Lighting and Reflectance

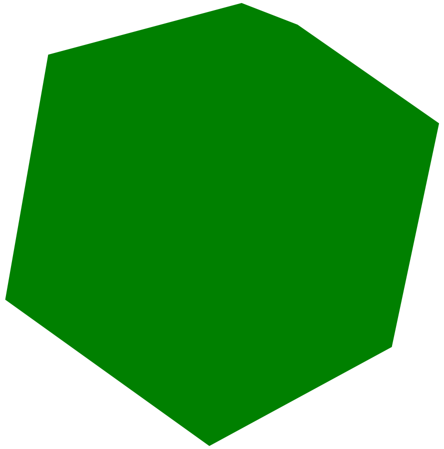
COS 426, Fall 2022

PRINCETON UNIVERSITY

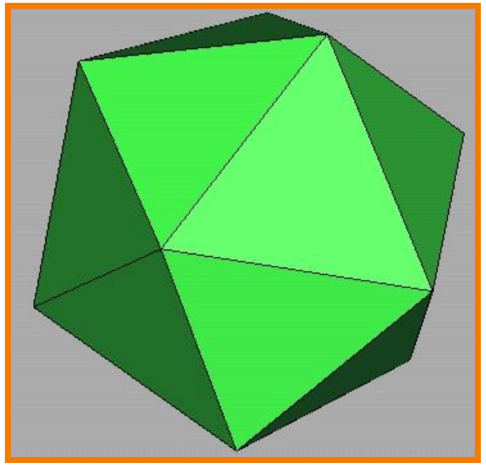
Ray Casting

```
R2Image *RayCast(R3Scene *scene, int width, int height)
{
    R2Image *image = new R2Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            R3Ray ray = ConstructRayThroughPixel(scene->camera, i, j);
            R3Rgb radiance = ComputeRadiance(scene,  &ray);
            image->SetPixel(i, j, radiance);
        }
    }
    return image;
}
```

```
R3Rgb ComputeRadiance(R3Scene *scene, R3Ray *ray)
{
    R3Intersection intersection = ComputeIntersection(scene, ray);
    return ComputeRadiance(scene, ray, intersection);
}
```



Without Illumination

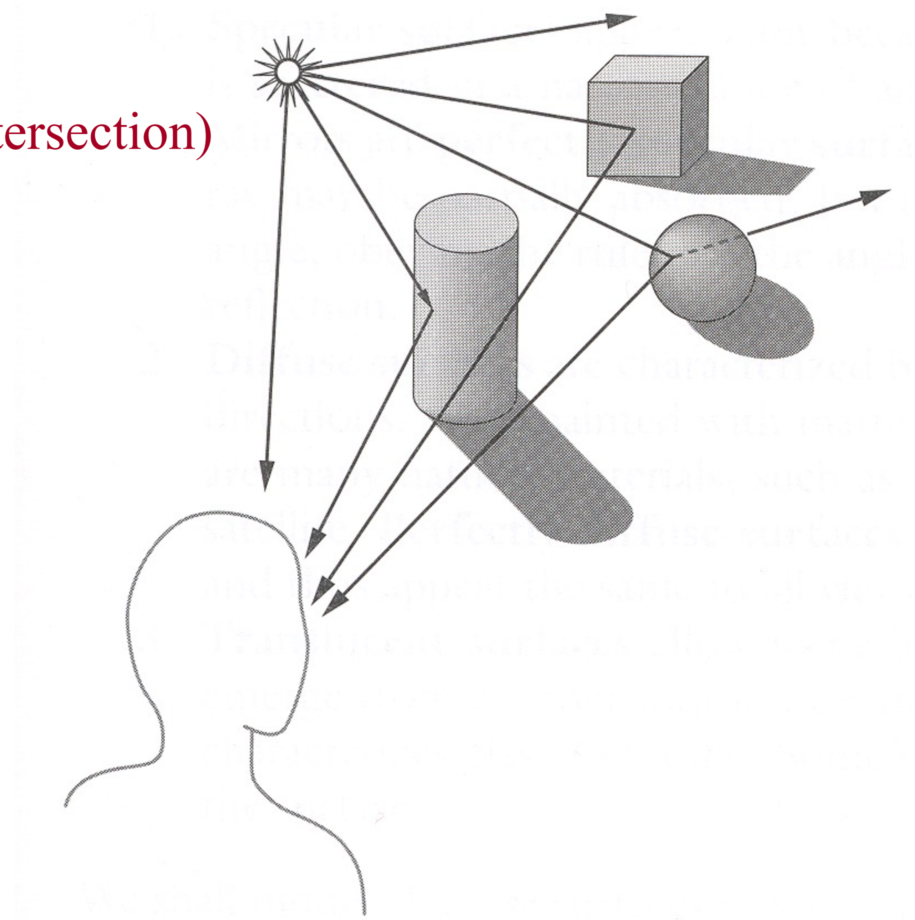


With Illumination

Illumination

- How do we compute radiance for a sample ray once we know what it hits?

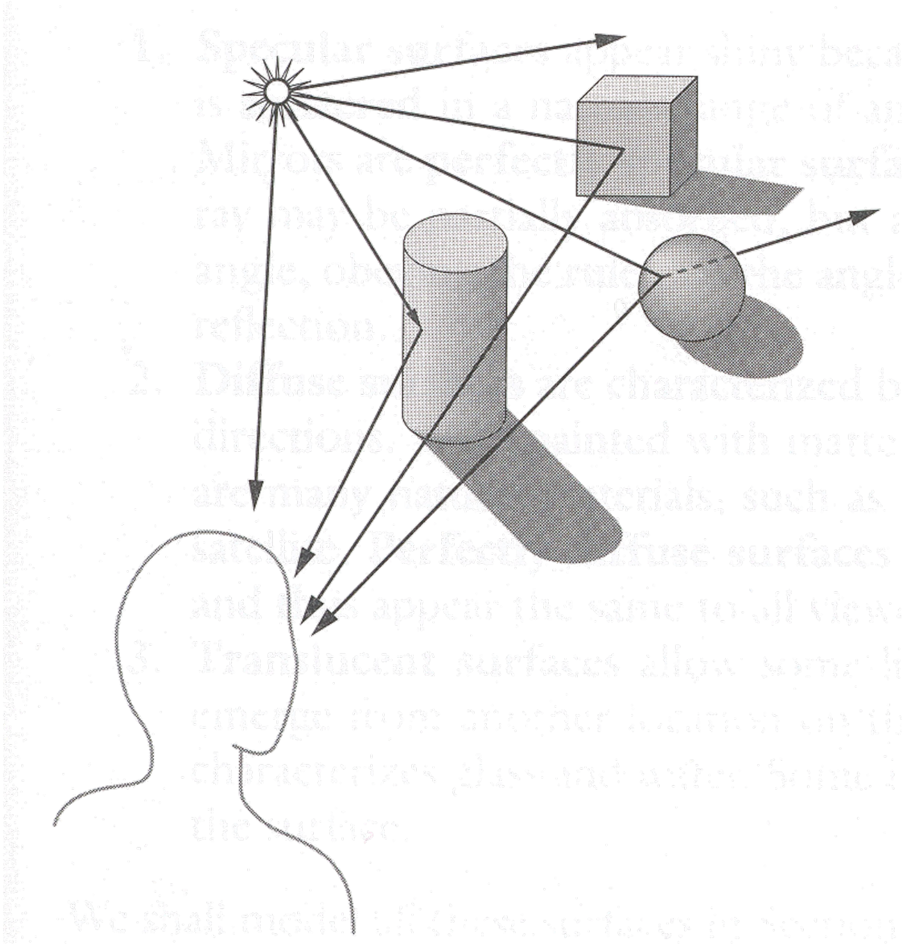
ComputeRadiance(scene, ray, intersection)



