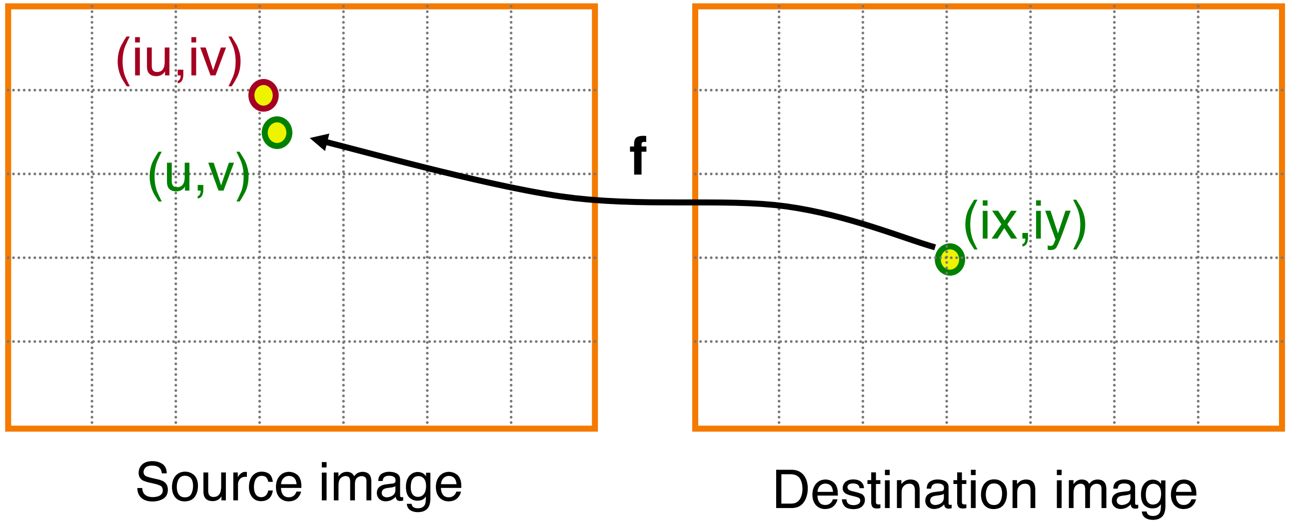


- Resampling: effectively (discrete) convolution to prevent artifacts
- Finite low-pass filters
  - Point sampling (bad)
  - Box filter
  - Triangle filter
  - Gaussian filter

Point Sampling

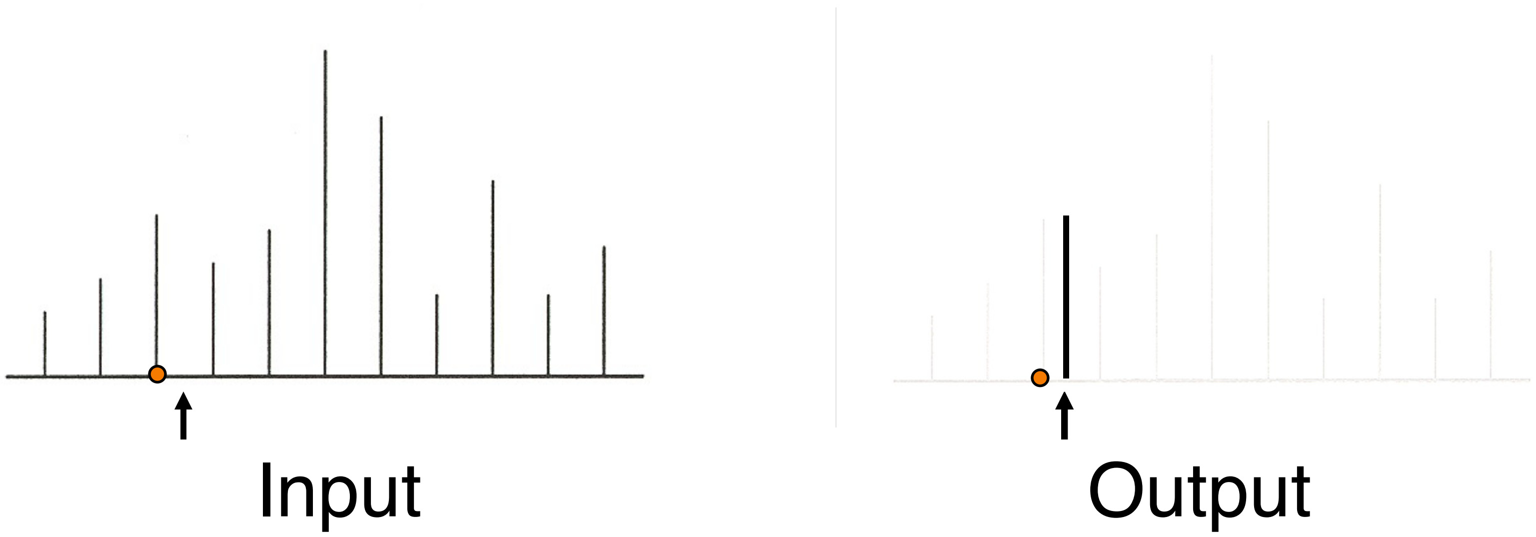
- Possible (poor) resampling implementation:

```
float Resample(src, u, v, k, w) {
   int iu = round(u);
   int iv = round(v);
   return src(iu,iv);
}
```



Point Sampling

- Use nearest sample



Point Sampling

Point Sampled: Aliasing!

