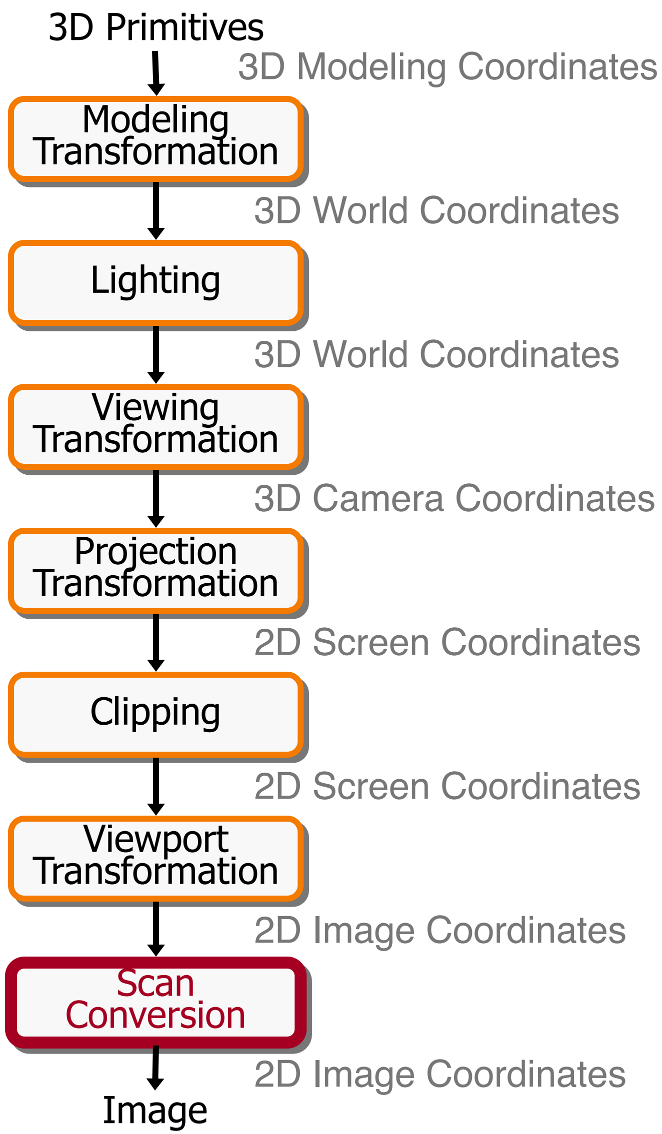
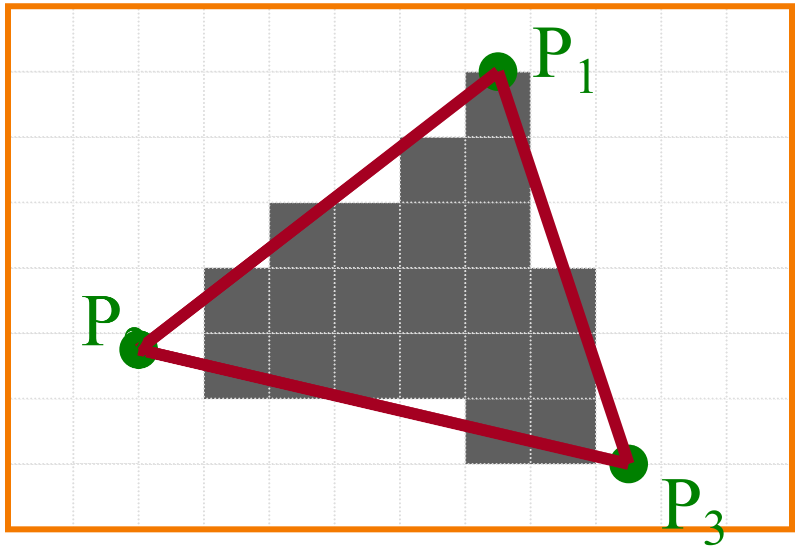
Rasterization: Shading and Visibility

COS 426, Fall 2022

PRINCETON UNIVERSITY

Rasterization Pipeline (for direct illumination)





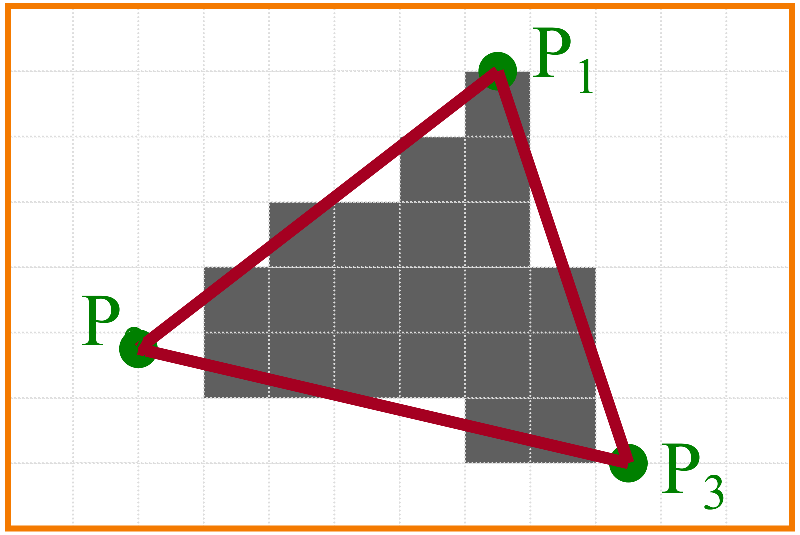
Scan Conversion

Rasterization

- Scan conversion (last time)
  - Determine which pixels to fill
- ➢ Shading
  - Determine a color for each filled pixel
- Texture mapping
  - Describe shading variation within polygon interiors
- Visible surface determination
  - Figure out which surface is front-most at every pixel

Shading

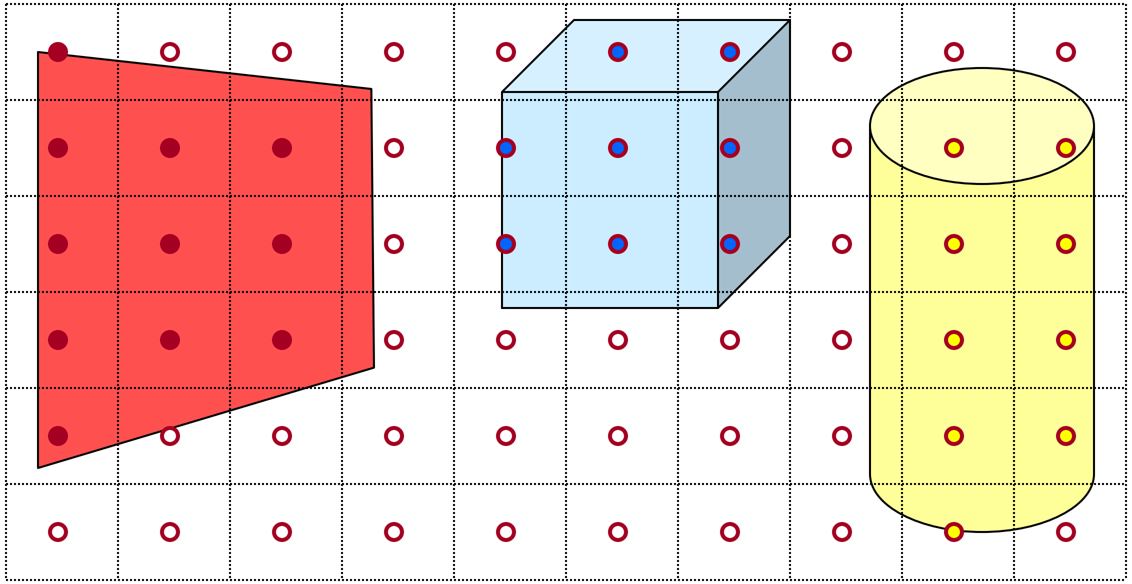
- How do we choose a color for each filled pixel?



Emphasis on methods that can be implemented in hardware…

Taking Inspiration from Ray Casting

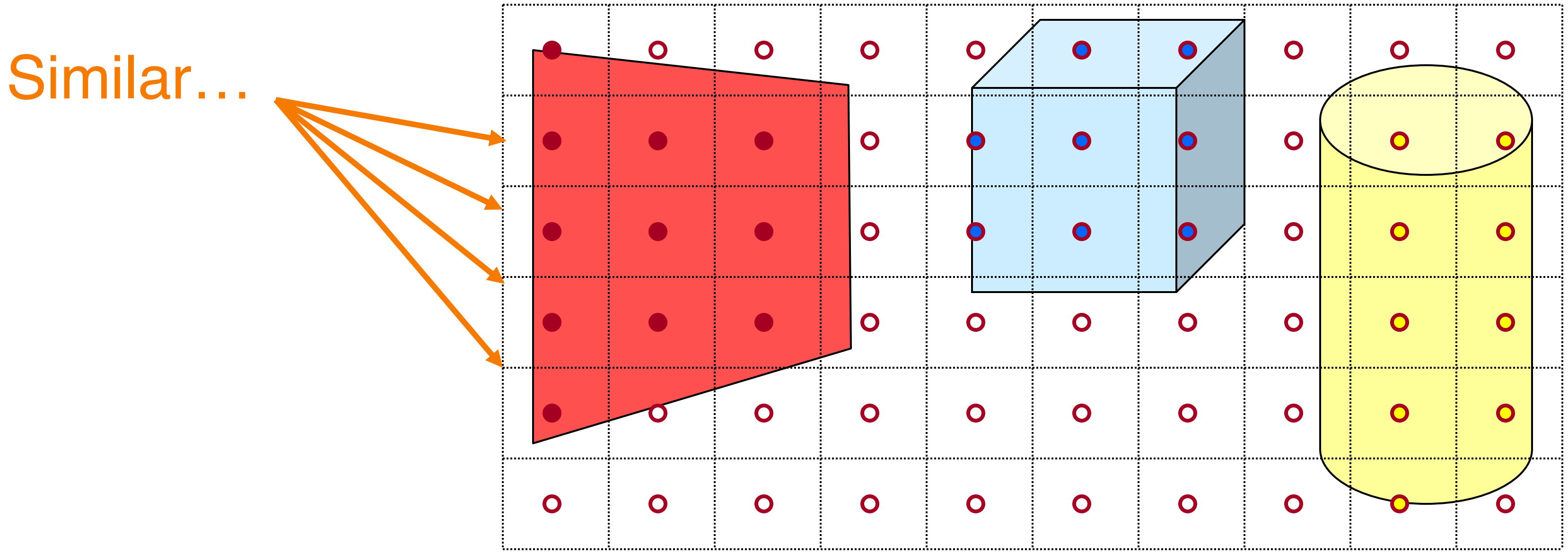
- Simplest shading approach is to perform *independent* lighting calculation for every pixel



$$I = I_E + K_A I_{AL} + \sum_i \left(K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i \right)$$

Polygon Shading

- Increase efficiency by exploiting spatial coherence
    - Illumination calculations for pixels covered by same primitive are related to each other



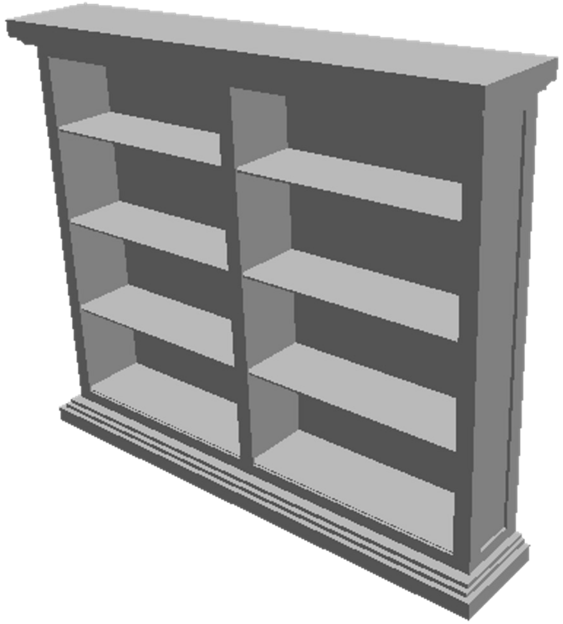
$$I = I_E + K_A I_{AL} + \sum_i \left(K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i \right)$$

Polygon Shading Algorithms

- **Flat Shading**
- Gouraud Shading
- Phong Shading

Flat Shading

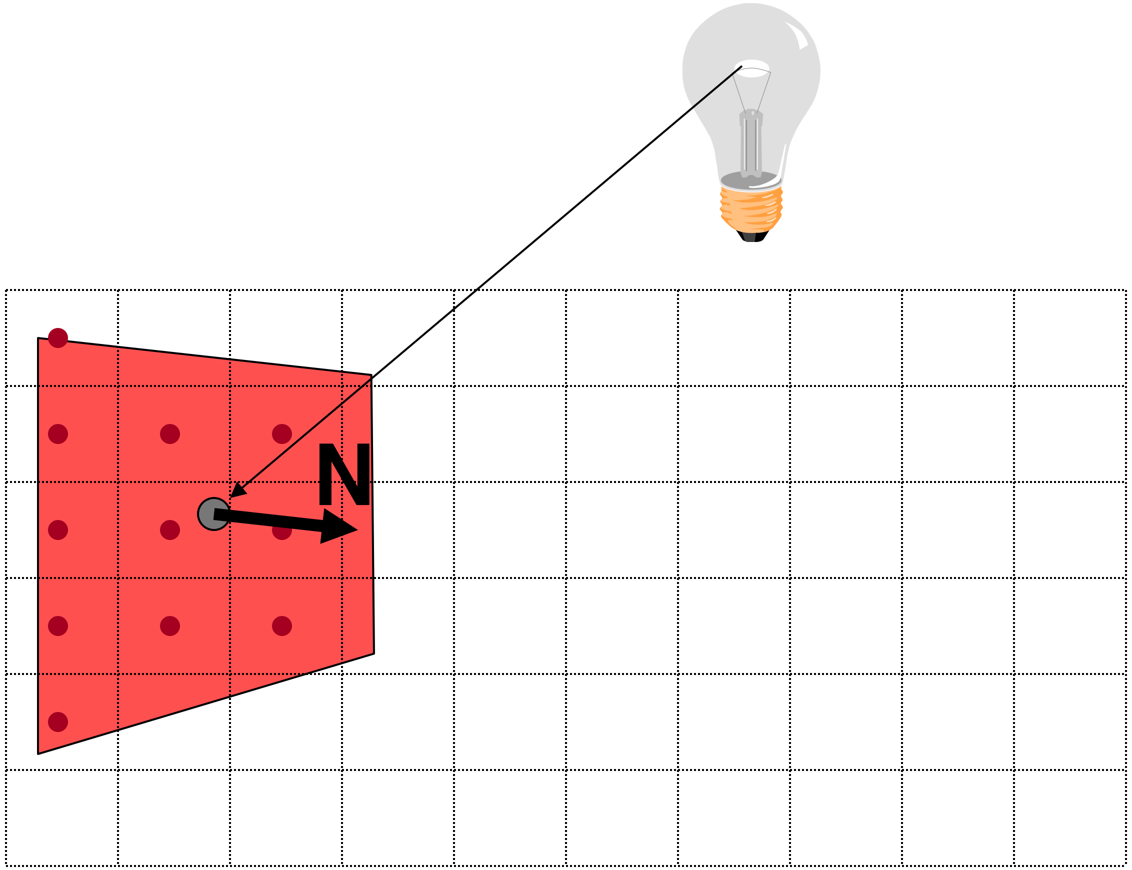
- What if a faceted object is illuminated only by directional light sources and is viewed from infinitely far away



$$I = I_E + K_A I_{AL} + \sum_i \left(K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i \right)$$

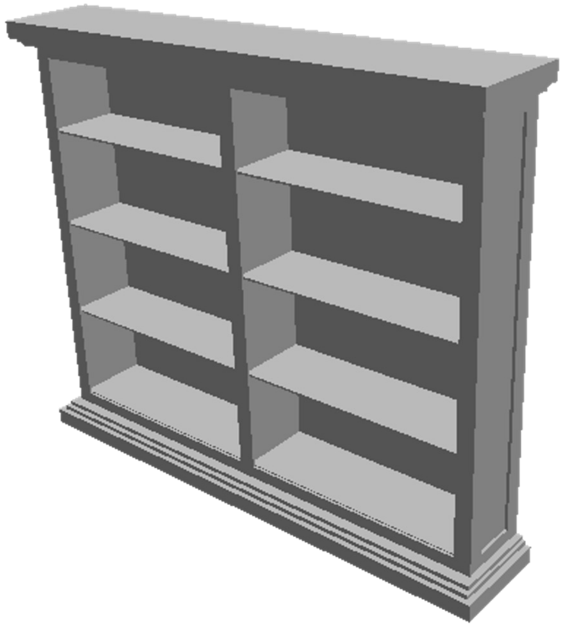
Flat Shading

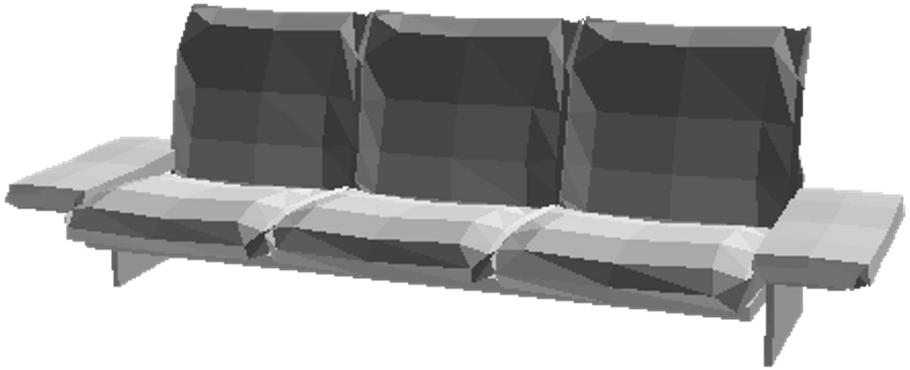
- One illumination calculation per polygon is enough
  - Assign all pixels inside each polygon the same color



Flat Shading

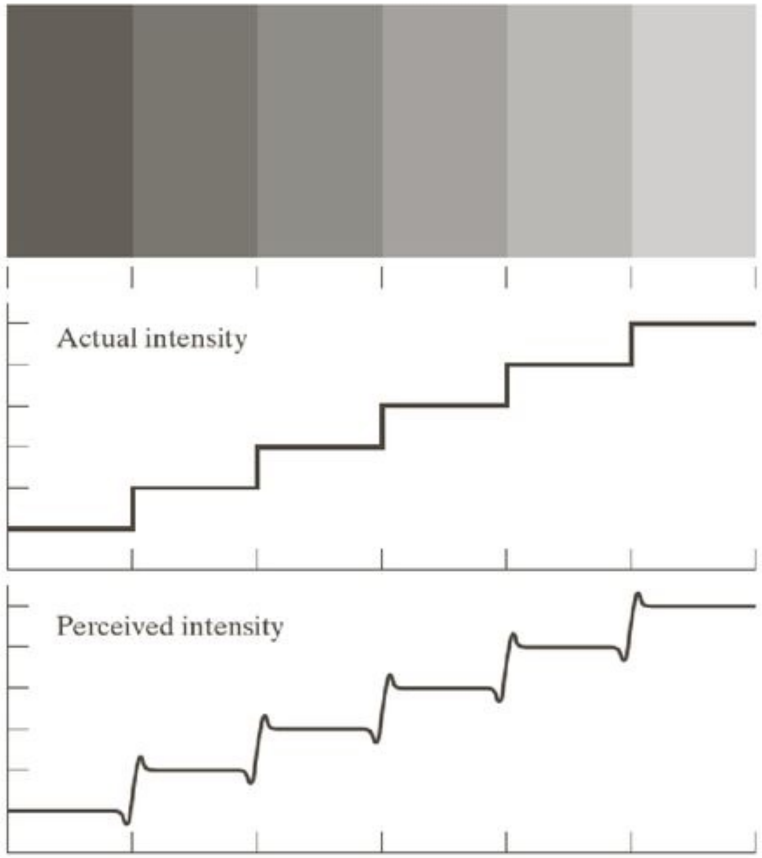
- Objects look like they are composed of polygons
  - OK for polyhedral objects
  - Not so good for smooth surfaces





Mach Band Effect

- Visual system perceives edges between adjacent shades of gray with exaggerated contrast

