The 3D Rasterization Pipeline

COS 426, Fall 2022

PRINCETON UNIVERSITY

3D Rendering Scenarios

- Offline
  - One image generated with as much quality as possible for a particular set of rendering parameters
    - » Take as much time as is needed (minutes)
    - » Targets photorealistism, movies, etc.

- ➢ Interactive
  - Images generated dynamically, in fraction of a second (e.g., 1/30) as user controls rendering parameters (e.g., camera)
    - » Achieve highest quality possible in given time
    - » Visualization, games, etc.

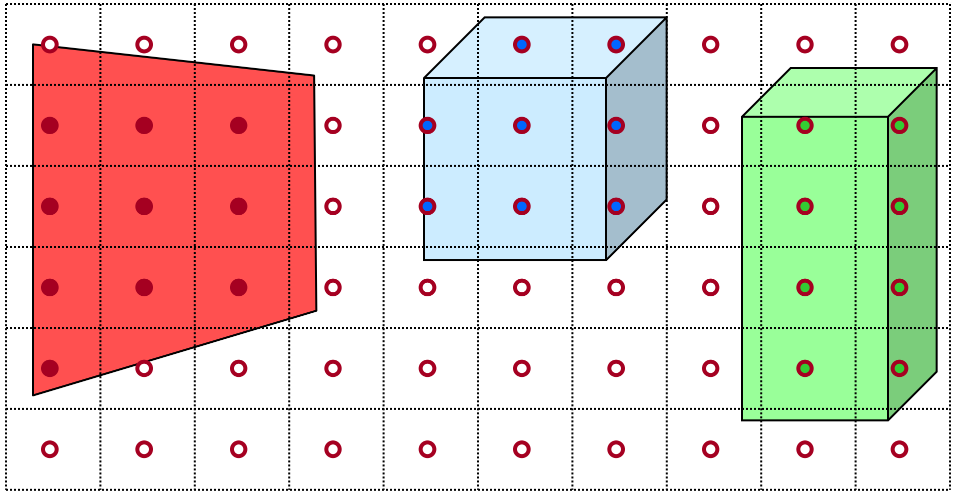
3D Polygon Rendering

- Many applications render 3D polygons with direct illumination

Valve

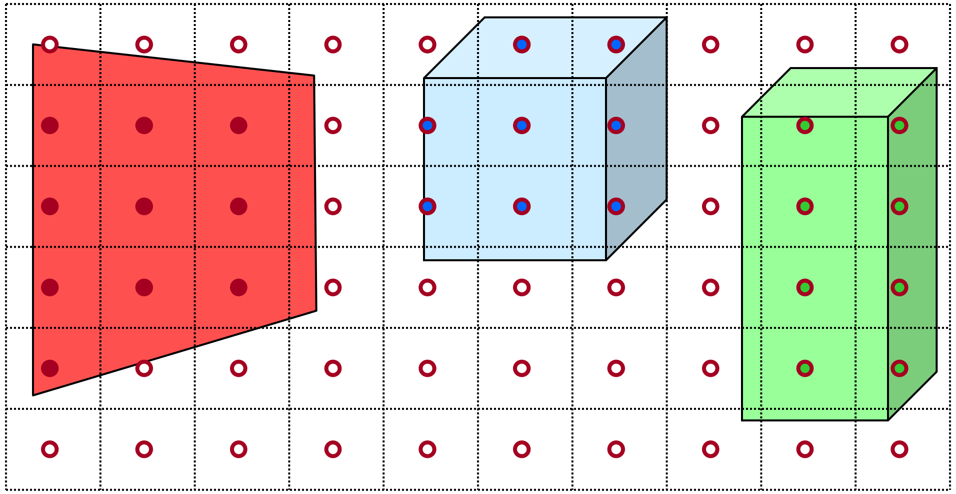
Ray Casting Revisited

- For each sample …
  - Construct ray from eye position through view plane
  - Find first surface intersected by ray through pixel
  - Compute color of sample based on illumination



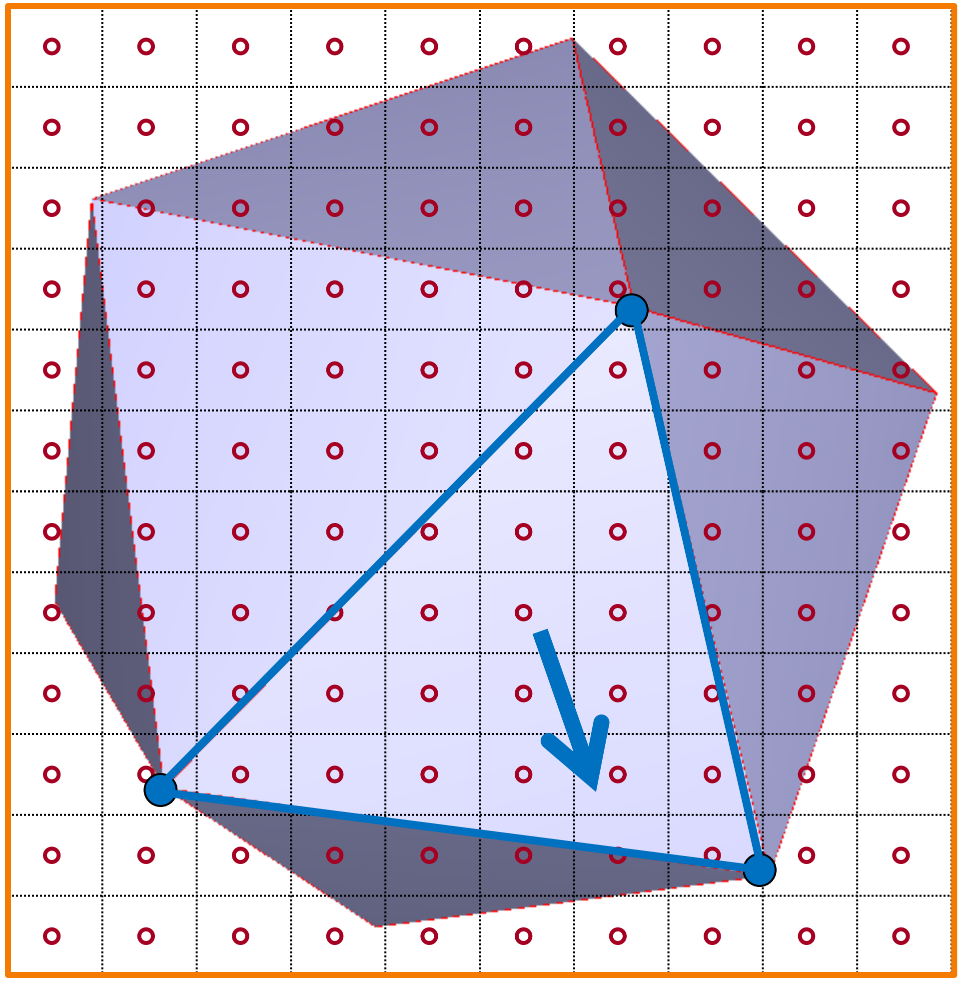
3D Polygon Rasterization

- We can render polygons faster if we take advantage of spatial coherence



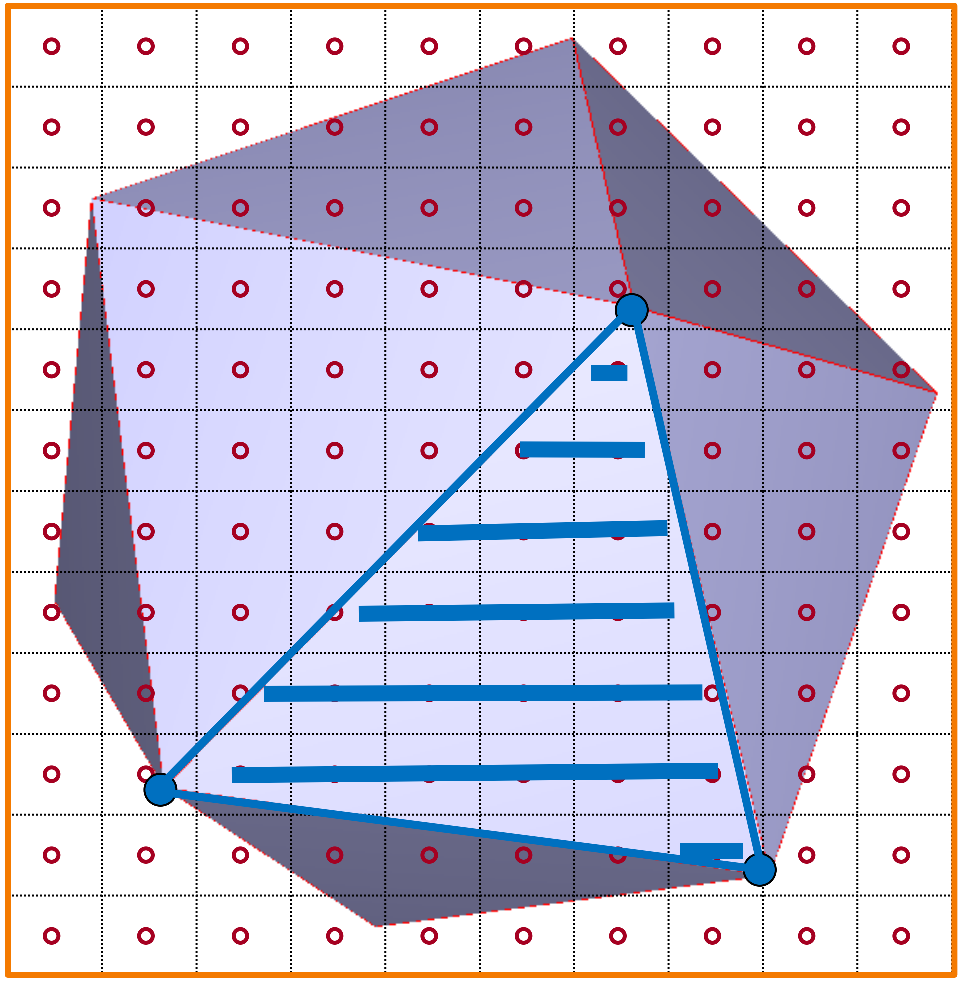
3D Polygon Rasterization

- How?

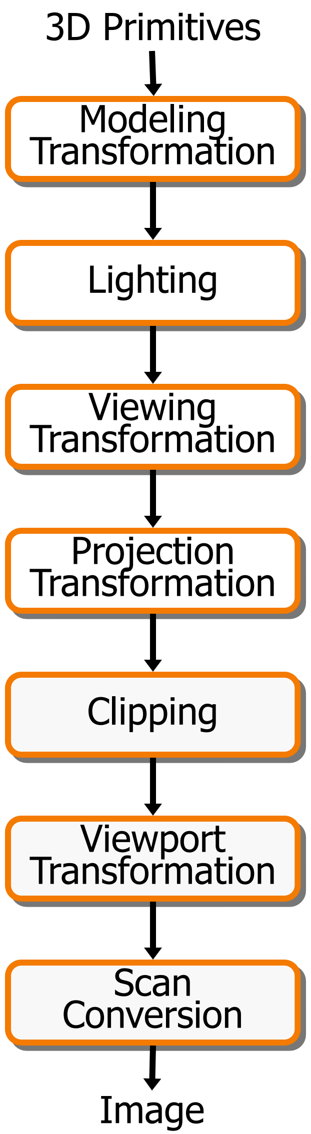


3D Polygon Rasterization

- How?

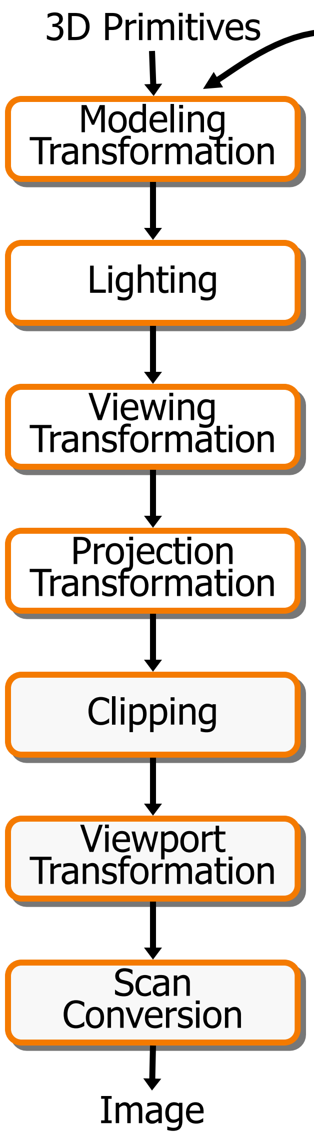


Rasterization Pipeline (for direct illumination)



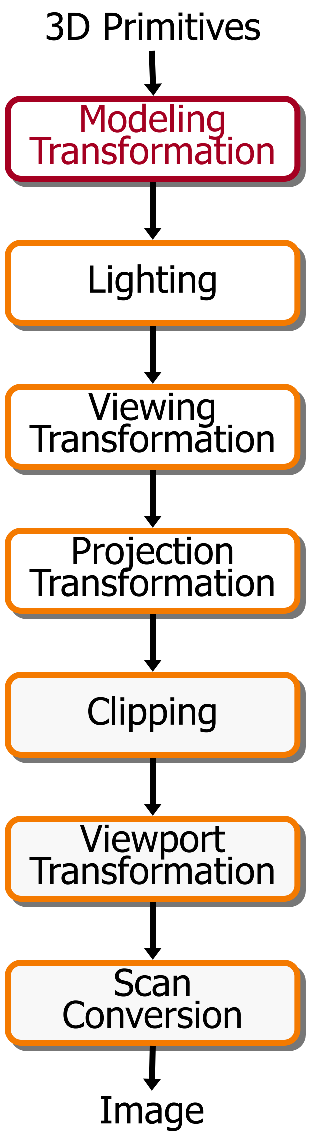
This is a pipelined sequence of operations to draw 3D primitives into a 2D image

Rasterization Pipeline (for direct illumination)



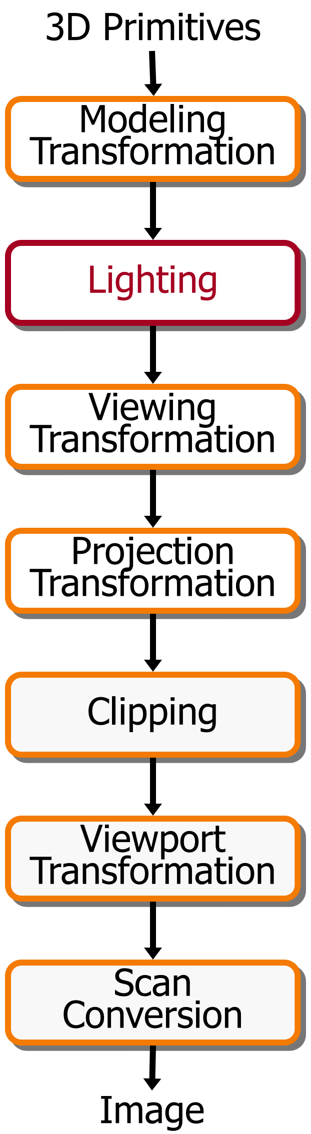
Vertex coordinates

Rasterization Pipeline (for direct illumination)



Transform into 3D world coordinate system

Rasterization Pipeline (for direct illumination)

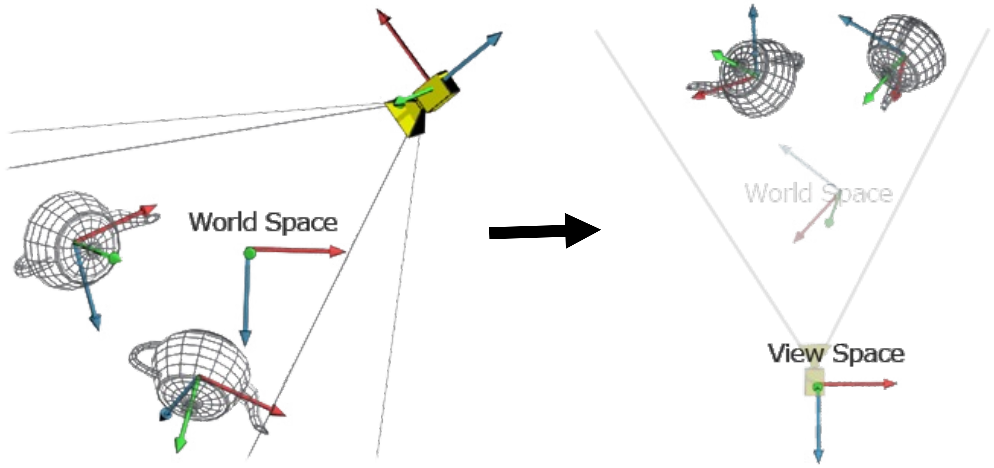


Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

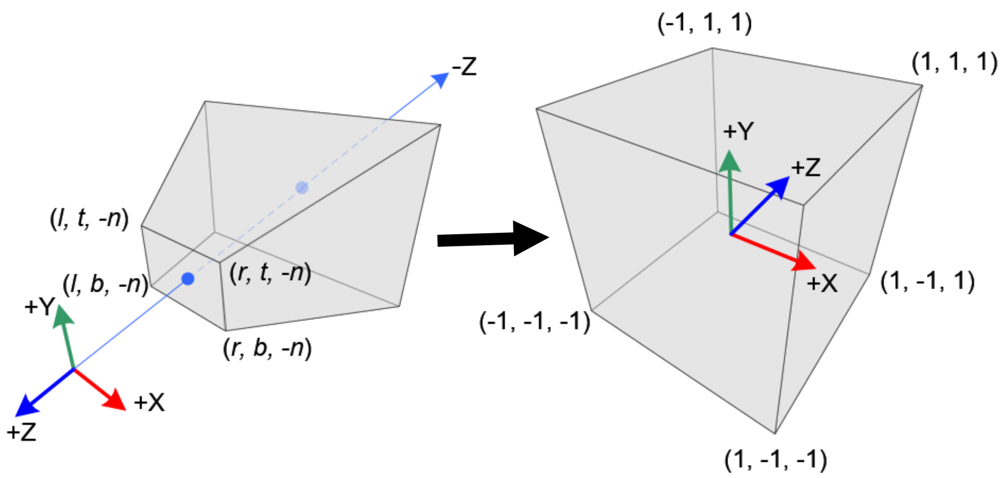
Rasterization Pipeline (for direct illumination)

3D Primitives
↓
Modeling Transformation — Transform into 3D world coordinate system
↓
Lighting — Illuminate according to lighting and reflectance
↓
Viewing Transformation — Transform into 3D camera coordinate system
↓
Projection Transformation
↓
Clipping
↓
Viewport Transformation
↓
Scan Conversion
↓
Image



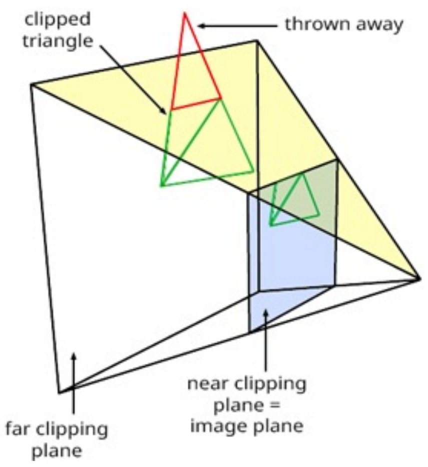
Rasterization Pipeline (for direct illumination)

3D Primitives
↓
Modeling Transformation — Transform into 3D world coordinate system
↓
Lighting — Illuminate according to lighting and reflectance
↓
Viewing Transformation — Transform into 3D camera coordinate system
↓
Projection Transformation — Transform into 2D camera coordinate system
↓
Clipping
↓
Viewport Transformation
↓
Scan Conversion
↓
Image



Rasterization Pipeline (for direct illumination)

3D Primitives
↓
Modeling Transformation — Transform into 3D world coordinate system
↓
Lighting — Illuminate according to lighting and reflectance
↓
Viewing Transformation — Transform into 3D camera coordinate system
↓
Projection Transformation — Transform into 2D camera coordinate system
↓
Clipping — Clip primitives outside camera's view
↓
Viewport Transformation
↓
Scan Conversion
↓
Image



Rasterization Pipeline (for direct illumination)

3D Primitives

↓

Modeling Transformation — Transform into 3D world coordinate system

↓

Lighting — Illuminate according to lighting and reflectance

↓

Viewing Transformation — Transform into 3D camera coordinate system

↓

Projection Transformation — Transform into 2D camera coordinate system

↓

Clipping — Clip primitives outside camera's view … in clip space

↓

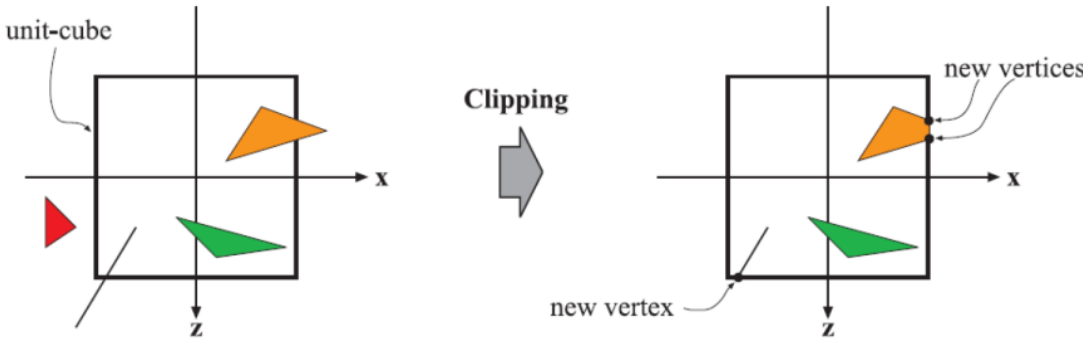
Viewport Transformation

↓

Scan Conversion

↓

Image



Rasterization Pipeline (for direct illumination)

3D Primitives

↓

Modeling Transformation — Transform into 3D world coordinate system

↓

Lighting — Illuminate according to lighting and reflectance

↓

Viewing Transformation

↓

Projection Transformation

↓

Clipping

↓

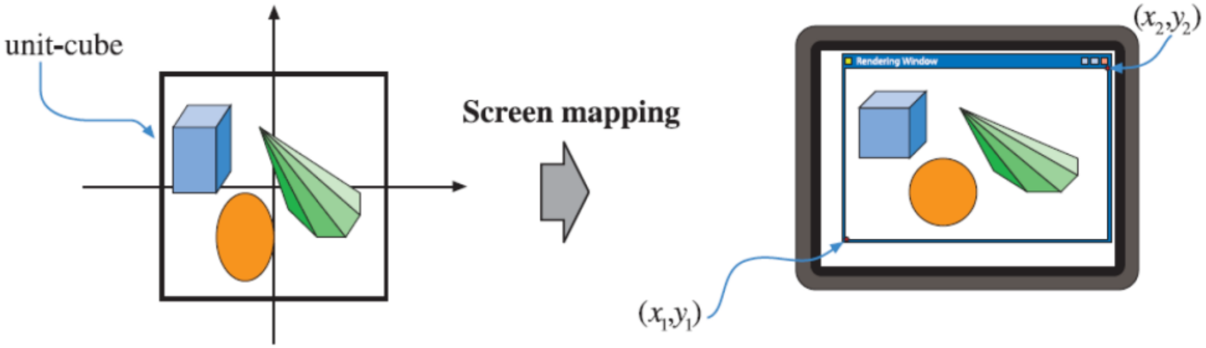
Viewport Transformation — Transform into image coordinate system

↓

Scan Conversion

↓

Image



Rasterization Pipeline (for direct illumination)

3D Primitives

↓

Modeling Transformation — Transform into 3D world coordinate system

↓

Lighting — Illuminate according to lighting and reflectance

↓

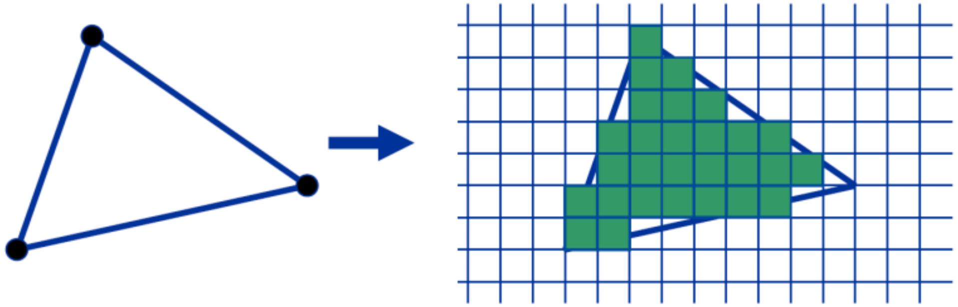
Viewing Transformation — Transform into 3D camera coordinate system

↓

Projection Transformation — Transform into 2D camera coordinate system

↓

Clipping



↓

Viewport Transformation

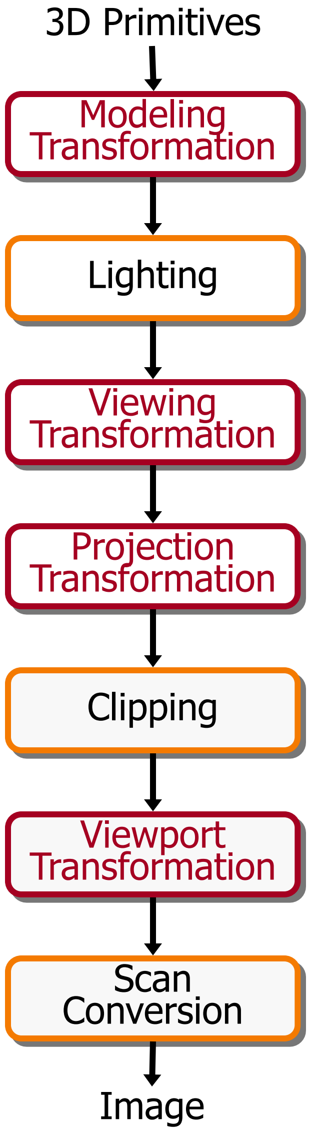
↓

Scan Conversion — Draw pixels (includes texturing, hidden surface, ...)