Correctly Bandlimited

Resampling with Filter

- Output is weighted average of inputs:

```
float Resample(src, u, v, k, w)
{
  float dst = 0;
  float ksum = 0;
  int ulo = u - w; etc.
  for (int iu = ulo; iu < uhi; iu++) {
    for (int iv = vlo; iv < vhi; iv++) {
      dst += k(u,v,iu,iv,w) * src(u,v);
      ksum += k(u,v,iu,iv,w);
    }
  }
  return dst / ksum;
}
```

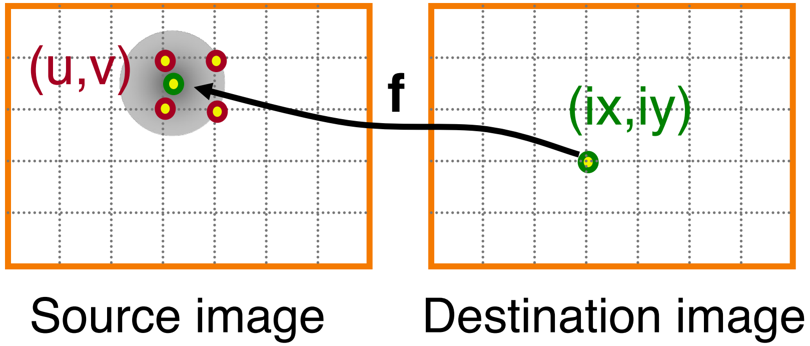
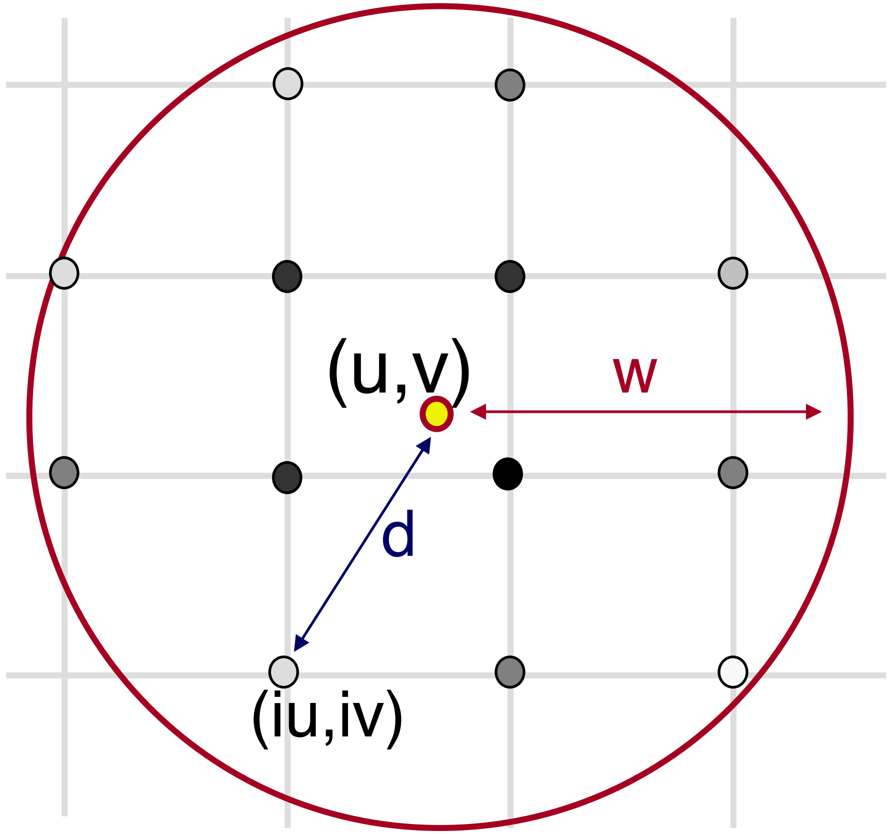