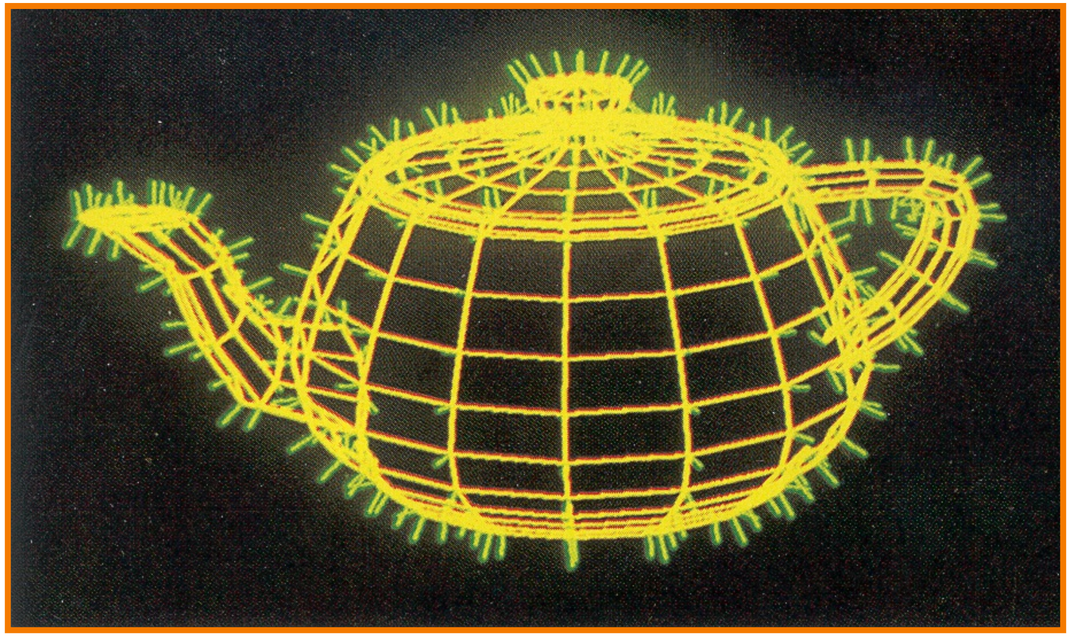


Polygon Shading Algorithms

- Flat Shading
- **Gouraud Shading**
- Phong Shading

Gouraud Shading

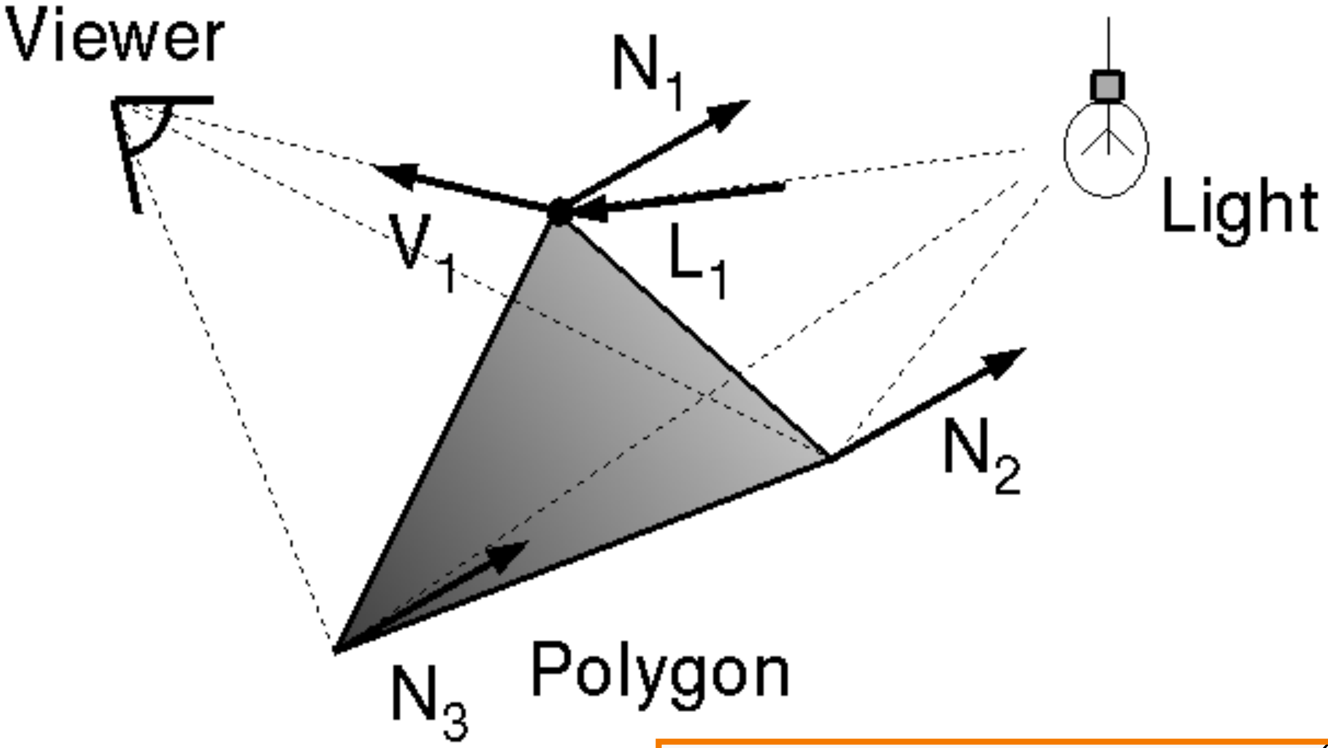
- Approximate smooth surface by polygonal mesh with a normal stored at each vertex
  - "Shared normals"
  - Calculated as (possibly area-weighted) average of normals of adjacent faces



Watt Plate 7

Gouraud Shading

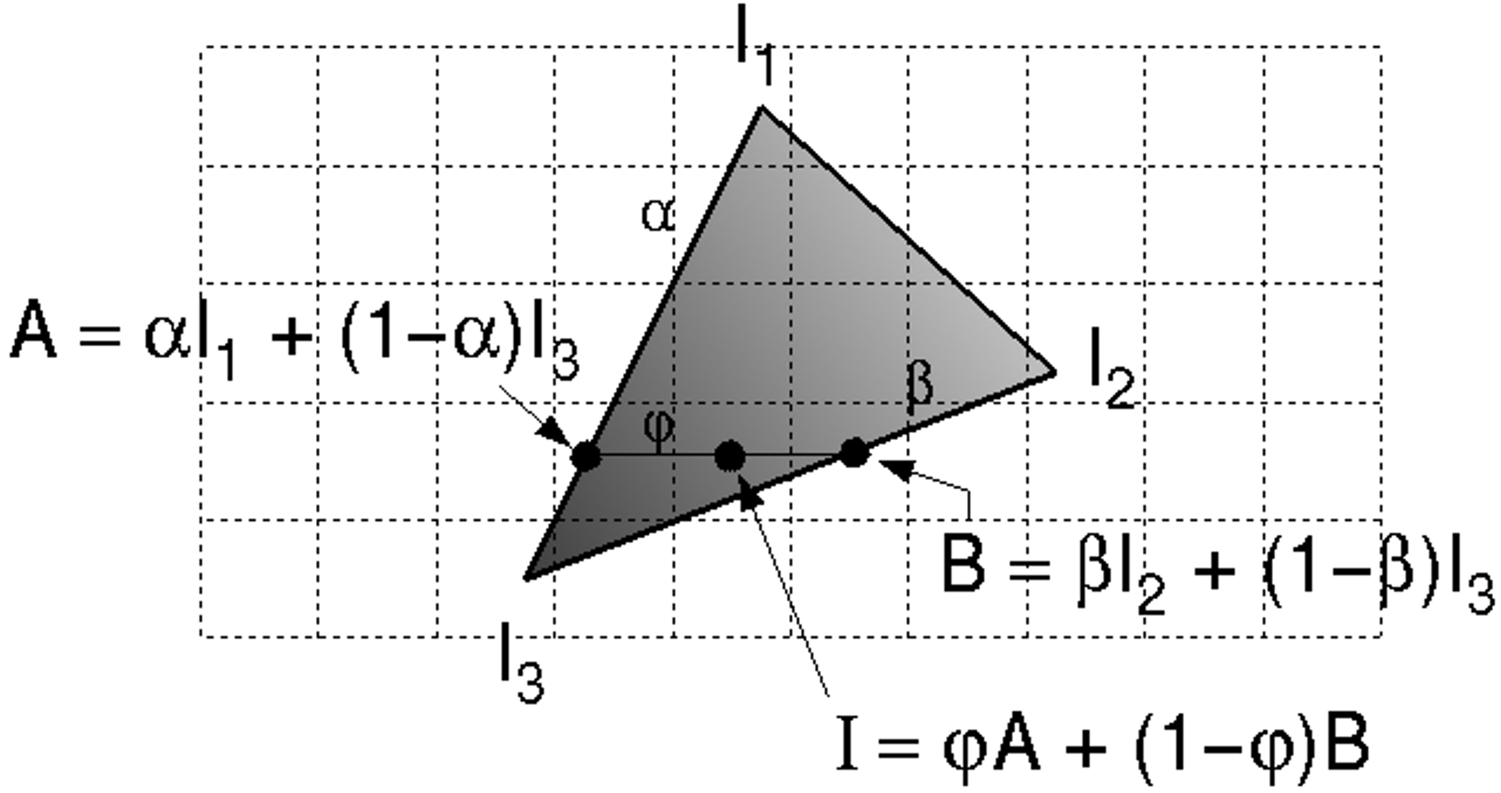
- One lighting calculation per vertex
  - Pixel colors inside polygon interpolated from colors computed at vertices



$$I = I_E + K_A I_{AL} + \sum_i \left(K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i \right)$$

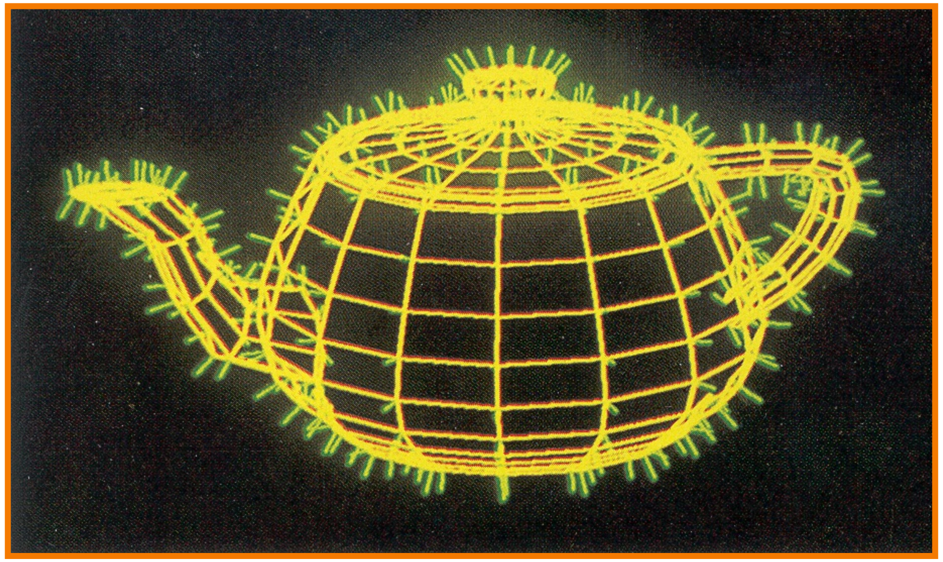
Gouraud Shading

- Bilinear interpolation of colors at vertices
  - Down and across scan lines = barycentric interpolation!
  - Specifically, linearly interpolate at left and right endpoints of each span, then linearly interpolate within scanlines



Gouraud Shading

- Smooth shading over adjacent polygons
  - Curved surfaces
  - Illumination highlights
  - Soft shadows

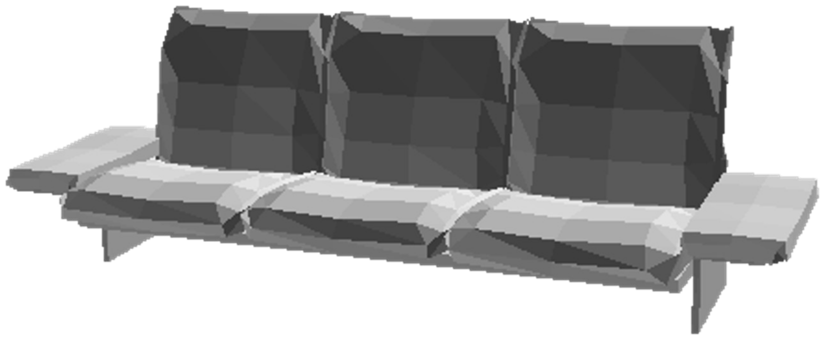


Watt Plate 7

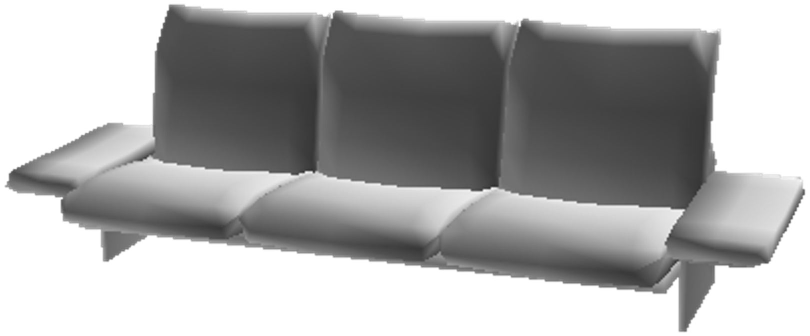
Mesh with shared normals at vertices

Gouraud Shading

- Produces smoothly shaded polygonal mesh
  - Piecewise linear (!) approximation
  - Need fine mesh to capture subtle lighting effects



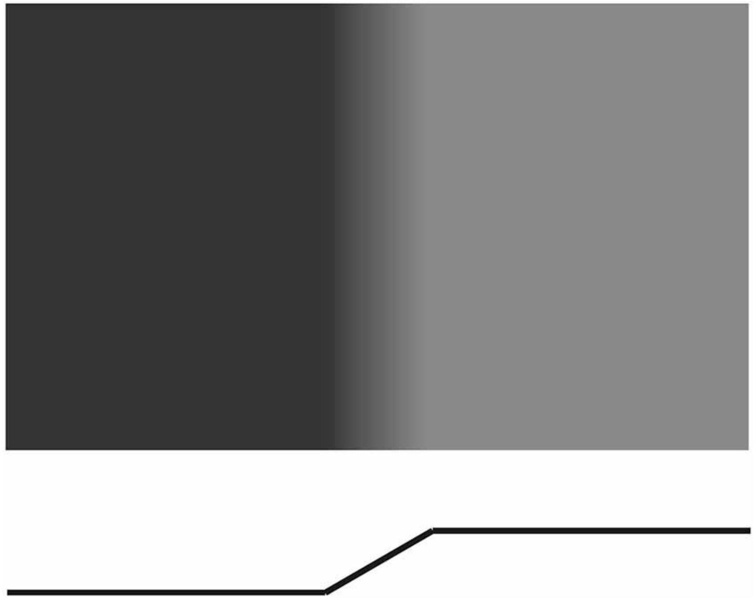
Flat Shading



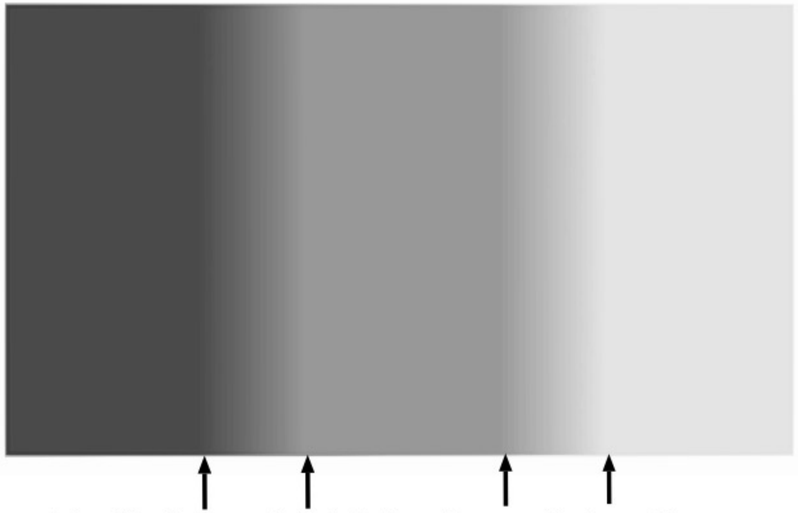
Gouraud Shading

Mach Band Effect

- Mach Band Effect also affects Gouraud Shading for piecewise linear interpolation



Actual Intensity



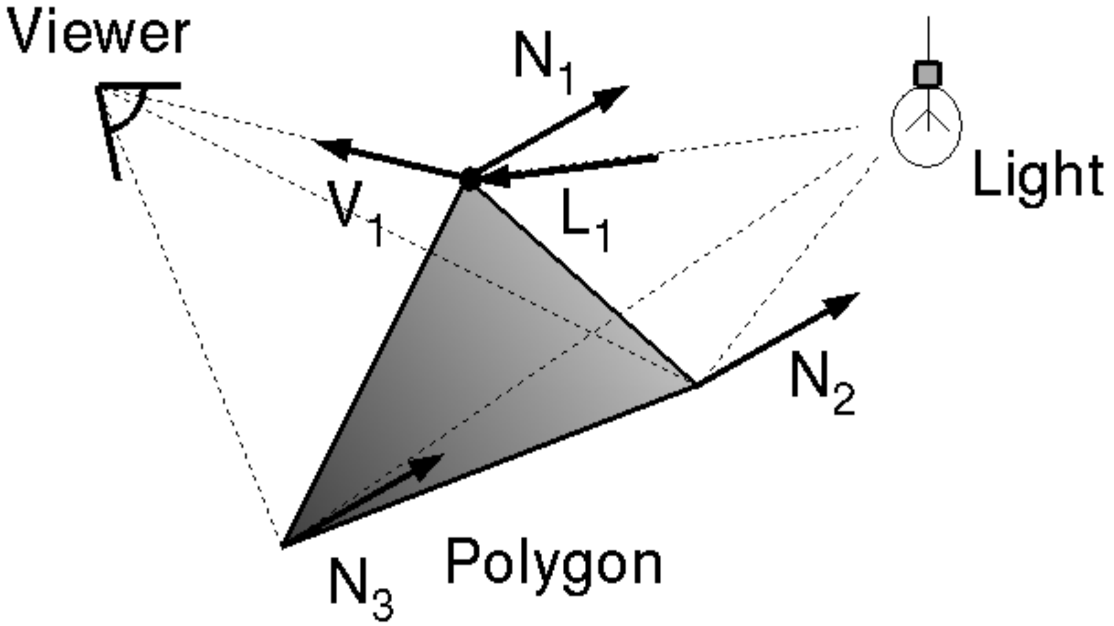
The thin lines or "bands" along the gradient are illusory.