↓

Image

Rasterization Pipeline (for direct illumination)



Transform into 3D world coordinate system

Illuminate according to lighting and reflectance

Transform into 3D camera coordinate system

Transform into 2D camera coordinate system

Clip primitives outside camera’s view

Transform into image coordinate system

Draw pixels (includes texturing, hidden surface, ...)

Transformations

p(x,y,z)

↓ 3D Object Coordinates

Modeling Transformation

↓ 3D World Coordinates

Viewing Transformation

↓ 3D Camera Coordinates

Projection Transformation

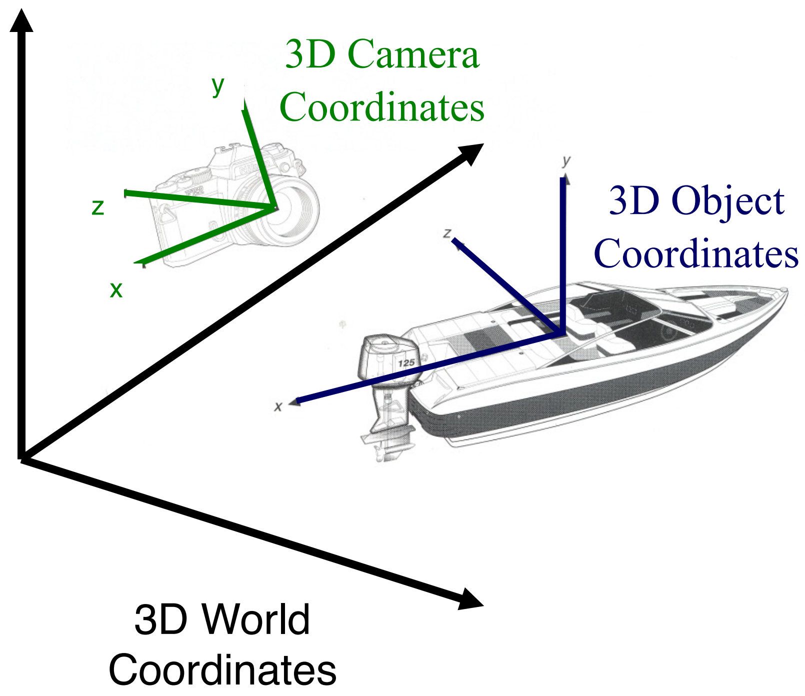
↓ 2D Screen Coordinates

Viewport Transformation

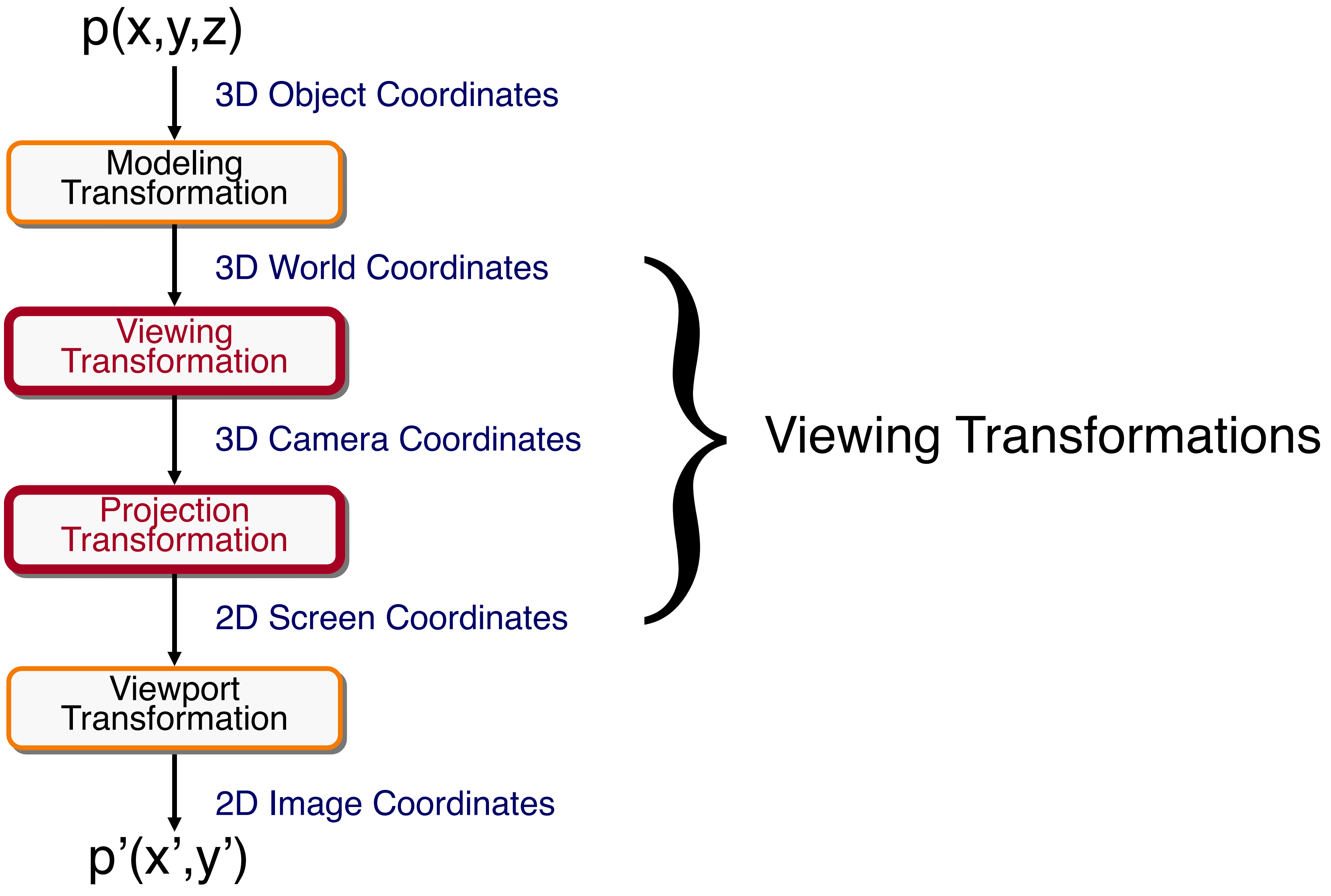
↓ 2D Image Coordinates

p'(x',y')

> Transformations map points from one coordinate system to another

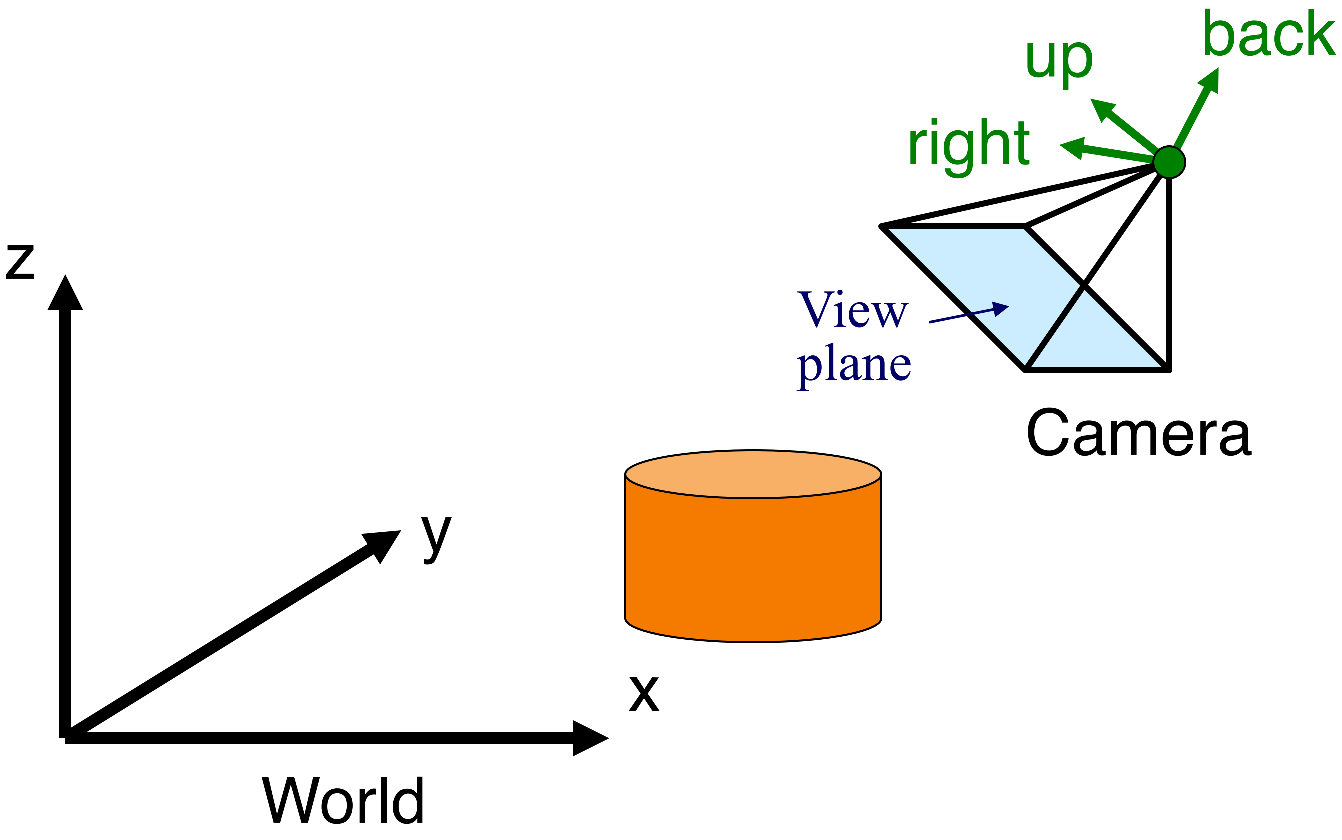


Viewing Transformations



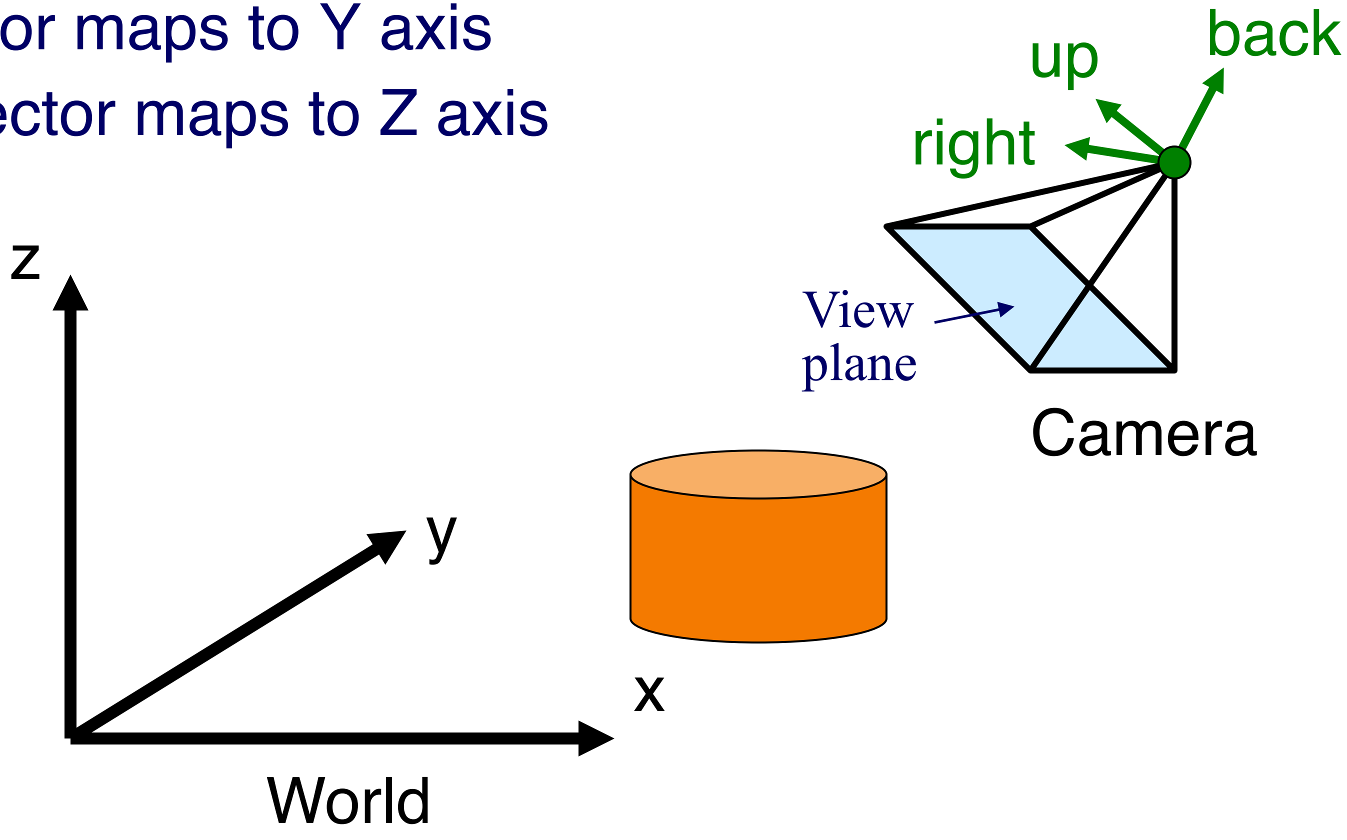
Review: Viewing Transformation

- Mapping from world to camera coordinates
  - Eye position maps to origin



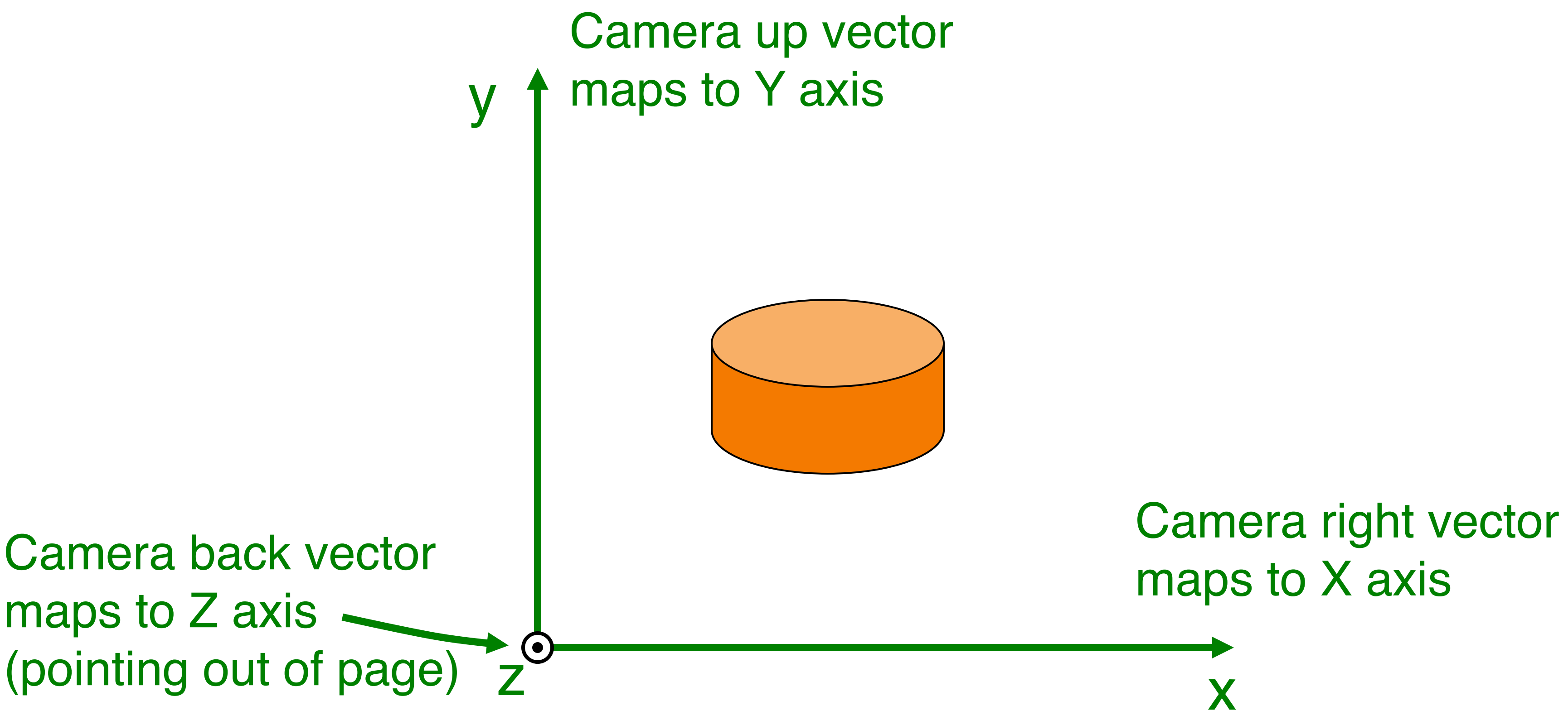
Review: Viewing Transformation

- Mapping from world to camera coordinates
  - Eye position maps to origin
  - Right vector maps to X axis
  - Up vector maps to Y axis
  - Back vector maps to Z axis



Review: Camera Coordinates

- Canonical coordinate system
  - Convention is right-handed (looking down -z axis)
  - Convenient for projection, clipping, etc.



Finding the viewing transformation

- We have the camera (in world coordinates)
- We want T taking objects from world to camera

$$p^c = T\ p^w$$

- Trick: find T^{-1} taking objects in camera to world

$$p^w = T^{-1} p^c$$

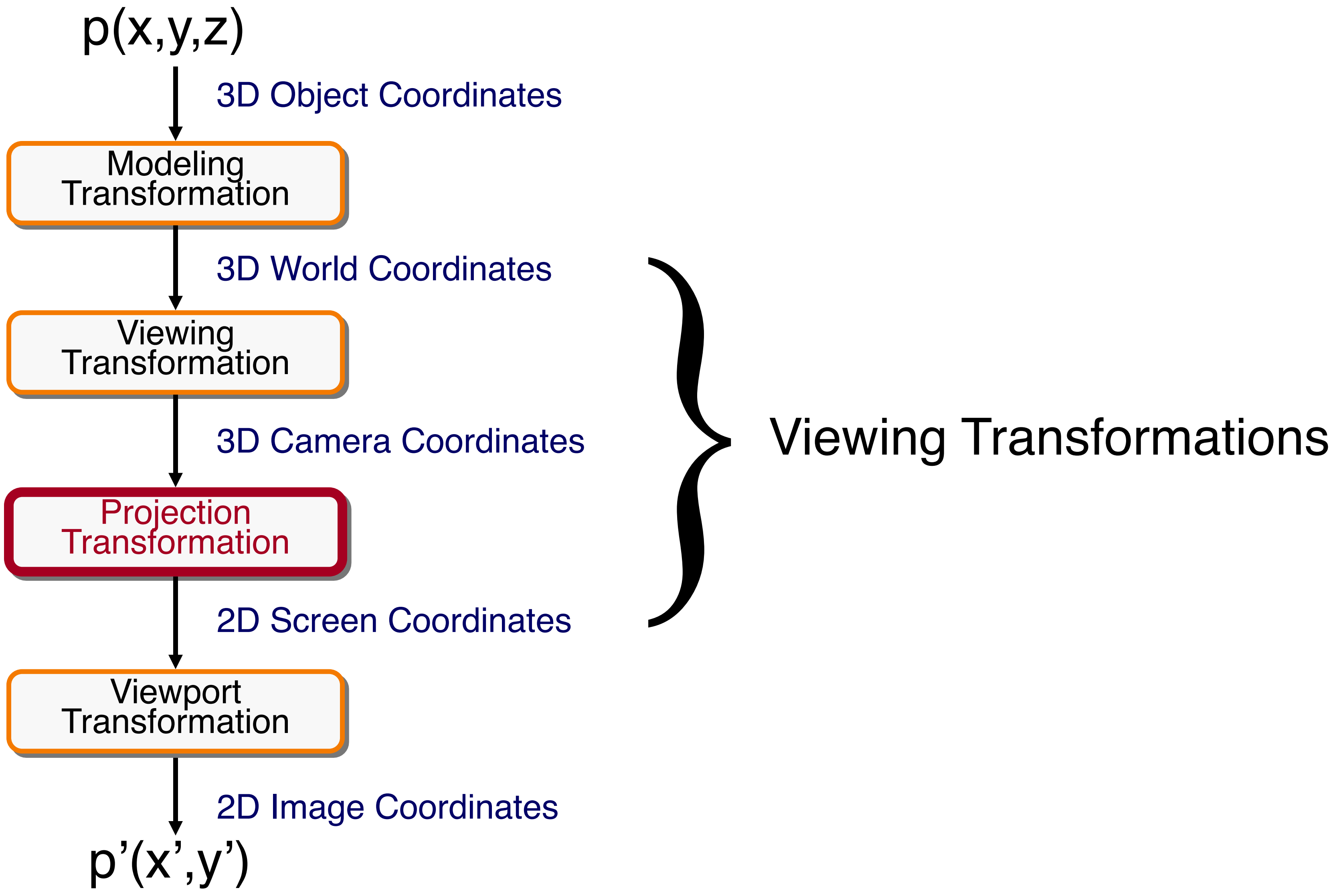
Finding the Viewing Transformation

- Trick: map from camera coordinates to world
  - Origin maps to eye position
  - Z axis maps to Back vector
  - Y axis maps to Up vector
  - X axis maps to Right vector

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} R_x & U_x & B_x & E_x \\ R_y & U_y & B_y & E_y \\ R_z & U_z & B_z & E_z \\ R_w & U_w & B_w & E_w \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

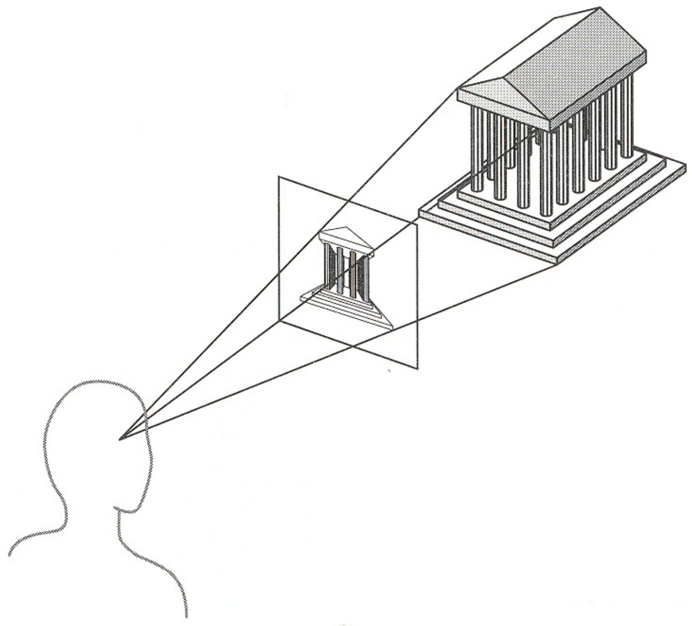
- This matrix is T^{-1} so we invert it to get T … easy!