Angel Figure 6.2

Goal

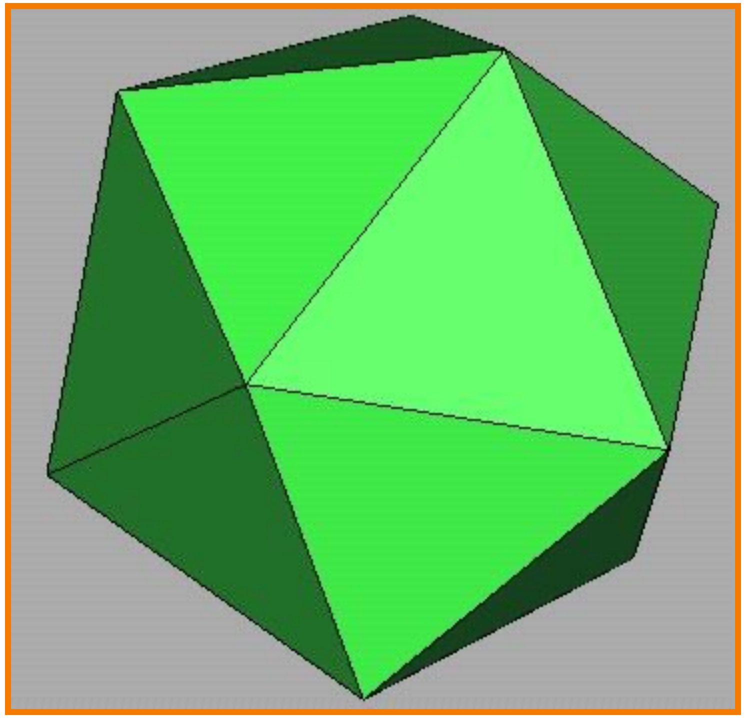
- Must derive computer models for ...
  - Emission at light sources
  - Scattering at surfaces
  - Reception at the camera



- Desirable features …
  - Concise
  - Efficient to compute
  - “Accurate”

Overview

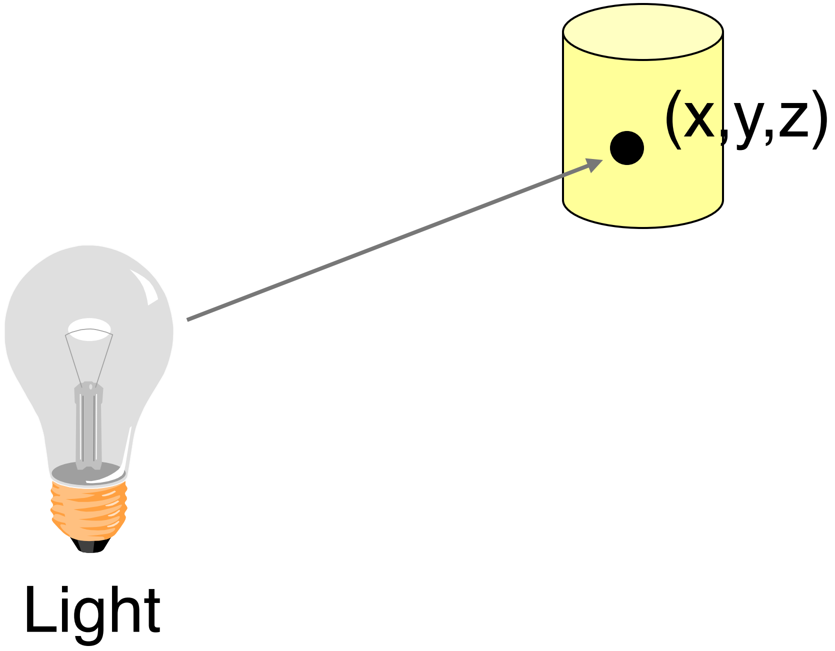
- Direct Illumination
  - Emission at light sources
  - Scattering at surfaces
- Global illumination
  - Shadows
  - Refractions
  - Inter-object reflections



Direct Illumination

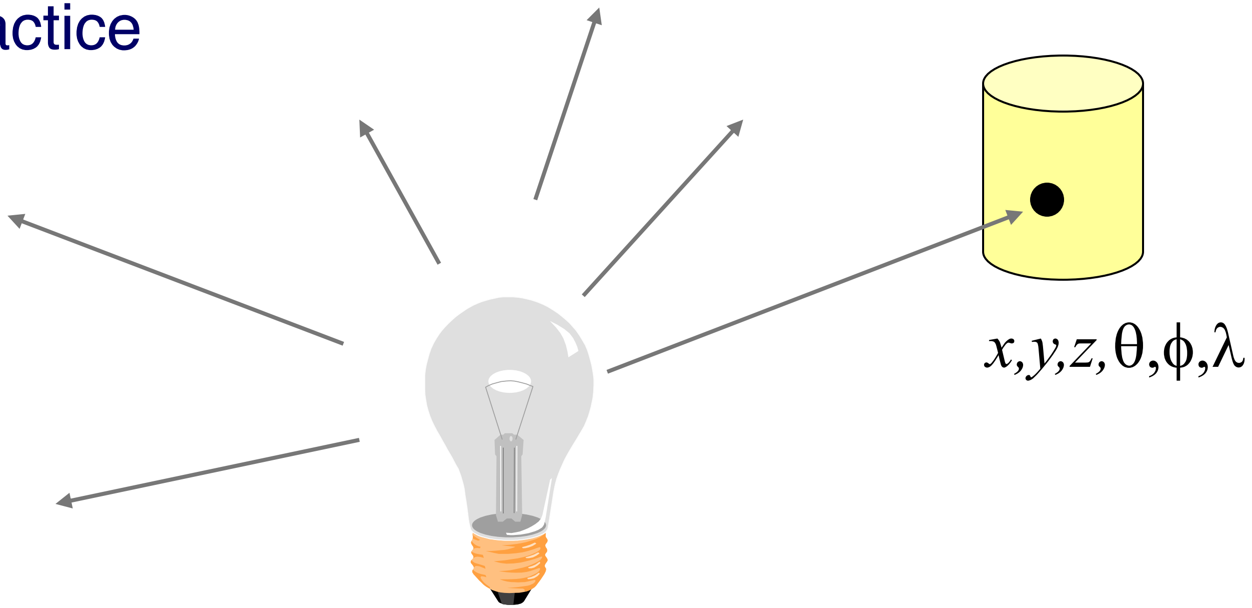
Emission at Light Sources

- $I_L(x,y,z,\theta,\phi,\lambda)$ ...
  - describes the intensity of energy,
  - leaving a light source, …
  - arriving at location(x,y,z), ...
  - in direction (θ,ϕ), ...
  - with wavelength λ