Polygon Shading Algorithms

- Flat Shading
- Gouraud Shading
- **Phong Shading** ($\neq$ Phong reflectance model)

Phong Shading

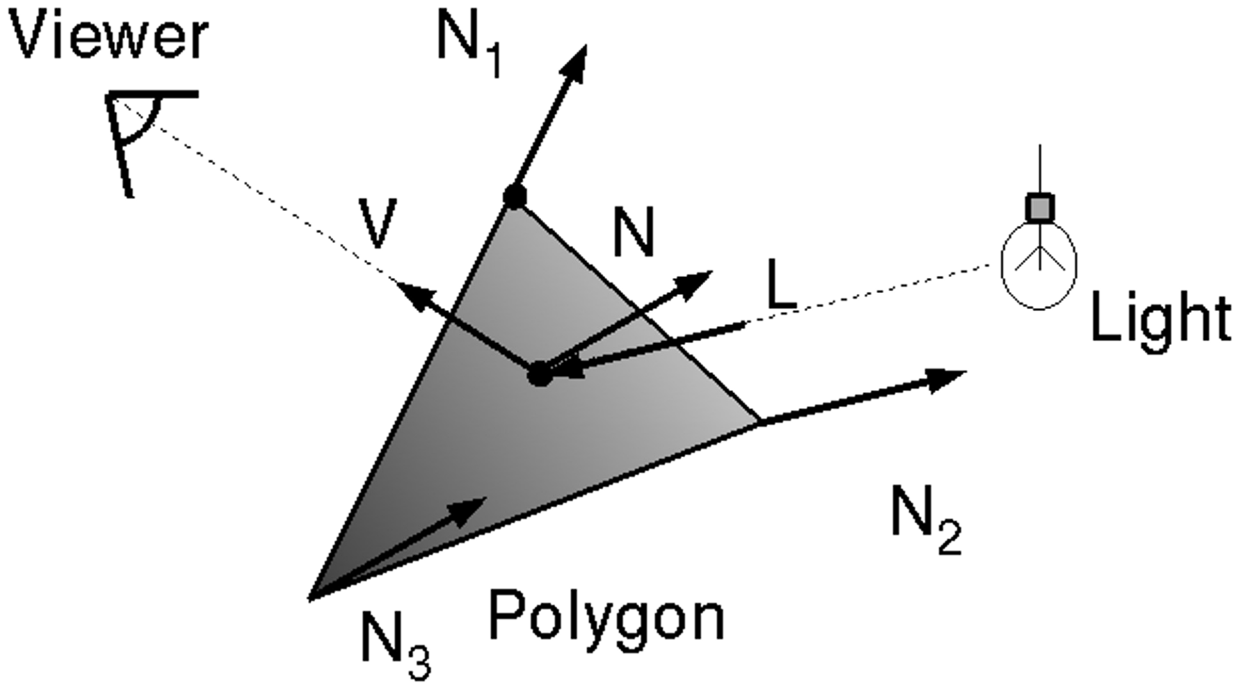
- What if polygonal mesh is too coarse to capture illumination effects in polygon interiors?



$$I = I_E + K_A I_{AL} + \sum_i \left(K_D (N \cdot L_i) I_i + K_S (V \cdot R_i)^n I_i \right)$$

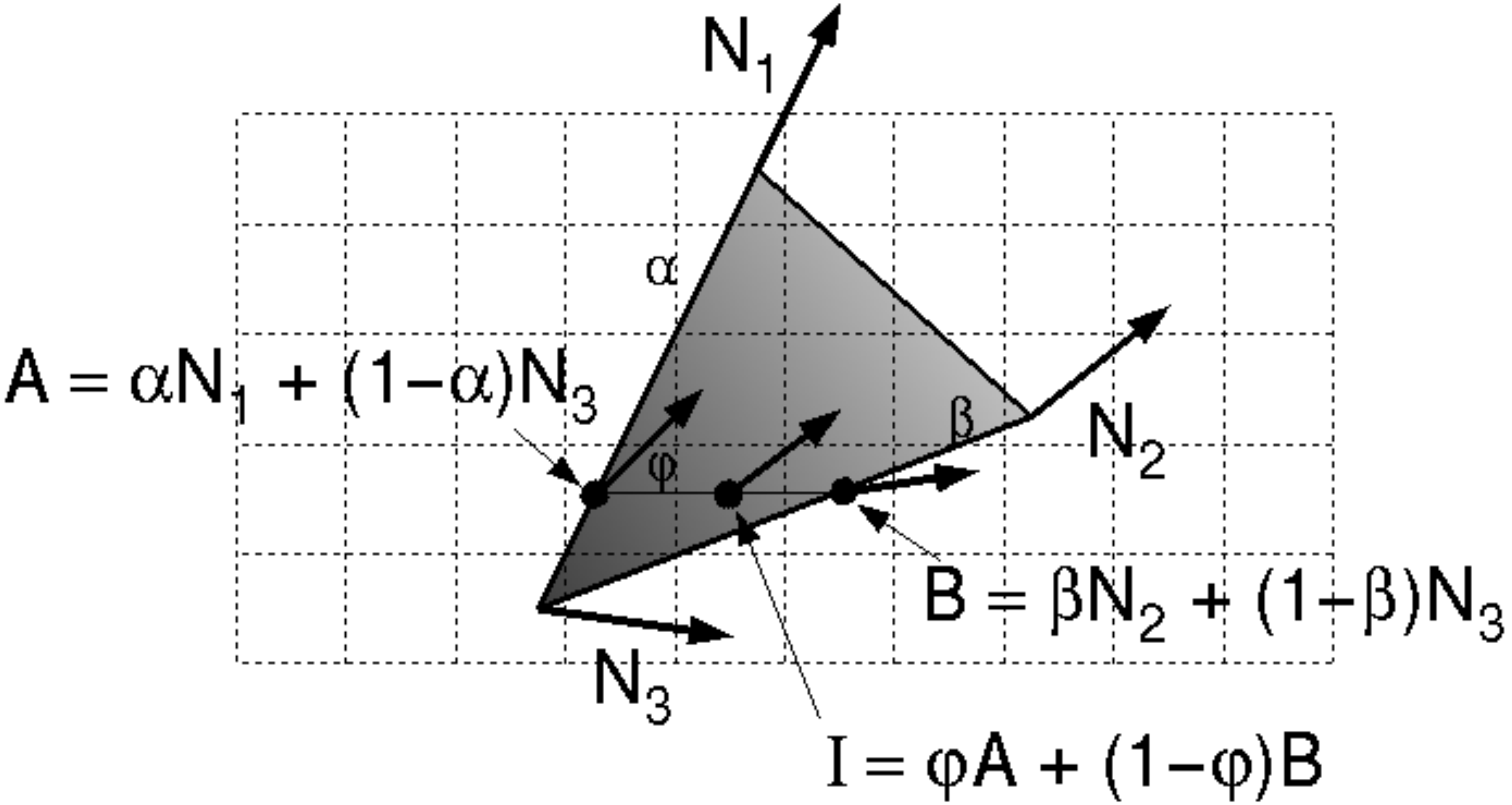
Phong Shading

- One lighting calculation per pixel
  - Approximate surface normals for points inside polygons by bilinear interpolation of normals from vertices
  - Normalize interpolated normal to unit length
  - Finally, do per-pixel lighting calculation using interpolated normal



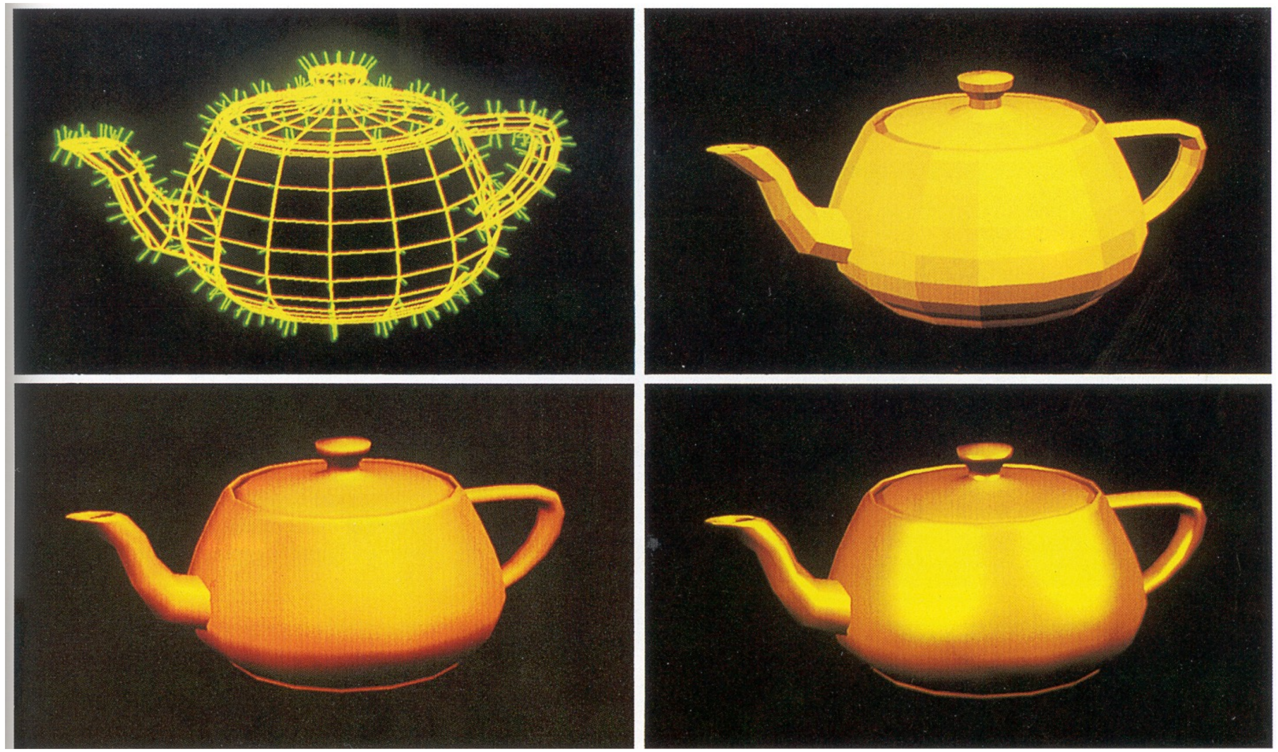
Phong Shading

- Bilinear interpolation of surface normals at vertices



Polygon Shading Algorithms

Wireframe Flat



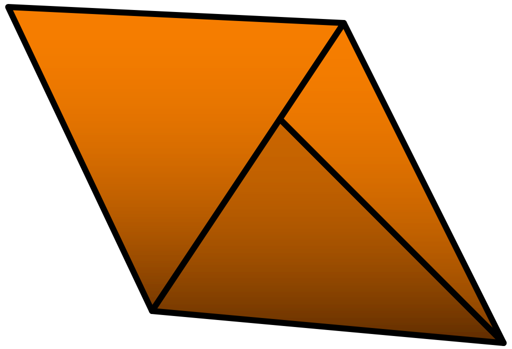
Gouraud Phong

Demo: https://threejs.org/docs/scenes/material-browser.html#MeshPhongMaterial

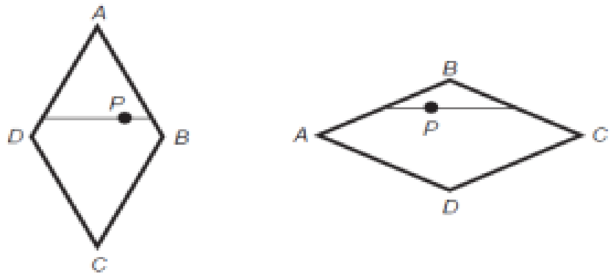
Watt Plate 7

Shading Issues

- Problems with interpolated shading:
  - Polygonal *silhouettes* still obvious
  - Perspective distortion (due to *screen-space interpolation*)
  - Problems at T-junctions



The results of interpolated-shading is not independent of the projected polygons position (Foley Figure 14.22).



Rasterization

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  - Determine which pixels to fill
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  - Determine a color for each filled pixel
- ➢ Texture mapping
  - Describe shading variation within polygon interiors
- Visible surface determination
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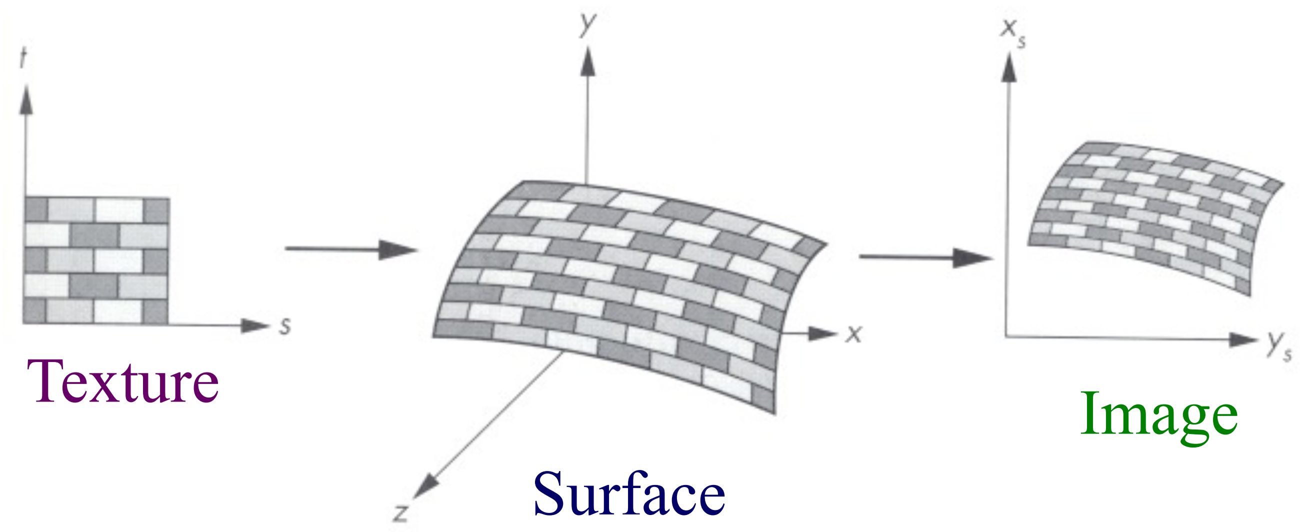
Textures

- Describe **color variation** in interior of 3D polygon
  - When scan converting a polygon, **vary pixel colors** according to values fetched from a texture image



Angel Figure 9.3

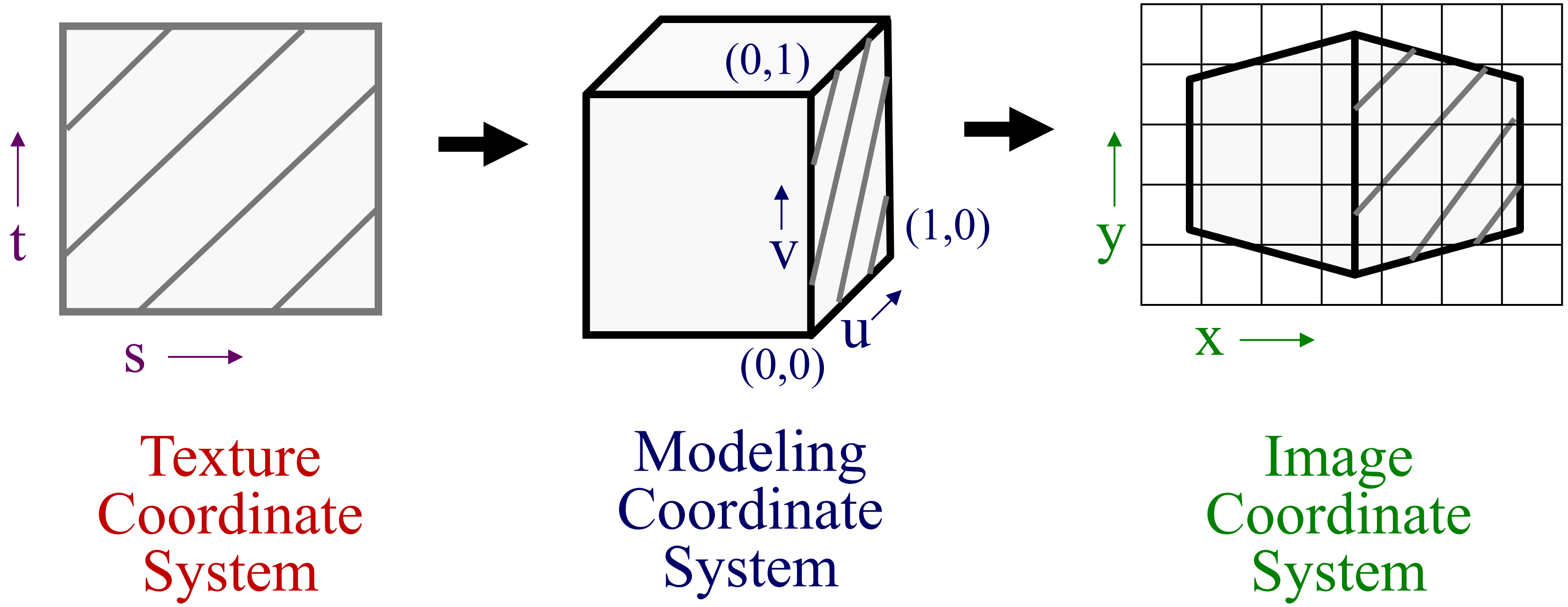
Textures

- Add visual detail to surfaces of 3D objects

[Daren Horley]

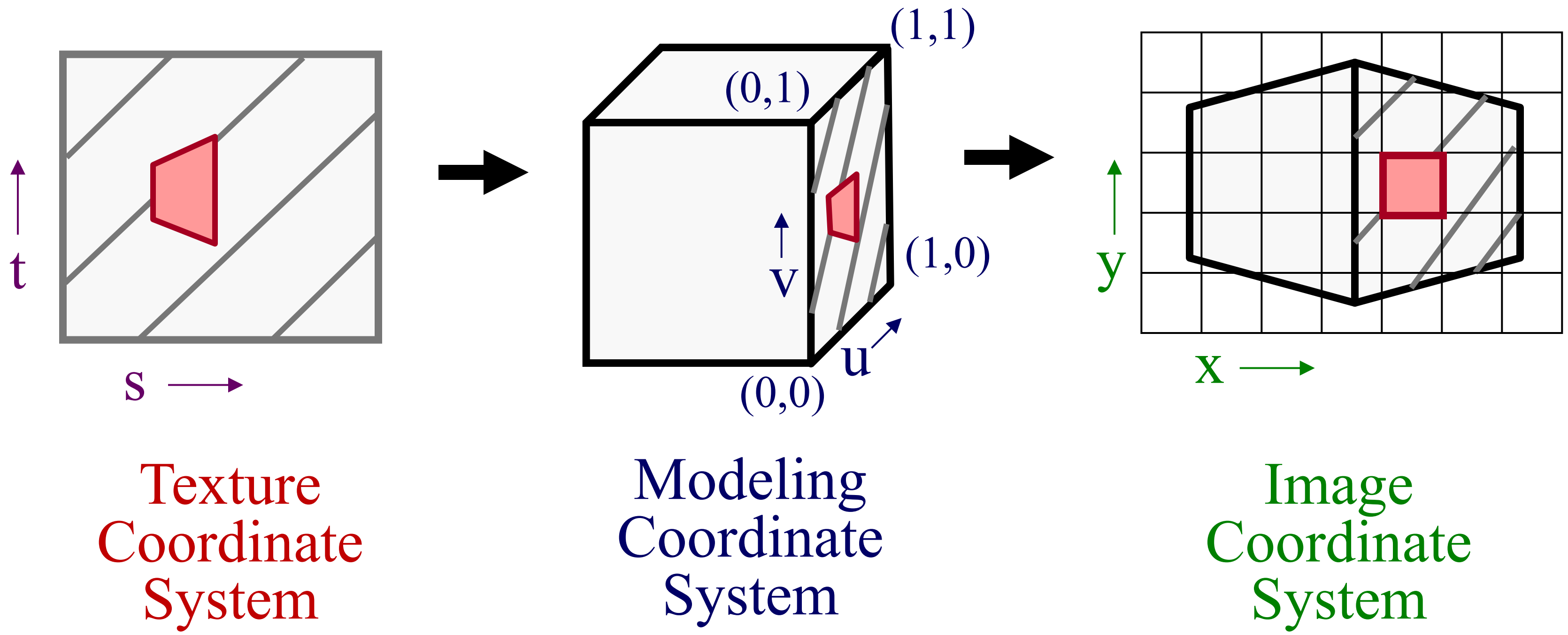
Texture Mapping

- Steps:
  1. Define texture image
  2. Specify mapping from texture to surface
  3. Look up texture values during scan conversion



Texture Mapping

- When scan converting, map from …
  - image coordinate system (x,y) to
  - modeling coordinate system (u,v) to
  - texture image (s,t)



Texture Overview

- Texture mapping stages
  - Parameterization
  - Mapping
  - Filtering
- Texture mapping applications
  - Modulation textures
  - Illumination mapping
  - Bump mapping
  - Environment mapping
  - Image-based rendering
  - Non-photorealistic rendering