Viewing Transformations



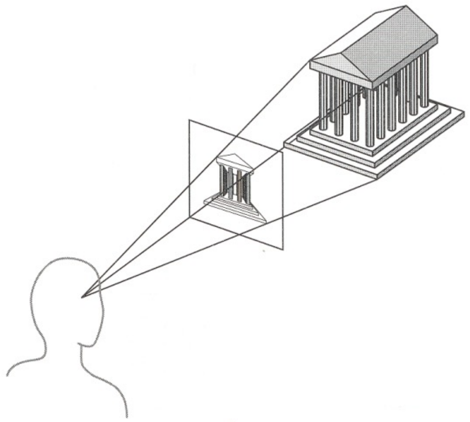
Projection

- General definition:
  - Transform points in n-space to m-space (m<n)
- In computer graphics:
  - Map 3D camera coordinates to 2D screen coordinates

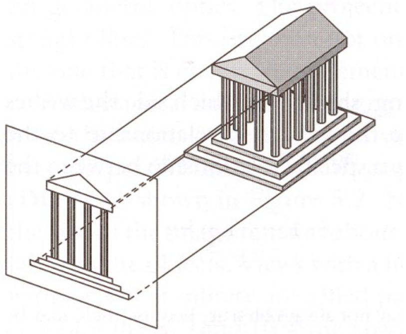


Perspective vs. Parallel

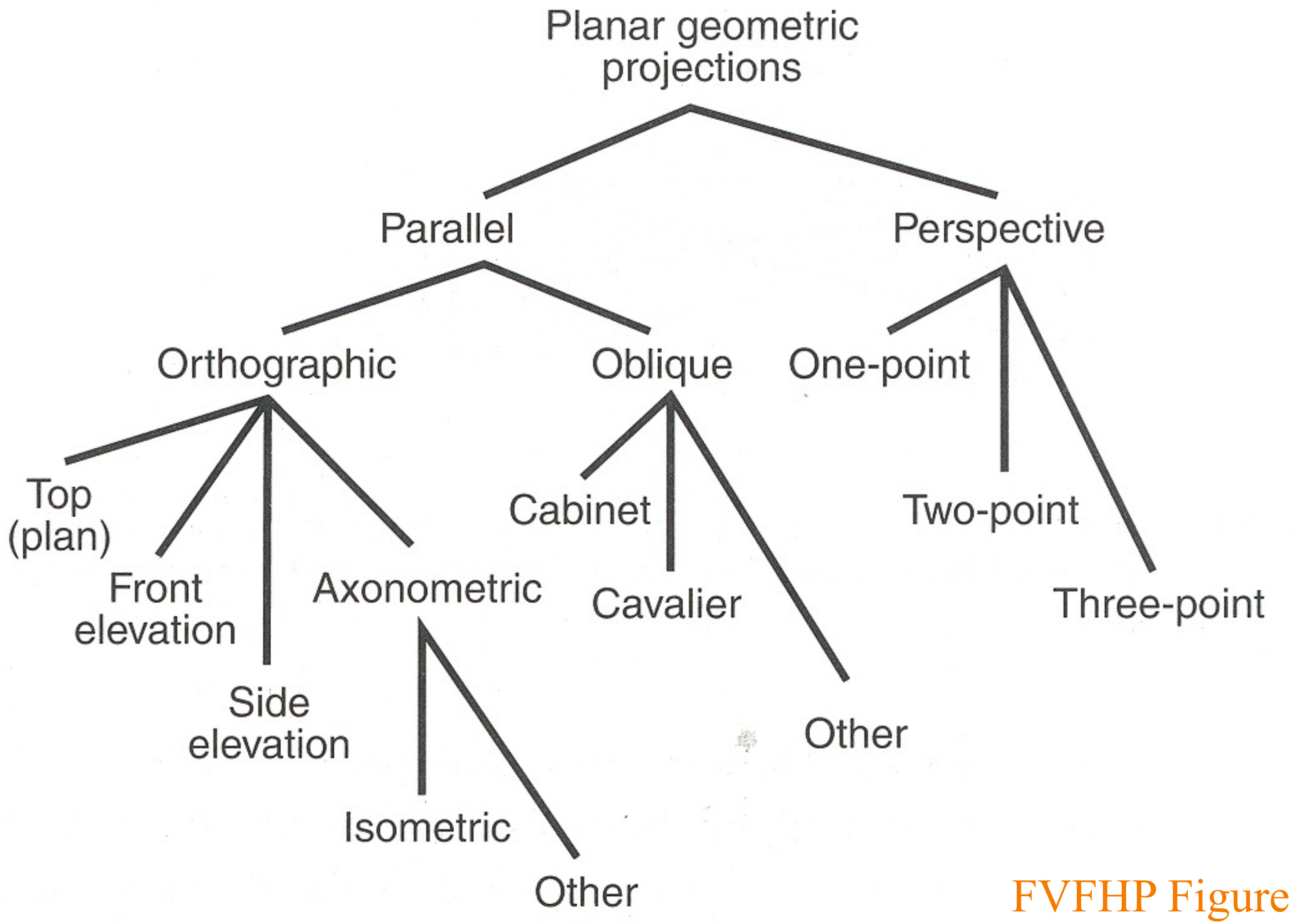
- Perspective projection
  - + Size varies inversely with distance - looks realistic
  - – Distance and angles are not (in general) preserved
  - – Parallel lines do not (in general) remain parallel



- Parallel projection
  - + Good for exact measurements
  - + Parallel lines remain parallel
  - – Angles are not (in general) preserved
  - – Less realistic looking

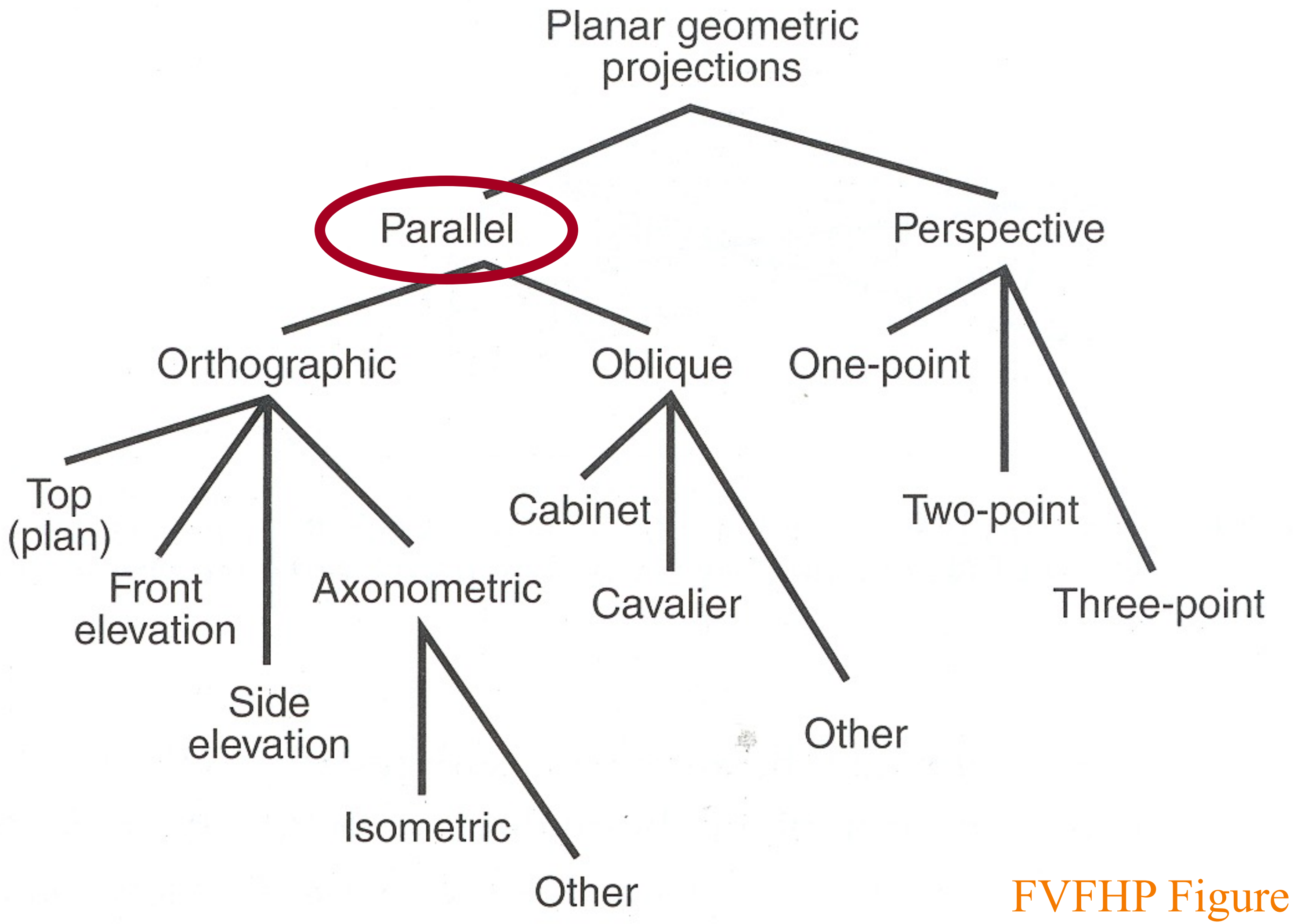


Taxonomy of Projections



FVFHP Figure 6.10

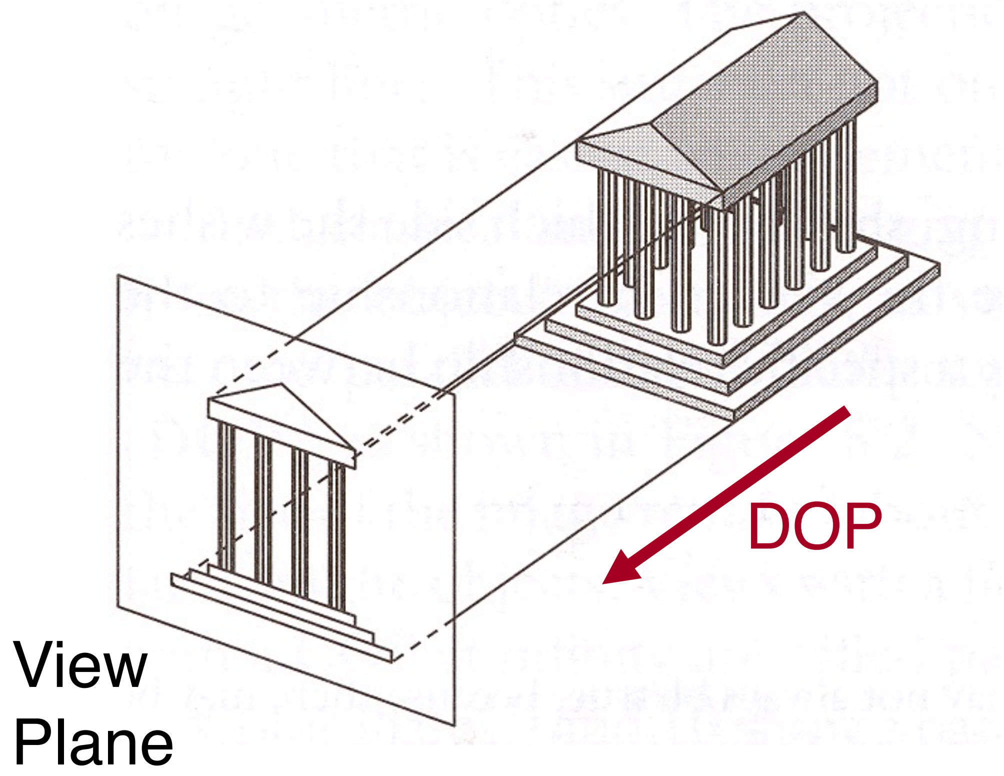
Taxonomy of Projections



FVFHP Figure 6.10

Parallel Projection

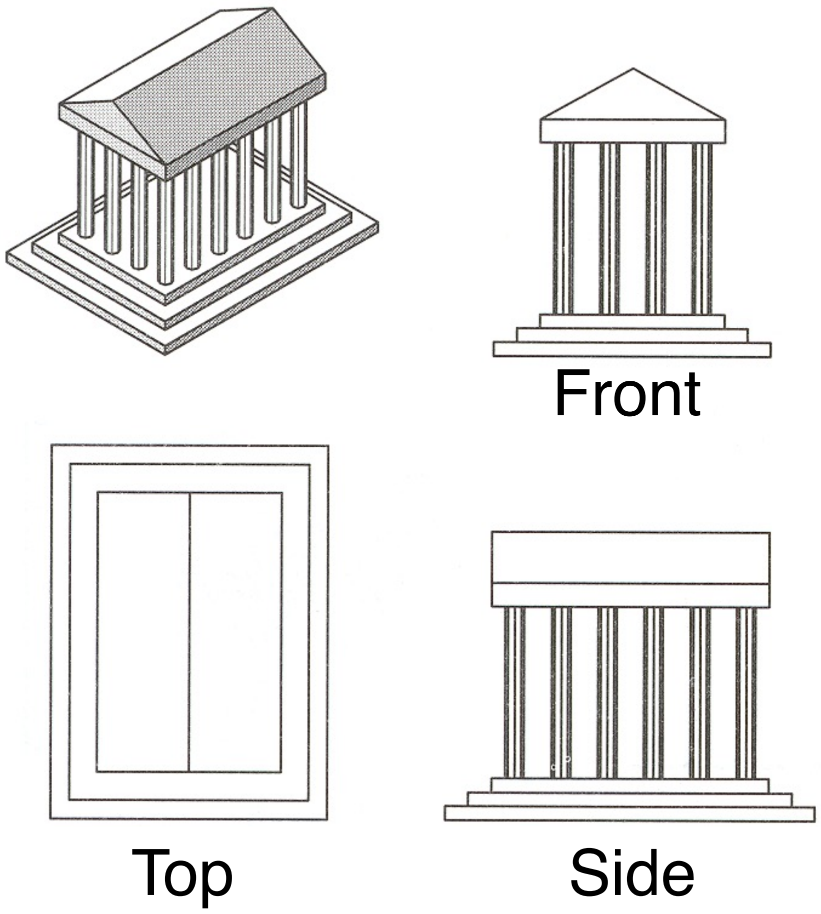
- Center of projection is at infinity
  - Direction of projection (DOP) same for all points



Angel Figure 5.4

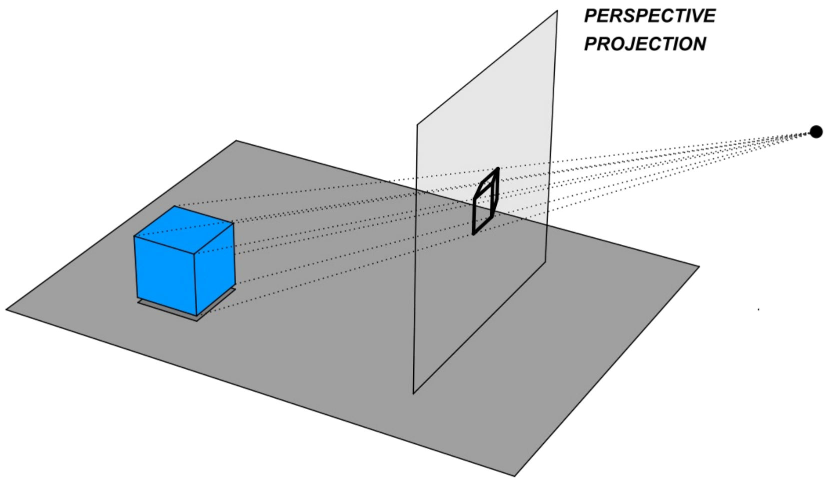
Orthographic Projections

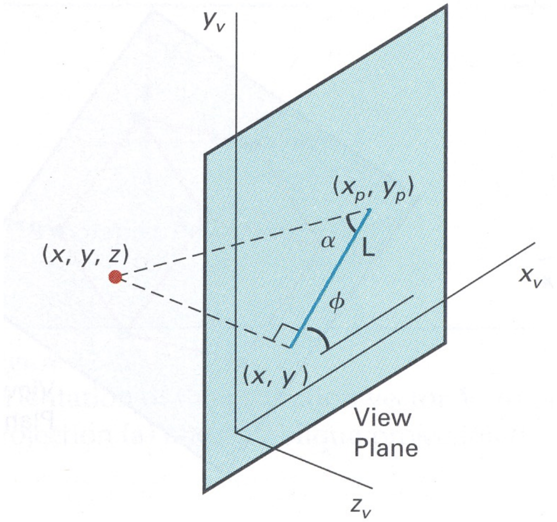
- DOP perpendicular to view plane

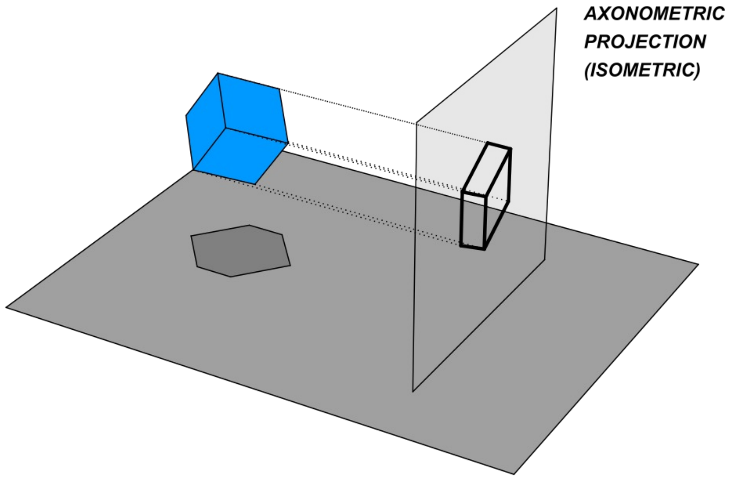


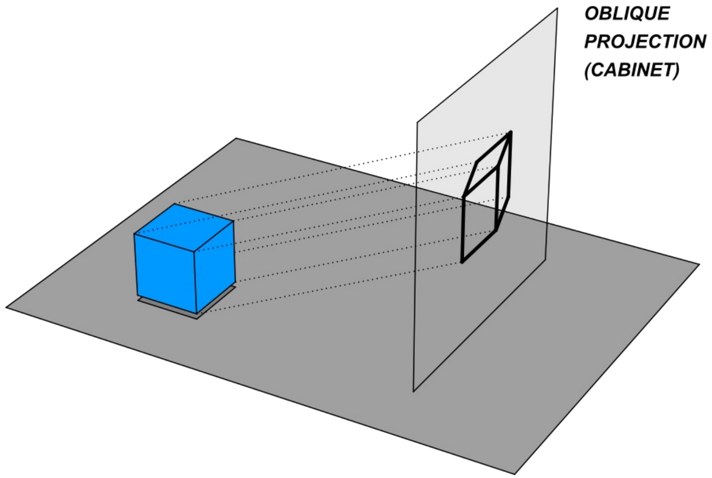
Angel Figure 5.5

Parallel Projection Matrix



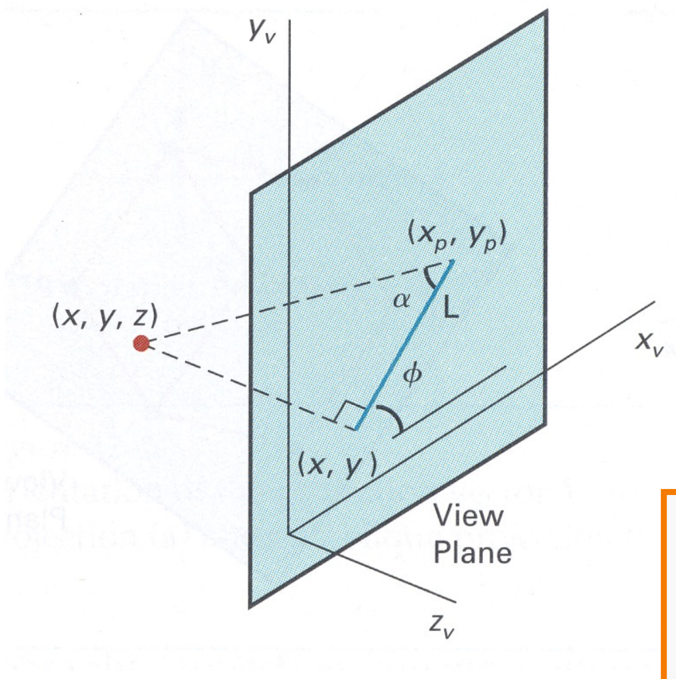






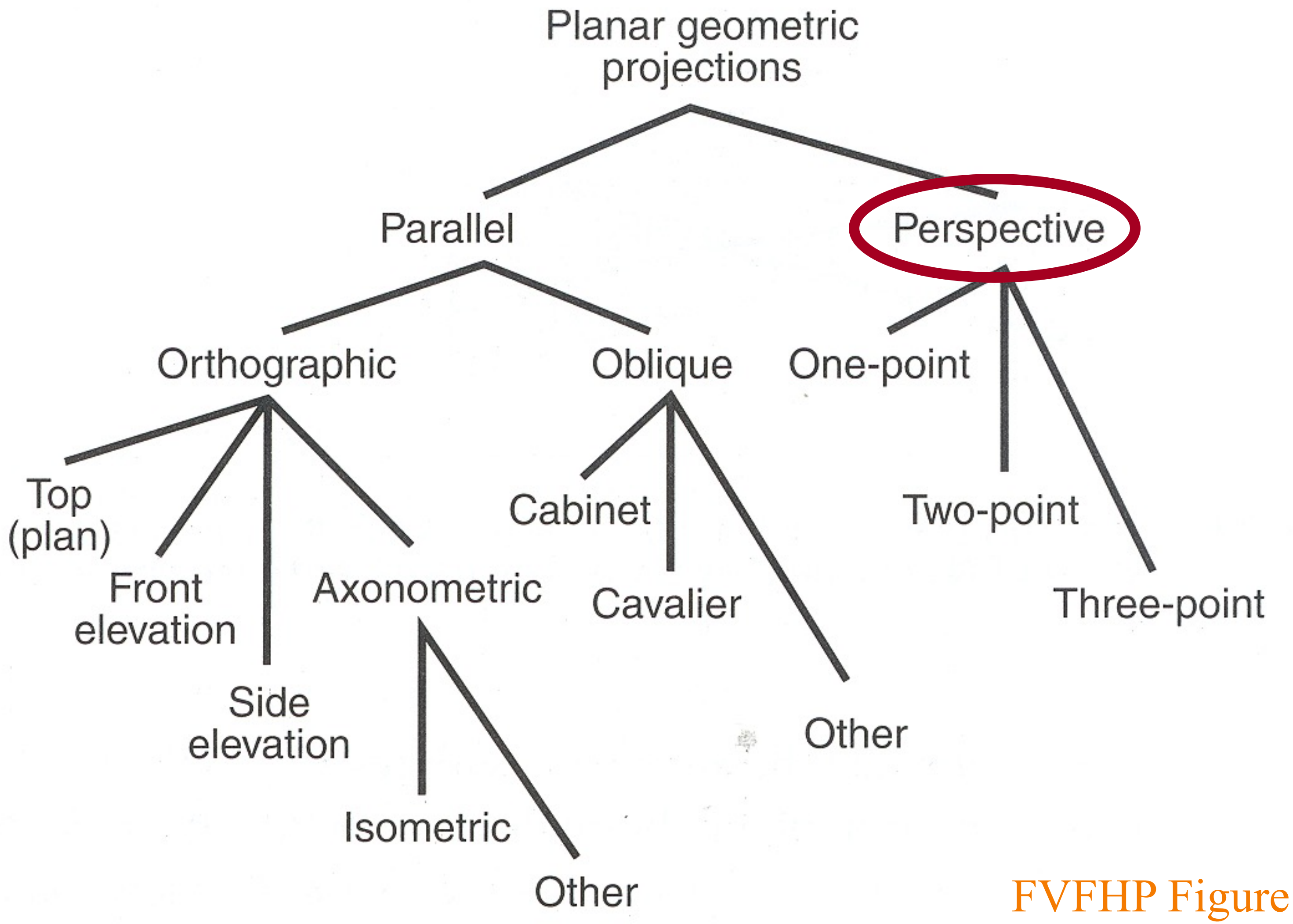
Parallel Projection Matrix

- General parallel projection transformation:



$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & L\cos\phi & 0 \\ 0 & 1 & L\sin\phi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

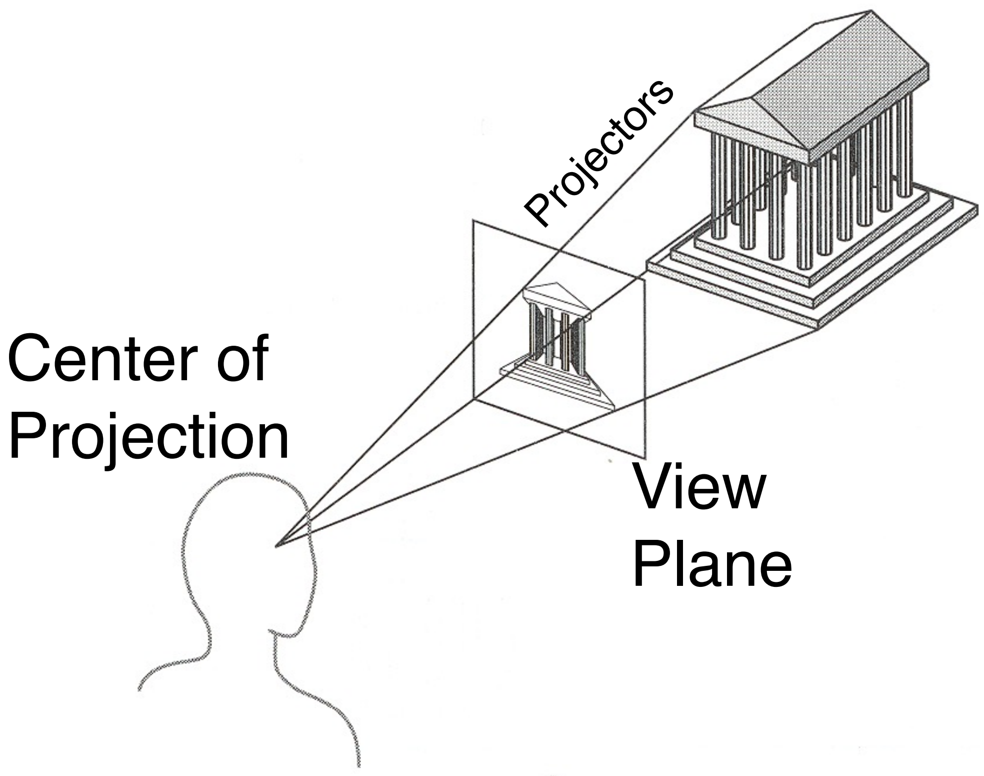
Taxonomy of Projections



FVFHP Figure 6.10

Return to Perspective Projection

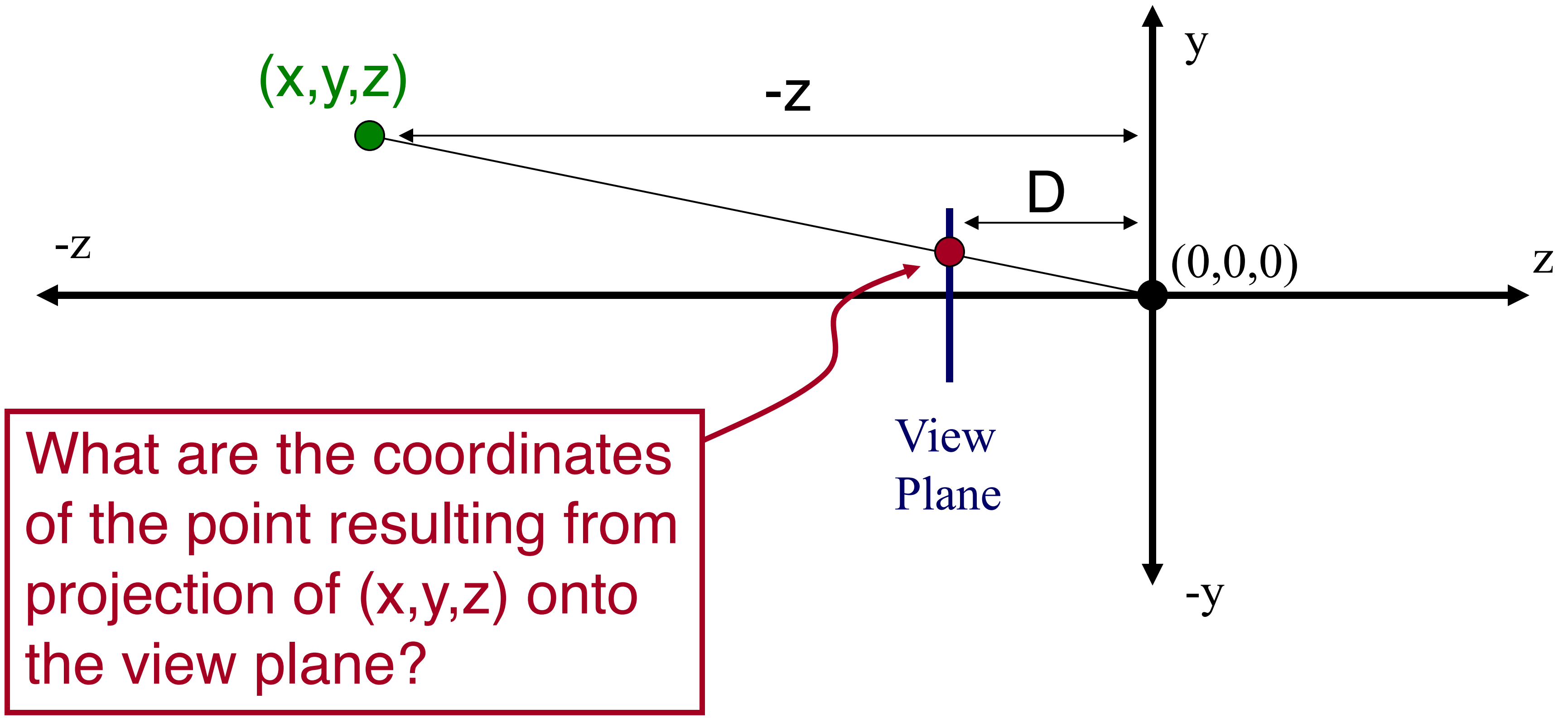
- Map points onto “view plane” along “projectors” emanating from “center of projection” (COP)



Angel Figure 5.9

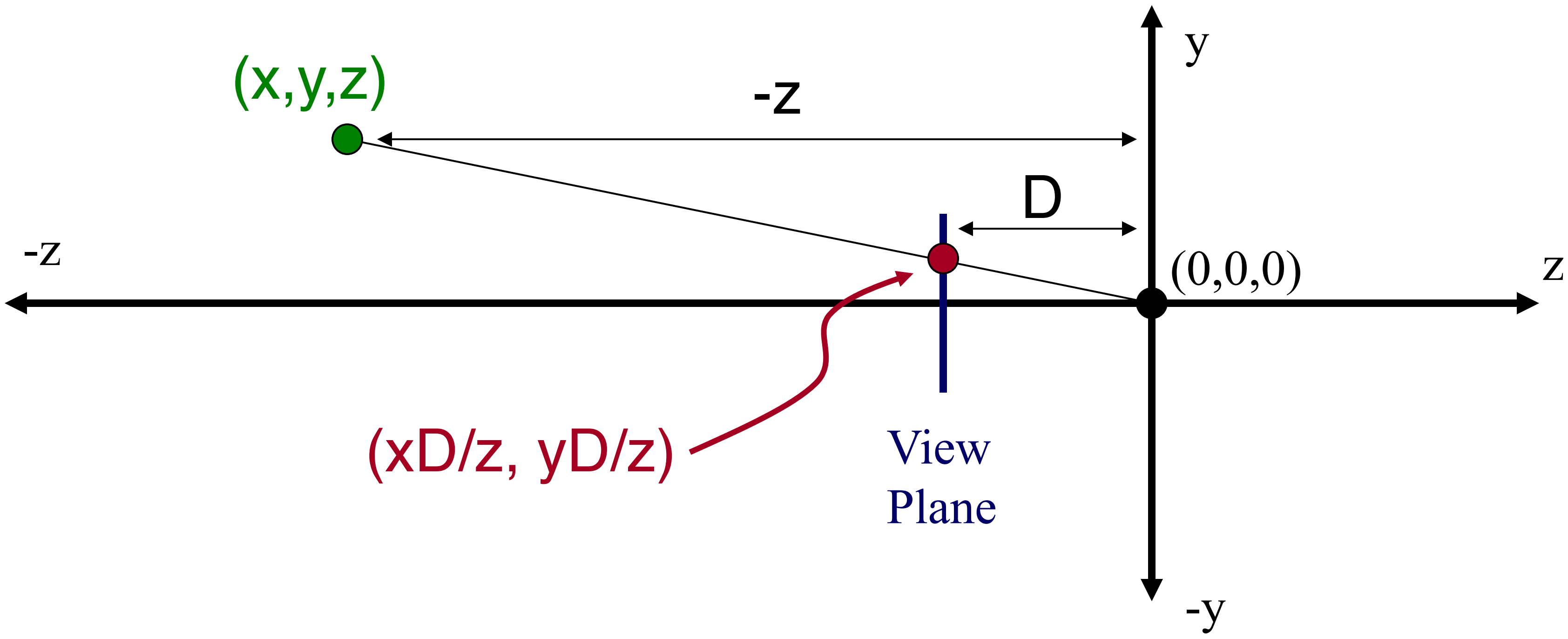
Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles



Perspective Projection

- Compute 2D coordinates from 3D coordinates with similar triangles



Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c$$
$$y_s = y_c D / z_c$$
$$z_s = D$$
$$w_s = 1$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Perspective Projection Matrix

- 4x4 matrix representation?

$$
\begin{aligned}
x_s &= x_c D / z_c \\
y_s &= y_c D / z_c \\
z_s &= D \\
w_s &= 1
\end{aligned}
\qquad
\begin{aligned}
x_s &= x' / w' \\
y_s &= y' / w' \\
z_s &= z' / w'
\end{aligned}
\qquad
\begin{aligned}
x' &= x_c \\
y' &= y_c \\
z' &= z_c \\
w' &= z_c / D
\end{aligned}
$$

$$
\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}
$$

Perspective Projection Matrix

- 4x4 matrix representation?

$$x_s = x_c D / z_c \qquad x_s = x' / w' \qquad x' = x_c$$
$$y_s = y_c D / z_c \qquad y_s = y' / w' \qquad y' = y_c$$
$$z_s = D \qquad z_s = z' / w' \qquad z' = z_c$$
$$w_s = 1 \qquad w' = z_c / D$$

$$\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/D & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

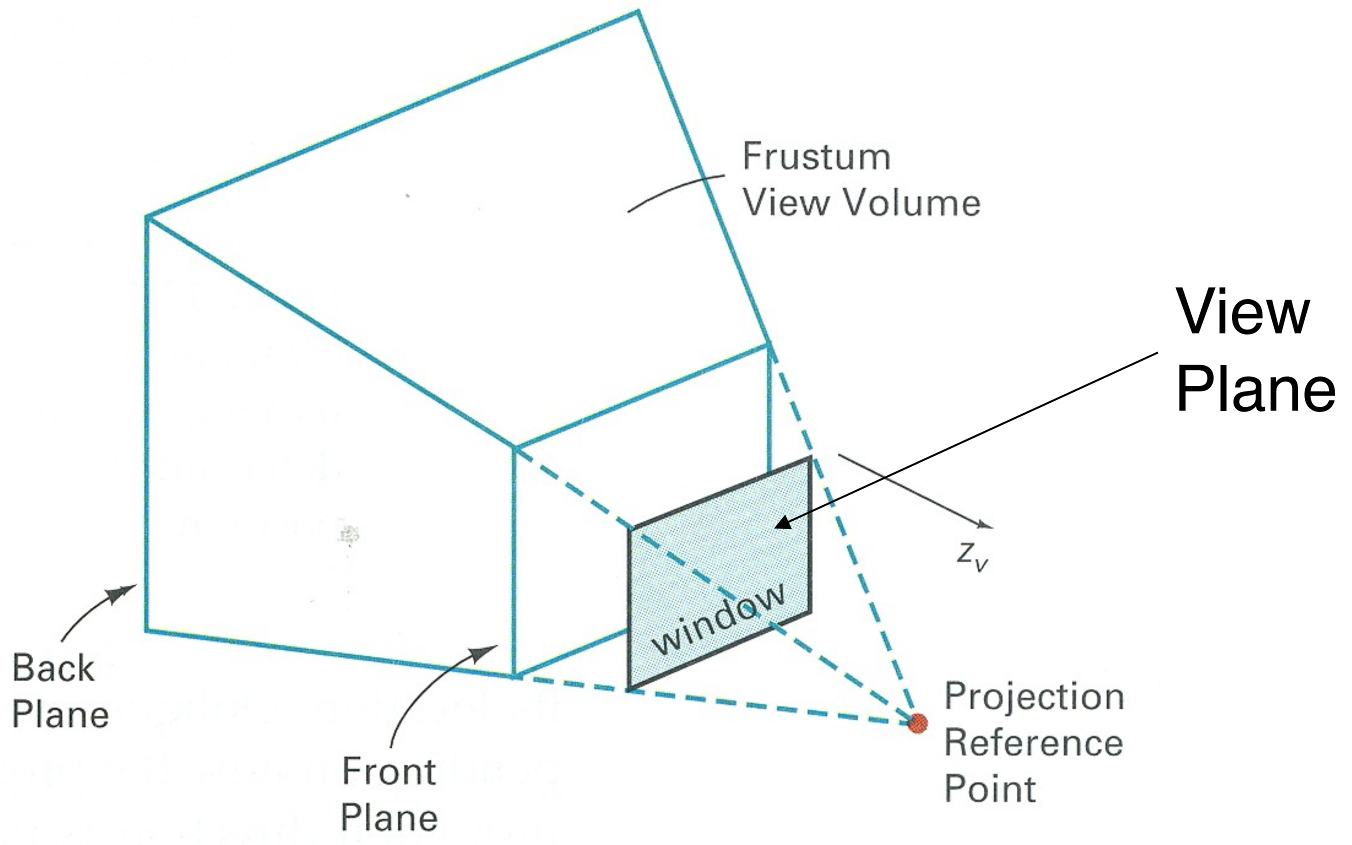
Perspective Projection Matrix

- In practice, want to compute a value related to depth to include in *z*-buffer

$$\begin{aligned} x_s &= x_c D / z_c \\ y_s &= y_c D / z_c \\ z_s &= -D / z_c \\ w_s &= 1 \end{aligned} \qquad \begin{aligned} x_s &= x' / w' \\ y_s &= y' / w' \\ z_s &= z' / w' \end{aligned} \qquad \begin{aligned} x' &= x_c \\ y' &= y_c \\ z' &= -1 \\ w' &= z_c / D \end{aligned}$$

$$\boxed{\begin{bmatrix} x_s \\ y_s \\ z_s \\ w_s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1/D & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}}$$

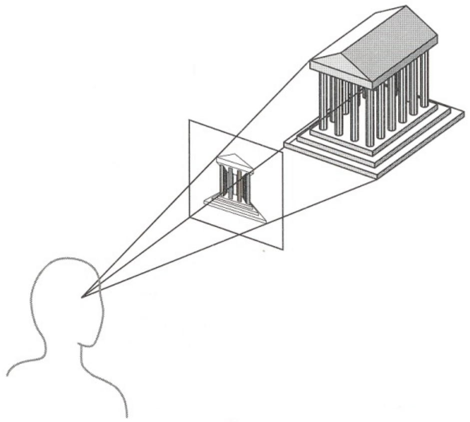
Perspective Projection View Volume



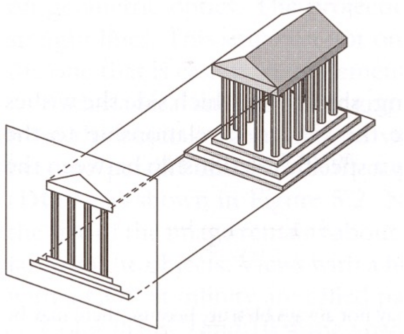
H&B Figure 12.30

Perspective vs. Parallel

- Perspective projection
  + Size varies inversely with distance - looks realistic
  – Distance and angles are not (in general) preserved
  – Parallel lines do not (in general) remain parallel



- Parallel projection
  + Good for exact measurements
  + Parallel lines remain parallel
  – Angles are not (in general) preserved
  – Less realistic looking



Transformations

p(x,y,z)

↓ 3D Object Coordinates

Modeling Transformation

↓ 3D World Coordinates

Viewing Transformation

↓ 3D Camera Coordinates

Projection Transformation

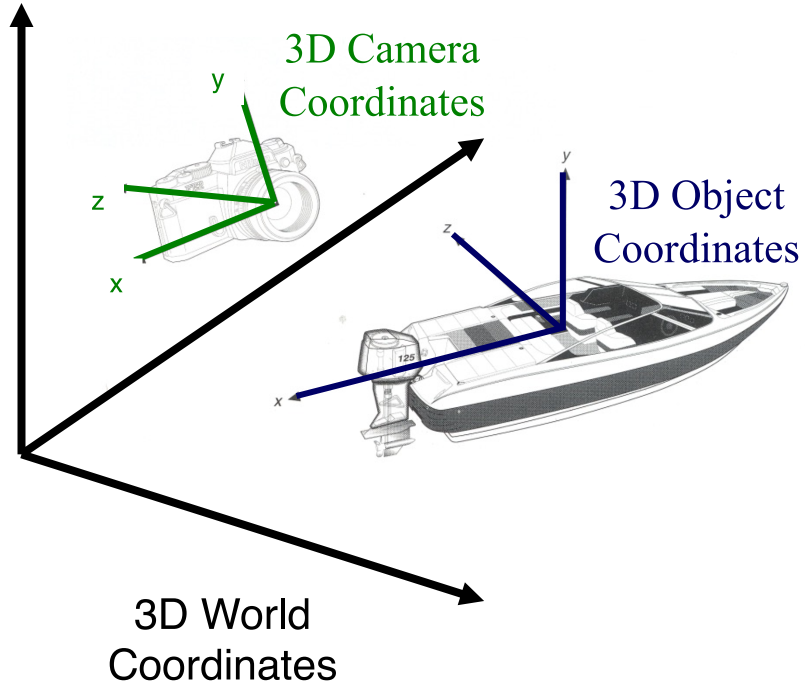
↓ 2D Screen Coordinates

Viewport Transformation

↓ 2D Image Coordinates

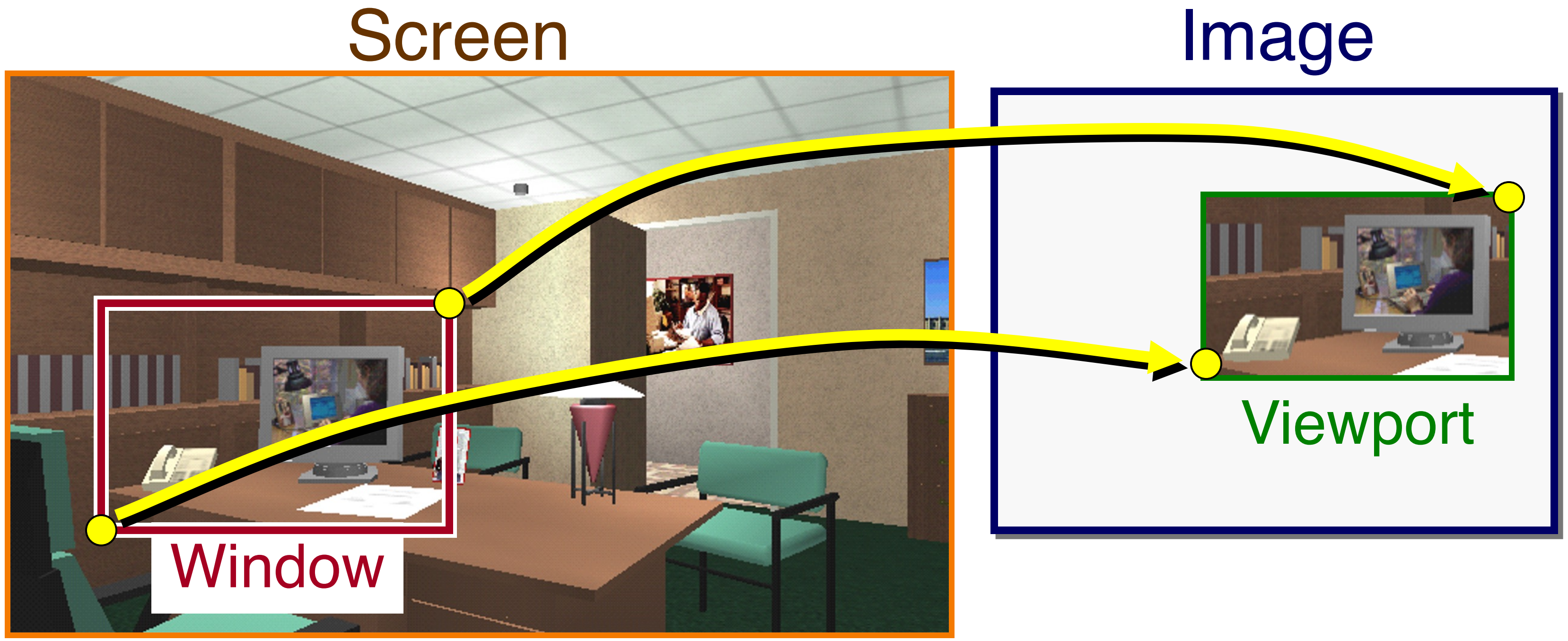
p'(x',y')