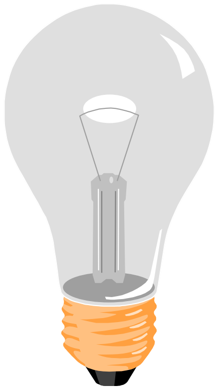
Empirical Models

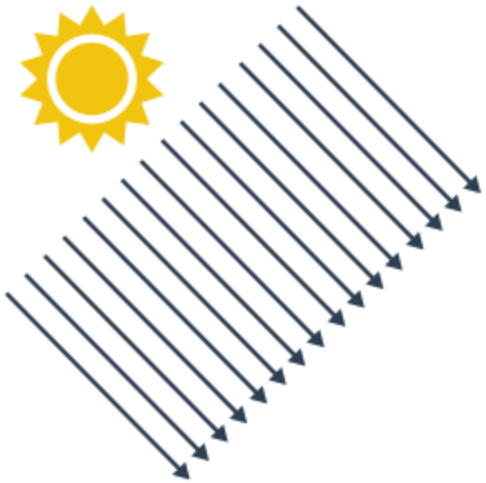
- Ideally measure irradiant energy for "all" situations
  - Too much storage
  - Difficult in practice



OpenGL Light Source Models

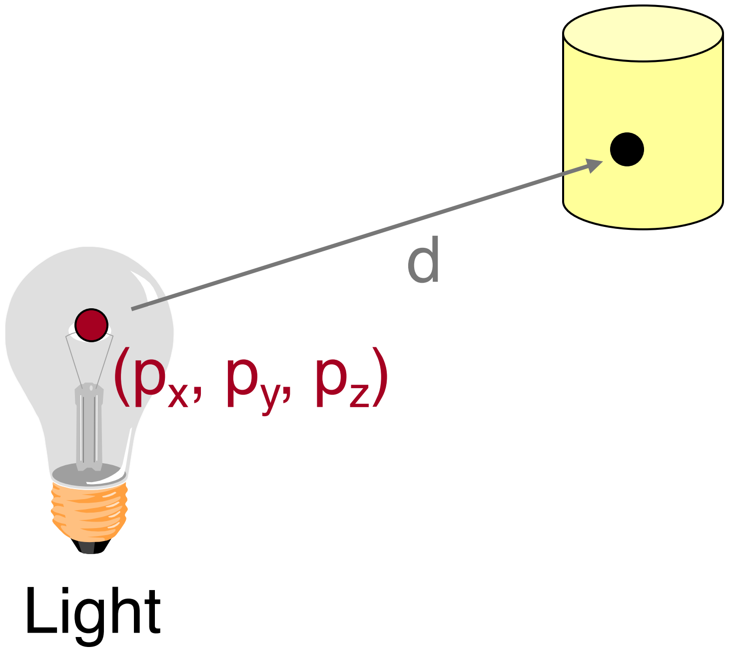
- Simple mathematical models:
  - Point light
  - Directional light
  - Spot light





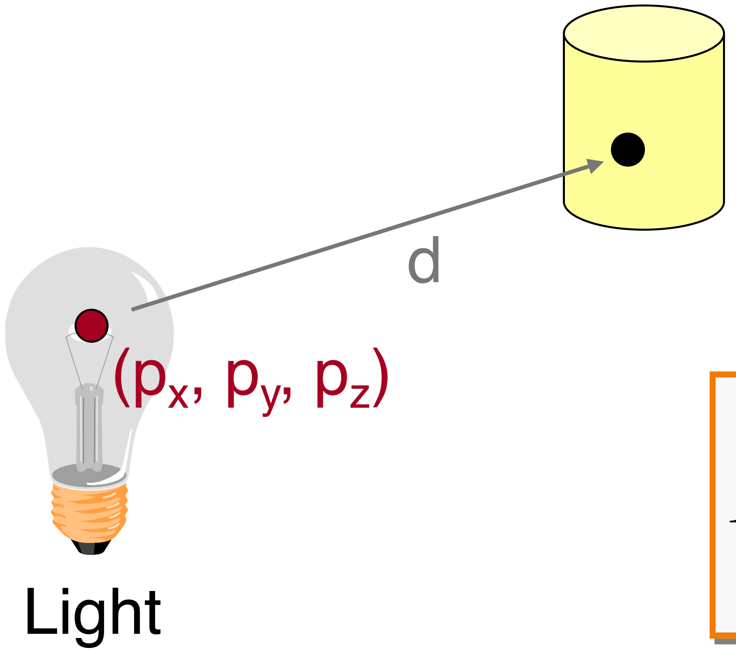
Point Light Source

- Models omni-directional point source
  - intensity (I_0),
  - position (p_x, p_y, p_z),



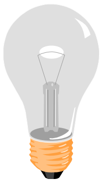
Point Light Source

- Models omni-directional point source
  - intensity (I_0),
  - position (p_x, p_y, p_z),
  - coefficients (c_a, l_a, q_a) for attenuation with distance (d)