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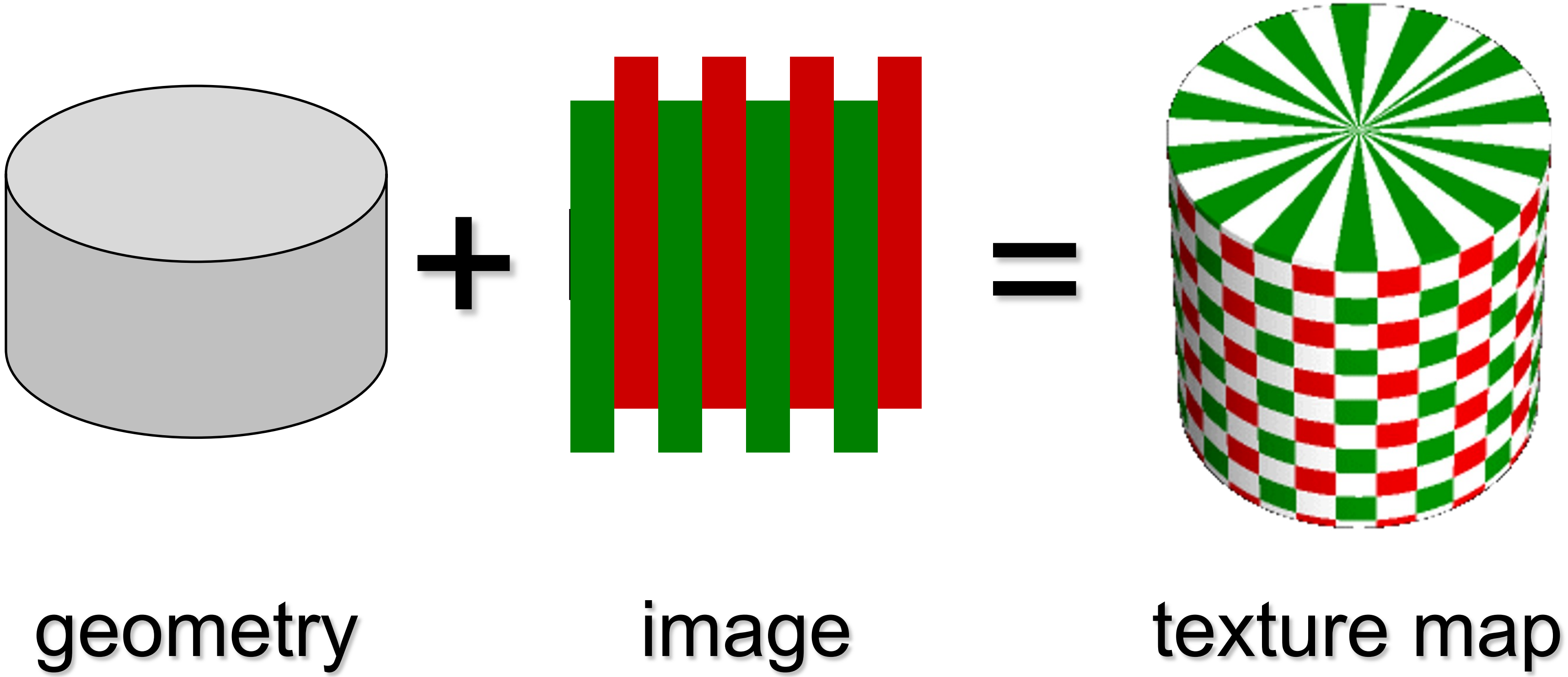
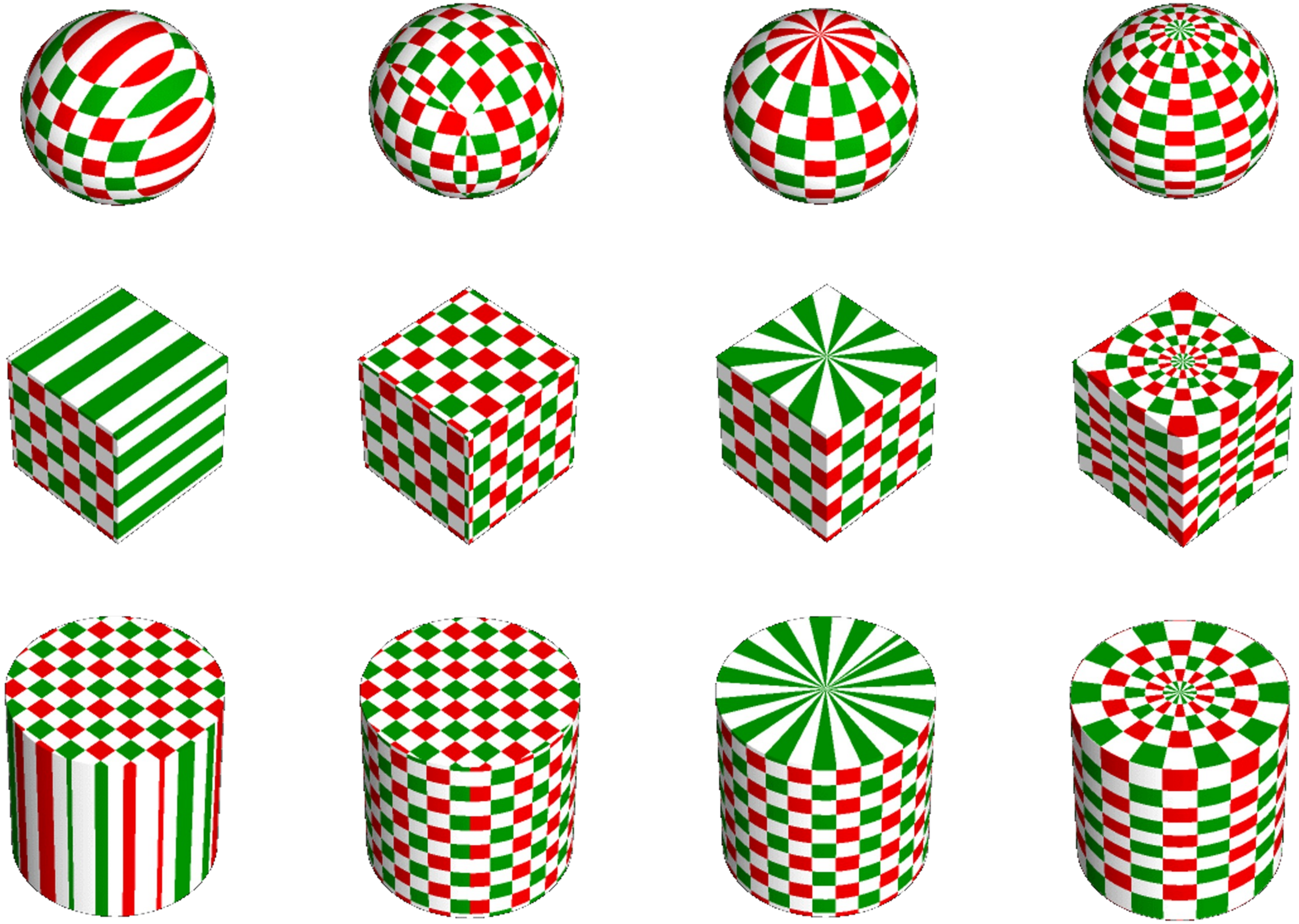
Texture Parameterization



- Q: How do we decide ***where*** **on the geometry** each color from the image should go?

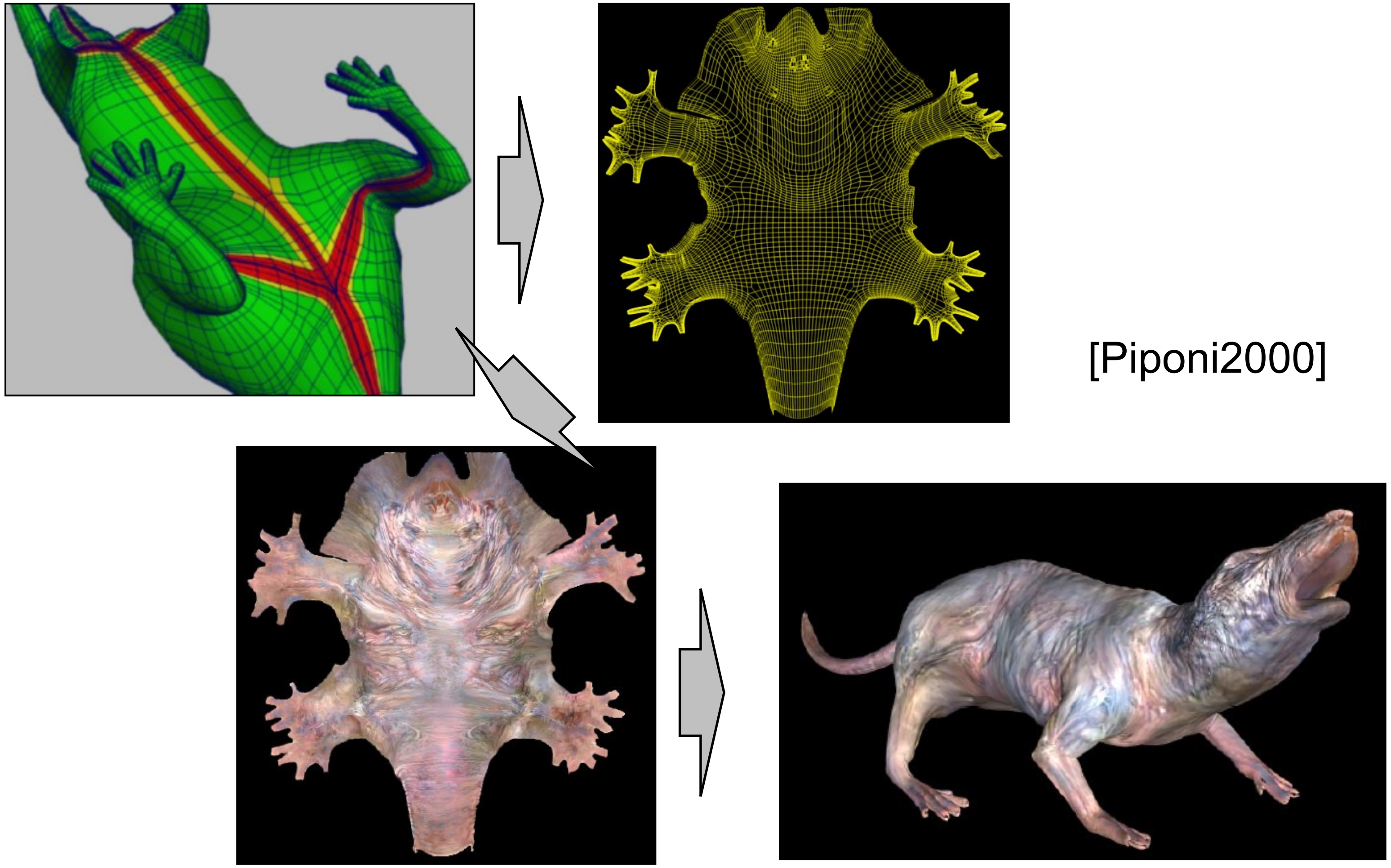
Texture Parameterization



[Paul Bourke]

Texture Parameterization

Option1: unfold the surface

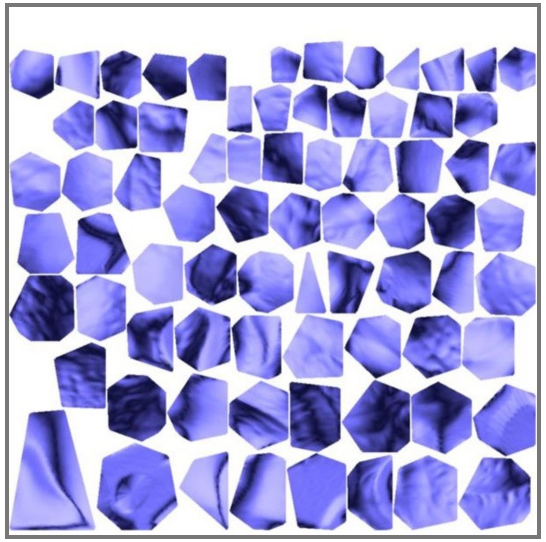


[Piponi2000]

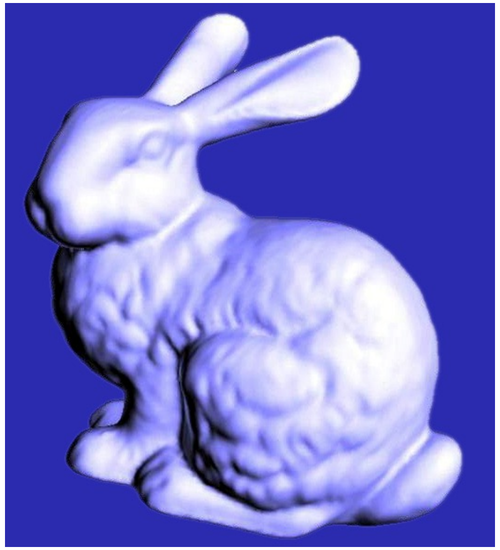
Texture Parameterization

Option2: make an atlas

charts



atlas



surface

[Sander2001]

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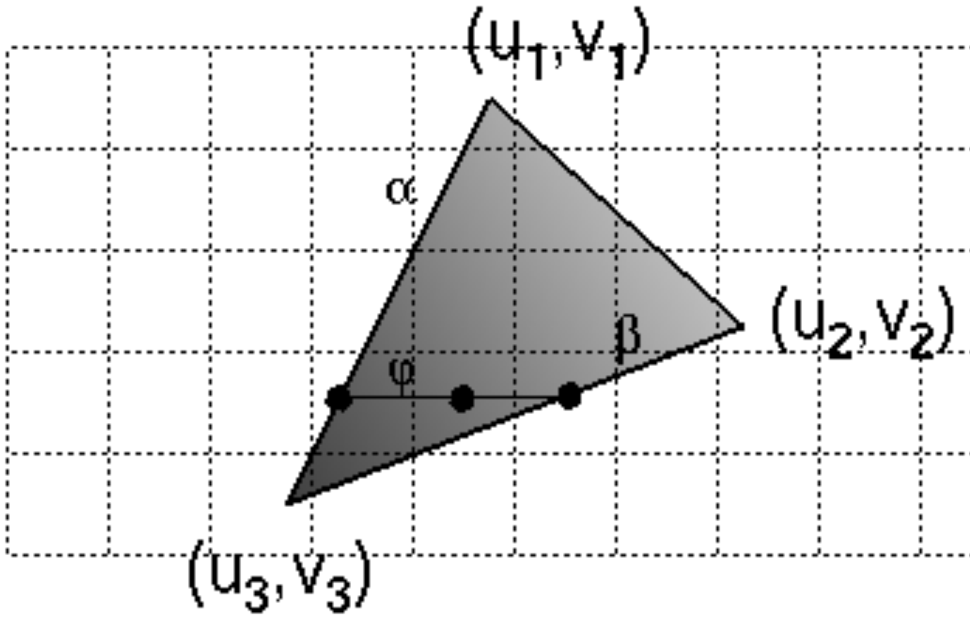
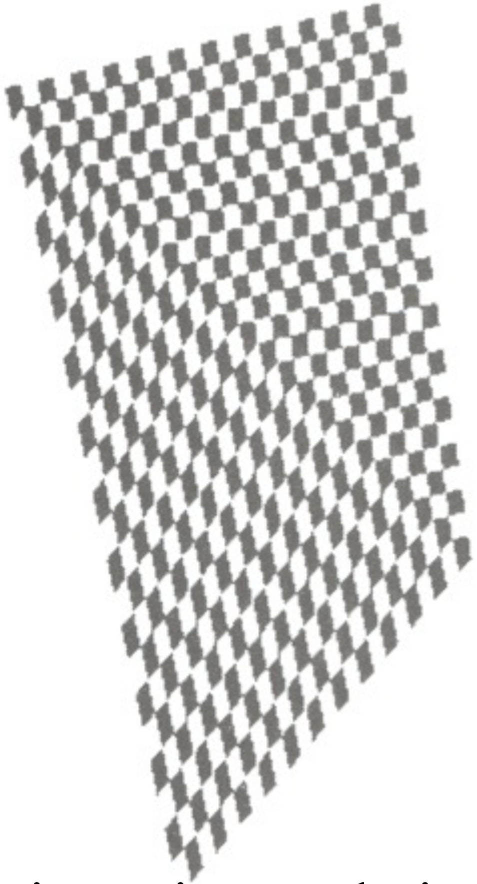
Texture Mapping

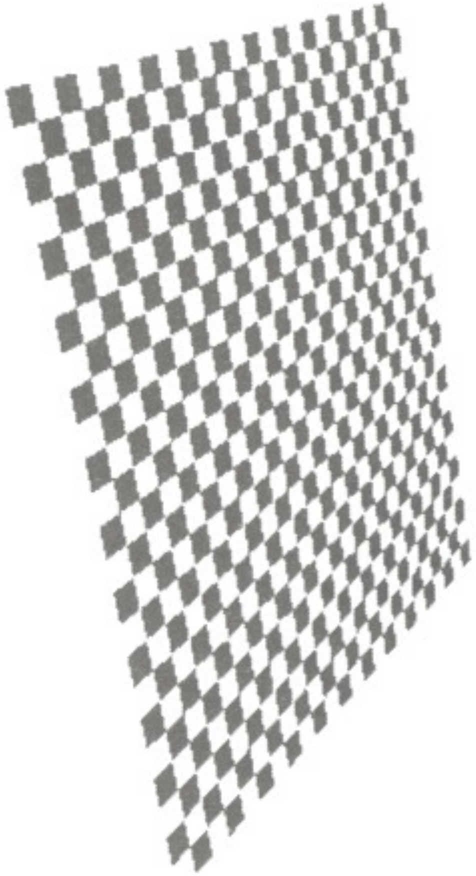
- Scan conversion
  - Interpolate texture coordinates down/across scan lines



Perspective Divide



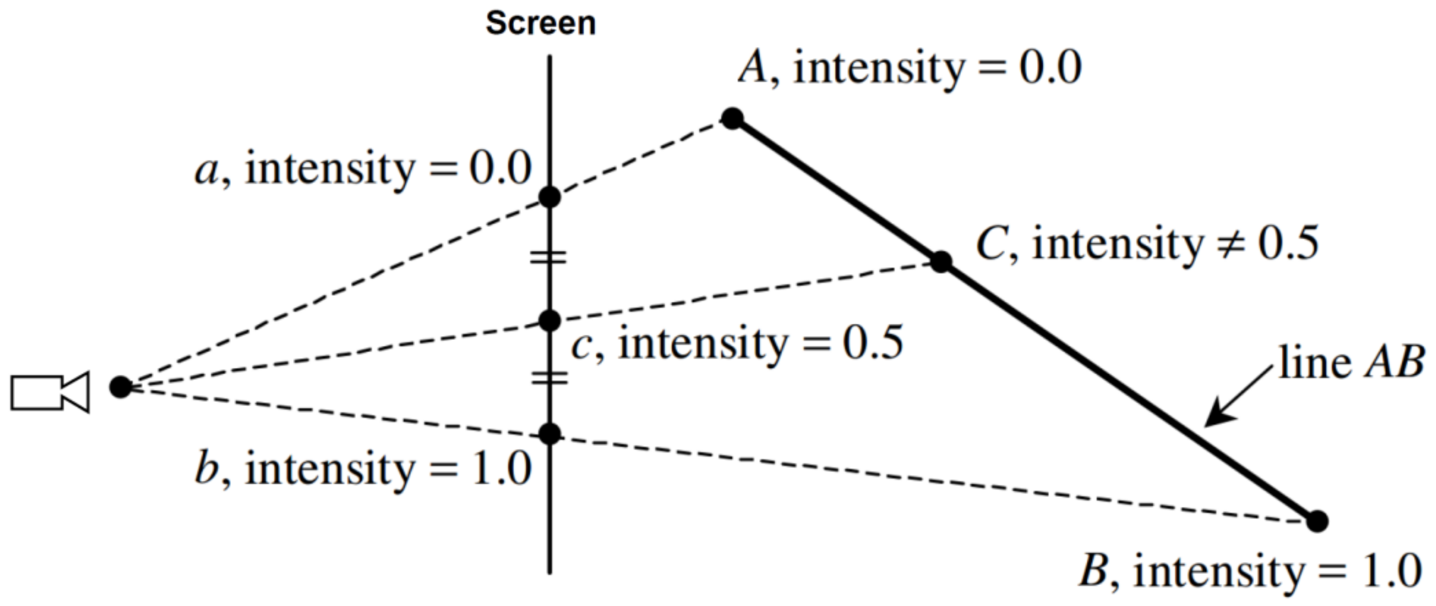
Linear interpolation of texture coordinates



Correct interpolation

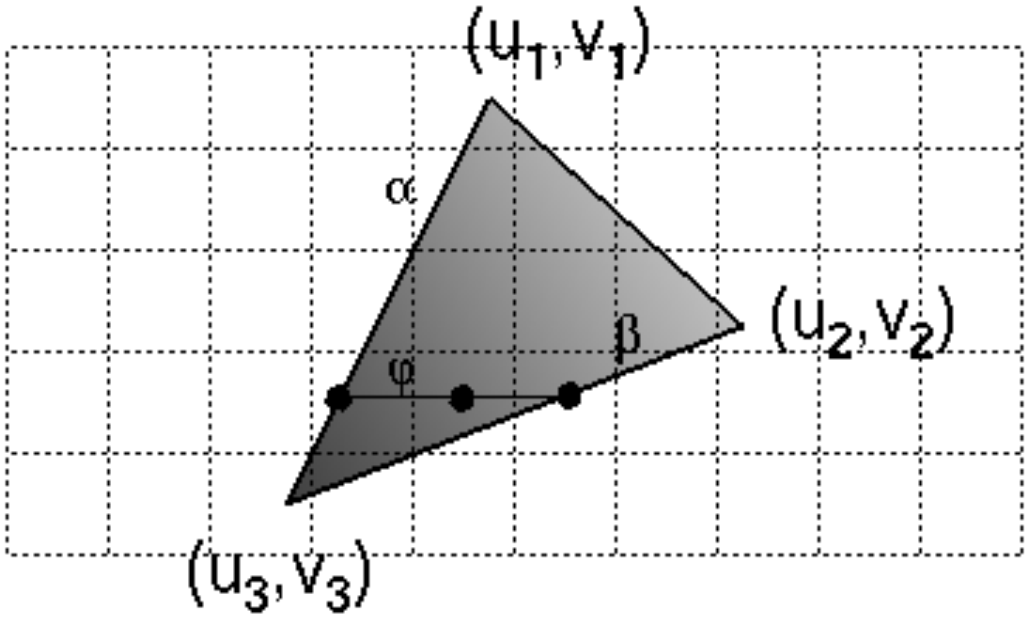
Hill Figure 8.42

Perspective Divide



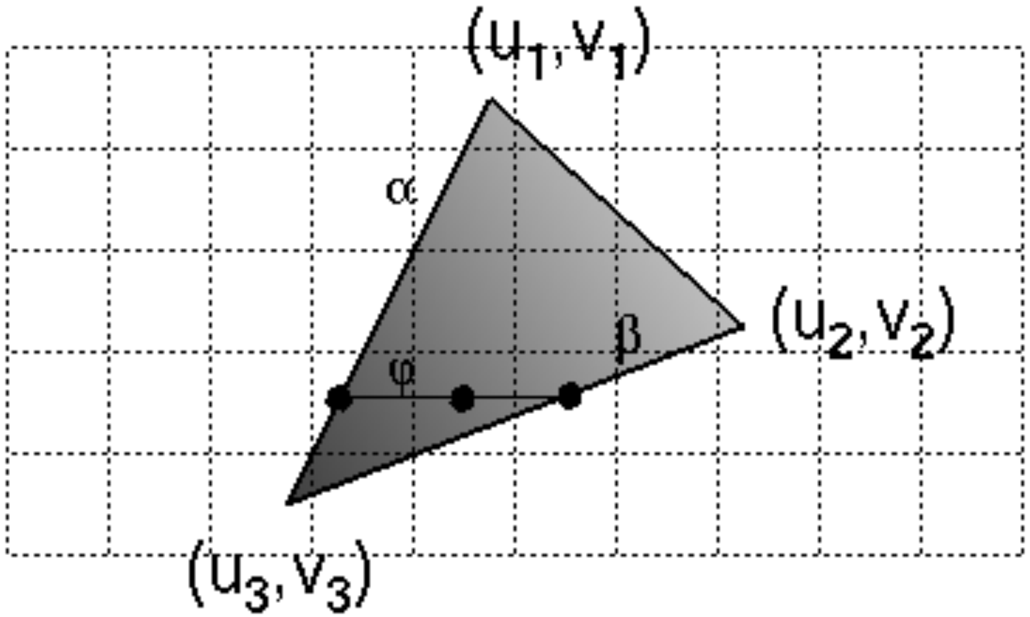
Texture Mapping

- Scan conversion
  - Interpolate texture coordinates down/across scan lines
  - Distortion due to bilinear interpolation approximation
    - » Cut polygons into smaller ones, or