Image Resampling

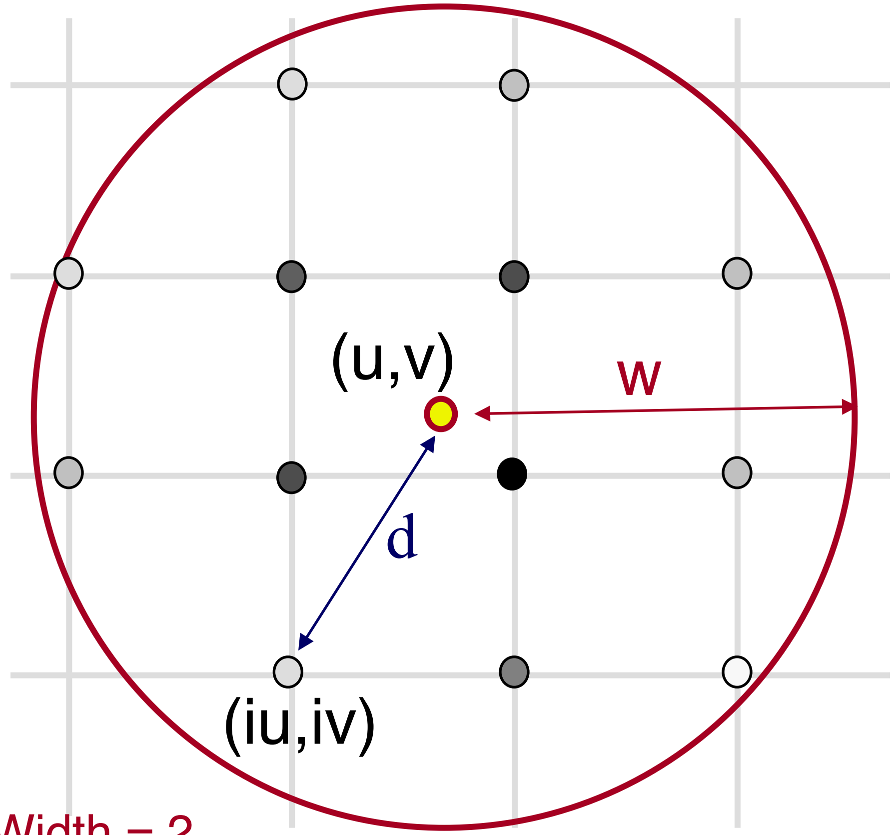
- Compute weighted sum of pixel neighborhood
  - Output is weighted average of input, where weights are normalized values of filter kernel (k)