$$I_L = \frac{I_0}{c_a + l_a d + q_a d^2}$$

Point Light Source

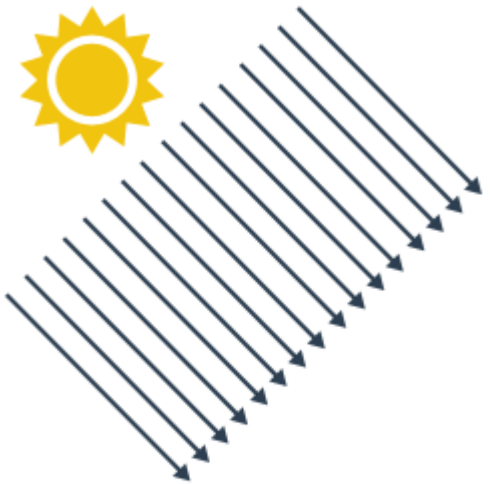


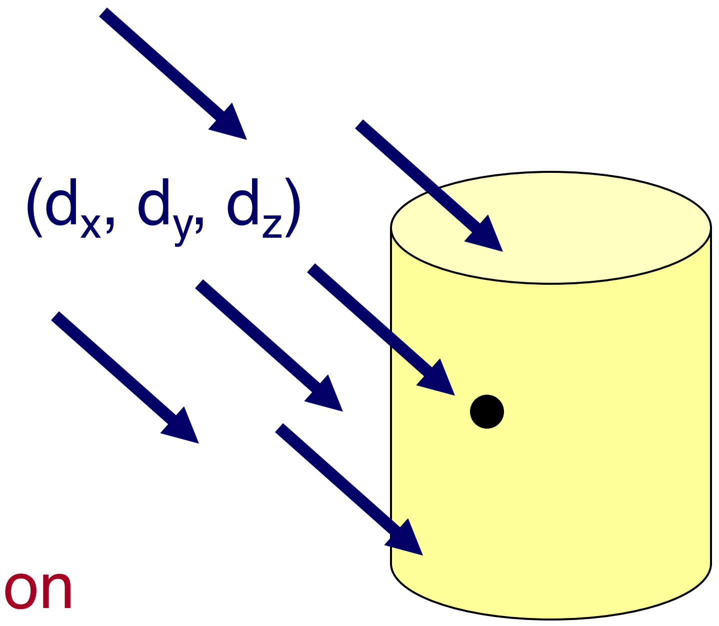
$$I_L = \frac{I_0}{c_a + l_a d + q_a d^2}$$

- Physically-based: “inverse square law”
  - $c_a = l_a = 0$
- Use c_a and $l_a \neq 0$ for non-physical effects
  - Better control of the look (artistic)

Directional Light Source

- Models point light source at infinity
  - intensity (I_0),
  - direction (d_x,d_y,d_z)



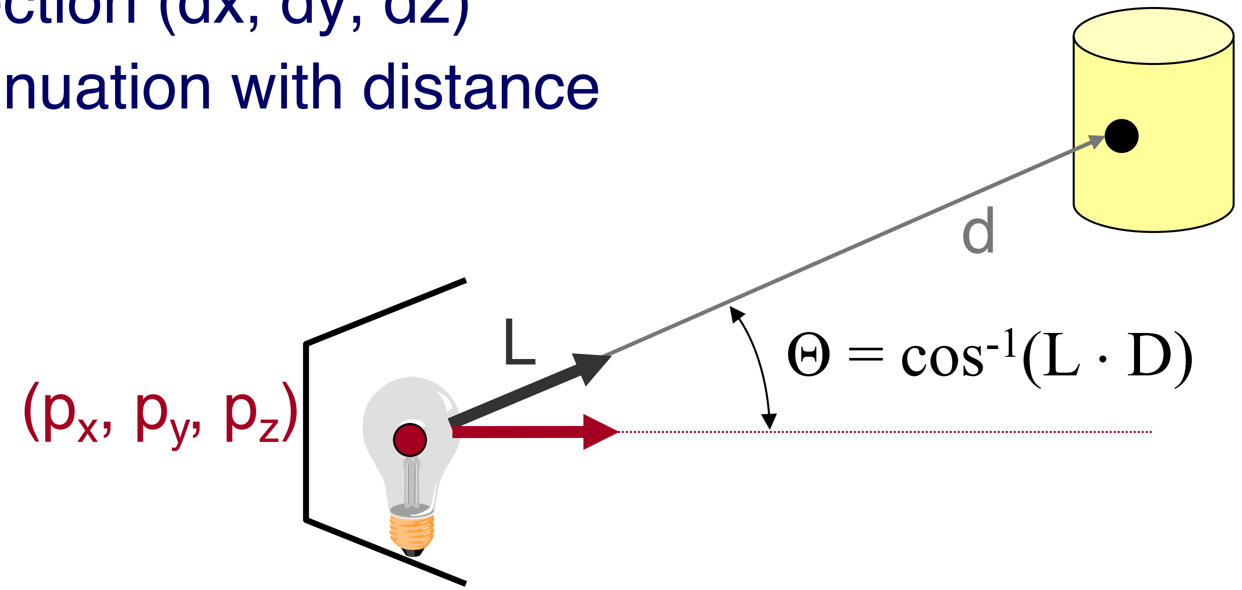


No attenuation with distance

$$I_L = I_0$$

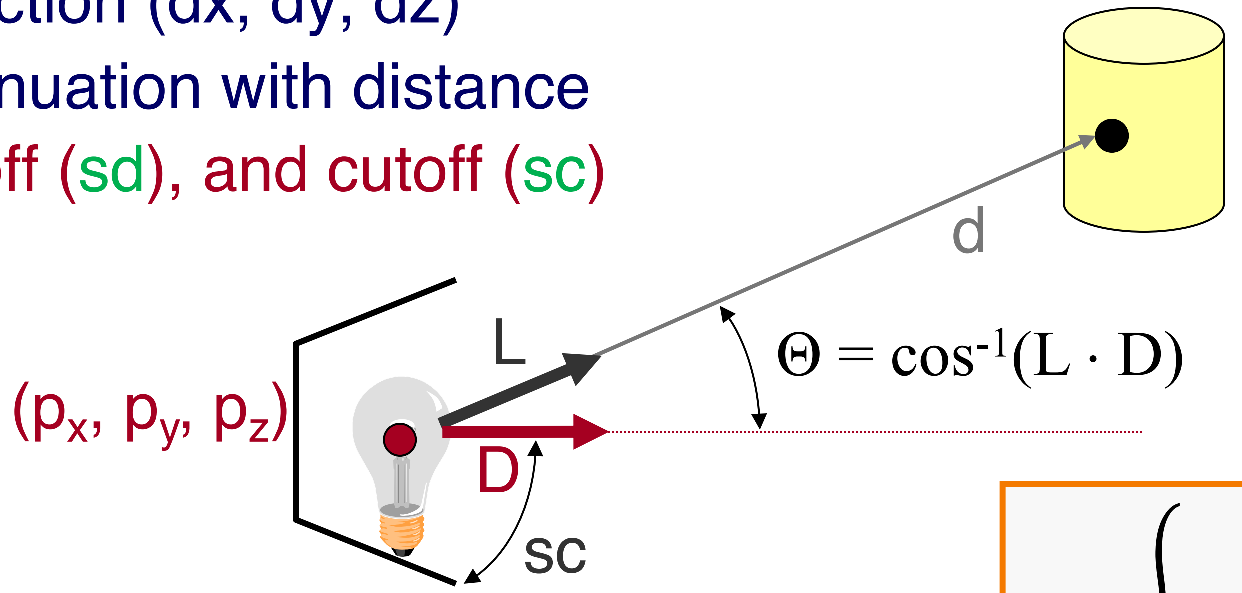
Spot Light Source

- Models point light source with direction
  - intensity (I0),
  - position (px, py, pz),
  - direction (dx, dy, dz)
  - attenuation with distance