Texture Mapping

- Scan conversion
  - Interpolate texture coordinates down/across scan lines
  - Distortion due to bilinear interpolation approximation
    - » Cut polygons into smaller ones, or
    - » Perspective divide at each pixel



Perspective Divide

Assume triangle attribute varies linearly across the triangle

Attribute's value at 3D (non-homogeneous) point $P = \begin{bmatrix} x & y & z \end{bmatrix}^T$ **is then:**

$$f(x, y, z) = ax + by + cz$$

Get 2D homogeneous representation : $\begin{bmatrix} x_{\text{2D-H}} & y_{\text{2D-H}} & w \end{bmatrix}^T = \begin{bmatrix} x & y & z \end{bmatrix}^T$

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Rewrite attribute equation for f **in terms of 2D homogeneous coordinates:**

$$f = ax_{\text{2D-H}} + by_{\text{2D-H}} + cw$$

Perspective Divide

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Rewrite attribute equation for f **in terms of 2D homogeneous coordinates:**

$$f = ax_{\text{2D-H}} + by_{\text{2D-H}} + cw$$

$$\frac{f}{w} = a\frac{x_{\text{2D-H}}}{w} + b\frac{y_{\text{2D-H}}}{w} + c$$

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$$\frac{f}{w} = ax_{\text{2D}} + by_{\text{2D}} + c$$

Where $\begin{bmatrix} x_{\text{2D}} & y_{\text{2D}} \end{bmatrix}^T$ **are projected screen 2D coordinates (after homogeneous divide)**

Perspective Divide

Assume triangle attribute varies linearly across the triangle

Attribute's value at 3D (non-homogeneous) point $P = \begin{bmatrix} x & y & z \end{bmatrix}^T$ is then:

$$f(x, y, z) = ax + by + cz$$

Get 2D homogeneous representation : $\begin{bmatrix} x_{\text{2D-H}} & y_{\text{2D-H}} & w \end{bmatrix}^T = \begin{bmatrix} x & y & z \end{bmatrix}^T$

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$$\frac{f}{w} = a\frac{x_{\text{2D-H}}}{w} + b\frac{y_{\text{2D-H}}}{w} + c$$

$$\frac{f}{w} = ax_{\text{2D}} + by_{\text{2D}} + c$$

Where $\begin{bmatrix} x_{\text{2D}} & y_{\text{2D}} \end{bmatrix}^T$ are projected screen 2D coordinates (after homogeneous divide)

So … $\frac{f}{w}$ is affine function of 2D screen coordinates…

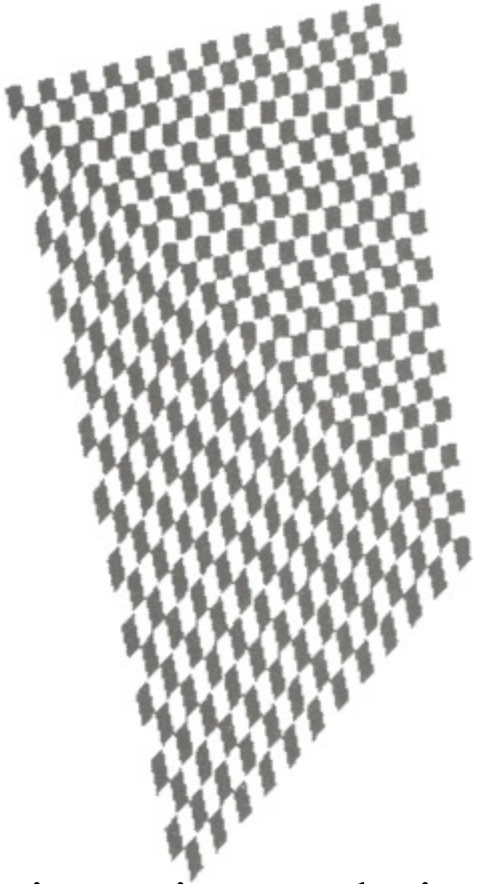
Perspective Divide

- Compute at each vertex *after perspective transformation*:
  - “Numerators” s/w, t/w
  - “Denominator” 1/w

- Linearly interpolate s/w, and t/w and 1/w across the polygon

- At each pixel:
  - Perform perspective division of interpolated texture coordinates (s/w, t/w) by interpolated 1/w (i.e., numerator over denominator) to get (s, t)

Perspective Divide



Linear interpolation
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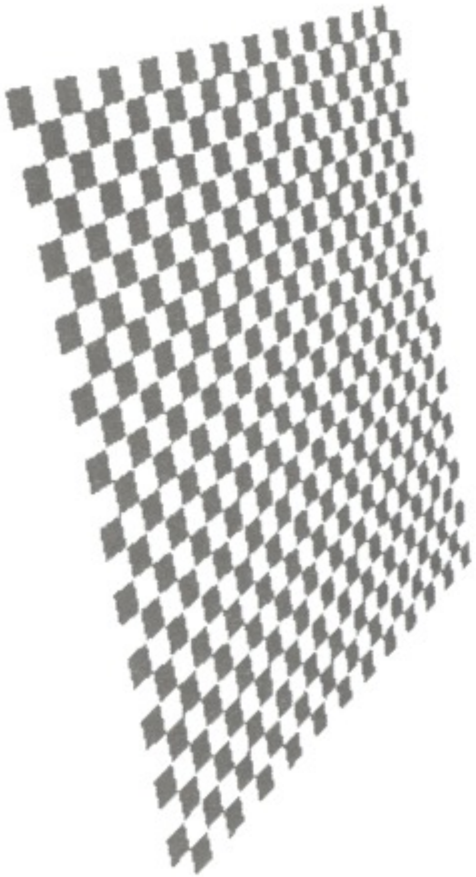

Correct interpolation

Hill Figure 8.42

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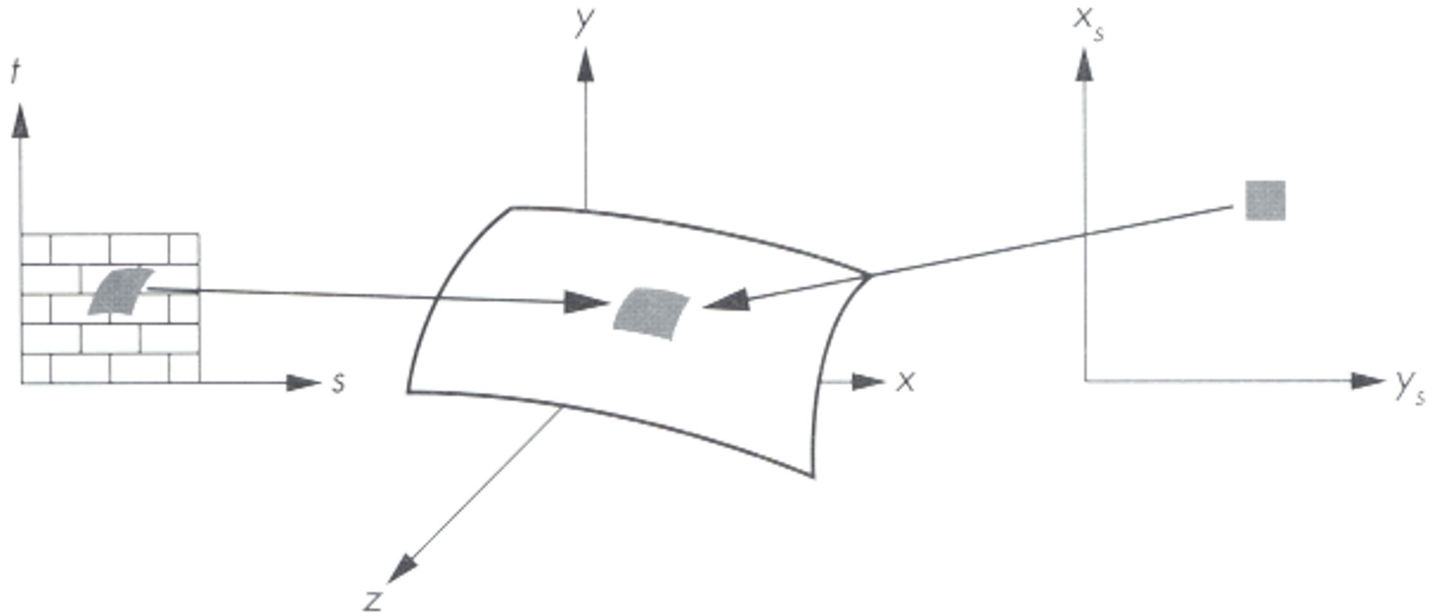
Texture Filtering

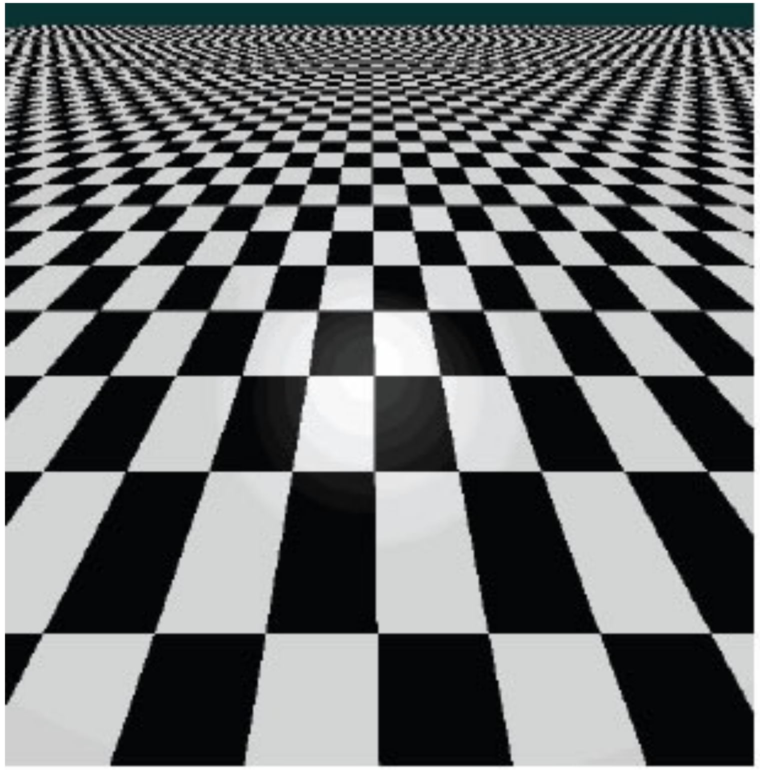
- Must sample texture to determine color at each pixel in image



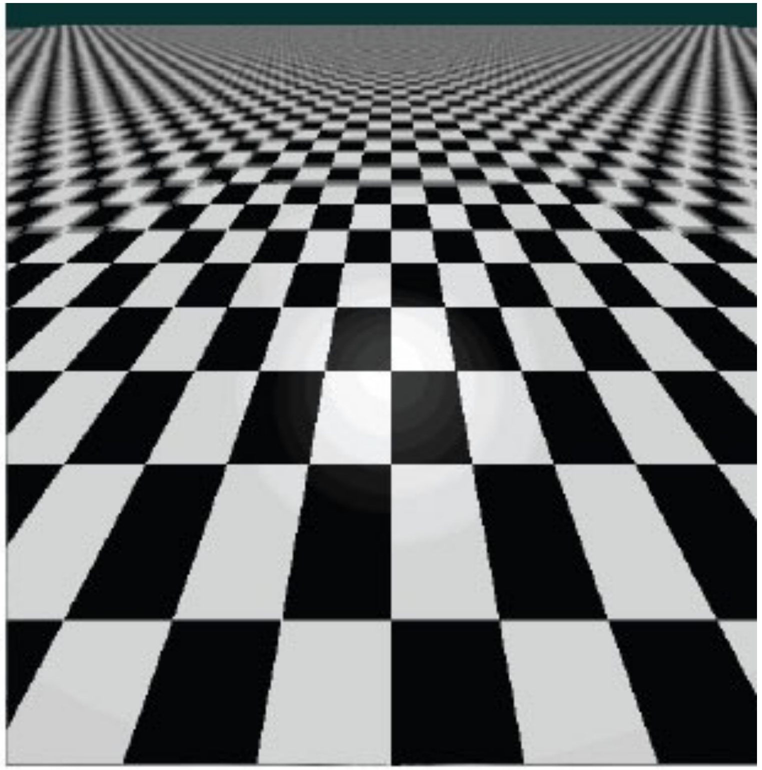
Angel Figure 9.4

Texture Filtering

- Aliasing is a problem



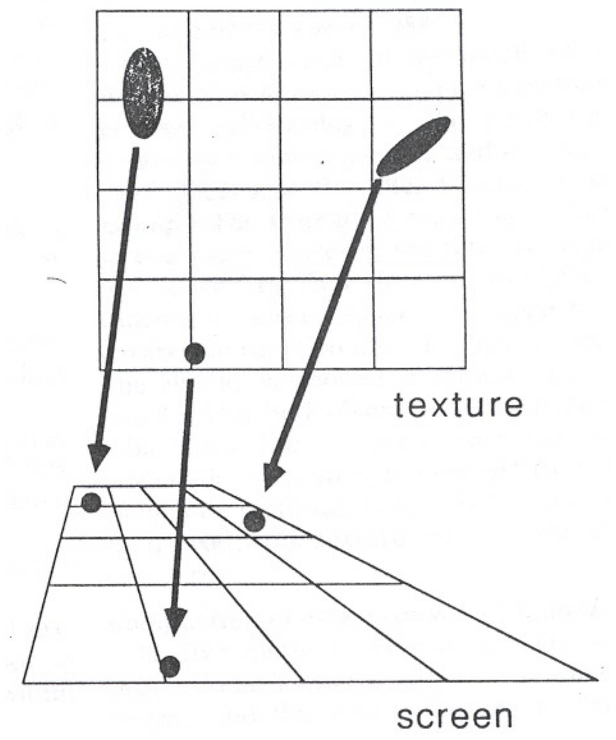
Point sampling



Area filtering

Texture Filtering

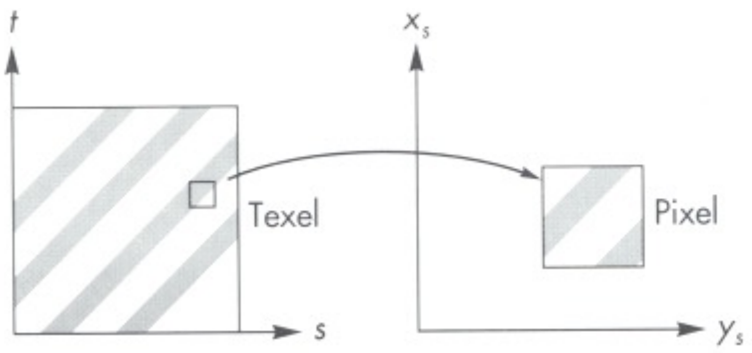
- Ideally, use elliptically shaped convolution filters



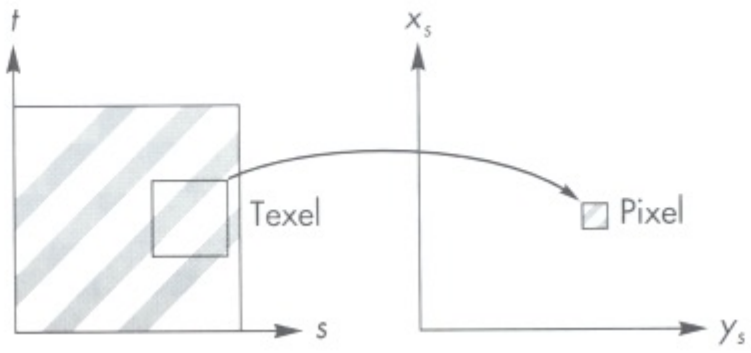
In practice, *use rectangles or squares*

Texture Filtering

- Size of filter depends on projective warp
  - Compute prefiltered images to avoid run-time cost
    - » Mipmaps
    - » Summed area tables