Transformations map points from one coordinate system to another



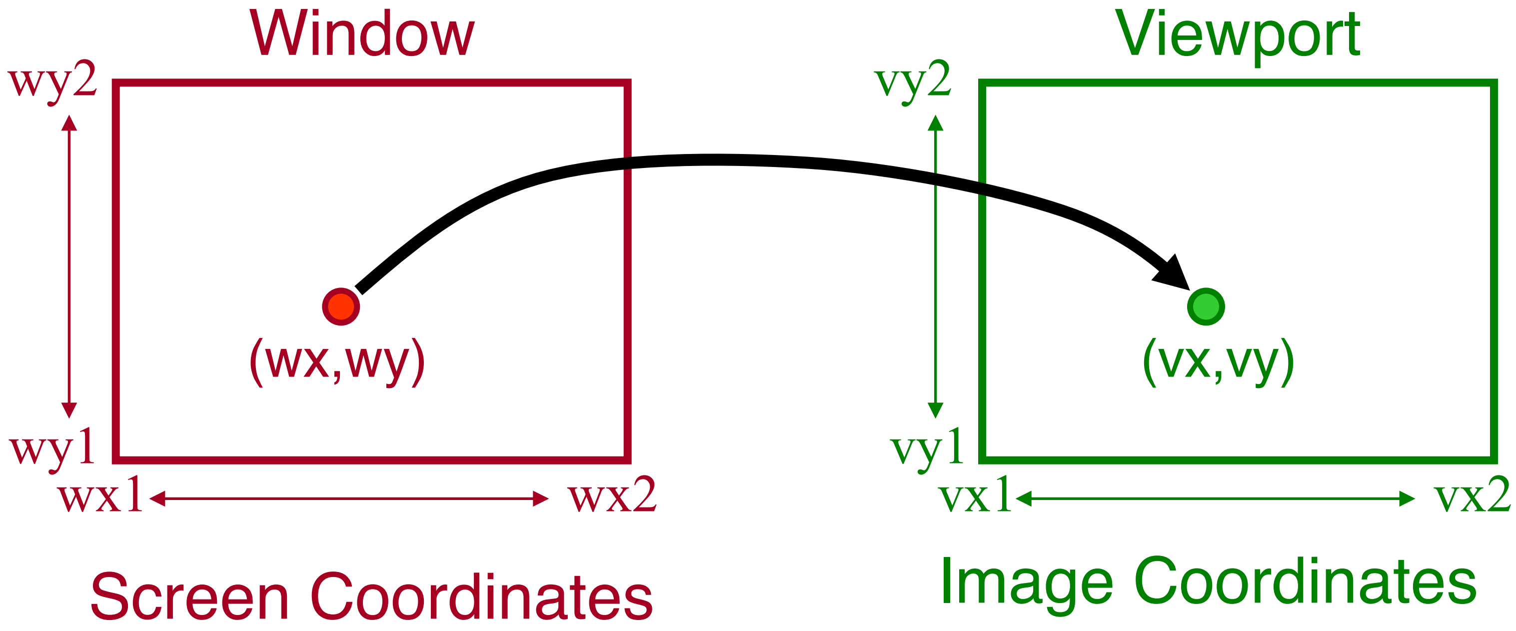
Viewport Transformation

- Transform 2D geometric primitives from screen coordinate system (normalized device coordinates) to image coordinate system (pixels)



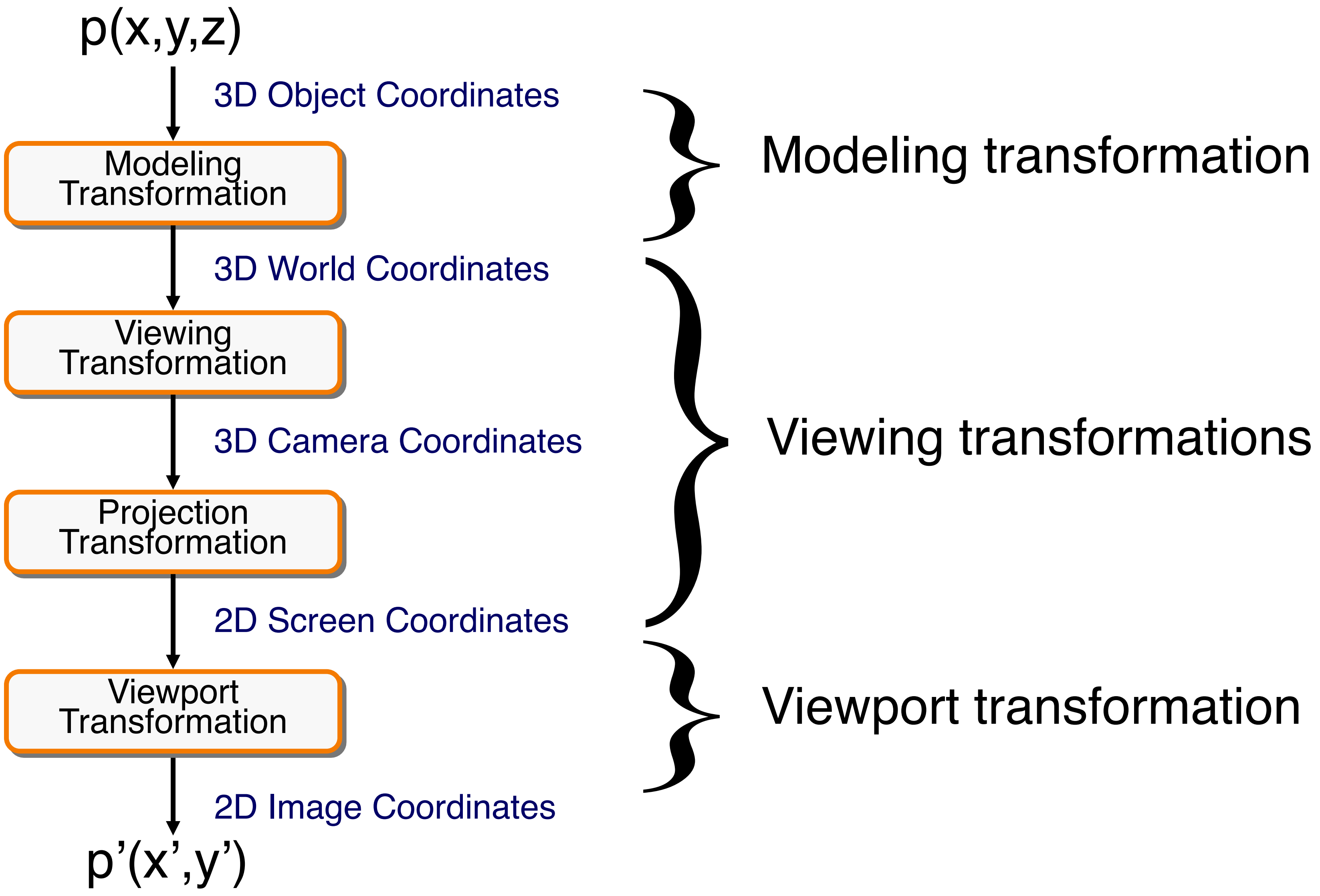
Viewport Transformation

- Window-to-viewport mapping




```
vx = vx1 + (wx - wx1) * (vx2 - vx1) / (wx2 - wx1);
vy = vy1 + (wy - wy1) * (vy2 - vy1) / (wy2 - wy1);
```

Summary of Transformations



Rasterization Pipeline (for direct illumination)

3D Primitives

↓ 3D Modeling Coordinates

Modeling Transformation

↓ 3D World Coordinates

Lighting

↓ 3D World Coordinates

Viewing Transformation

↓ 3D Camera Coordinates

Projection Transformation

↓ 2D Screen Coordinates

Clipping

↓ 2D Screen Coordinates

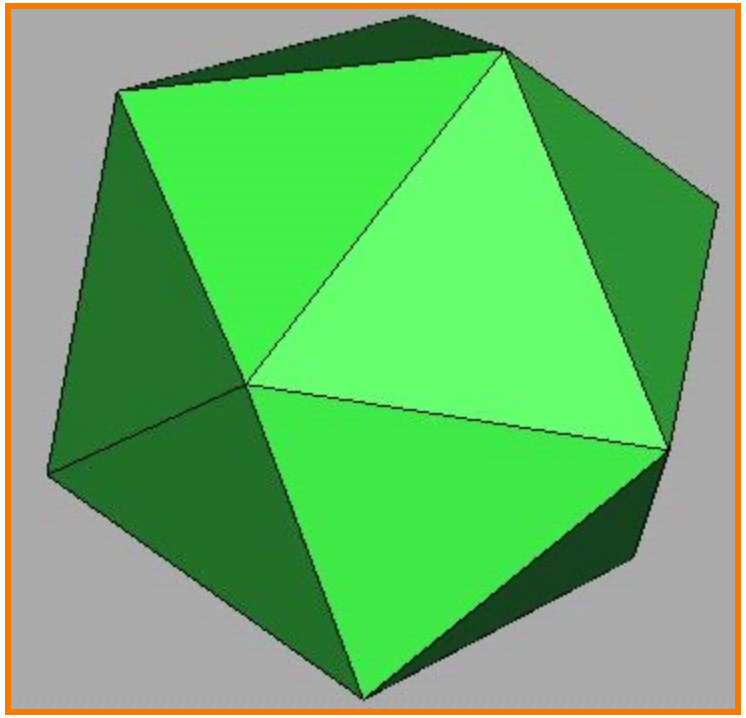
Viewport Transformation

↓ 2D Image Coordinates

Scan Conversion

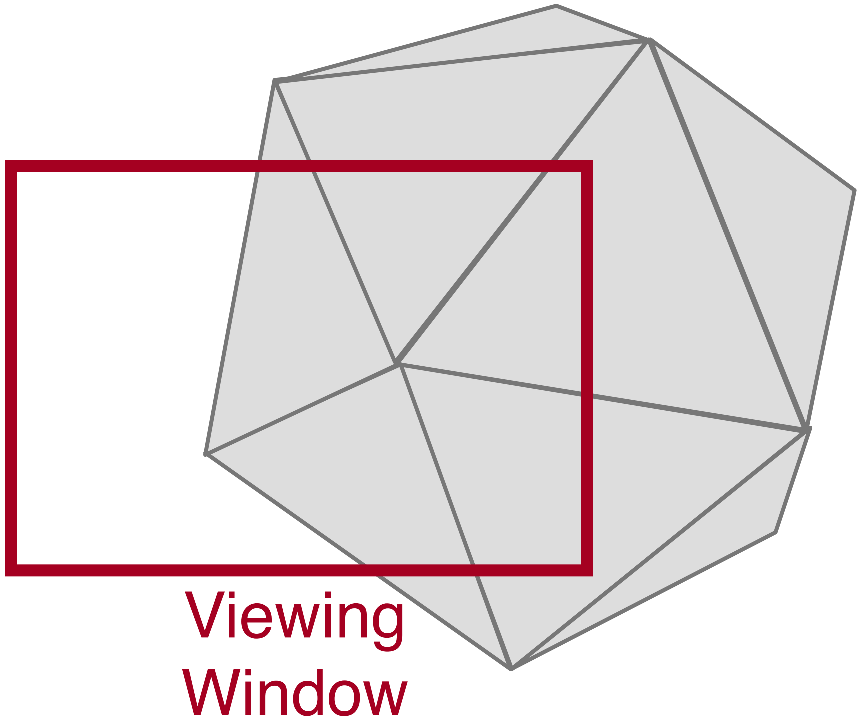
↓ 2D Image Coordinates

Image



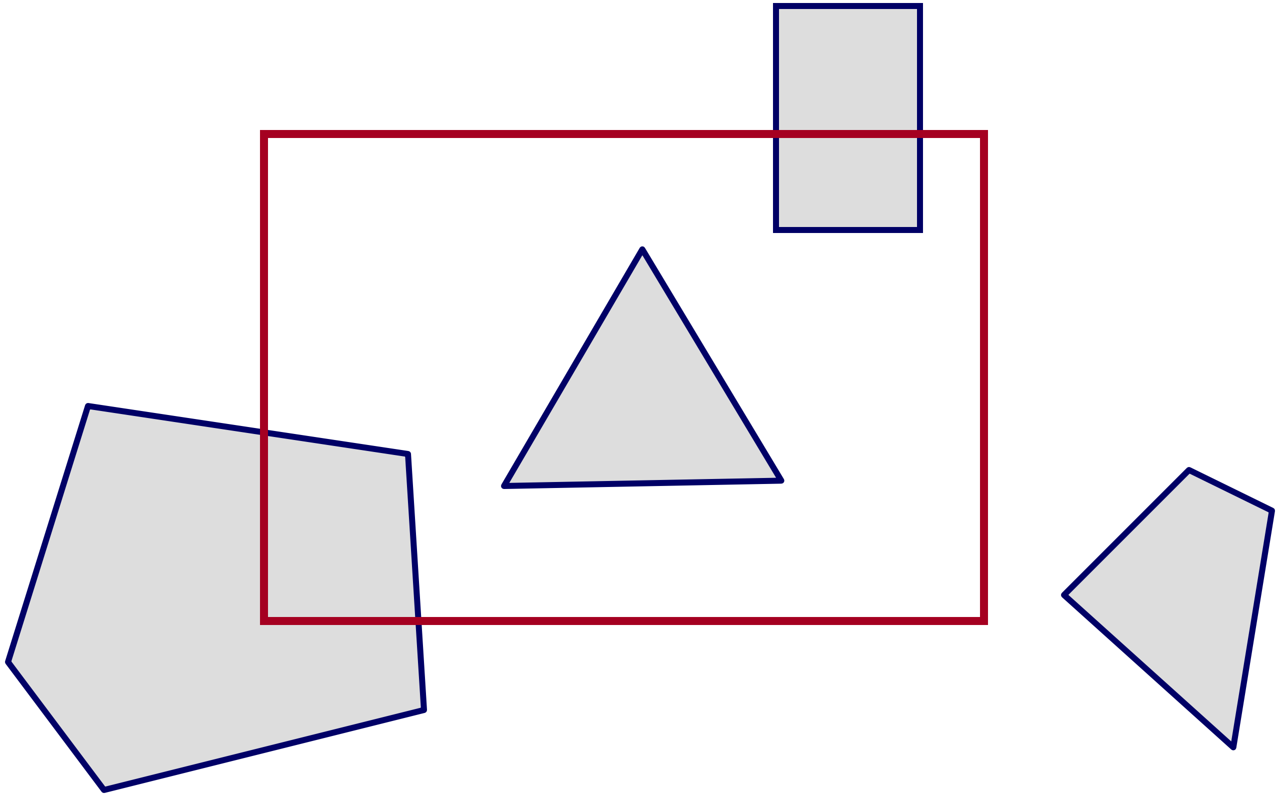
Clipping

- Avoid drawing parts of primitives outside window
  - Window defines part of scene being viewed
  - Must draw geometric primitives only inside window



Polygon Clipping

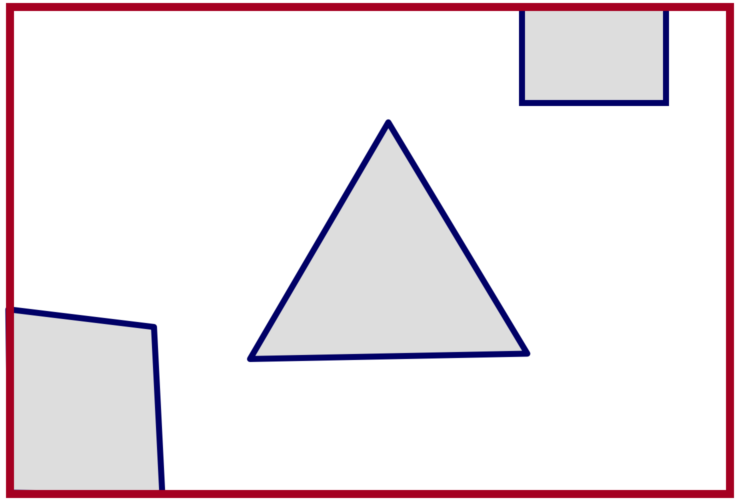
- Find the part of a polygon inside the clip window?



Before Clipping

Polygon Clipping

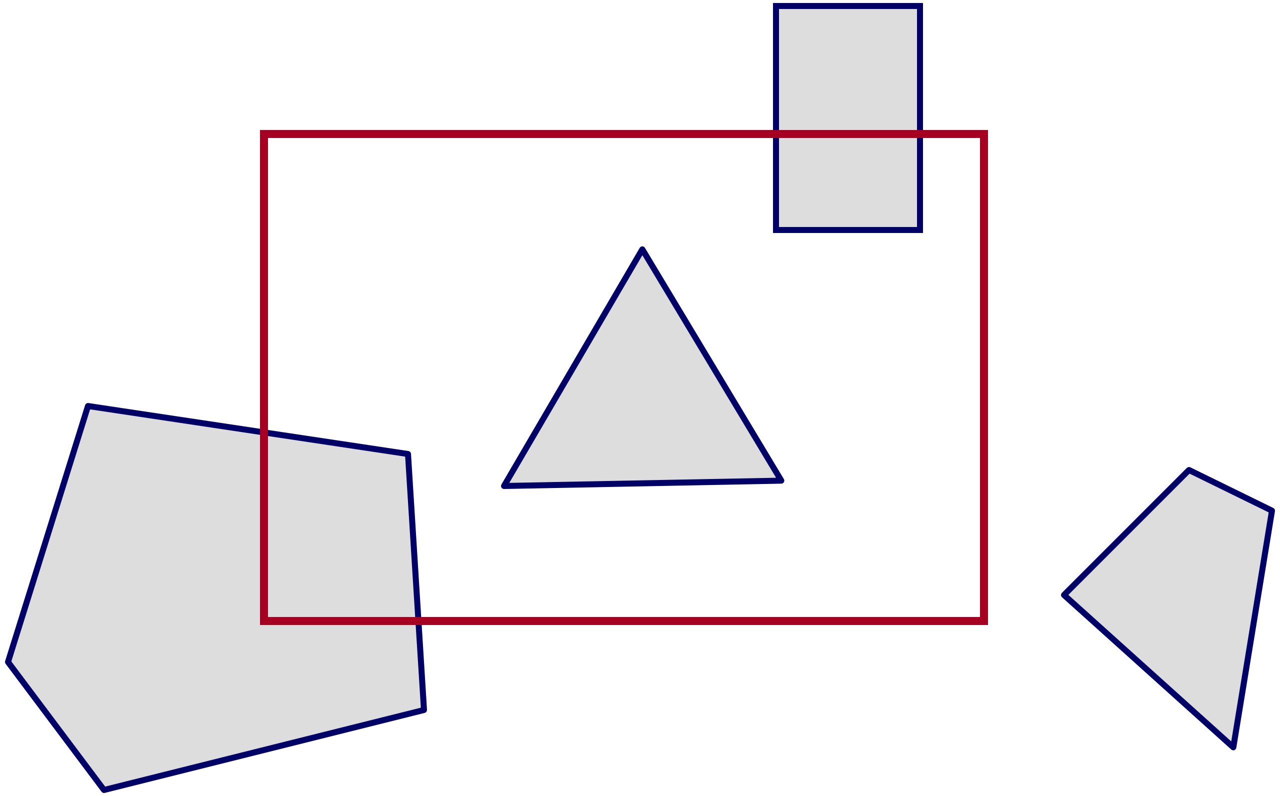
- Find the part of a polygon inside the clip window?



After Clipping

Sutherland Hodgeman Clipping

- Clip to each window boundary one at a time (*for convex polygons*)



Sutherland Hodgeman Clipping

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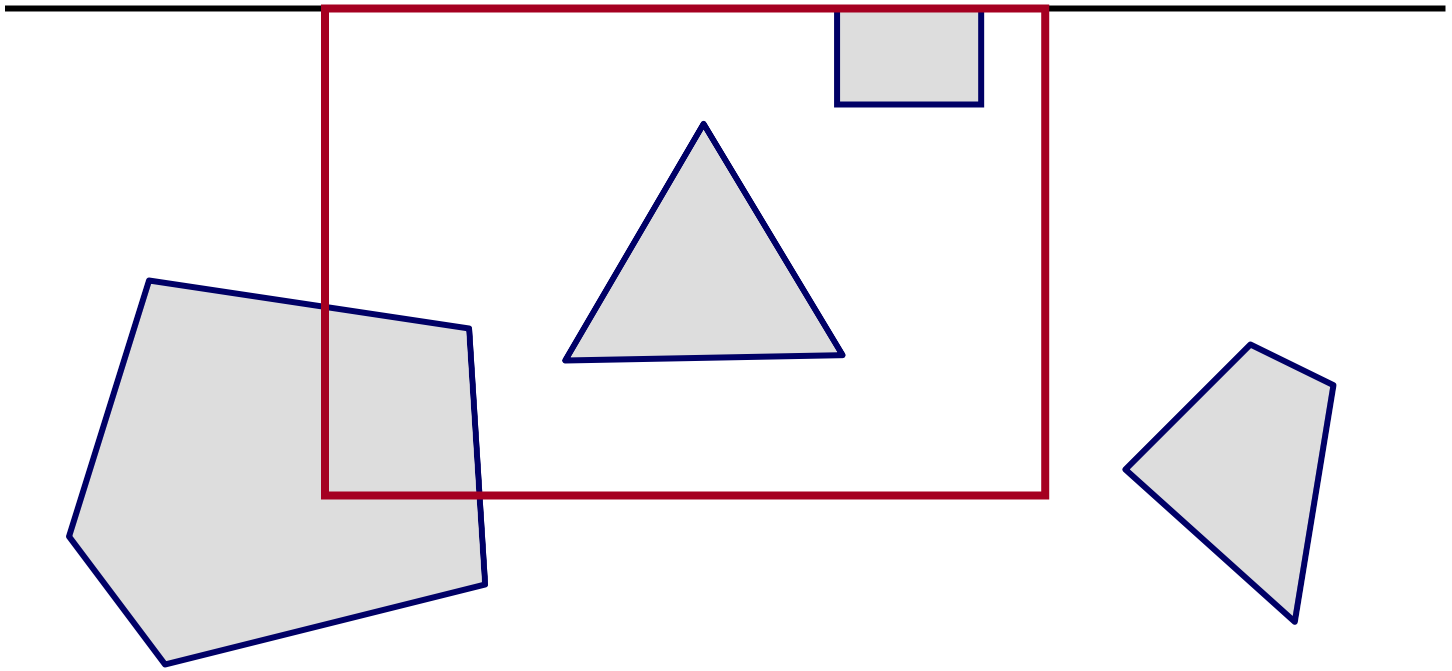