Spot Light Source

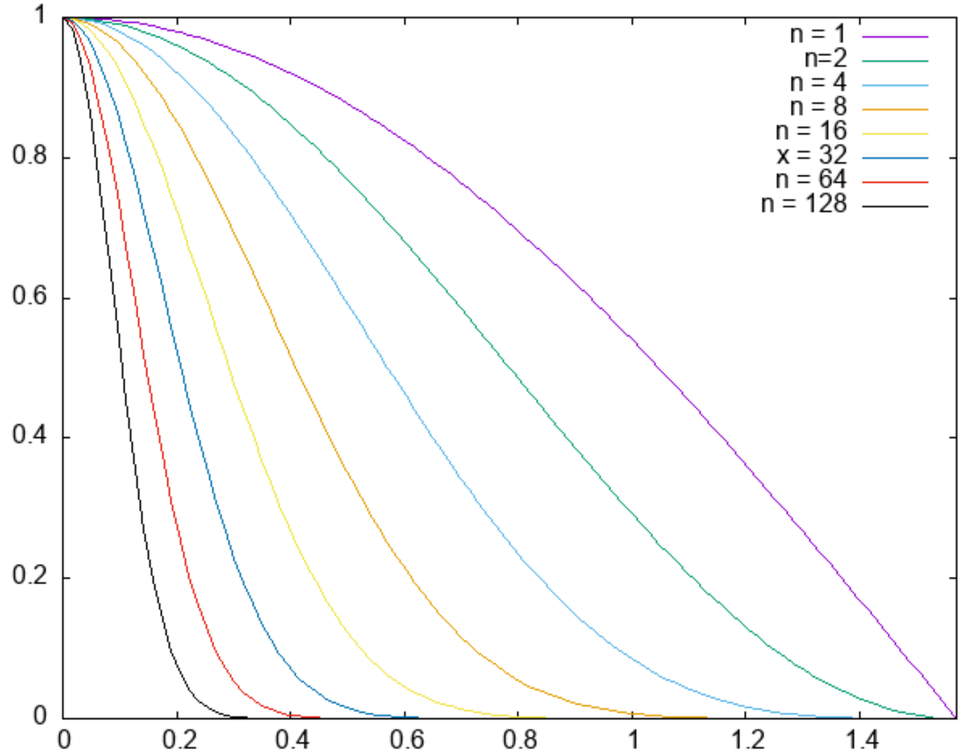
- Models point light source with direction
  - intensity (I0),
  - position (px, py, pz),
  - direction (dx, dy, dz)
  - attenuation with distance
  - falloff (sd), and cutoff (sc)



$$I_L = \begin{cases} \dfrac{I_0(\cos\Theta)^{sd}}{c_a + l_a d + q_a d^2} & \text{if } \Theta \leq sc, \\ 0 & \text{otherwise} \end{cases}$$

Power of Dot Product

- $(\cos\theta)^n = (a \cdot b)^n$
- Common form for “peaky” functions
- “Peakiness” depends on *n*
- We’ll see it later as well…



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  - Scattering at surfaces
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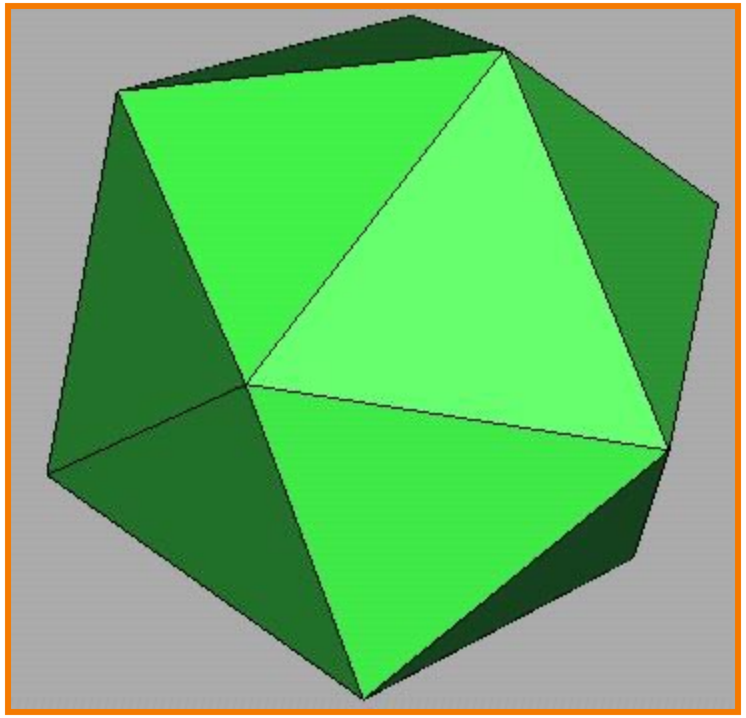
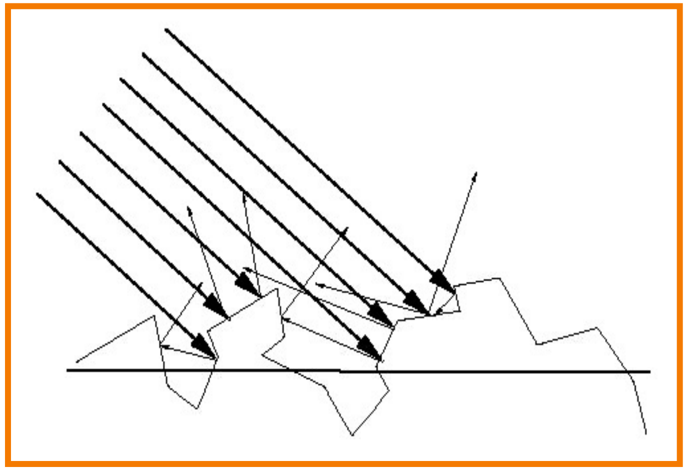