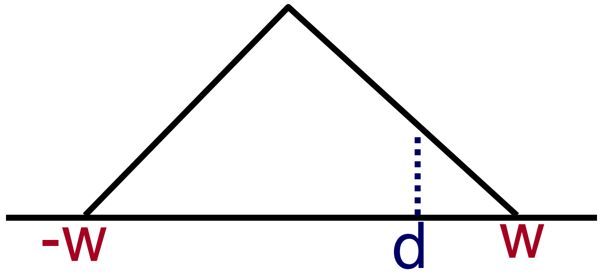
k(ix,iy) represented by gray value

Image Resampling

- For isotropic Triangle and Gaussian filters, k(ix,iy) is function of d and w



Filter Width = 2

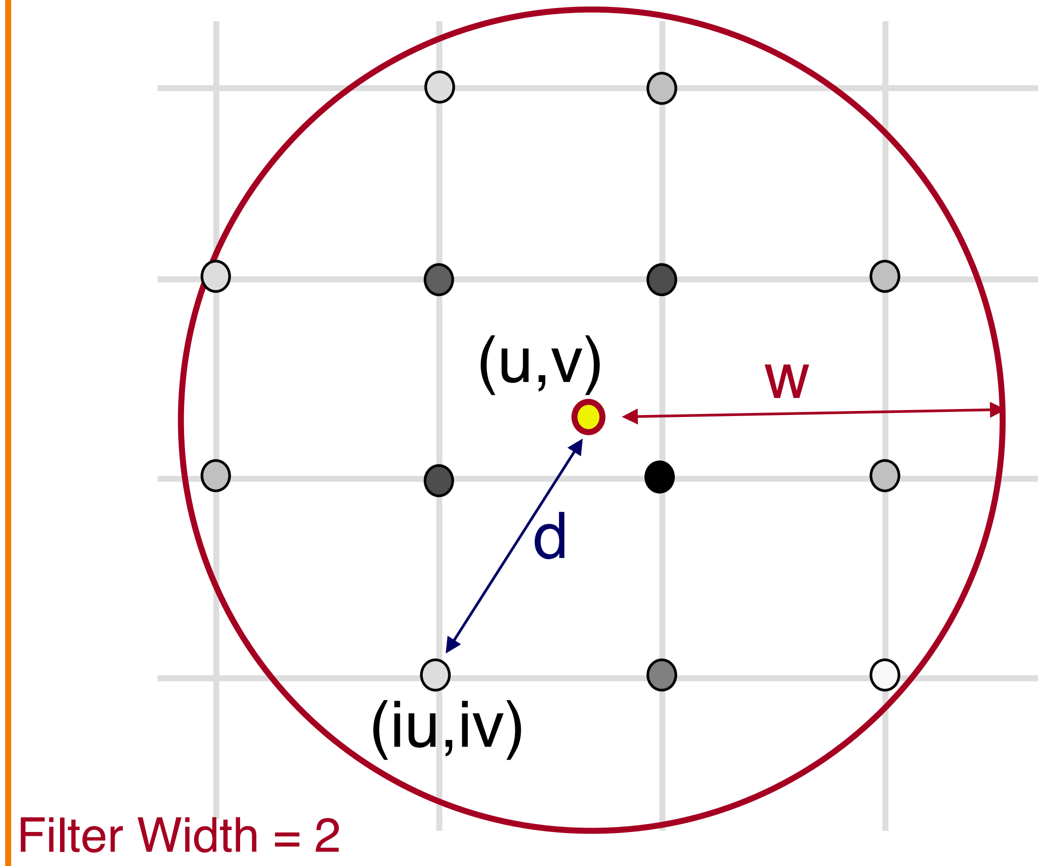


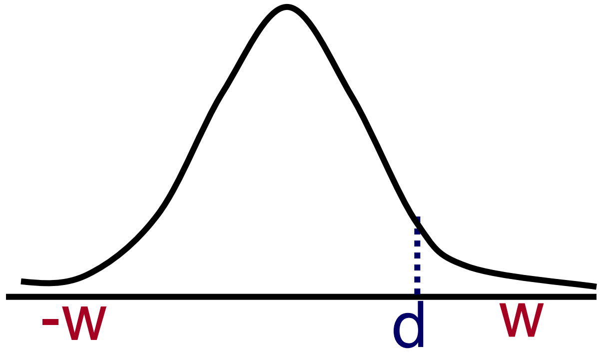
Triangle filter

k(i,j)=max(1 - d/w, 0)

Image Resampling

- For isotropic Triangle and Gaussian filters, k(ix,iy) is function of d and w





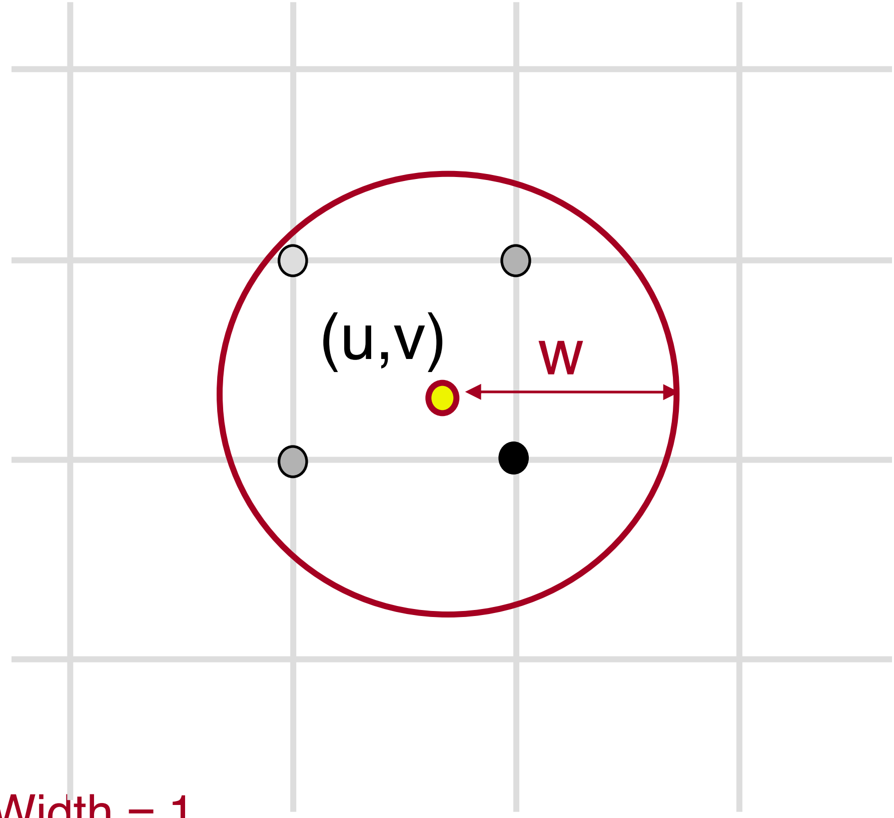
Gaussian filter

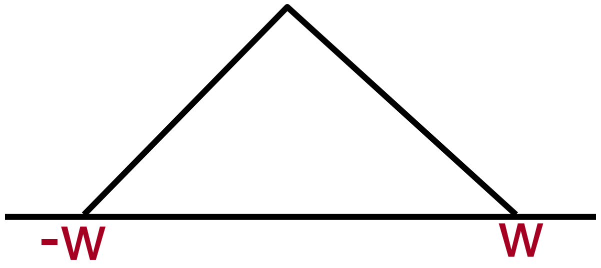
$$G(d,\sigma) = e^{-d^2/(2\sigma^2)}$$

- Drops off quickly, but never gets to exactly 0
- In practice: compute out to w ~ 2.5σ or 3σ

Image Resampling

- Filter width chosen based on scale factor (or blur)