Sutherland Hodgeman Clipping

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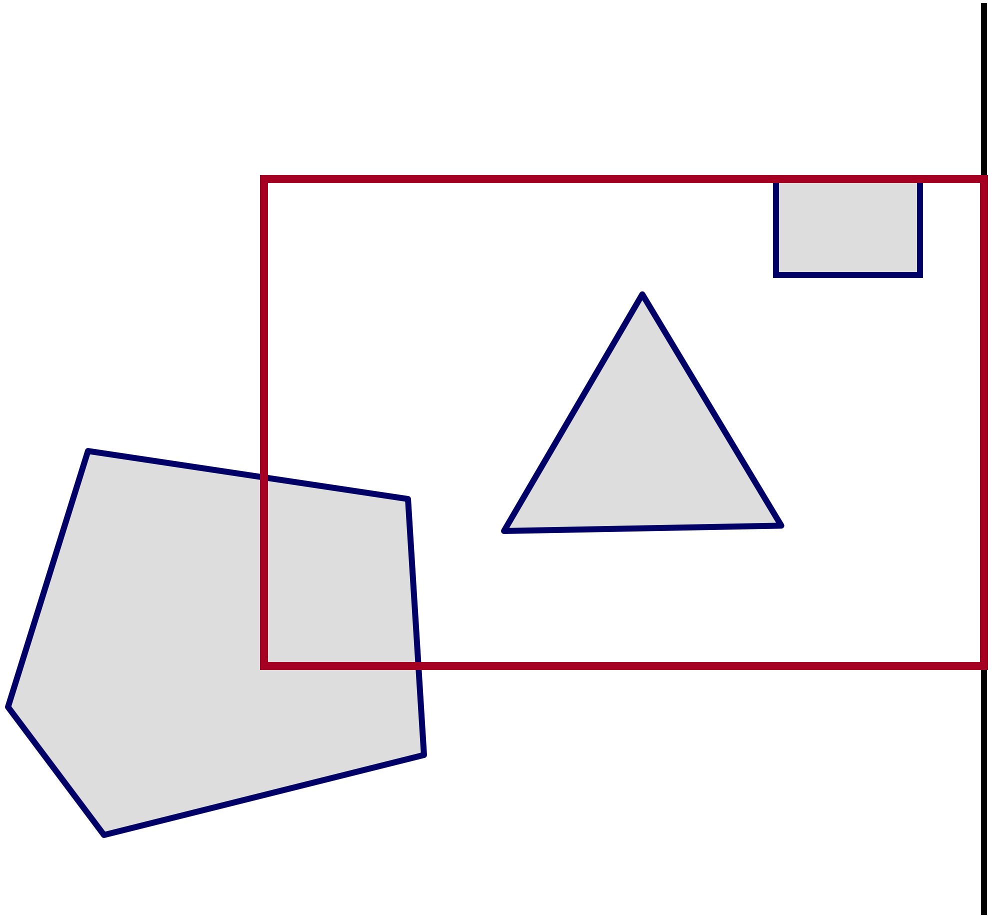

Sutherland Hodgeman Clipping

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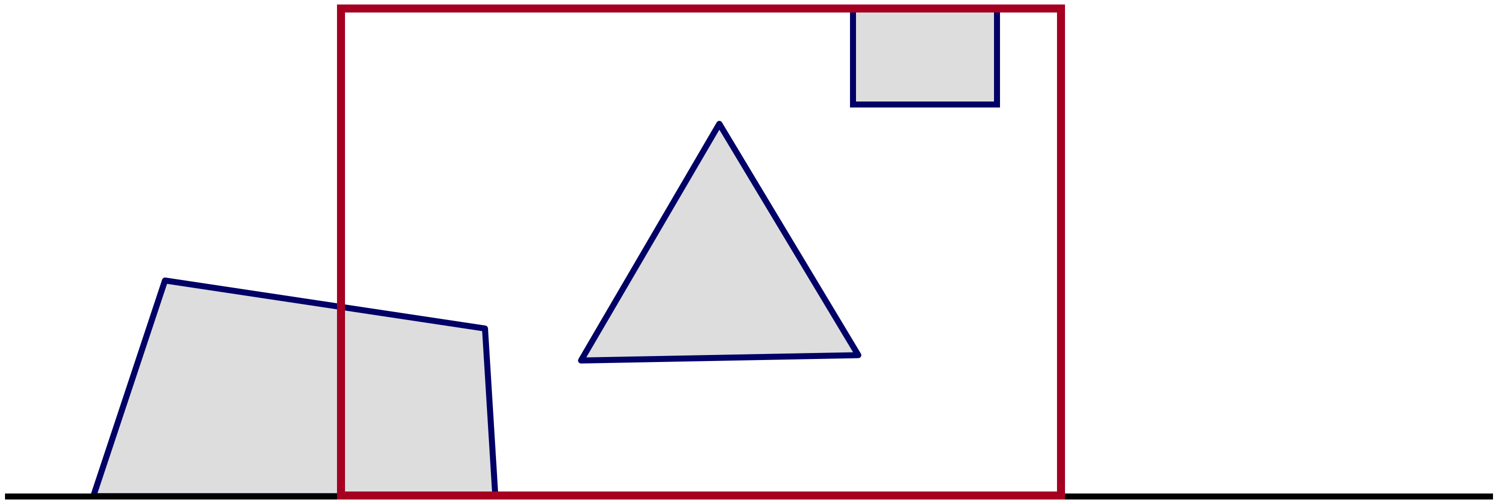

Sutherland Hodgeman Clipping

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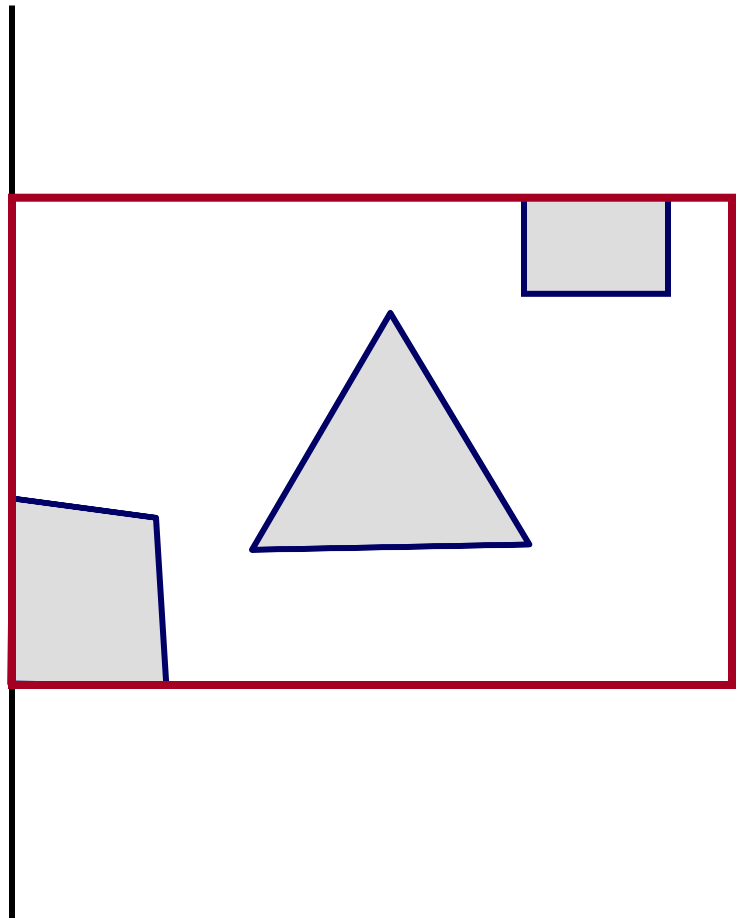
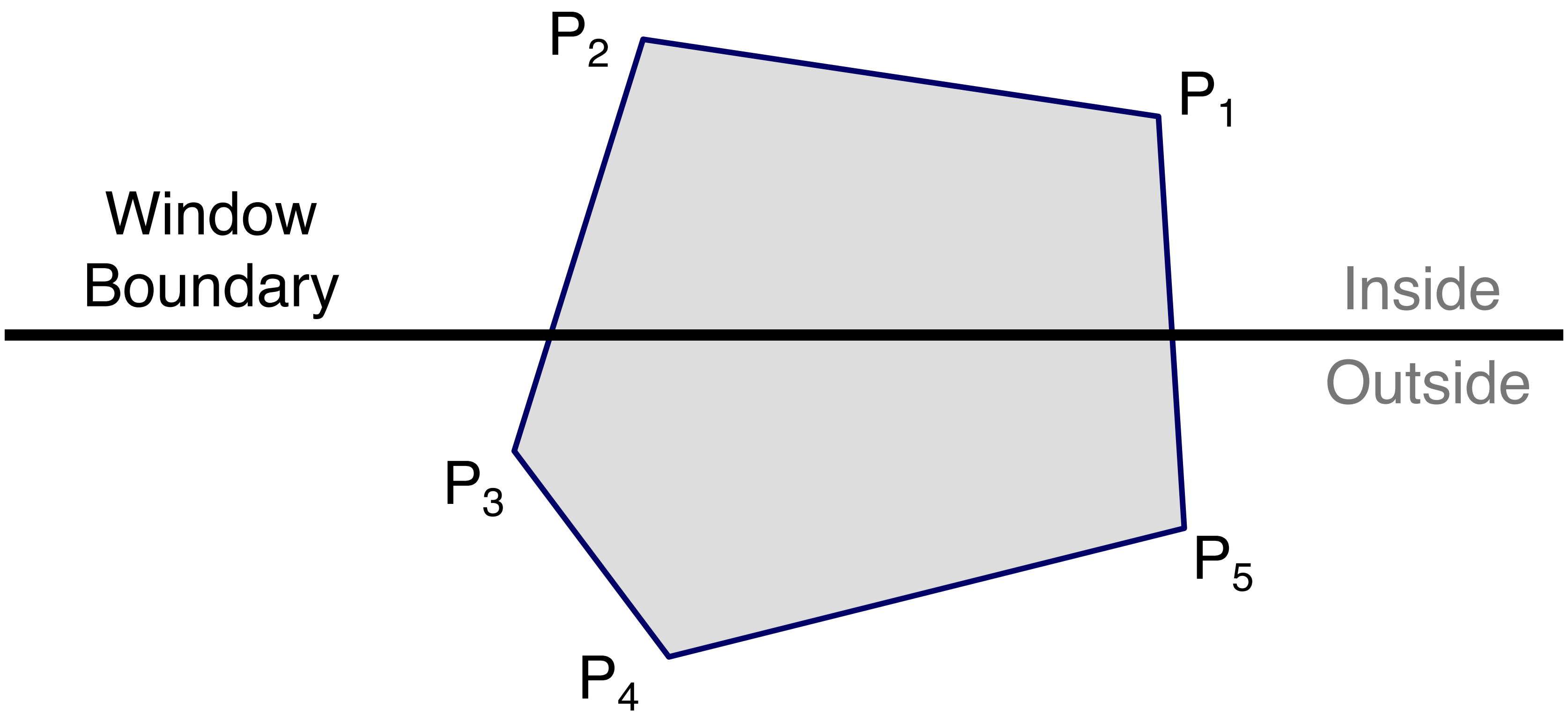


Clipping to a Boundary

- Do inside test for each point in sequence
  - Insert new points when crossing window boundary
  - Remove points outside window boundary



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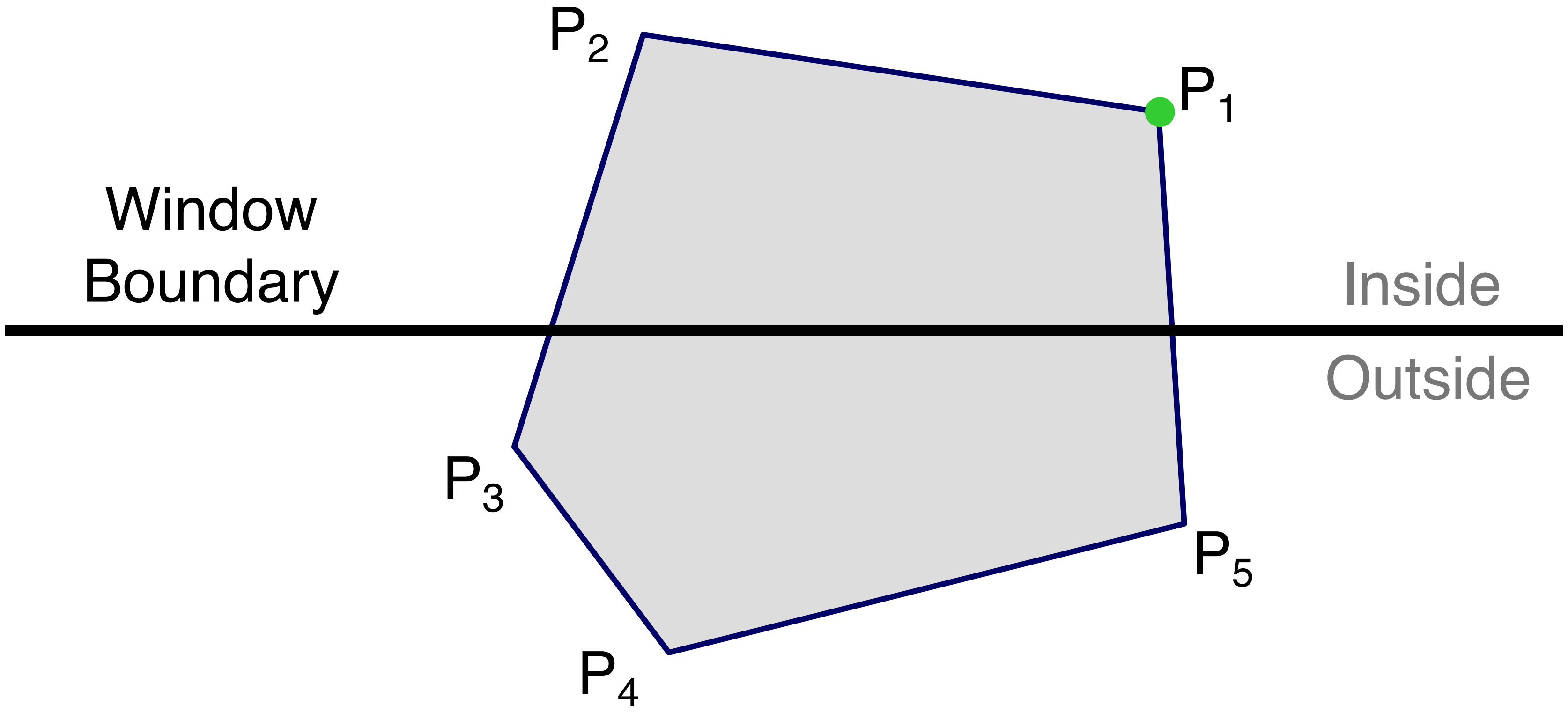

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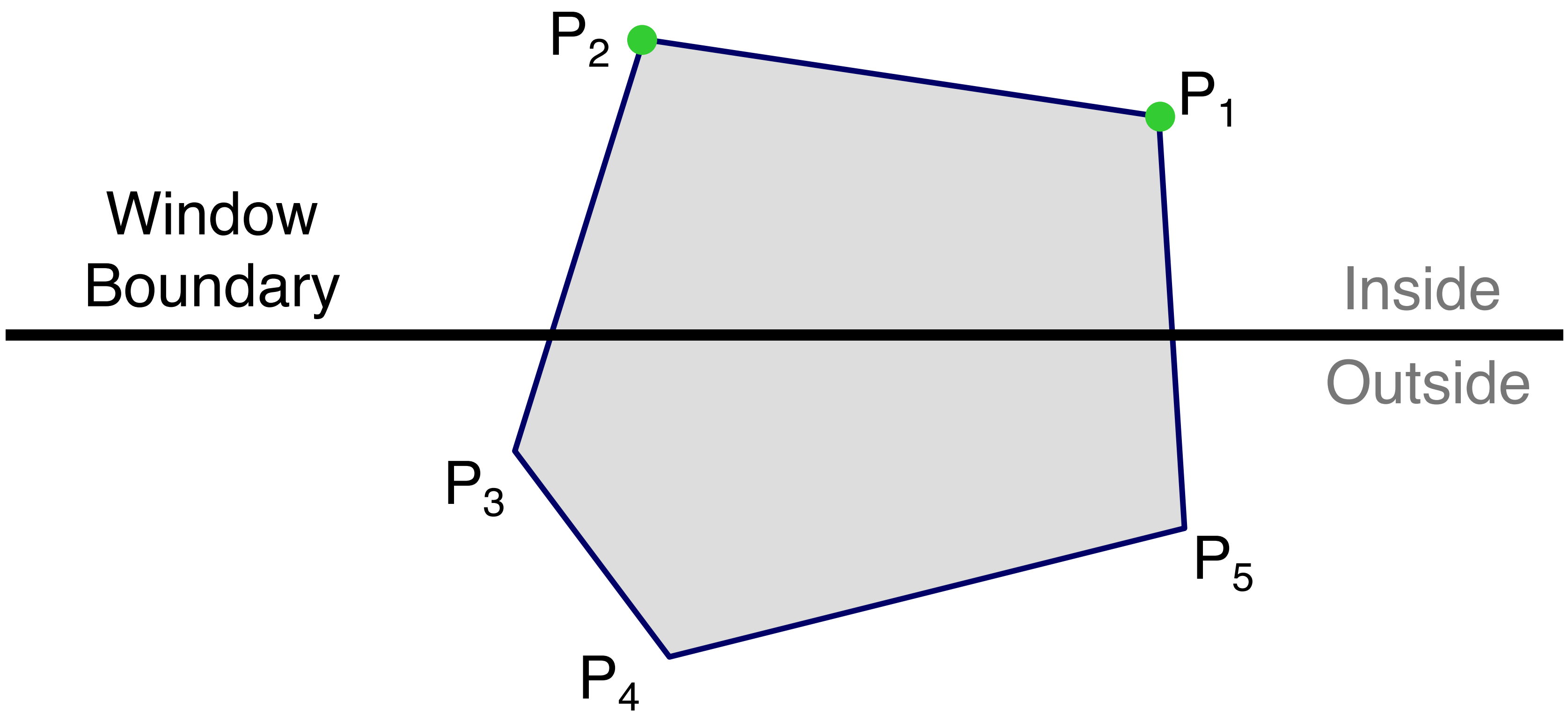

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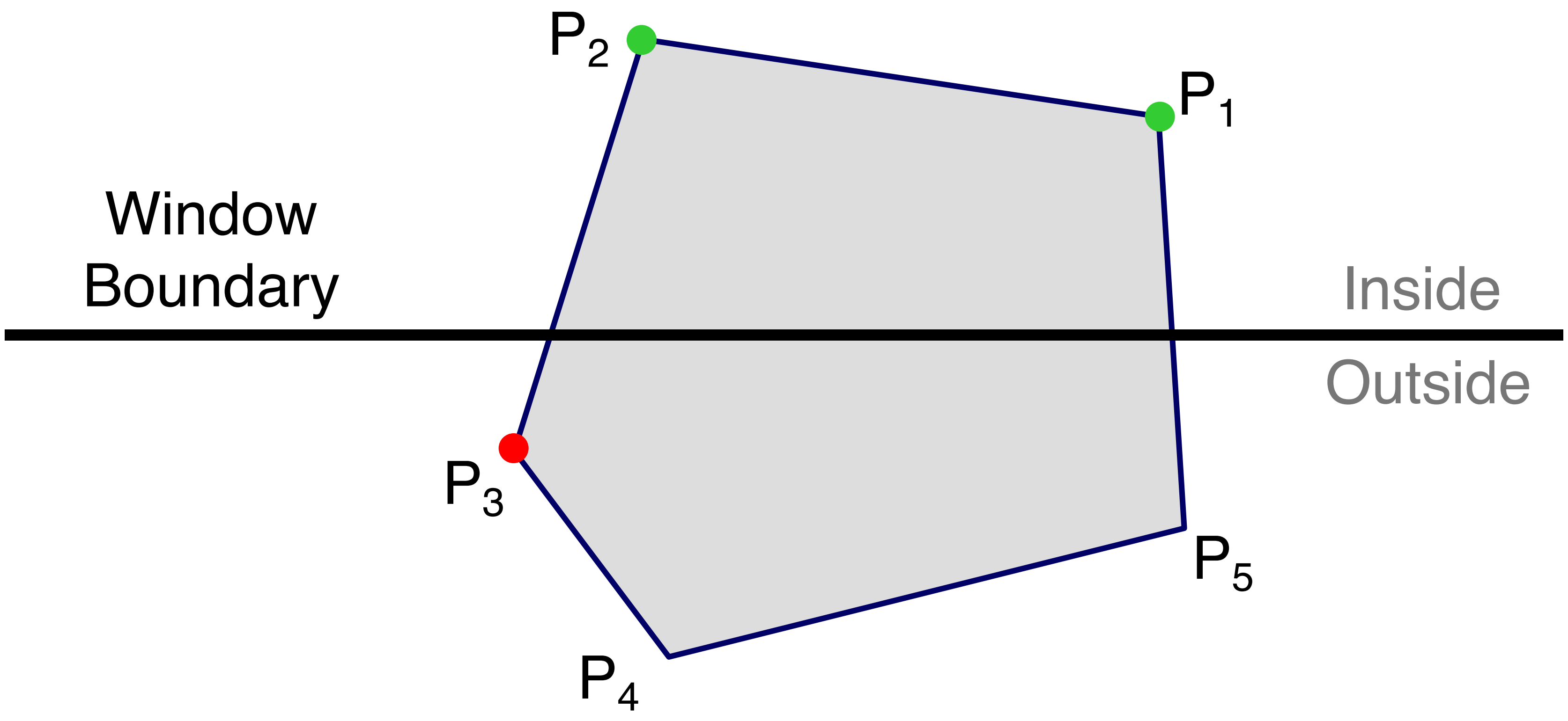

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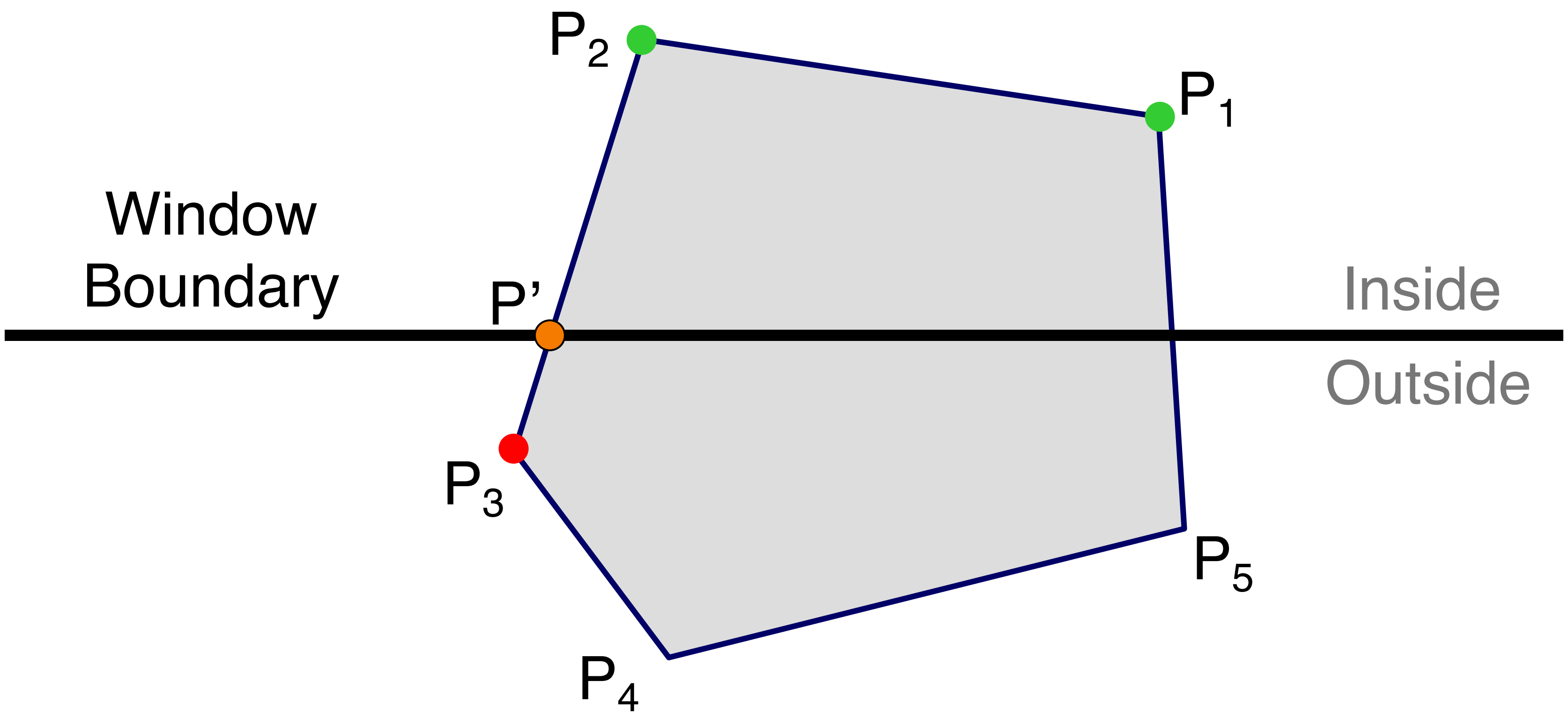