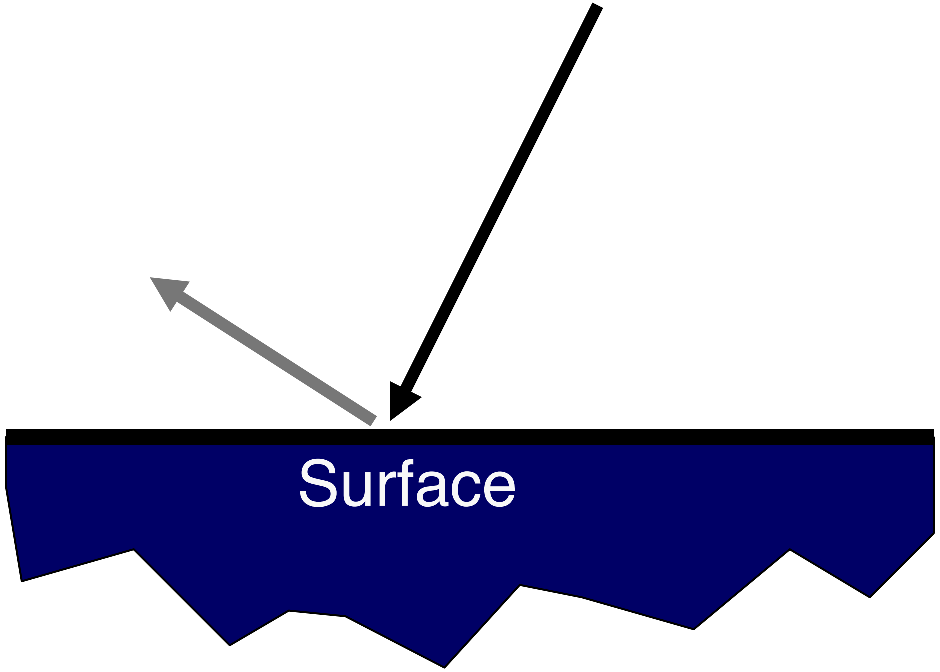
Direct Illumination

Scattering at Surfaces

Bidirectional Reflectance Distribution Function $f_r(\theta_i,\phi_i,\theta_o,\phi_o,\lambda)$ ...

- describes the aggregate fraction of incident energy,

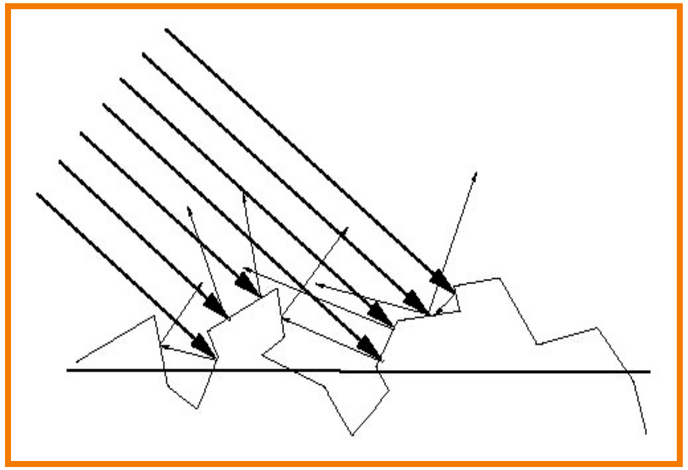


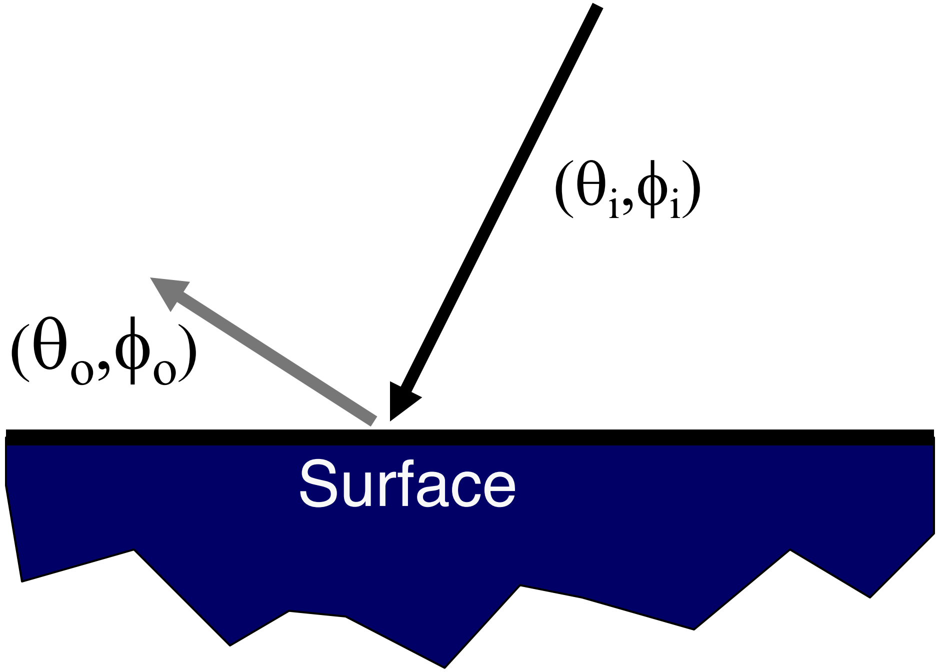


Scattering at Surfaces

Bidirectional **R**eflectance **D**istribution **F**unction $f_r(\theta_i,\phi_i,\theta_o,\phi_o,\lambda)$ ...

- describes the aggregate fraction of incident energy,
- arriving from direction (θ_i,ϕ_i), ...
- leaving in direction (θ_o,ϕ_o), …