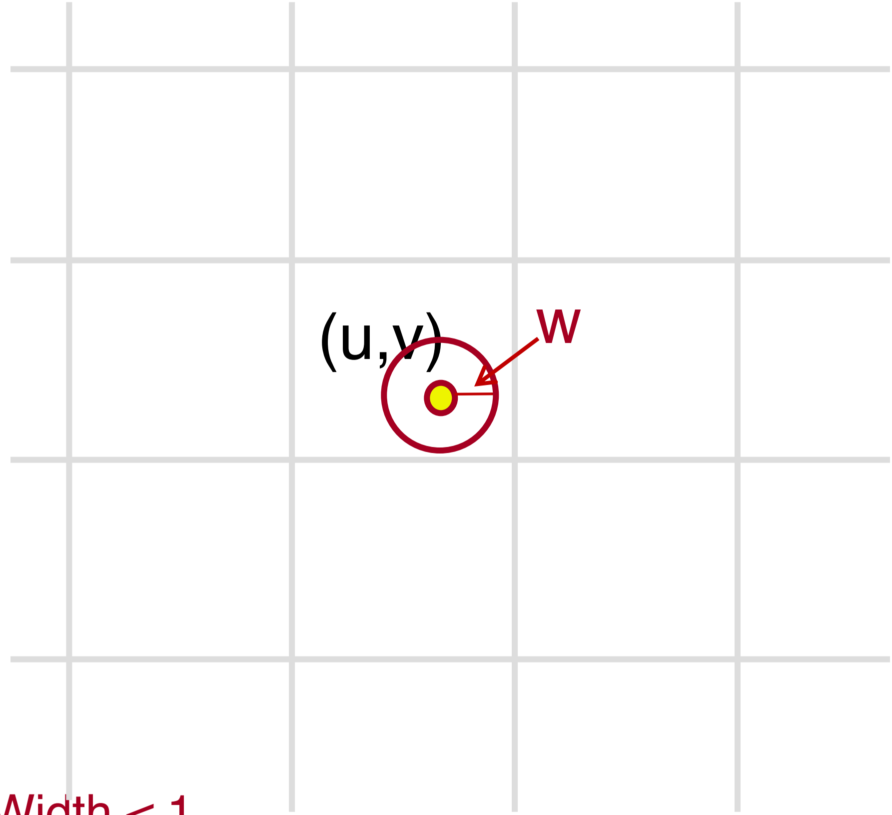
Triangle filter

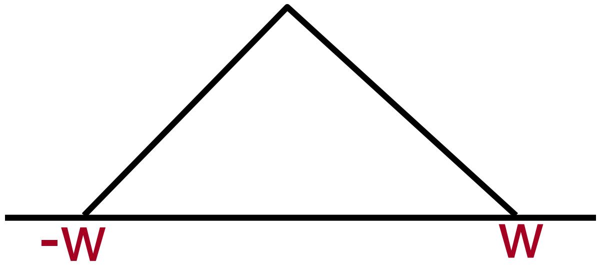
Width of filter affects blurriness

Filter Width = 1

Image Resampling

- What if width (w) is smaller than sample spacing?



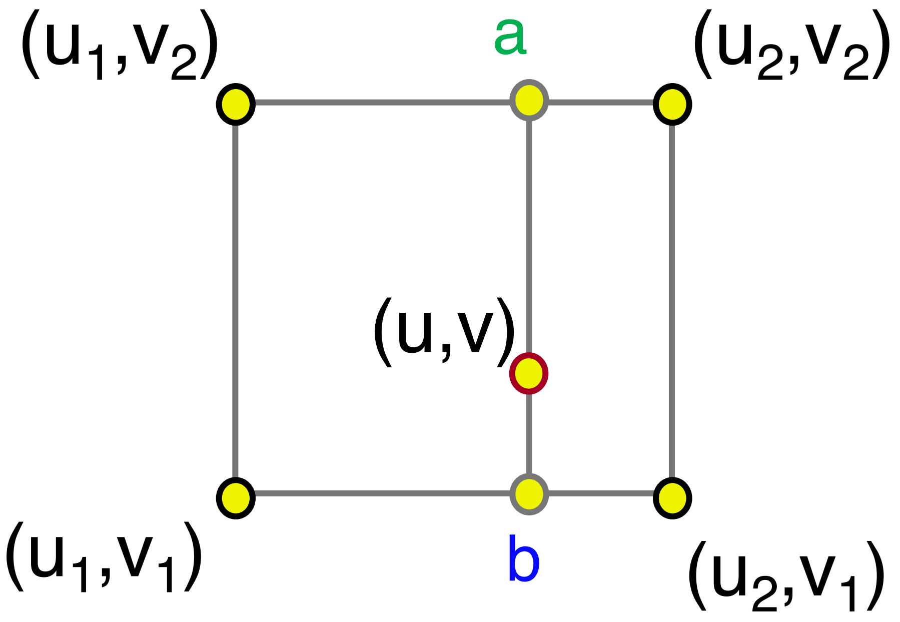


Triangle filter

Filter Width < 1

Image Resampling (with width < 1)