Magnification

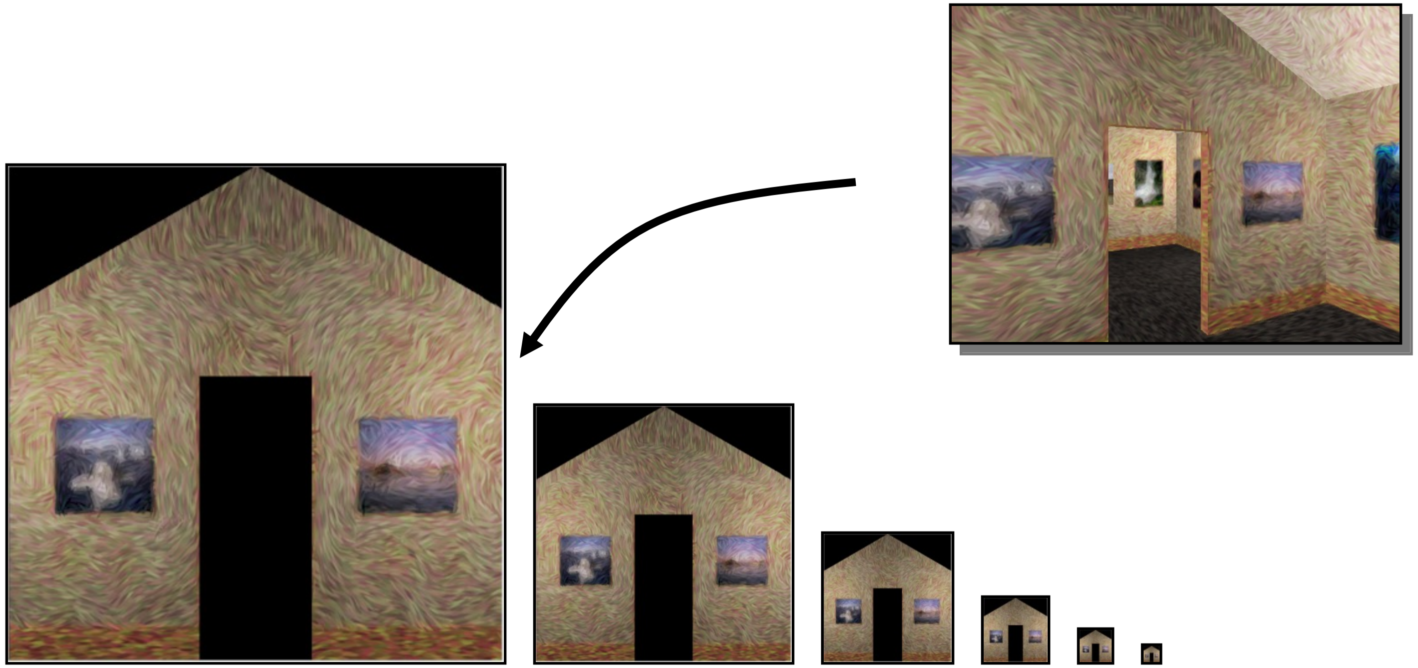


Minification

Angel Figure 9.14

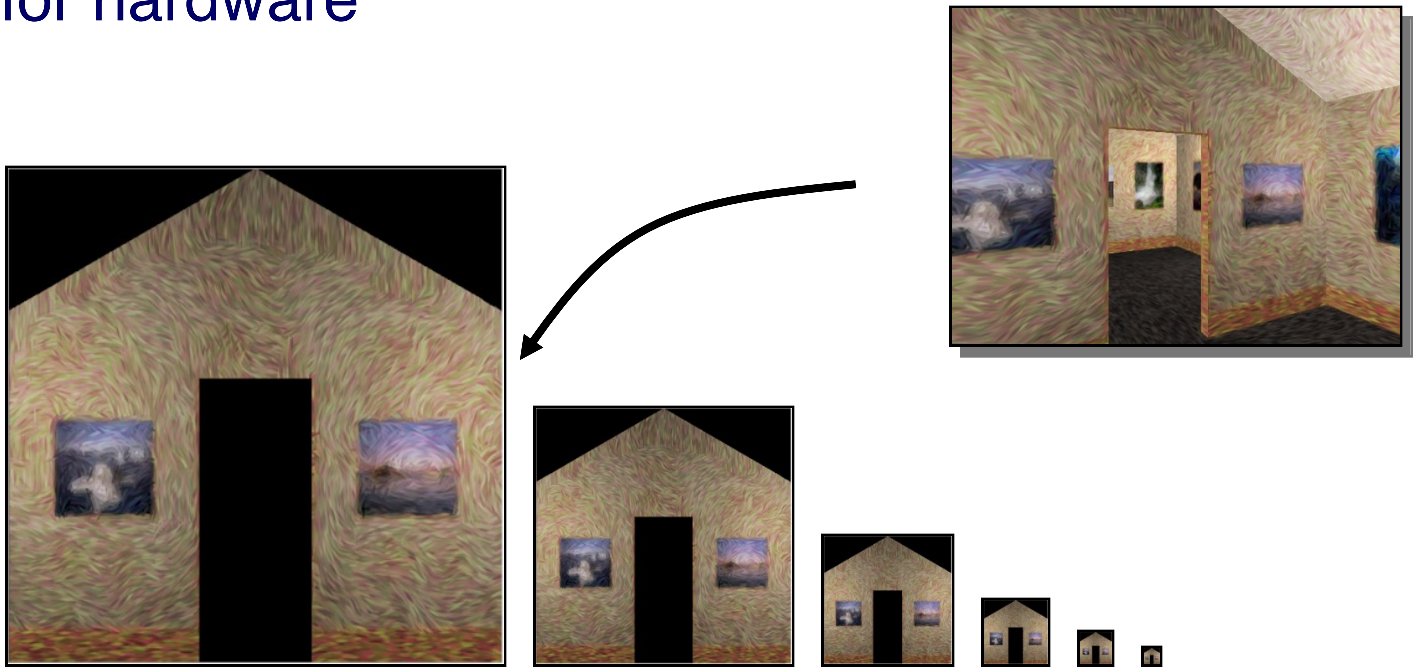
Mipmaps

- Keep textures prefiltered at multiple resolutions
  - Usually powers of 2



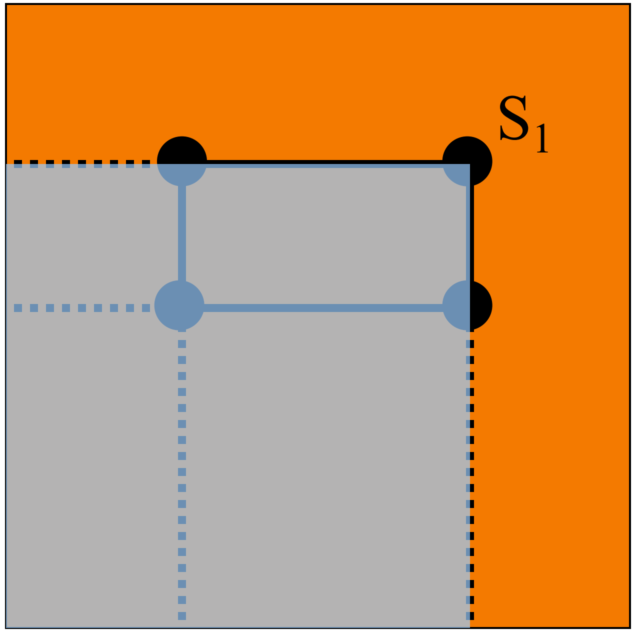
Mipmaps

- Keep textures prefiltered at multiple resolutions
  - Usually powers of 2
  - For each pixel, linearly interpolate between two closest levels (i.e., trilinear filtering)
  - Fast, easy for hardware



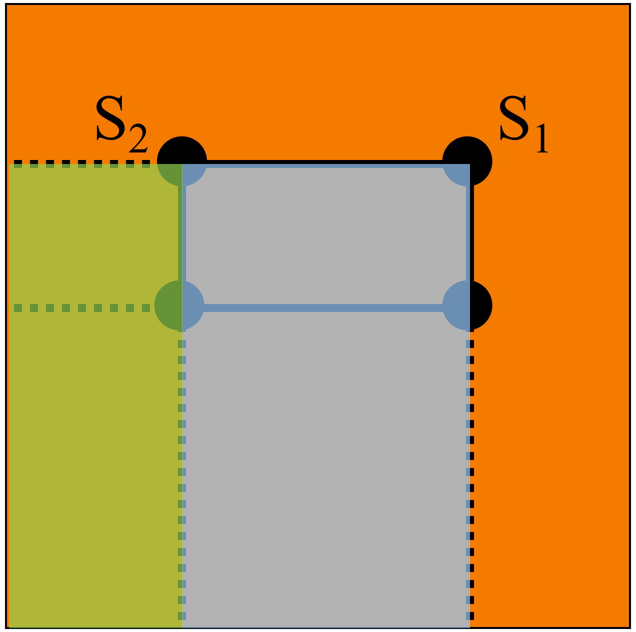
Summed-area tables

- At each texel keep sum of all values down & left
  - To compute sum of all values within a rectangle, simply combine four entries: S_1



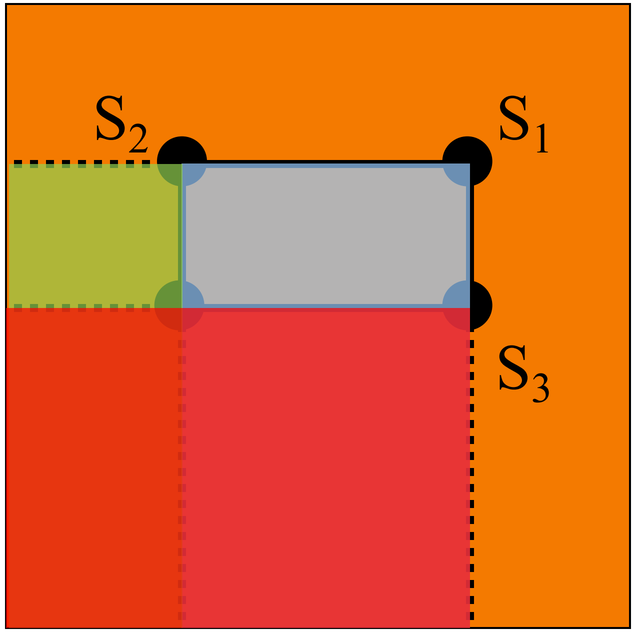
Summed-area tables

- At each texel keep sum of all values down & left
  - To compute sum of all values within a rectangle, simply combine four entries: $S_1 - S_2$



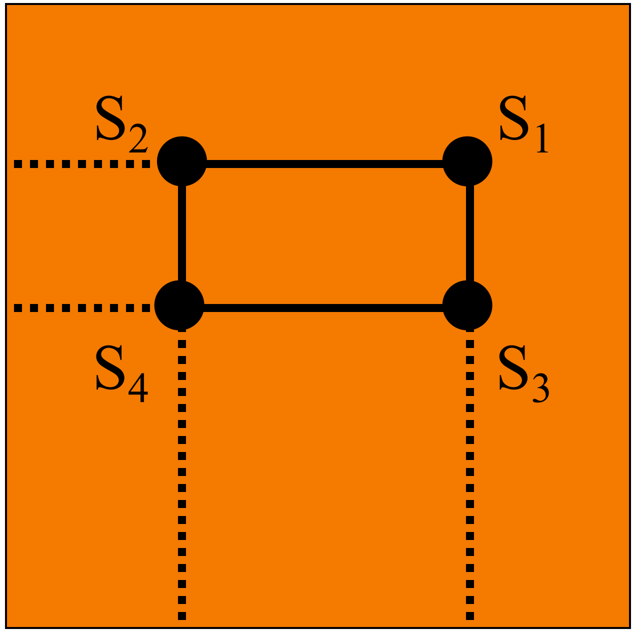
Summed-area tables

- At each texel keep sum of all values down & left
  - To compute sum of all values within a rectangle, simply combine four entries: $S_1 - S_2 - S_3$



Summed-area tables

- At each texel keep sum of all values down & left
  - To compute sum of all values within a rectangle, simply combine four entries: $S_1 - S_2 - S_3 + S_4$
  - Better ability to capture oblique projections, but still not perfect



  - (Mipmaps are more common.)

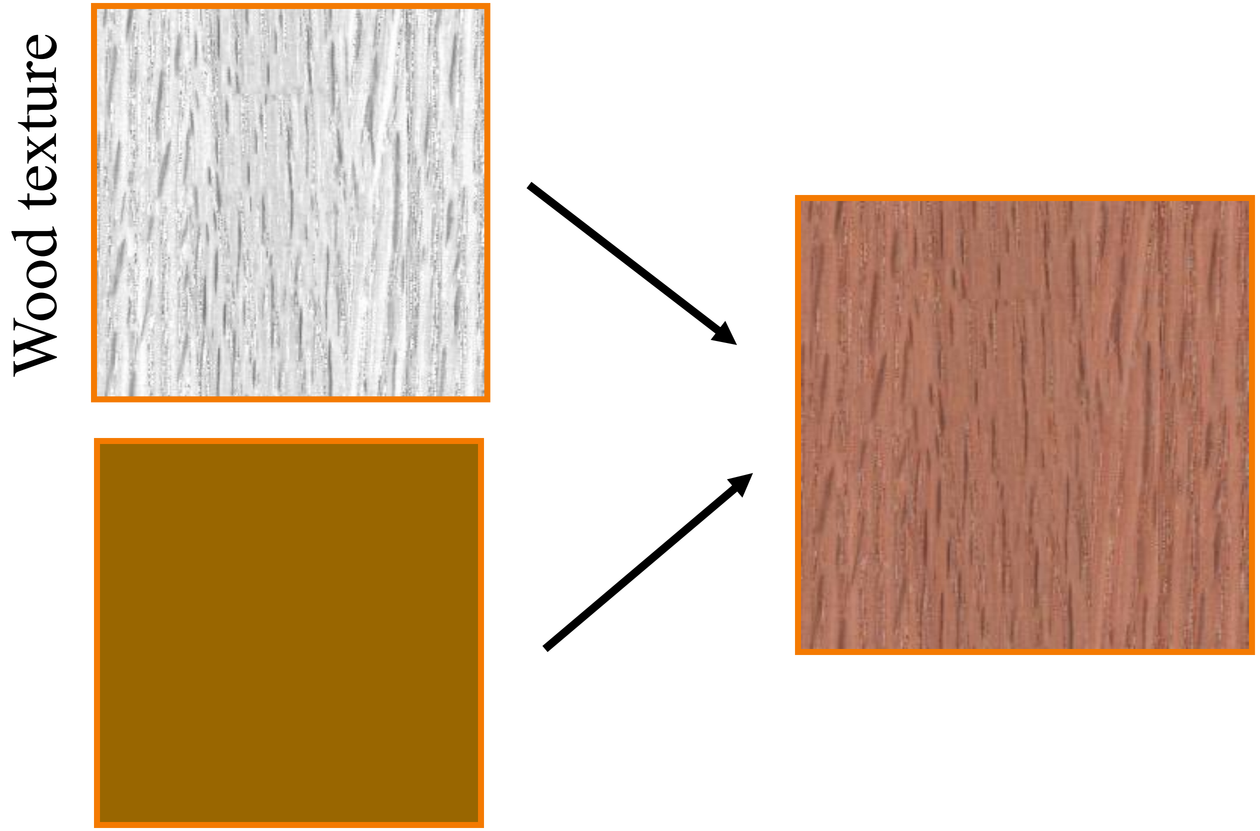
Texture Overview

- Texture mapping stages
  - Parameterization
  - Mapping
  - Filtering

- Texture mapping applications
  - Modulation textures
  - Illumination mapping
  - Bump mapping
  - Environment mapping
  - Image-based rendering

Modulation textures

- Texture values scale result of lighting calculation



Texture value

$$I = T(s,t)\left(I_E + K_A I_A + \sum_L \left(K_D (N \cdot L) + K_S (V \cdot R)^n\right) S_L I_L + K_T I_T + K_S I_S\right)$$

Illumination Mapping

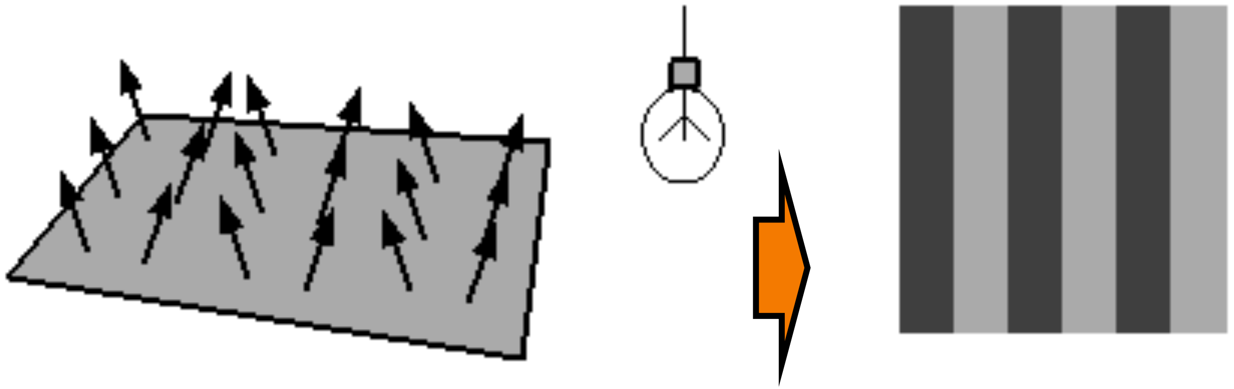
- Map texture values to surface material parameter
  - K_A
  - K_D
  - K_S
  - K_T
  - n

Texture value

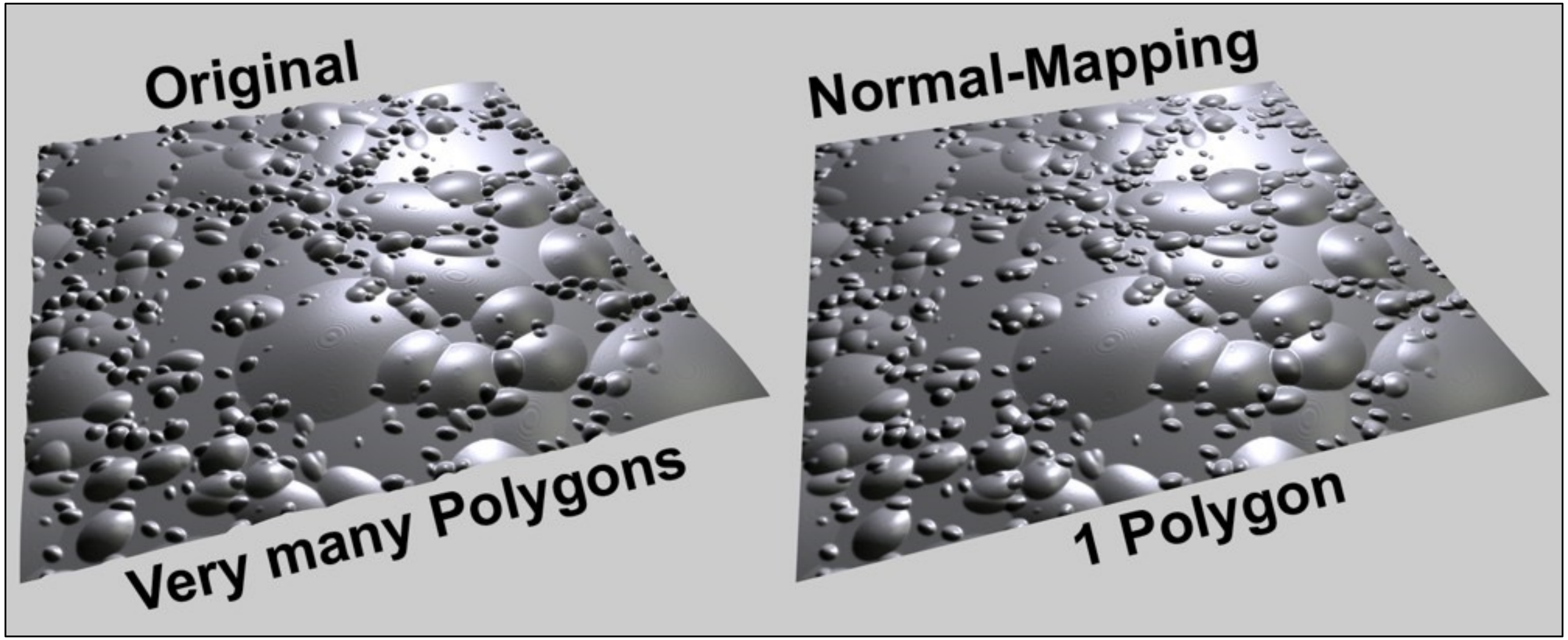
$$I = I_E + K_A I_A + \sum_L \left(K_D(s,t)(N \cdot L) + K_S (V \cdot R)^n\right) S_L I_L + K_T I_T + K_S I_S$$

Bump/Normal Mapping

- Texture values determine or perturb surface normals:
  - Encode normals in RGB (R → N_x, G → N_y, B → N_z, 0..255 → -1..1)
  - Or encode normal offsets in RGB
  - Or use gradient of grayscale image as normal offset ("bump mapping")