Scattering at Surfaces

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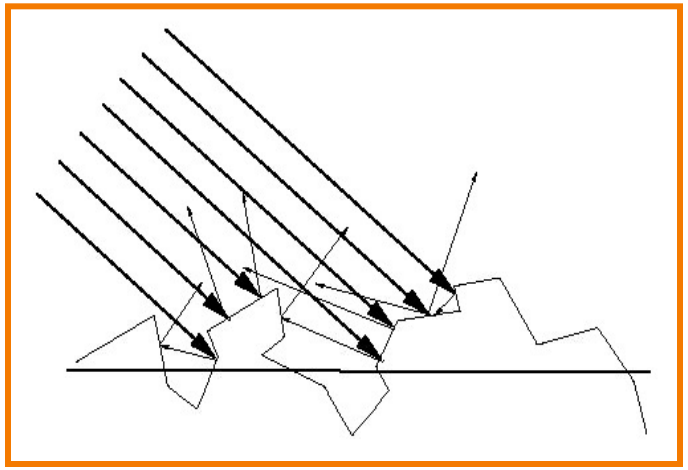
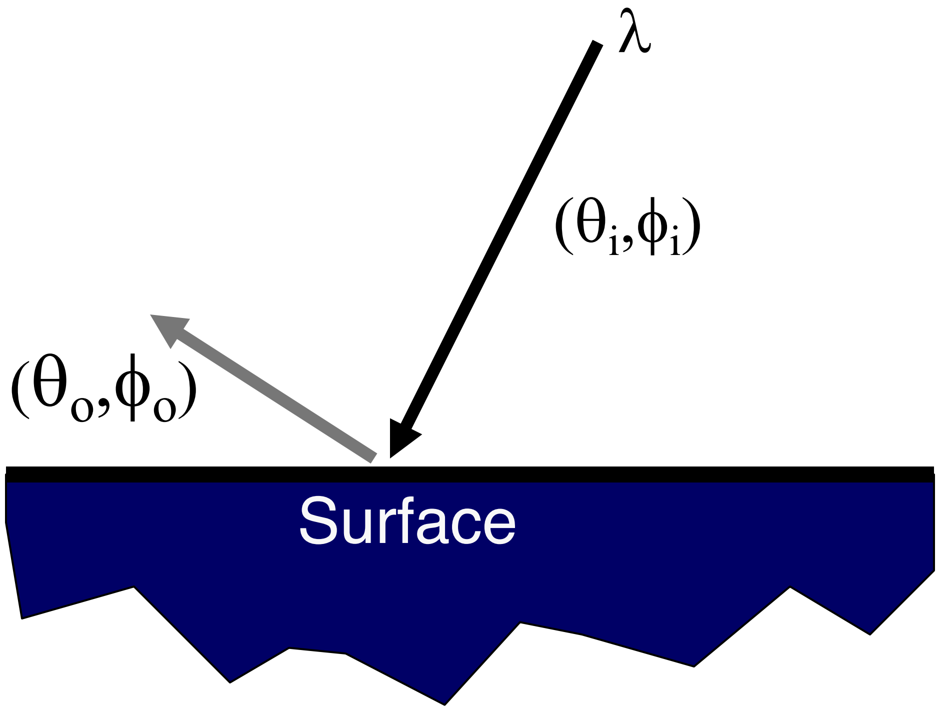
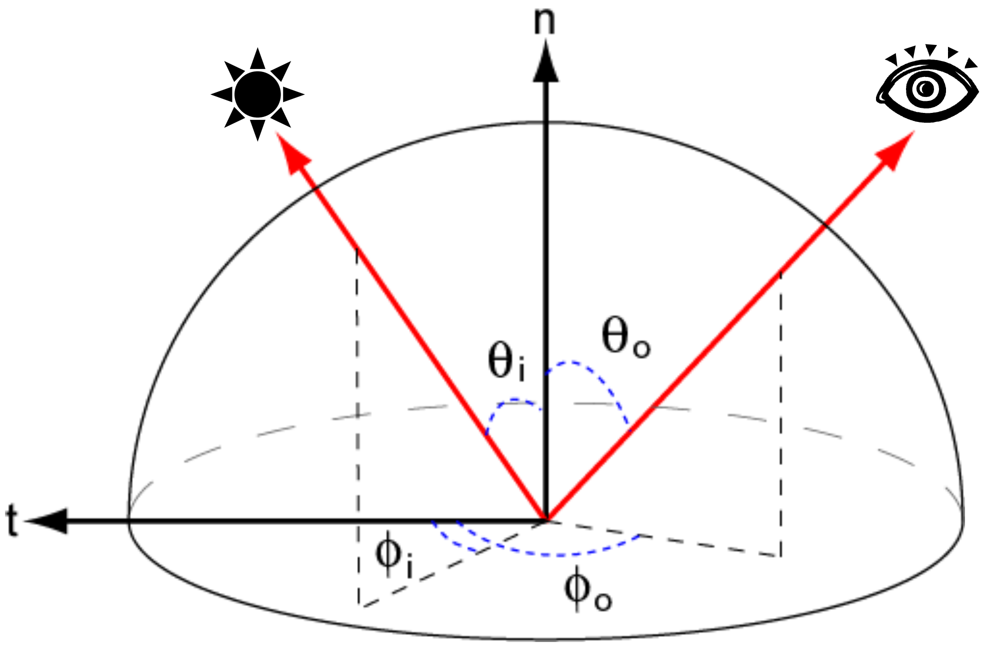
- with wavelength λ





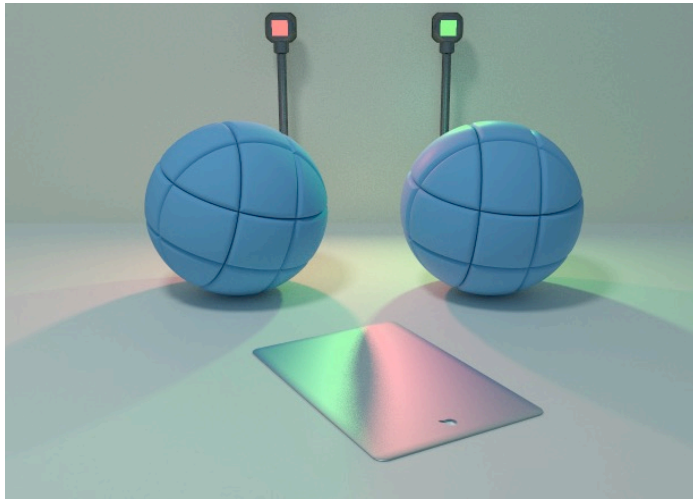
Empirical Models

- Ideally measure BRDF for “all” combinations of angles: $\theta_i, \phi_i, \theta_o, \phi_o$
  - Difficult in practice
  - Too much storage

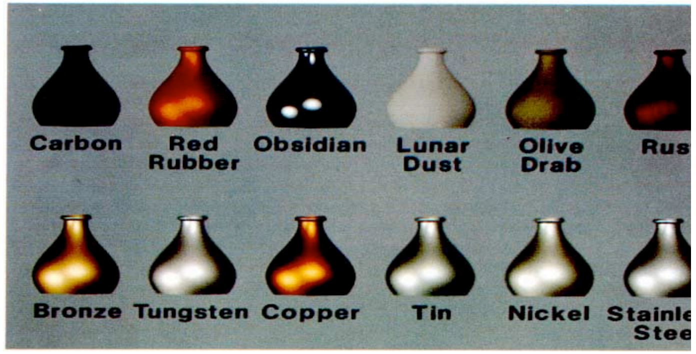


Parametric Models

- Approximate BRDF with simple parametric function that is fast to compute
  - Phong [75]
  - Blinn-Phong [77]
  - Cook-Torrance [81]
  - He et al. [91]
  - Ward [92]
  - Lafortune et al. [97]
  - Ashikhmin et al. [00]
  - etc.