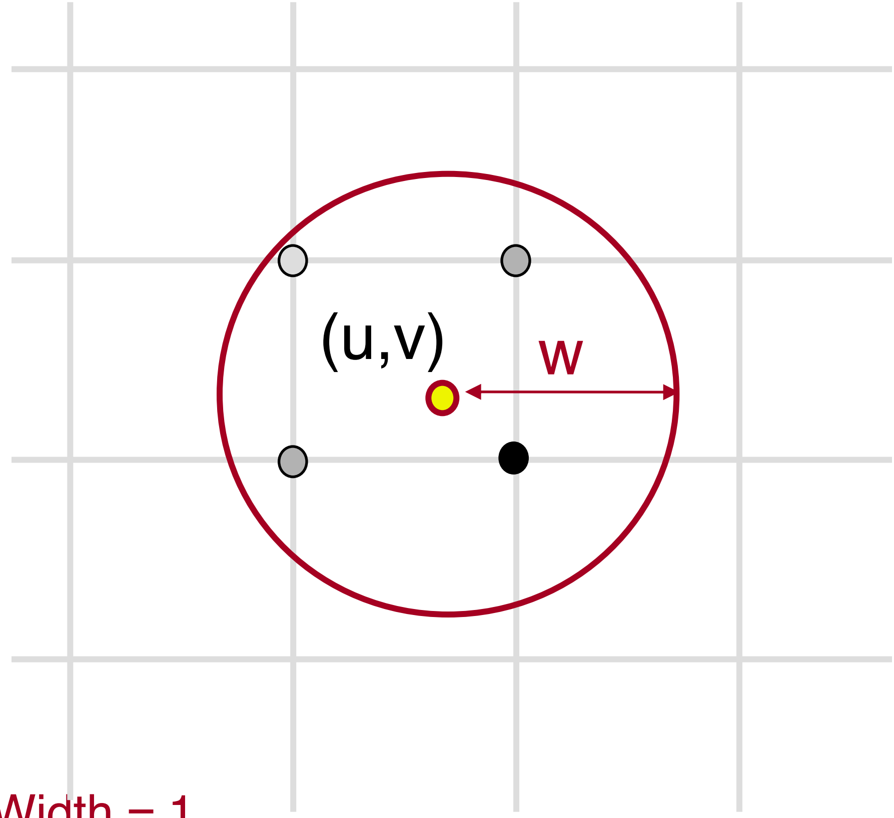
- Reconstruction filter: Bilinearly interpolate four closest pixels
  - a = linear interpolation of src(u_1,v_2) and src(u_2,v_2)
  - b = linear interpolation of src(u_1,v_1) and src(u_2,v_1)
  - dst(x,y) = linear interpolation of "a" and "b"

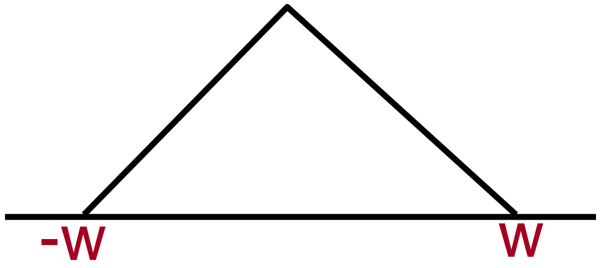


Filter Width < 1

Image Resampling (with width < 1)

- Alternative: force width to be at least 1





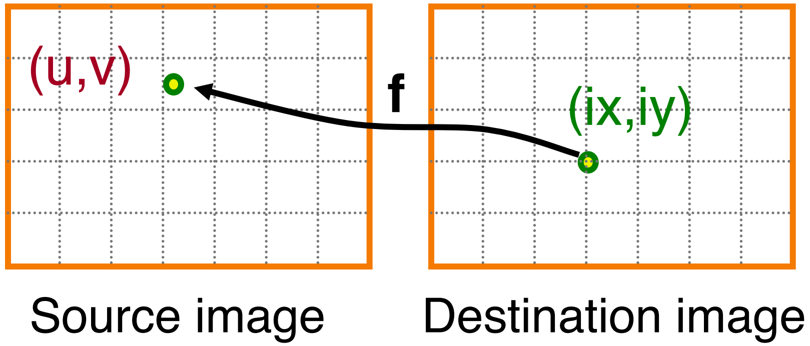
Triangle filter

Filter Width = 1

Putting it All Together

- Possible implementation of image scale:

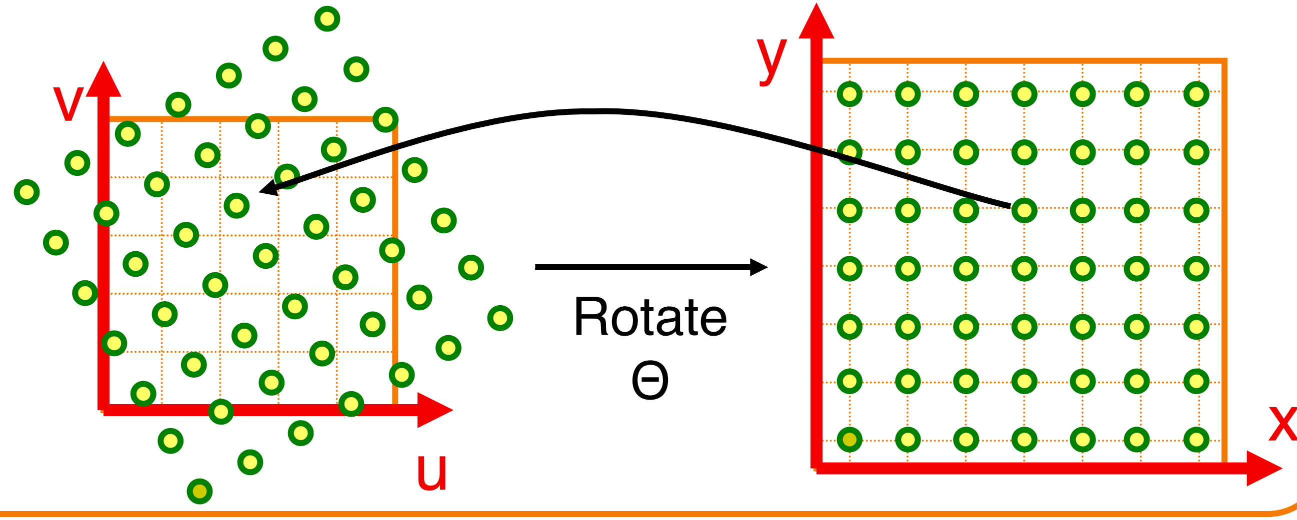
```
Scale(src, dst, sx, sy) {
  w ≈ max(1/sx,1/sy,1);
  for (int ix = 0; ix < xmax; ix++) {
    for (int iy = 0; iy < ymax; iy++) {
      float u = ix / sx;
      float v = iy / sy;
      dst(ix,iy) = Resample(src,u,v,k,w);
    }
  }
}
```



Putting it All Together

- Possible implementation of image rotation:

```
Rotate(src, dst, Θ) {
  w ≈ 1;
  for (int ix = 0; ix < xmax; ix++) {
    for (int iy = 0; iy < ymax; iy++) {
      float u = ix*cos(-Θ) – iy*sin(-Θ);
      float v = ix*sin(-Θ) + iy*cos(-Θ);
      dst(ix,iy) = Resample(src,u,v,k,w);
    }
  }
}
```



Sampling Method Comparison

- Trade-offs
  - Aliasing versus blurring
  - Computation speed

Point

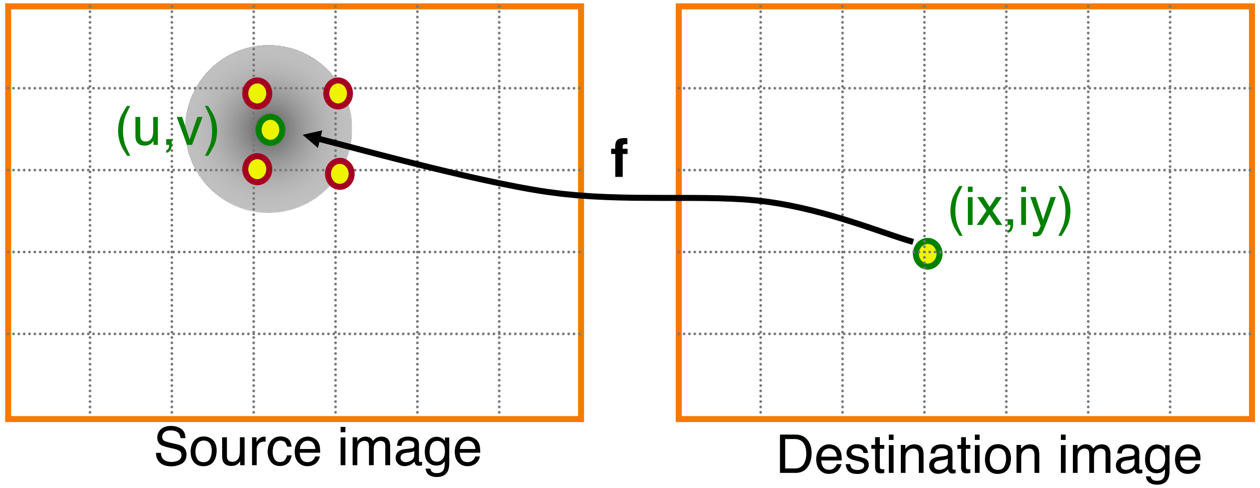
Triangle

Gaussian

Forward vs. Reverse Mapping

- Reverse mapping:

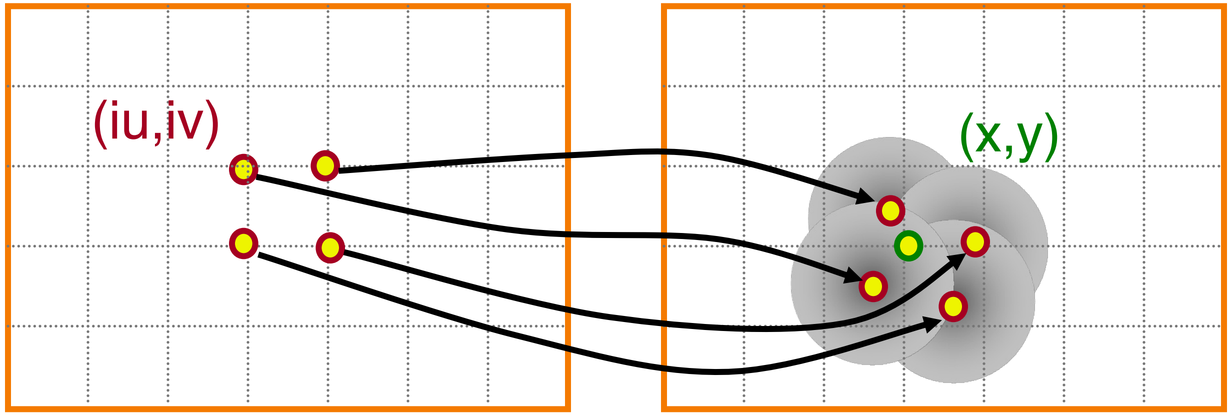
```
Warp(src, dst) {
  for (int ix = 0; ix < xmax; ix++) {
    for (int iy = 0; iy < ymax; iy++) {
      float w ≈ 1 / scale(ix, iy);
      float u = fx^-1(ix,iy);
      float v = fy^-1(ix,iy);
      dst(ix,iy) = Resample(src,u,v,w);
    }
  }
}
```



Forward vs. Reverse Mapping

- Forward mapping:

```
Warp(src, dst) {
   for (int iu = 0; iu < umax; iu++) {
      for (int iv = 0; iv < vmax; iv++) {
         float x = fx(iu,iv);
         float y = fy(iu,iv);
         float w ≈ 1 / scale(x, y);
         Splat(src(iu,iv),x,y,k,w);
      }
   }
}
```



Source image
Destination image

Forward vs. Reverse Mapping

- Forward mapping:

```
Warp(src, dst) {
  for (int iu = 0; iu < umax; iu++) {
    for (int iv = 0; iv < vmax; iv++) {
      float x = fx(iu,iv);
      float y = fy(iu,iv);
      float w ≈ 1 / scale(x, y);
      for (int ix = xlo; ix <= xhi; ix++) {
        for (int iy = ylo; iy <= yhi; iy++) {
          dst(ix,iy) += k(x,y,ix,iy,w) * src(iu,iv);
        }
      }
    }
  }
}
```

Problem?

Forward vs. Reverse Mapping

```
Warp(src, dst) {
  for (int iu = 0; iu < umax; iu++) {
    for (int iv = 0; iv < vmax; iv++) {
      float x = fx(iu,iv);
      float y = fy(iu,iv);
      float w ≈ 1 / scale(x, y);
      for (int ix = xlo; ix <= xhi; ix++) {
        for (int iy = ylo; iy <= yhi; iy++) {
          dst(ix,iy) += k(x,y,ix,iy,w) * src(iu,iv);
          ksum(ix,iy) += k(x,y,ix,iy,w);
        }
      }
    }
  }
  for (ix = 0; ix < xmax; ix++)
    for (iy = 0; iy < ymax; iy++)
      dst(ix,iy) /= ksum(ix,iy)
}
```

Forward vs. Reverse Mapping

- Tradeoffs?

Forward vs. Reverse Mapping

- Tradeoffs:
  - Forward mapping:
    - Requires separate buffer to store weights
  - Reverse mapping:
    - Requires inverse of mapping function, random access to original image

Summary

- Mapping
  - Forward vs. reverse
  - Parametric vs. correspondences
- Sampling, reconstruction, resampling
  - Frequency analysis of signal content
  - Filter to avoid undersampling: point, triangle, Gaussian
  - Reduce visual artifacts due to aliasing
    - » Blurring is better than aliasing

Next Time…

- Changing pixel values
  - Linear: scale, offset, etc.
  - Nonlinear: gamma, saturation, etc.
  - Histogram equalization
- Filtering over neighborhoods
  - Blur & sharpen
  - Detect edges
  - Median
  - Bilateral filter
- Moving image locations
  - Scale
  - Rotate
  - Warp
- Combining images
  - Composite
  - Morph
- Quantization
- Spatial / intensity tradeoff
  - Dithering