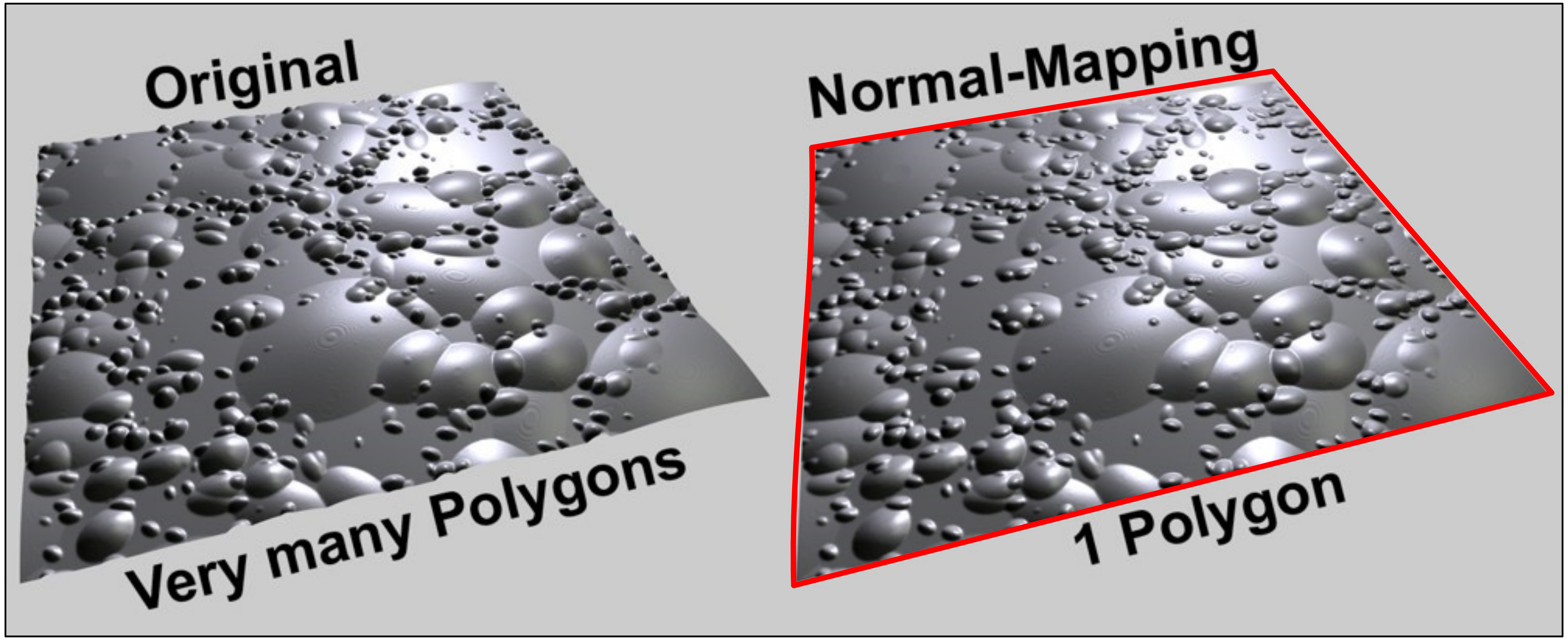


Normal Mapping



Graphisoft.com

Normal Mapping

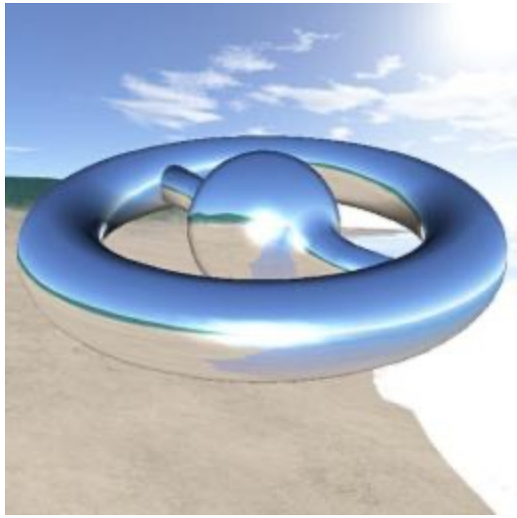


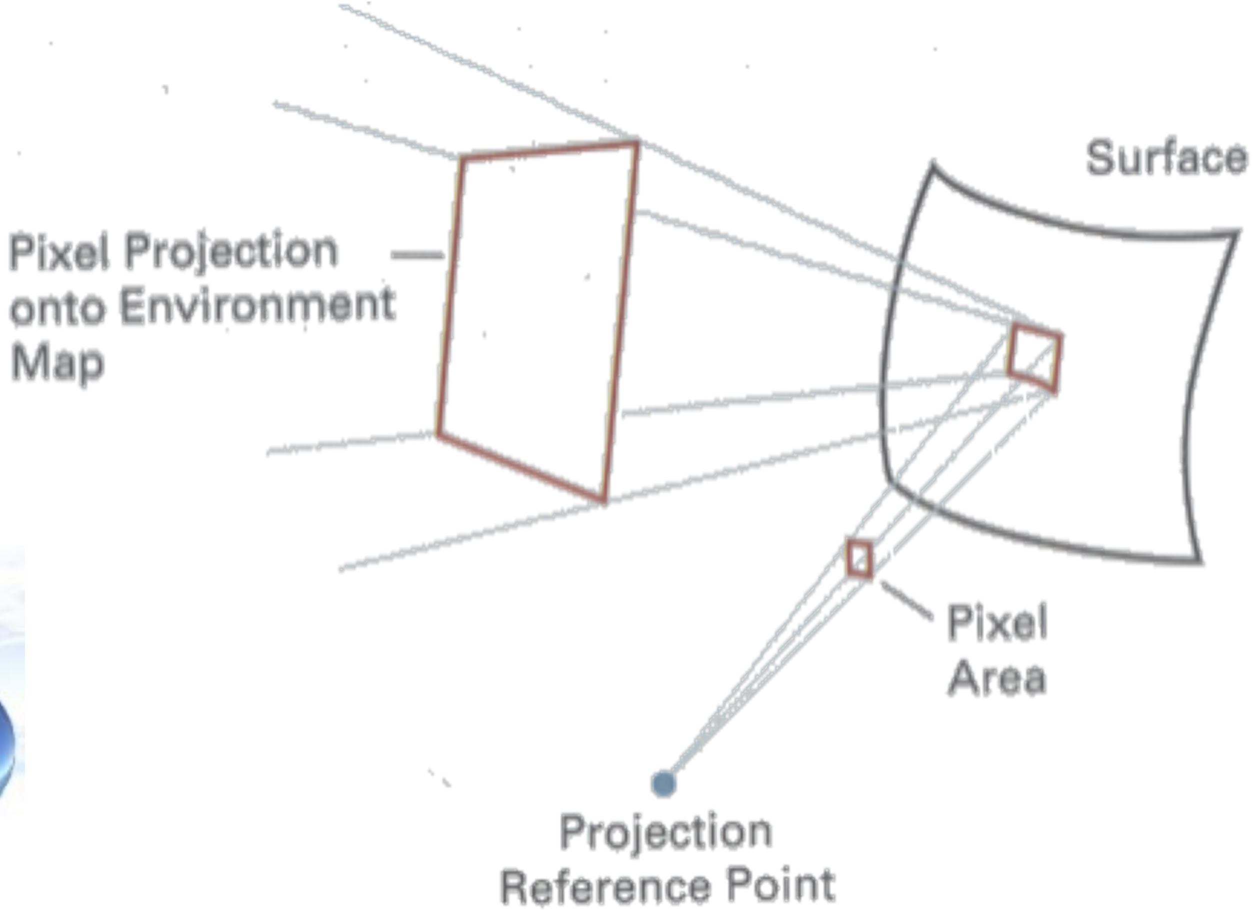
Graphisoft.com

Environment Mapping

- Texture values are reflected off surface patch

Gamer3D/Wikipedia

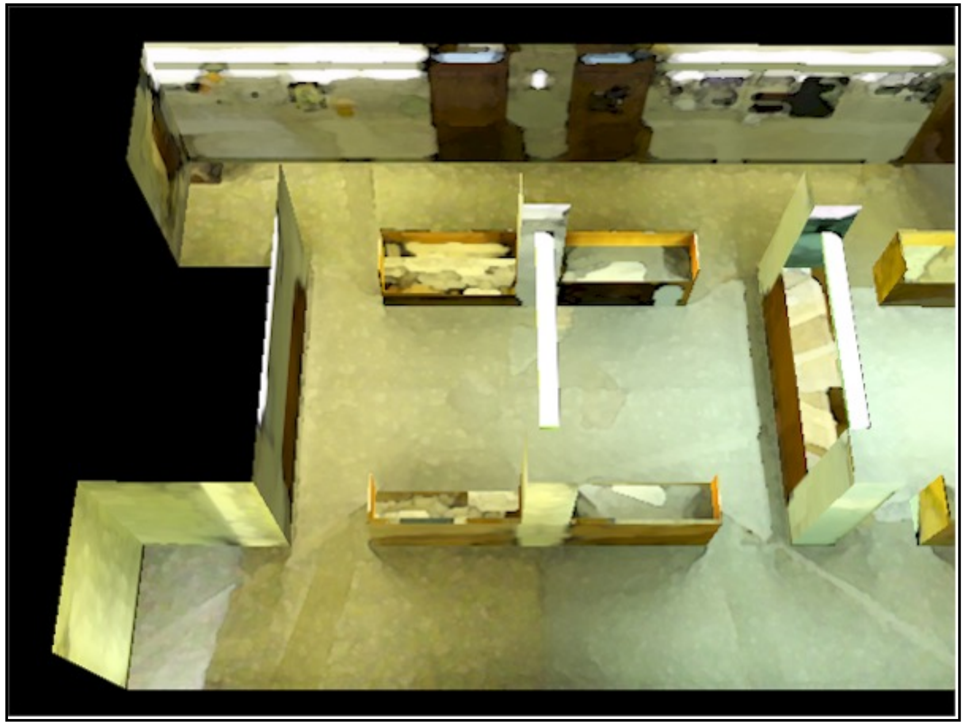




H&B Figure 14.93

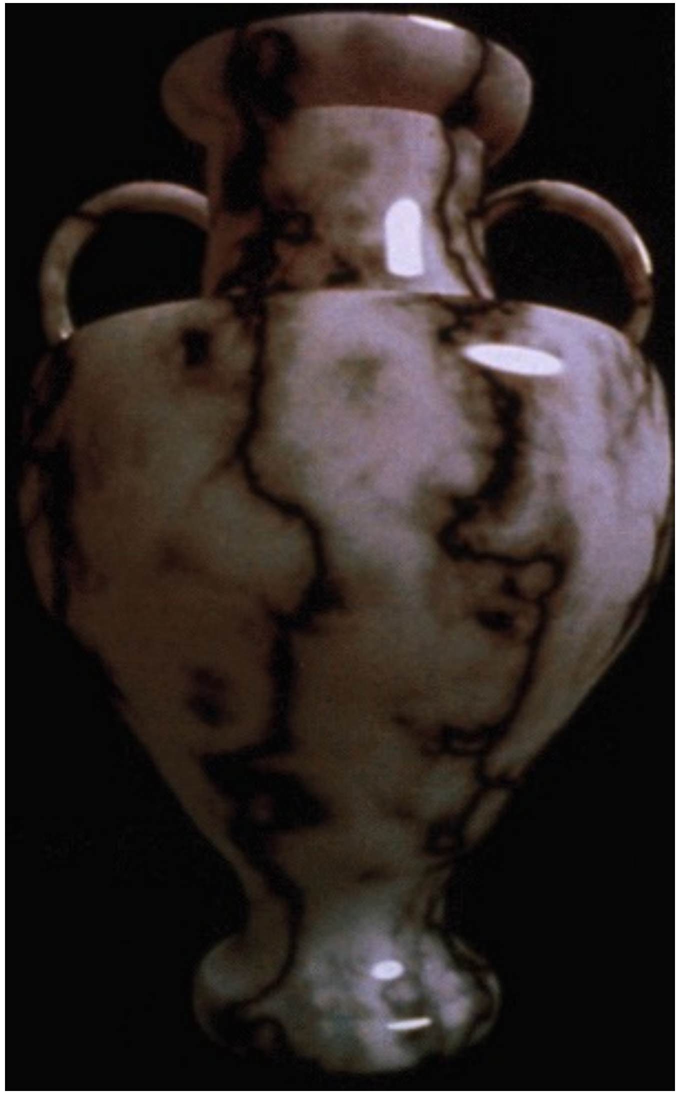
Image-Based Rendering

- Map photographic textures to provide details for coarsely detailed polygonal model



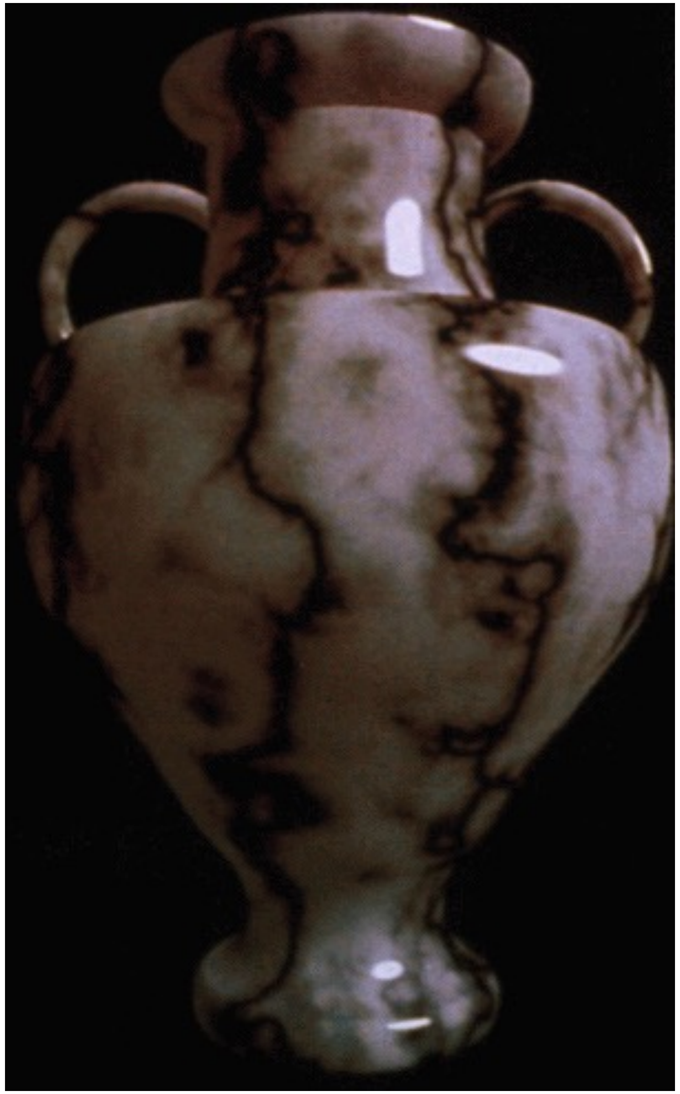
Solid textures

- Texture values indexed by 3D location (x,y,z)
  - Expensive storage, or



Solid textures

- Texture values indexed by 3D location (x,y,z)
  - Expensive storage, or
  - Compute on the fly, e.g. Perlin noise →

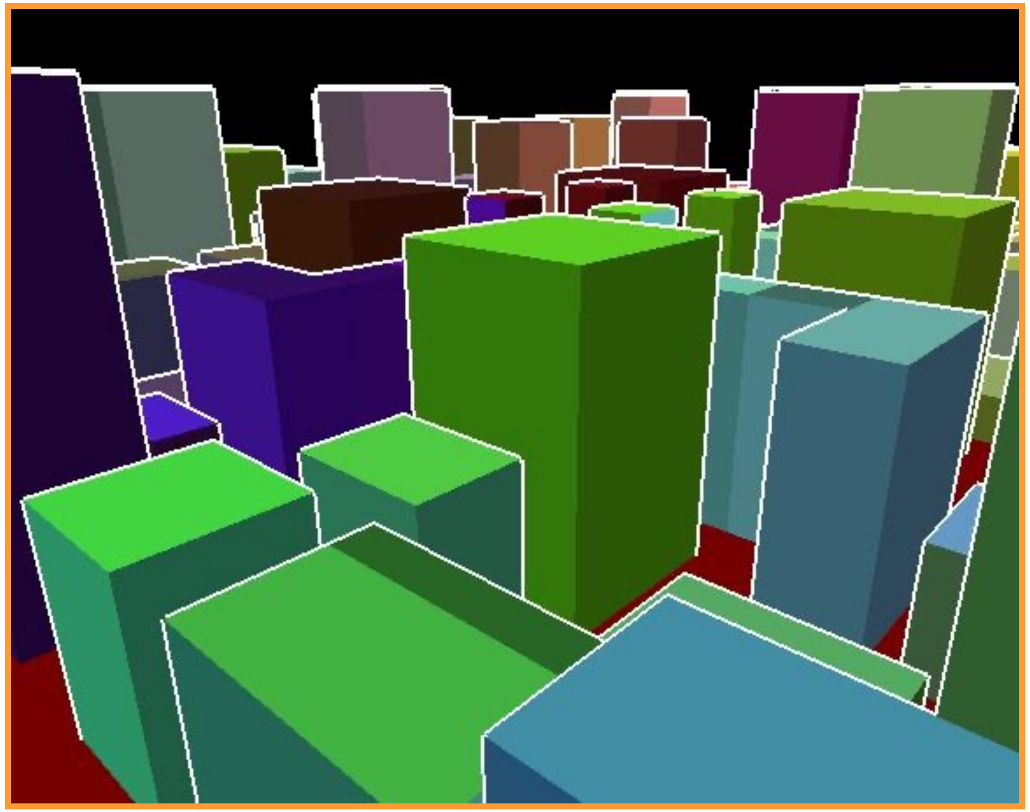


Rasterization

- Scan conversion
  - Determine which pixels to fill
- Shading
  - Determine a color for each filled pixel
- Texture mapping
  - Describe shading variation within polygon interiors
- ➢ Visible surface determination
  - Figure out which surface is front-most at every pixel

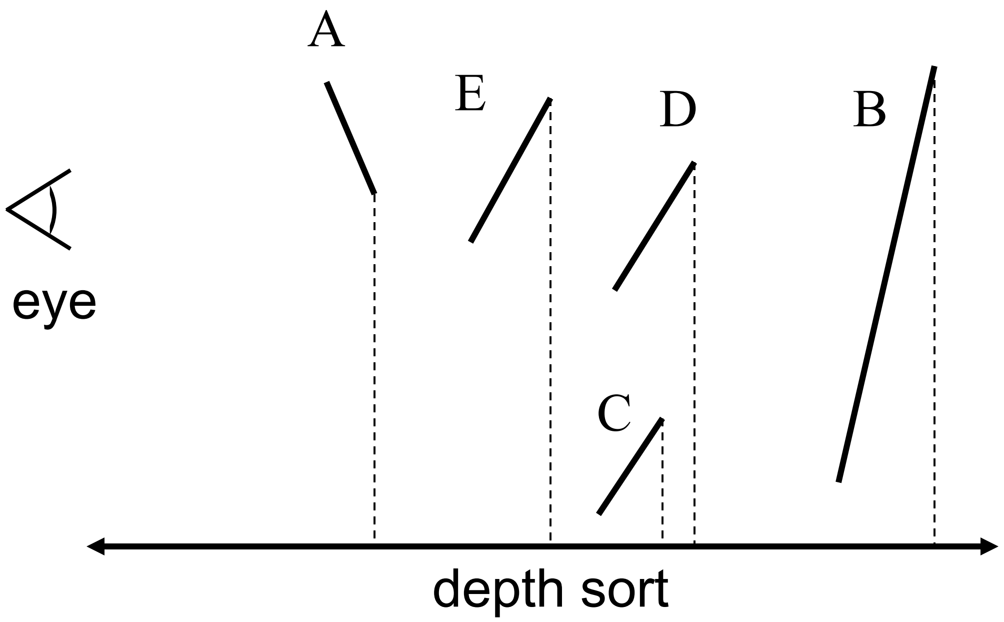
Visible Surface Determination

- Make sure only front-most surface contributes to color at every pixel



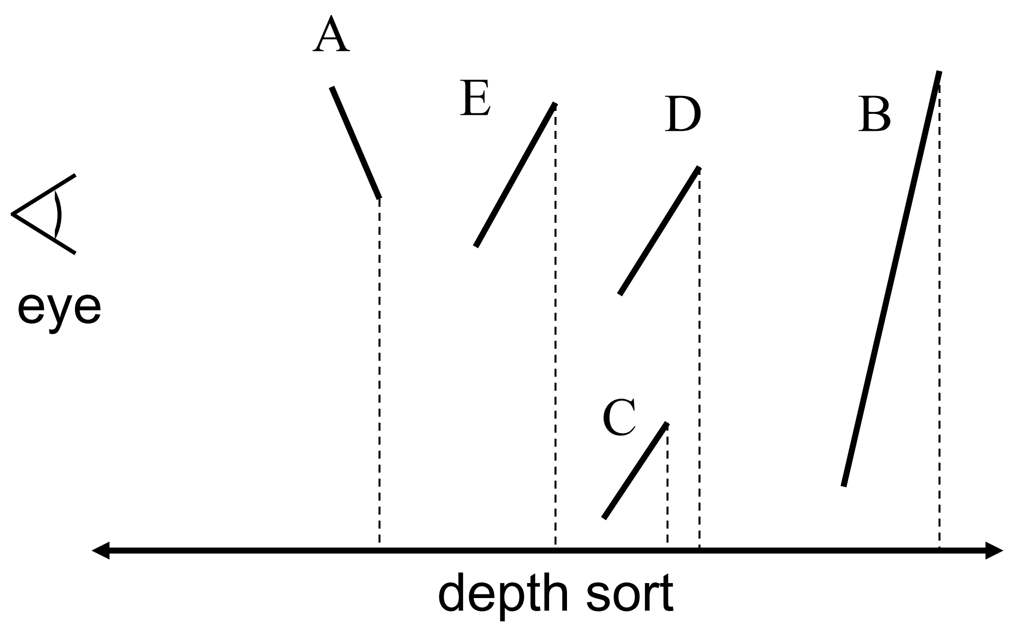
Depth sort

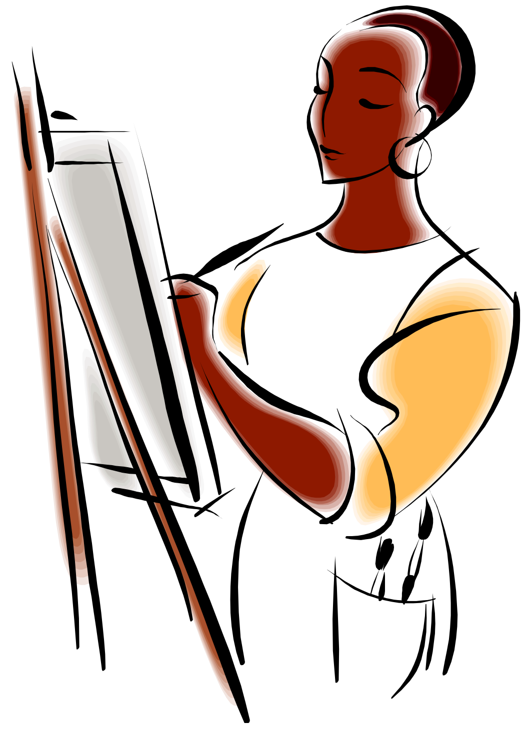
- "Painter's algorithm"
  - First sort surfaces in order of decreasing maximum depth