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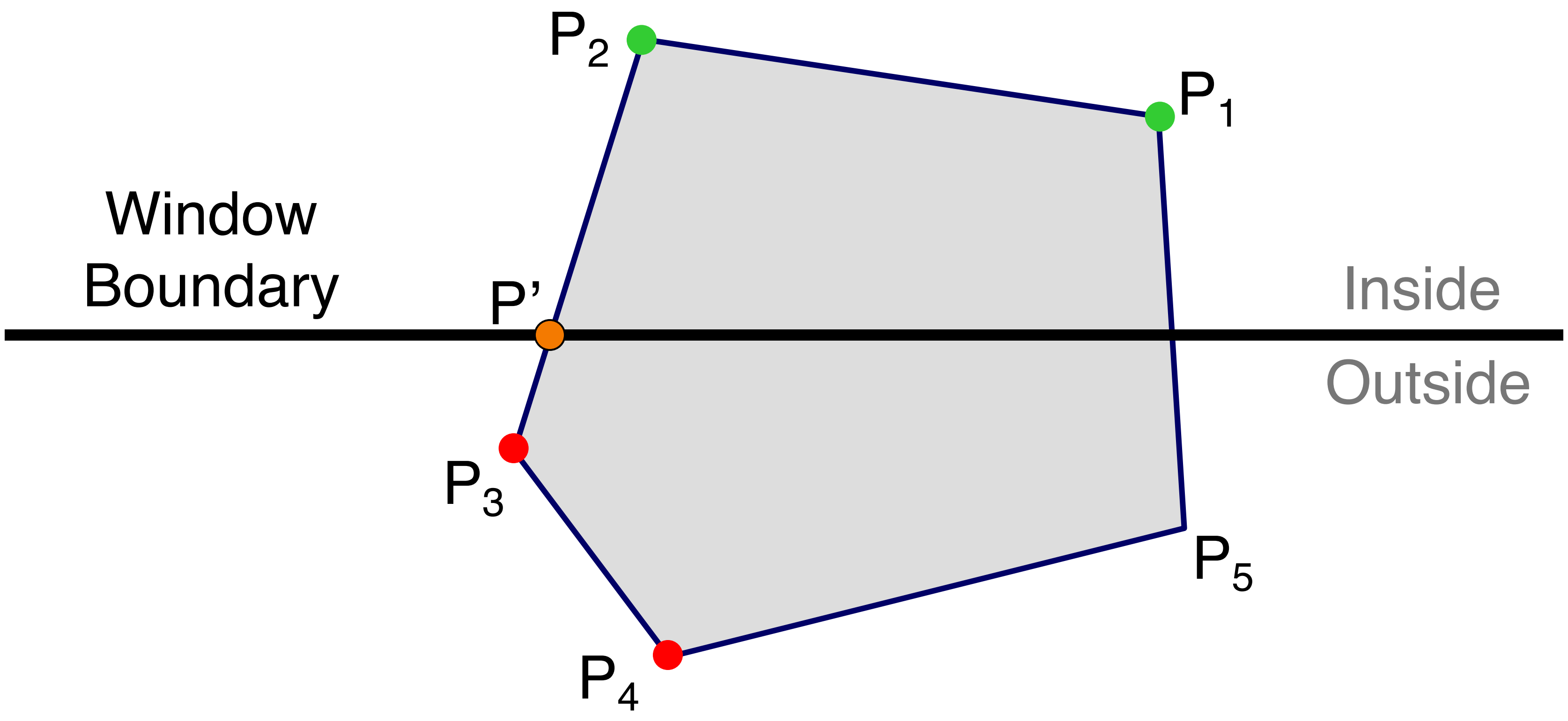

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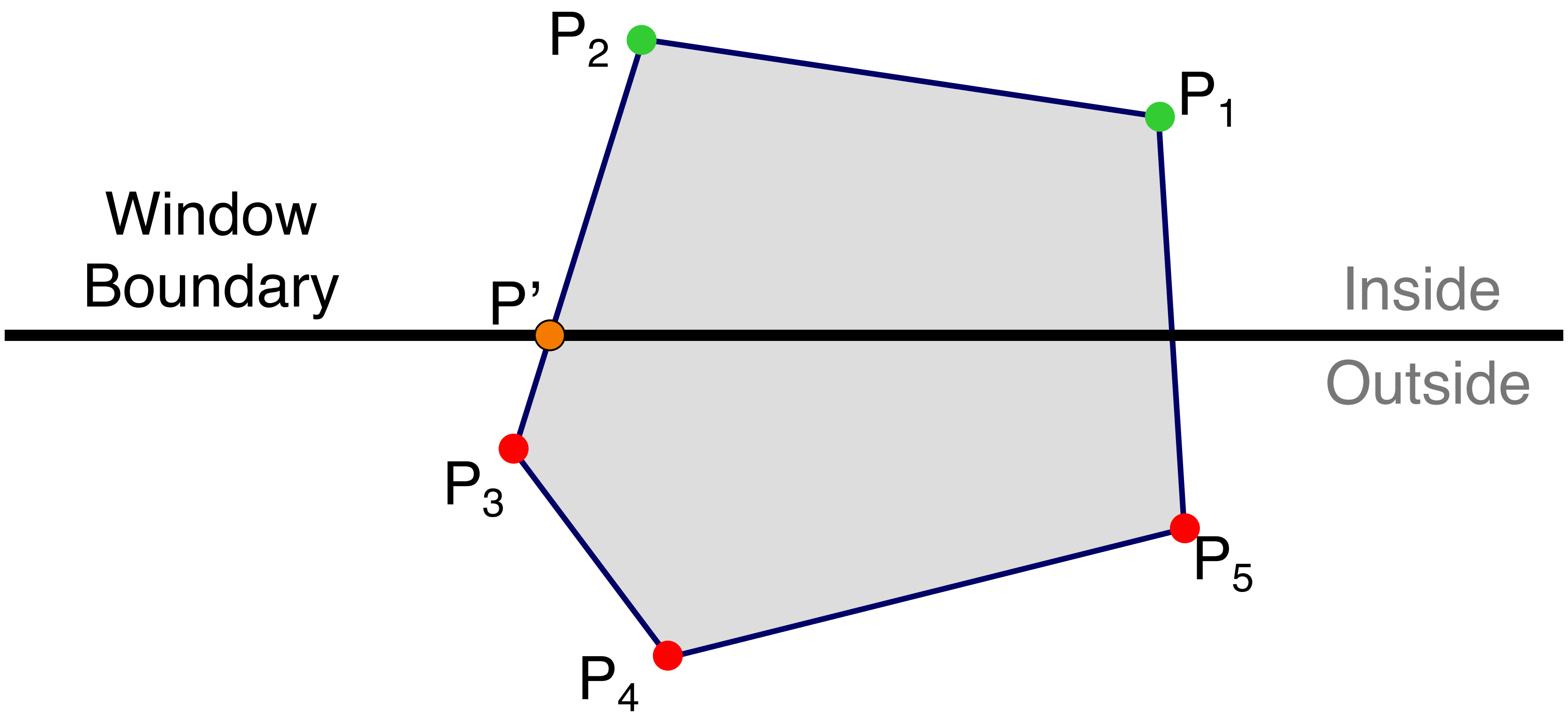

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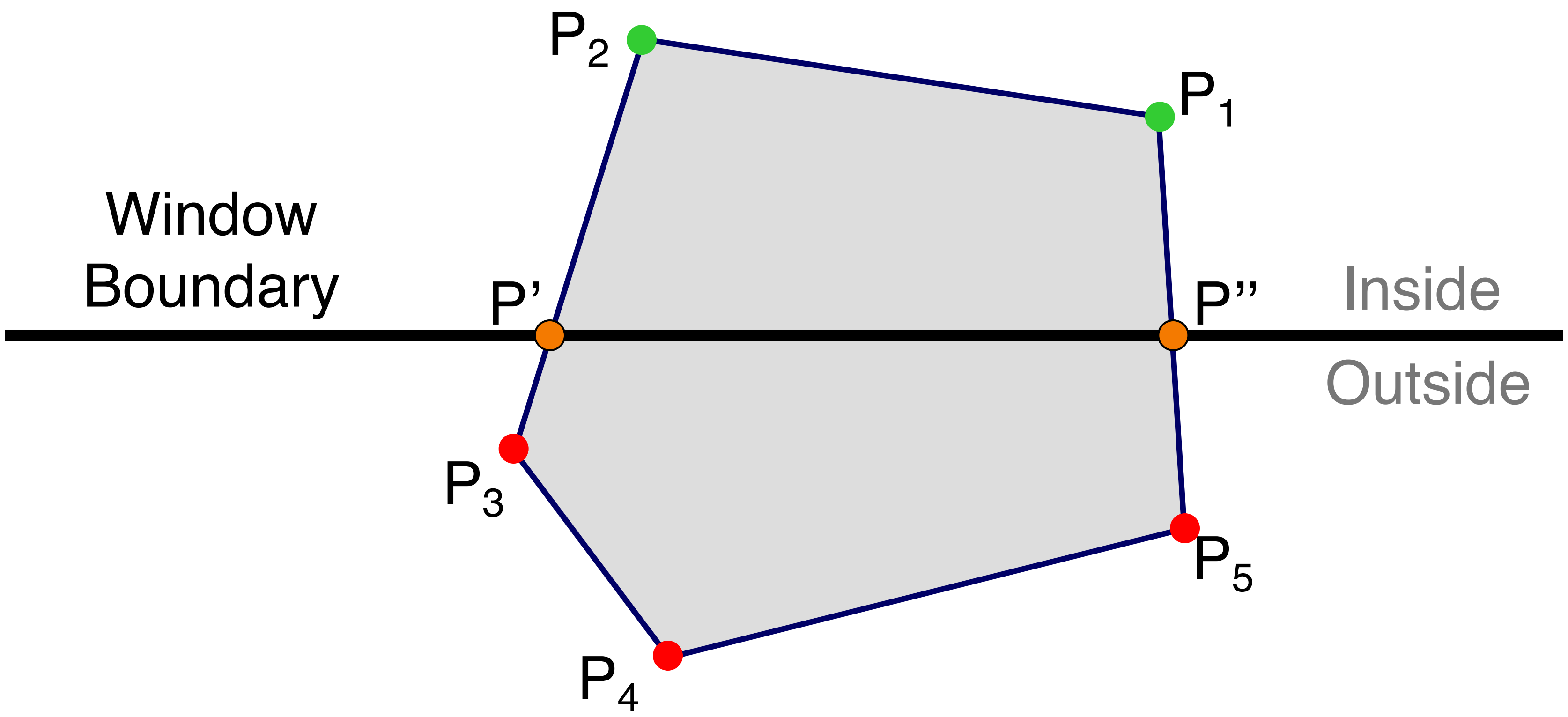

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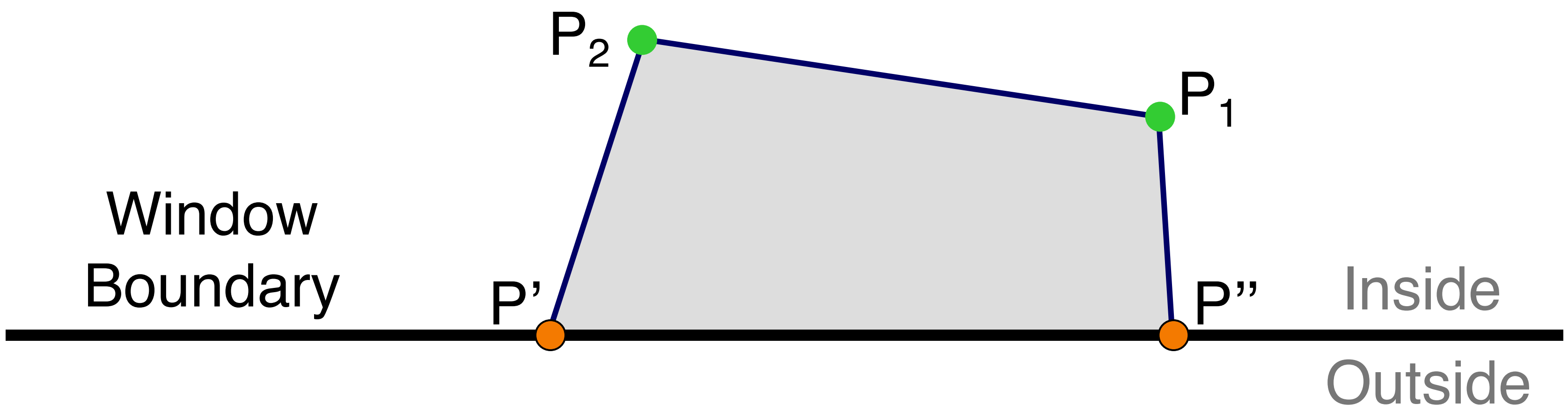
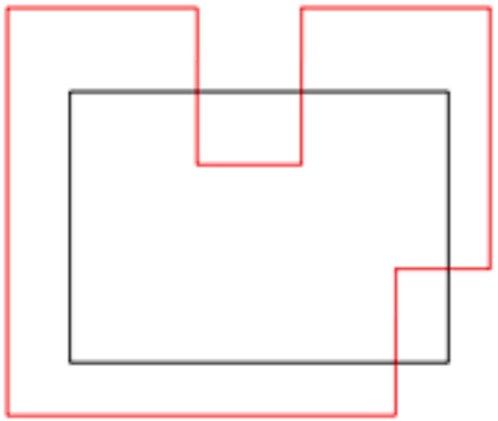


Sutherland Hodgeman Failure

- Concave Polygons



Sutherland Hodgeman Failure

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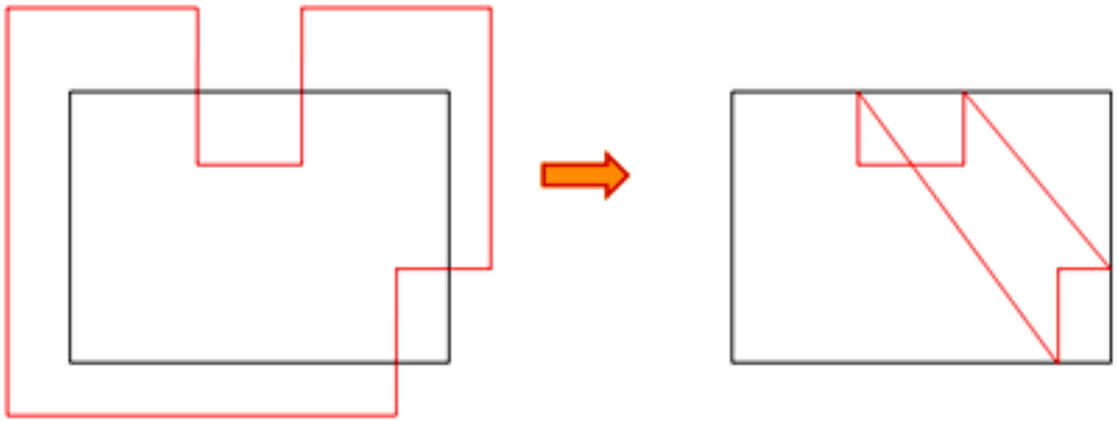
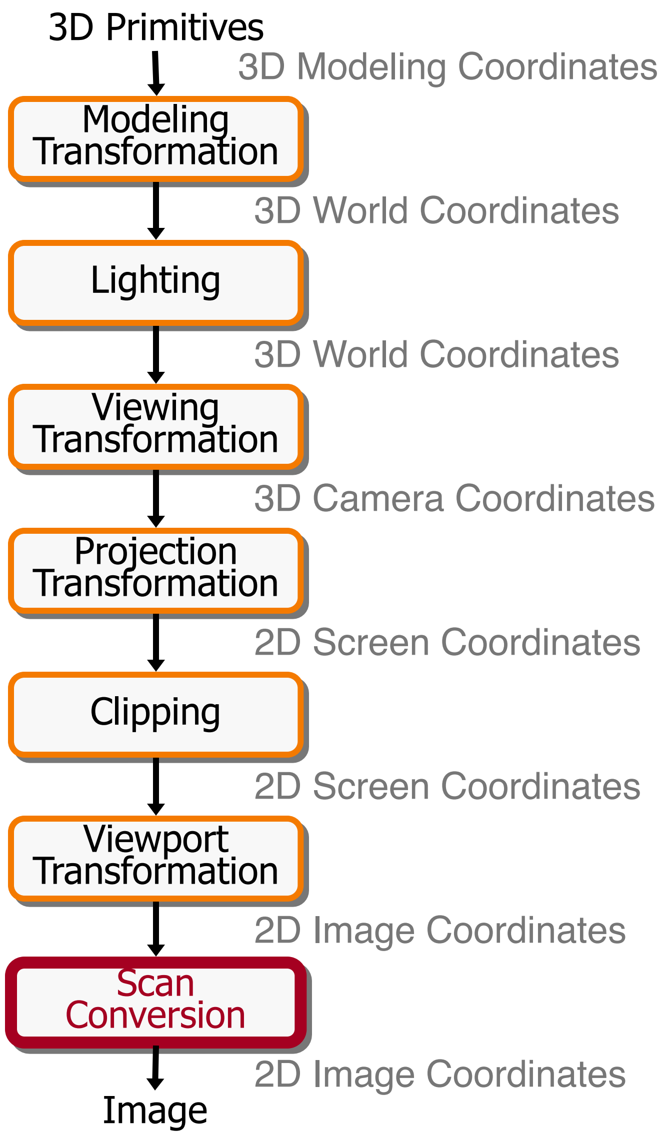
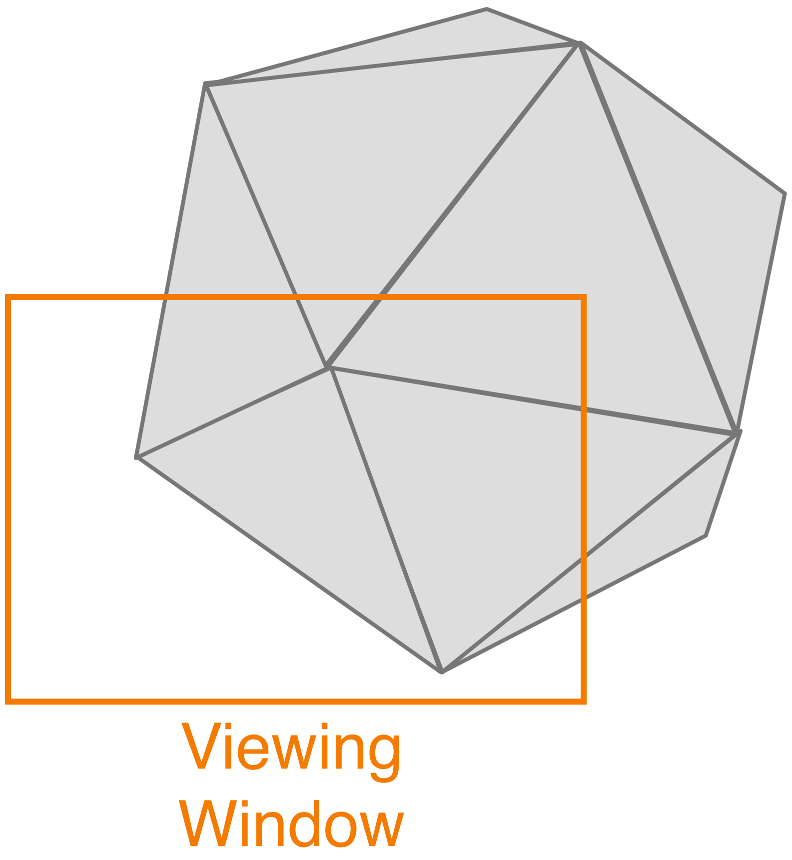
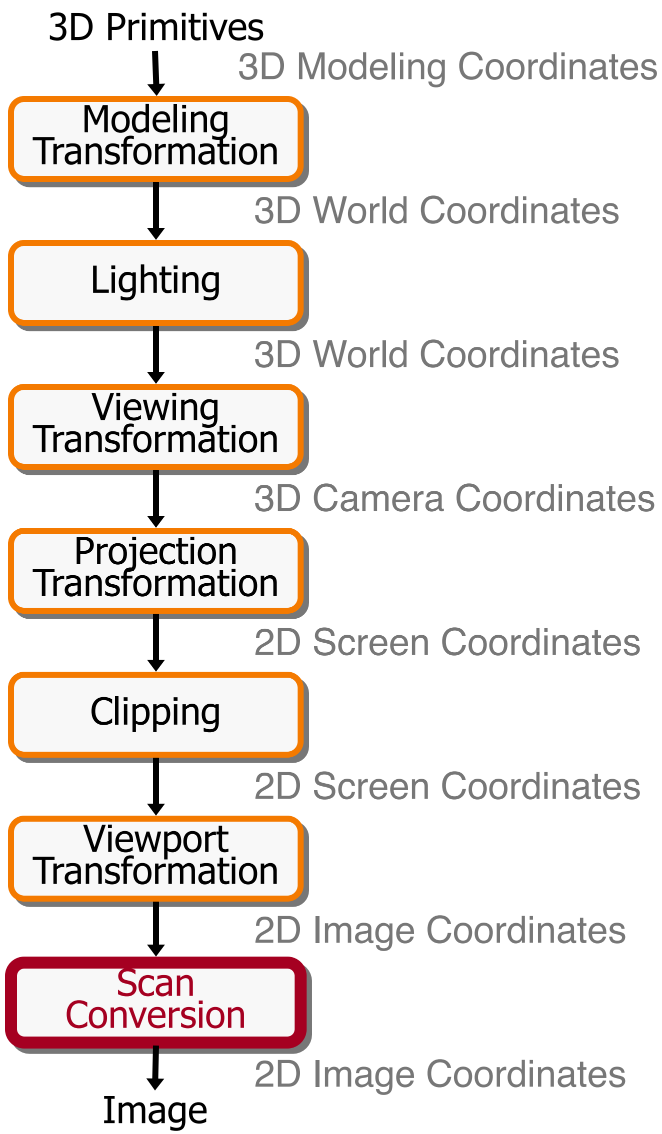


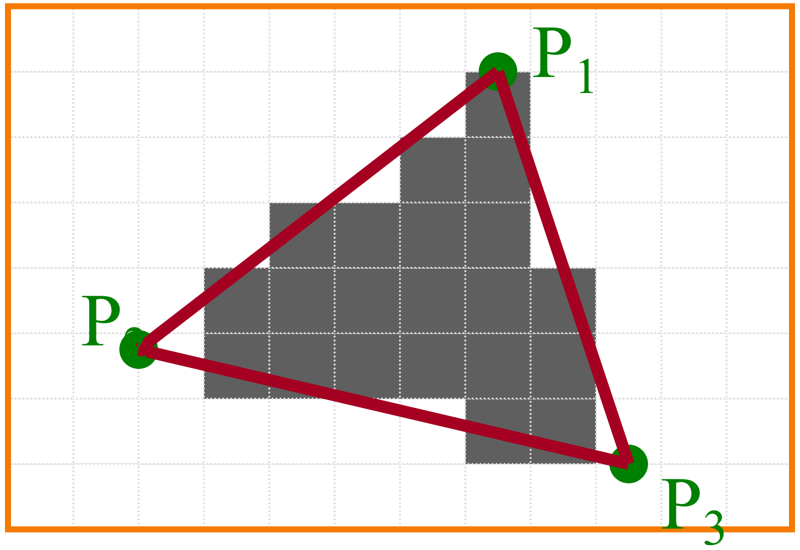
Rasterization Pipeline (for direct illumination)





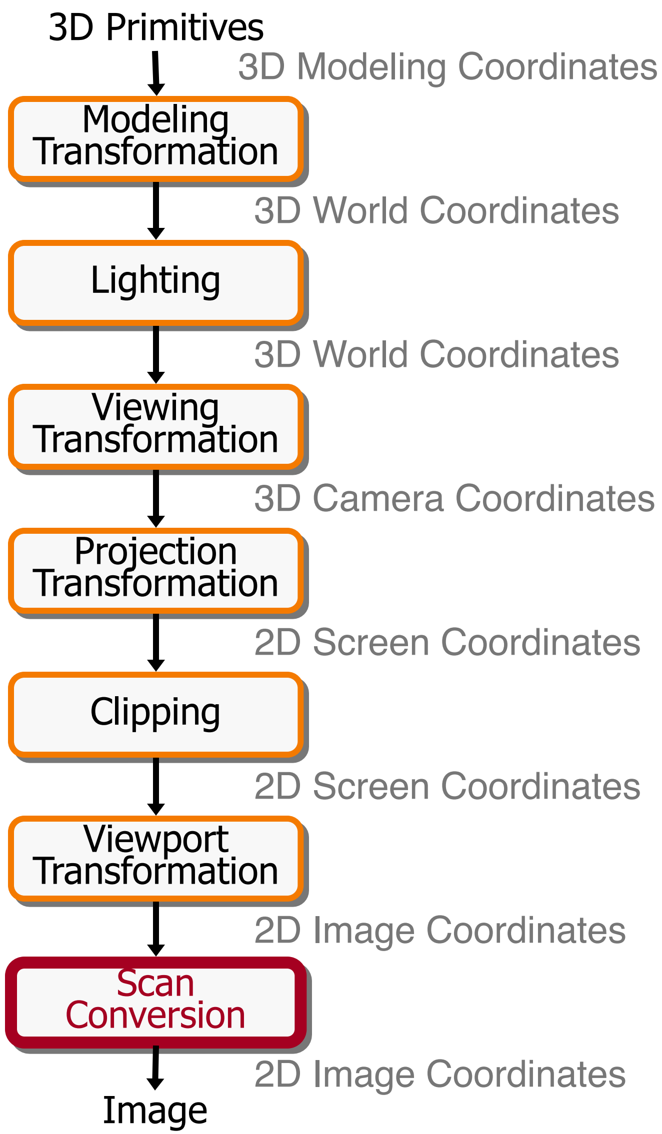
Rasterization Pipeline (for direct illumination)

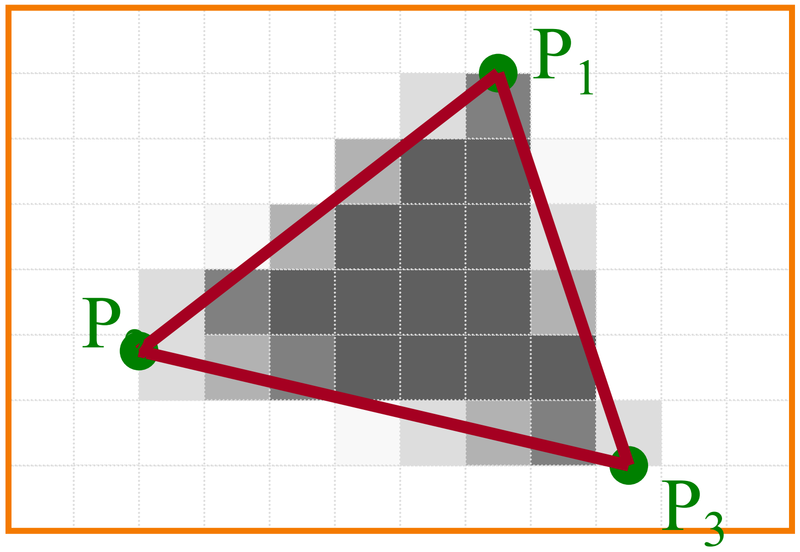




Standard (aliased)
Scan Conversion

Rasterization Pipeline (for direct illumination)





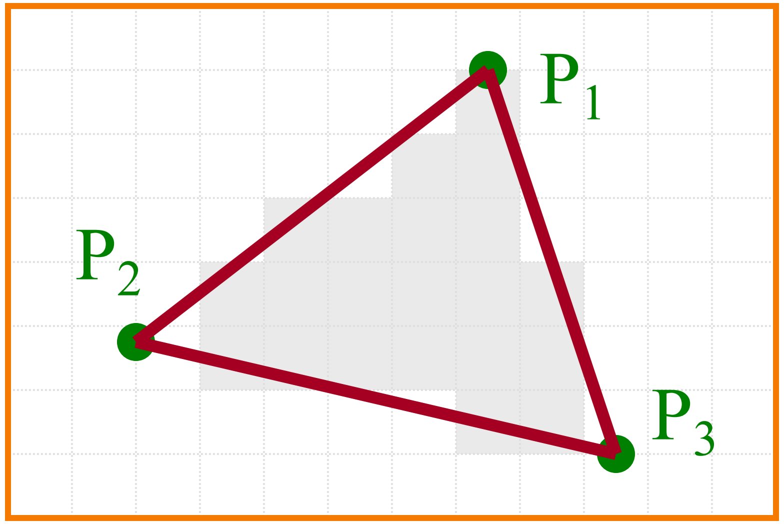
Antialiased
Scan Conversion

Scan Conversion

- Render an image of a geometric primitive by setting pixel colors

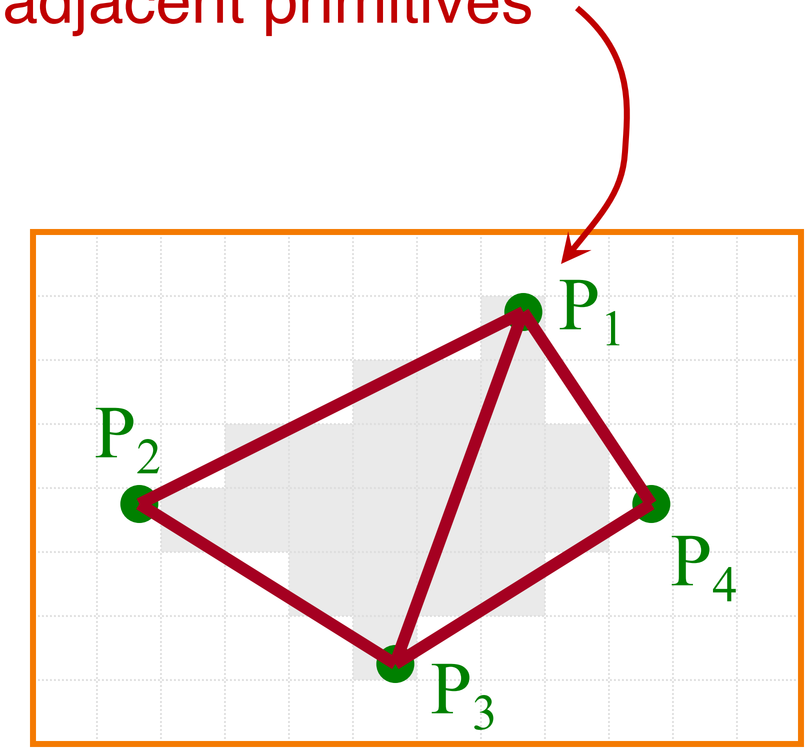
```
void SetPixel(int x, int y, Color rgba)
```

- Example: Filling the inside of a triangle



Triangle Scan Conversion

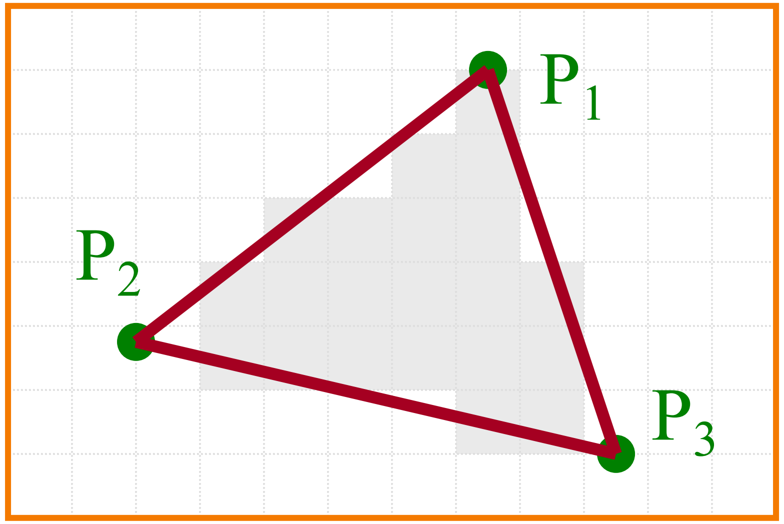
- Properties of a good algorithm
  - Symmetric
  - Straight edges
  - No cracks between adjacent primitives
  - (Antialiased edges)
  - FAST!



Simple Algorithm

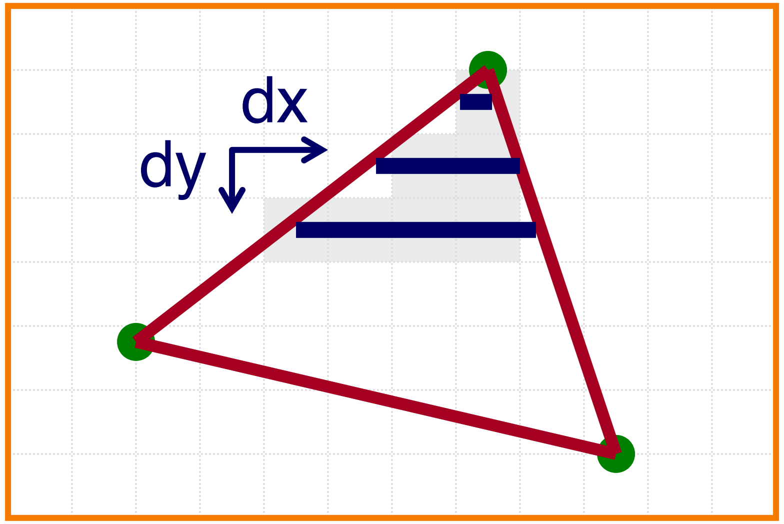
- Color all pixels inside triangle

```
void ScanTriangle(Triangle T, Color rgba){
   for each pixel P in bbox(T){
       if (Inside(T, P))
           SetPixel(P.x, P.y, rgba);
   }
}
```



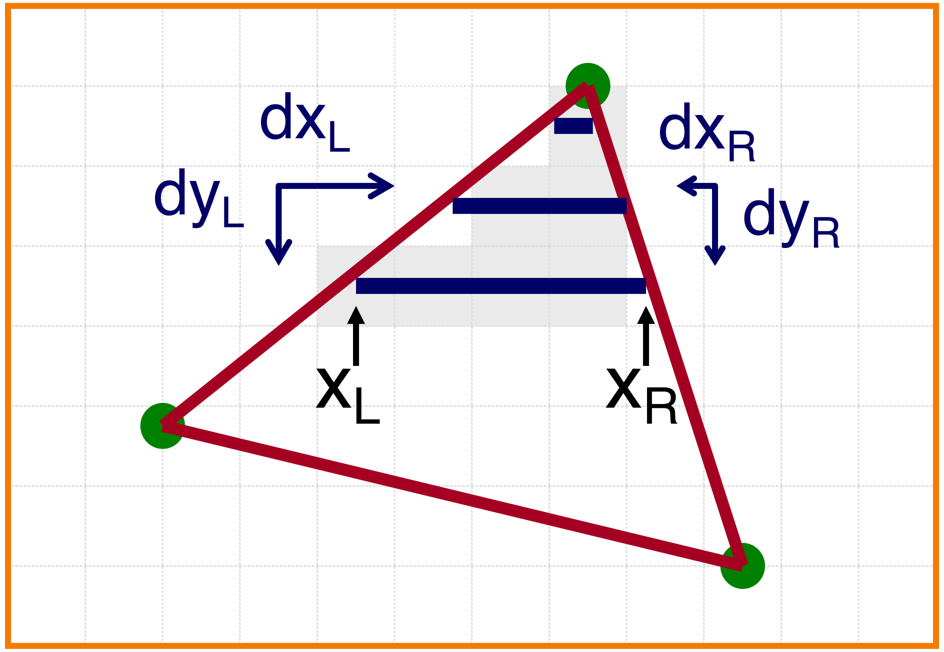
Triangle Sweep-Line Algorithm

- Take advantage of spatial coherence
  - Compute which pixels are inside using horizontal spans
  - Process horizontal spans in scan-line order
- Take advantage of edge linearity
  - Use edge slopes to update coordinates incrementally



Triangle Sweep-Line Algorithm

```
void ScanTriangle(Triangle T, Color rgba){
   for each edge pair {
      initialize xL, xR;
      compute dxL/dyL and dxR/dyR;
      for each scanline at y
         for (int x = xL; x <= xR; x++)
            SetPixel(x, y, rgba);
      xL += dxL/dyL;
      xR += dxR/dyR;
   }
}
```



Triangle Sweep-Line Algorithm

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void ScanTriangle(Triangle T, Color rgba){
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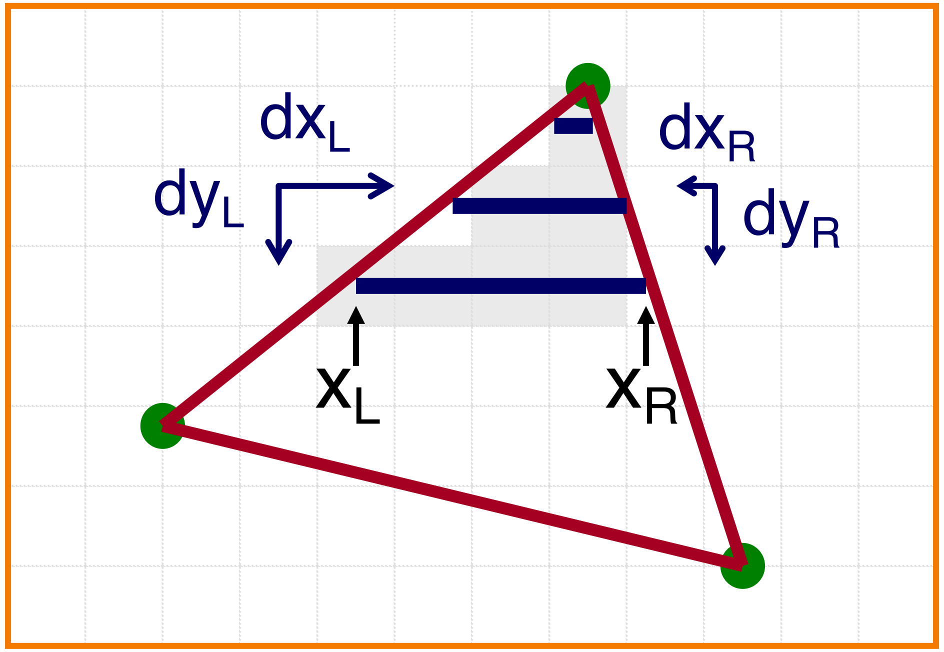
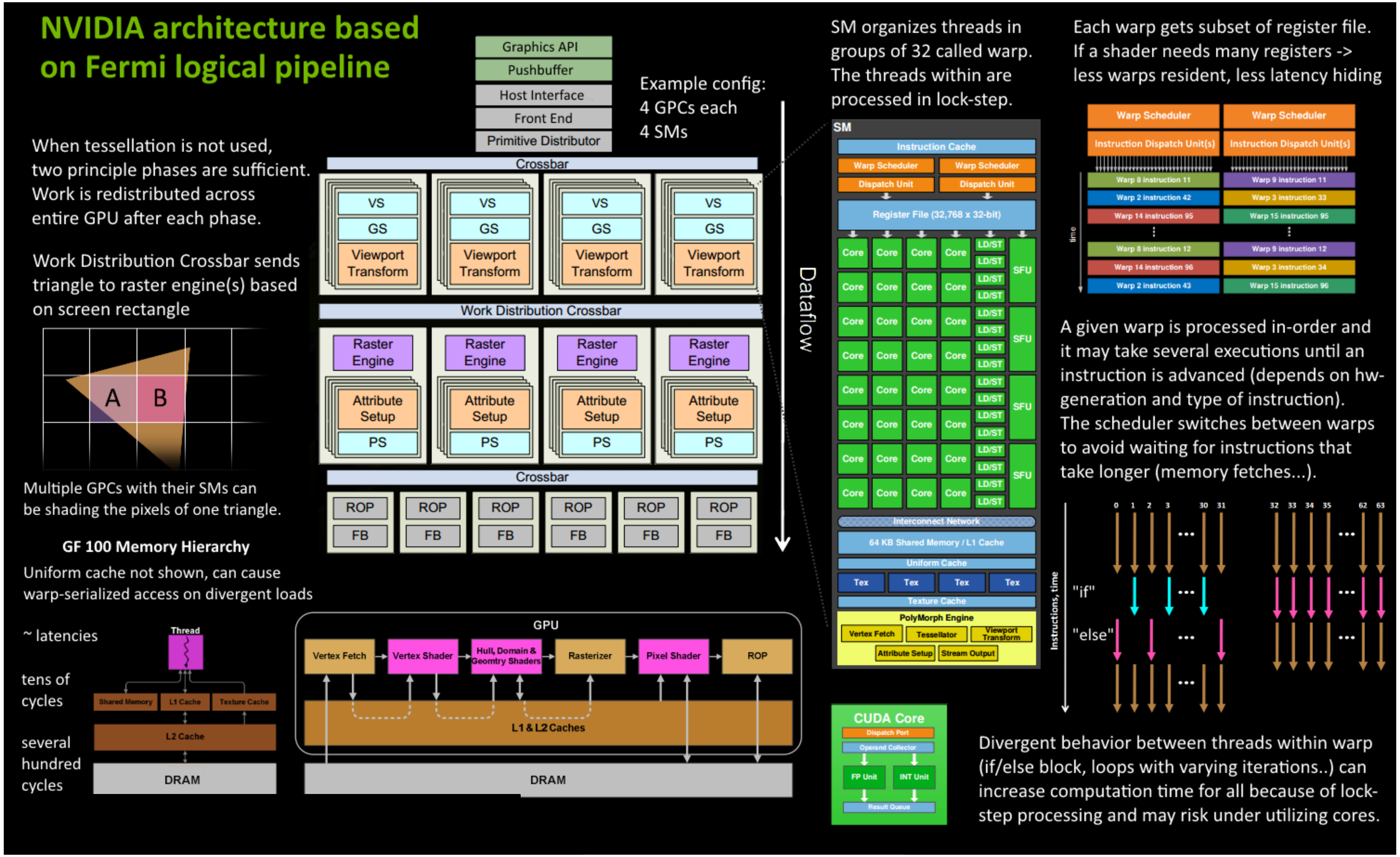
         for (int x = xL; x <= xR; x++)
            SetPixel(x, y, rgba);
      xL += dxL/dyL;
      xR += dxR/dyR;
   }
}
```

Minimize computation in inner loops



GPU Architecture



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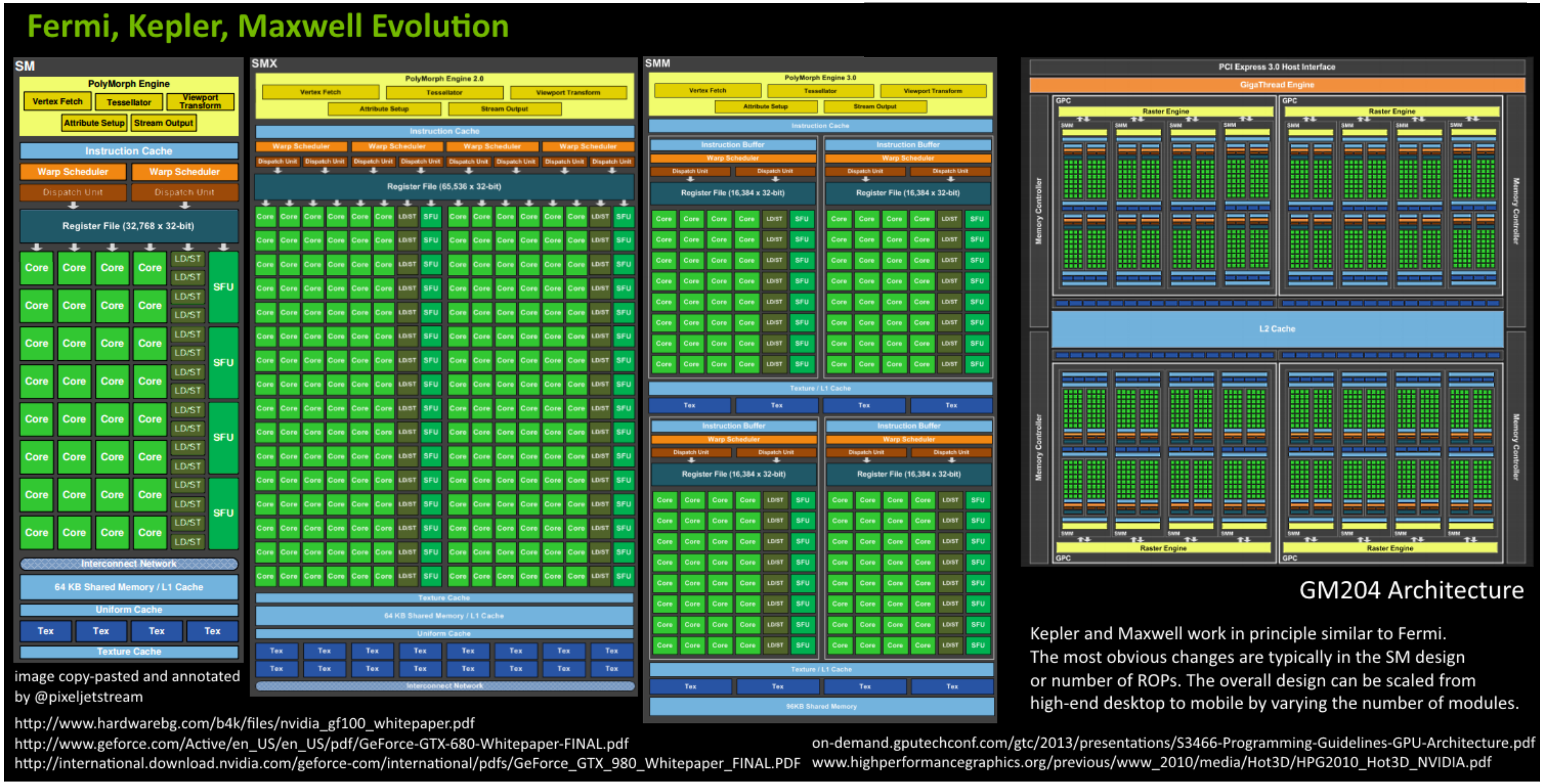

GPU Architecture