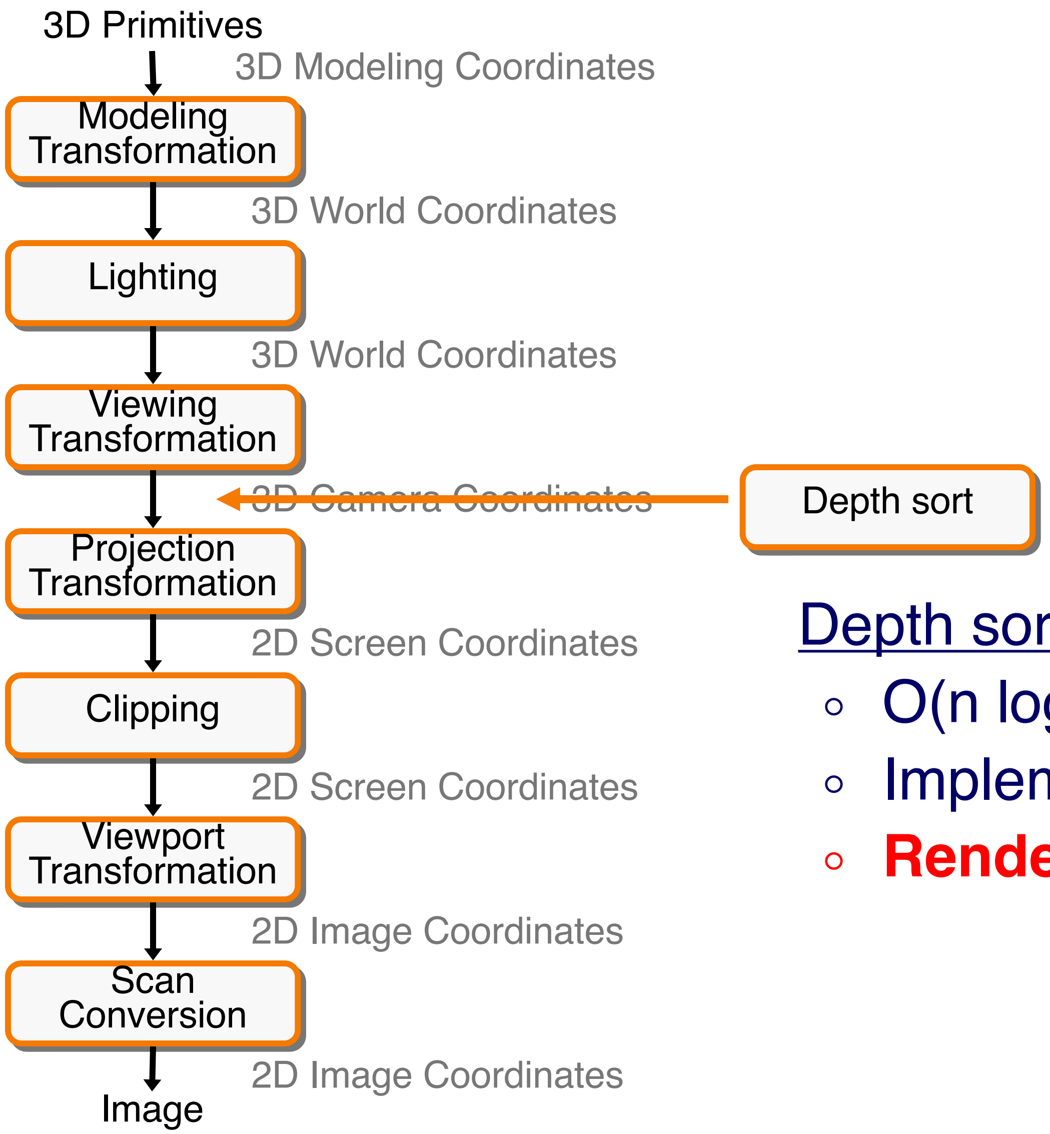
Depth sort

- "Painter's algorithm"
  - First sort surfaces in order of decreasing maximum depth
  - Scan convert surfaces in back-to-front order, always overwriting pixels





3D Rasterization Pipeline

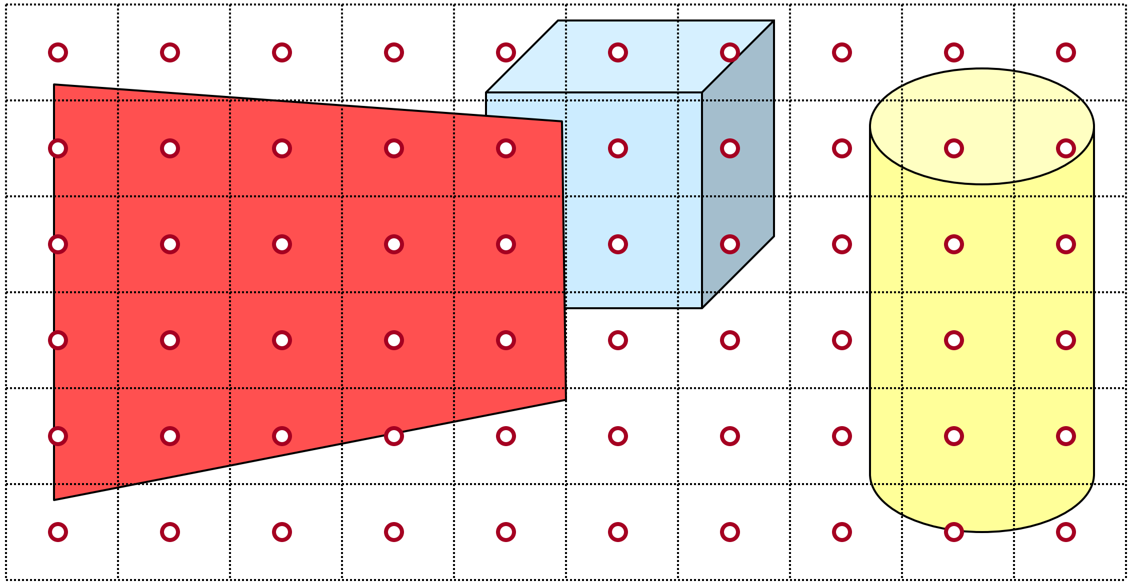


Depth sort comments

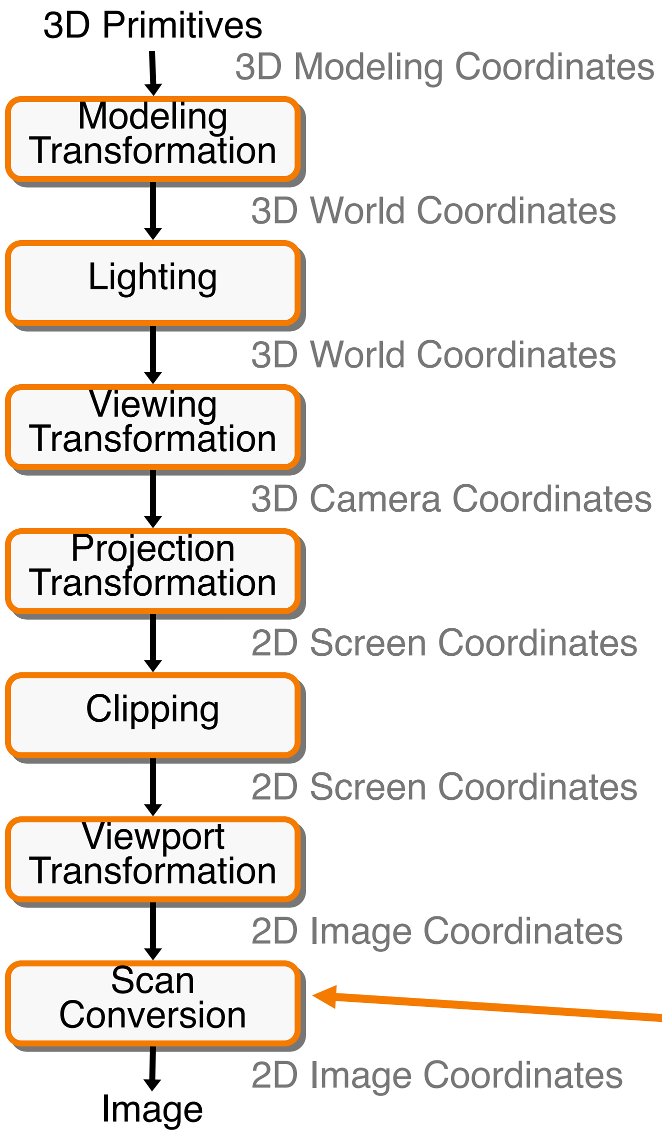
- O(n log n)
- Implemented in software
- **Render pixels of every polygon**

Z-Buffer

- Maintain color & depth of closest object per pixel
  - Framebuffer now RGBAz – initialize z to far plane
  - Update only pixels with depth closer than currently in z-buffer
  - Depths are interpolated for in-primitive pixels from vertices, just like colors

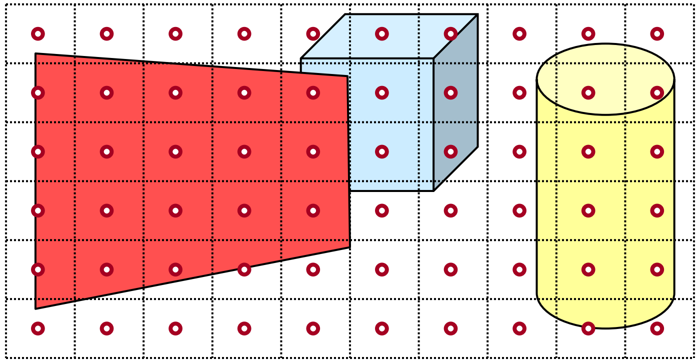


Z-Buffer

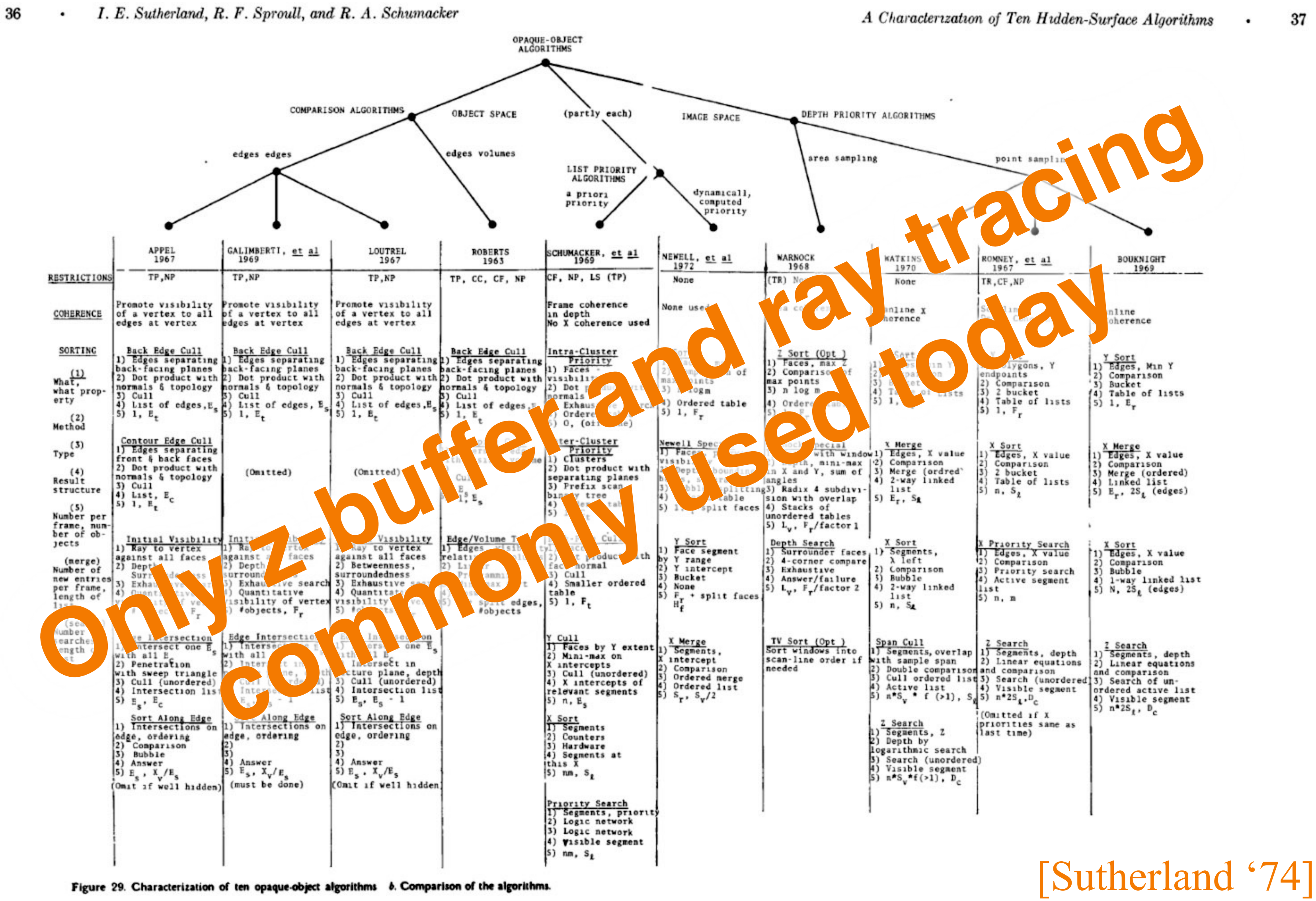


Z-buffer comments

+ Polygons rasterized in any order
+ Process one polygon at a time
+ Suitable for hardware pipeline
- Requires extra memory for z-buffer
◦ Commonly in hardware



Hidden Surface Removal Algorithms



Only z-buffer and ray tracing commonly used today

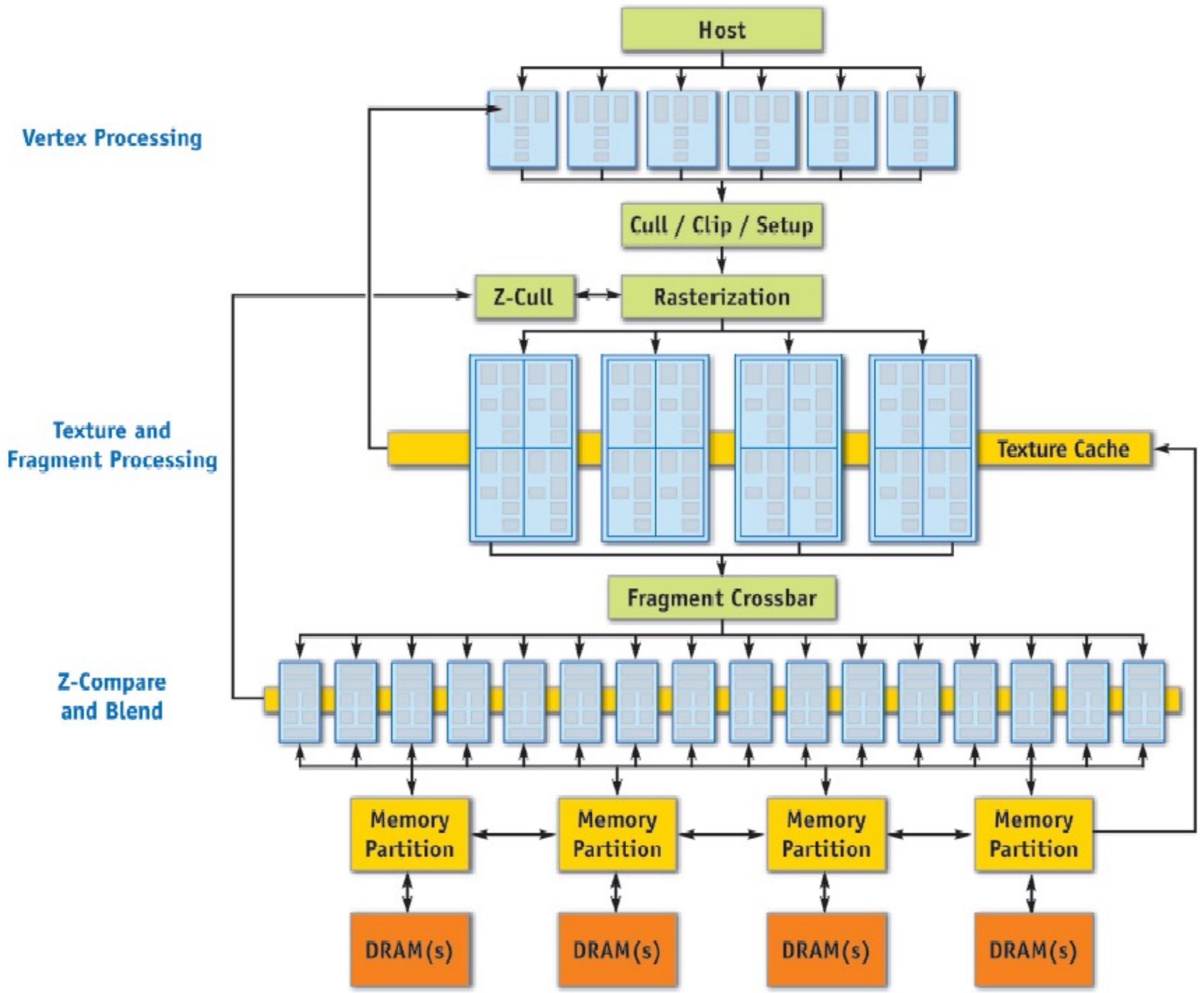
[Sutherland '74]

Rasterization Summary

- Scan conversion
  - Sweep-line algorithm
- Shading algorithms
  - Flat, Gouraud, Phong
- Texture mapping
  - Mipmaps
- Visibiliity determination
  - Z-buffer

This is all in hardware

GPU Architecture

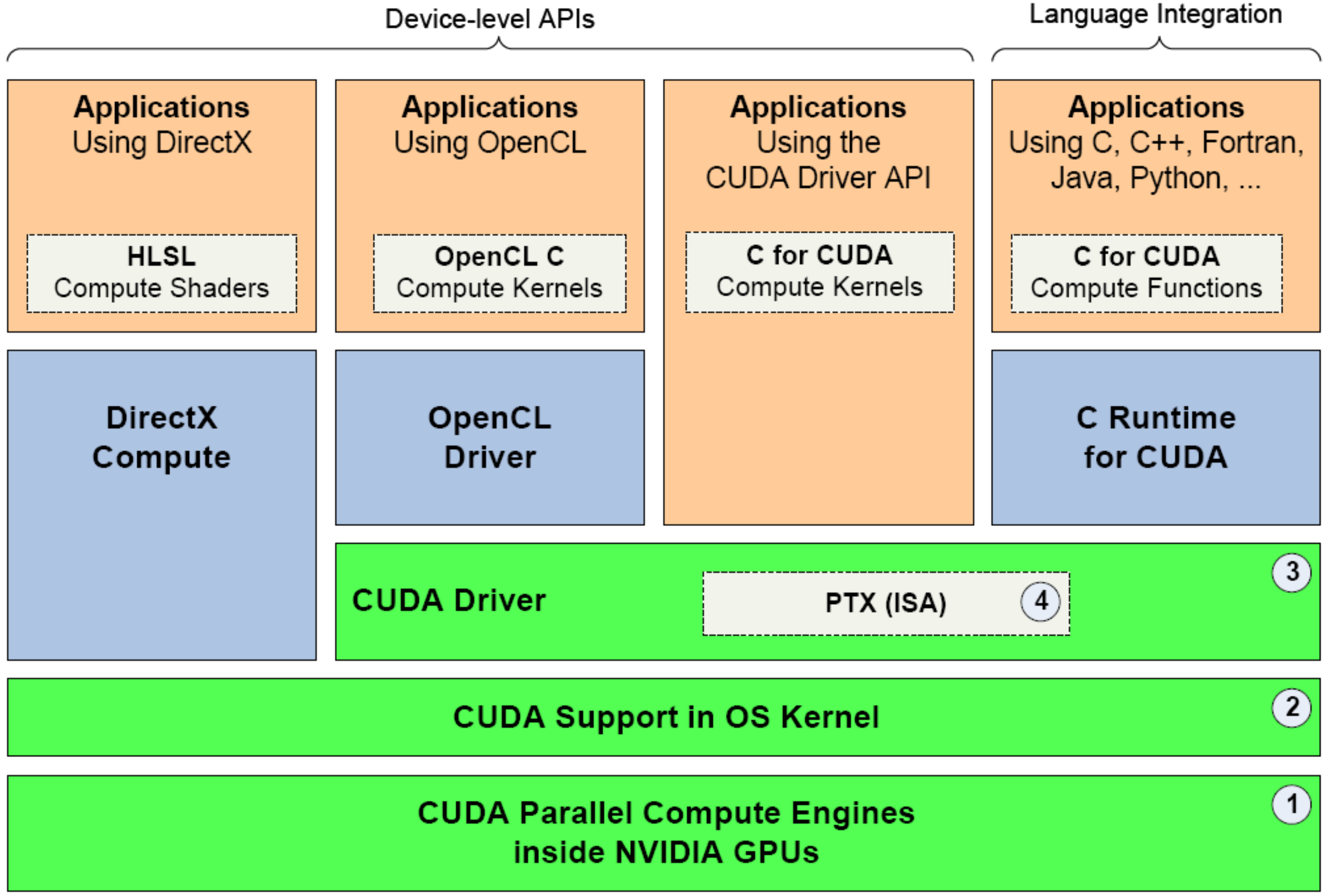


GeForce 6 Series Architecture

GPU Gems 2, NVIDIA

Actually …

- Modern graphics hardware is programmable



www.nvidia.com/cuda

Trend …

- GPU is general-purpose parallel computer

www.nvidia.com/cuda