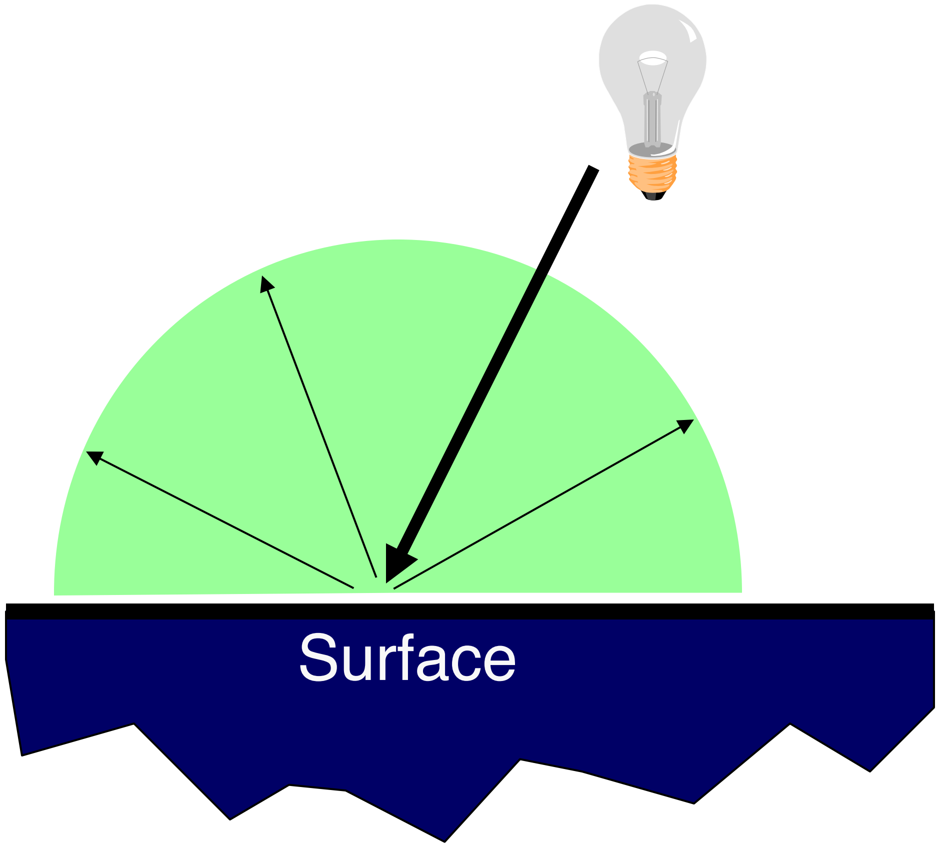
Lafortune [97]



Cook-Torrance [81]

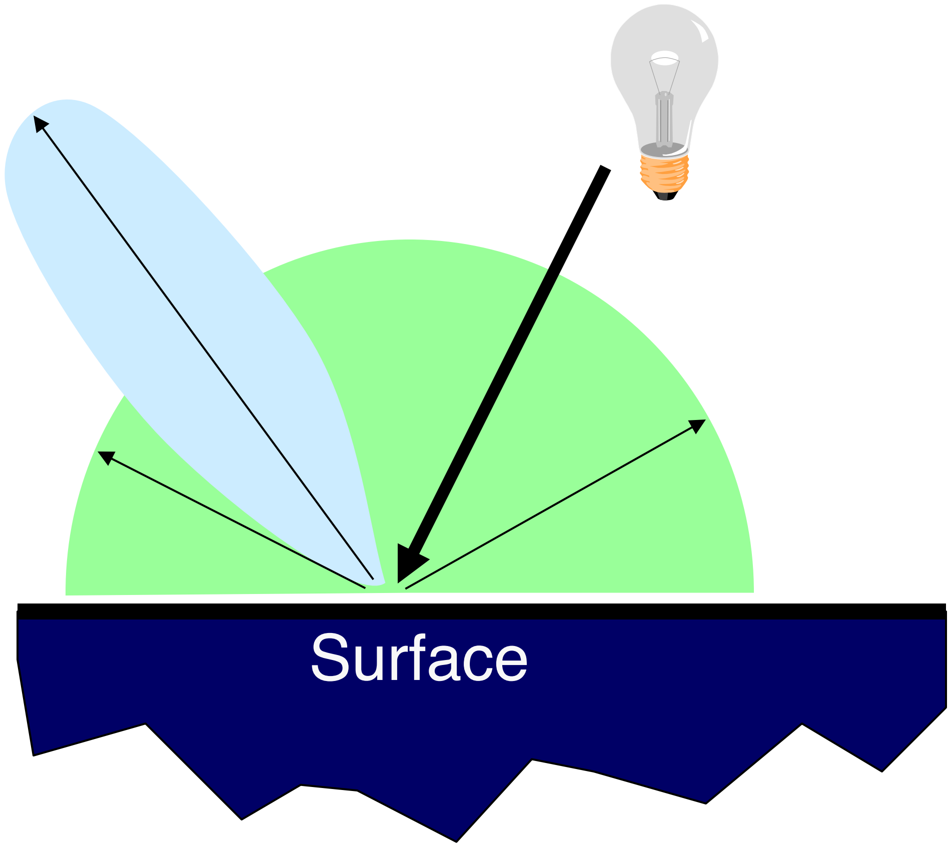
OpenGL Reflectance Model

- Simple analytic model:
  - diffuse reflection +



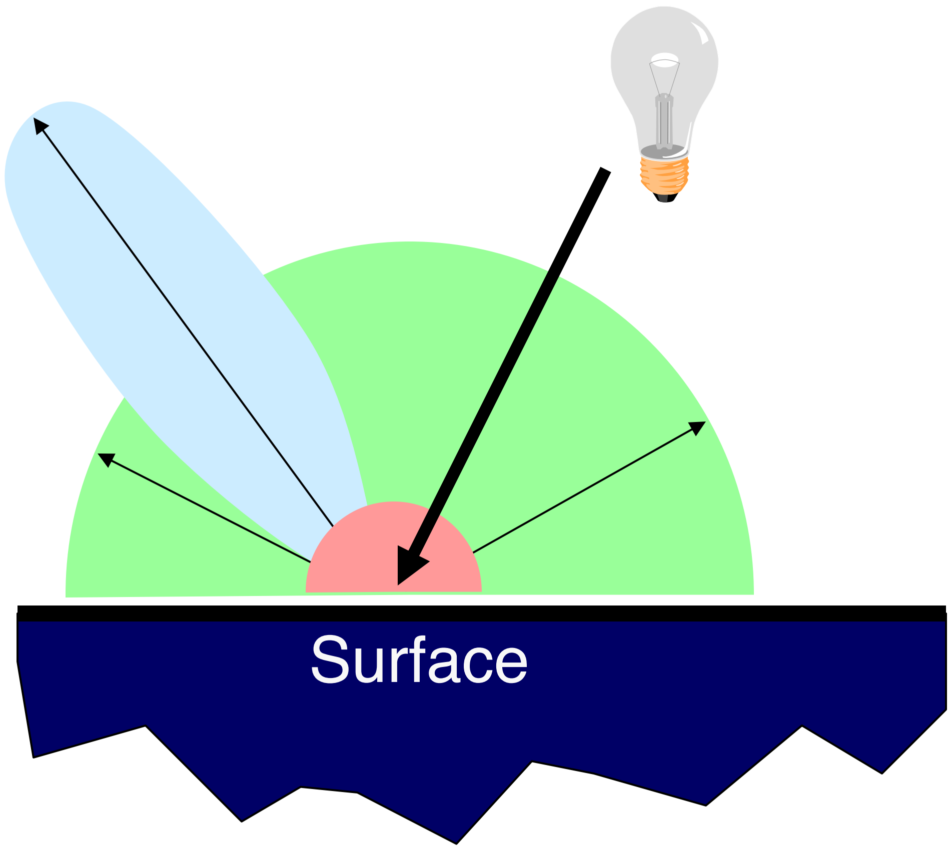
OpenGL Reflectance Model

- Simple analytic model:
  - diffuse reflection +
  - specular reflection +



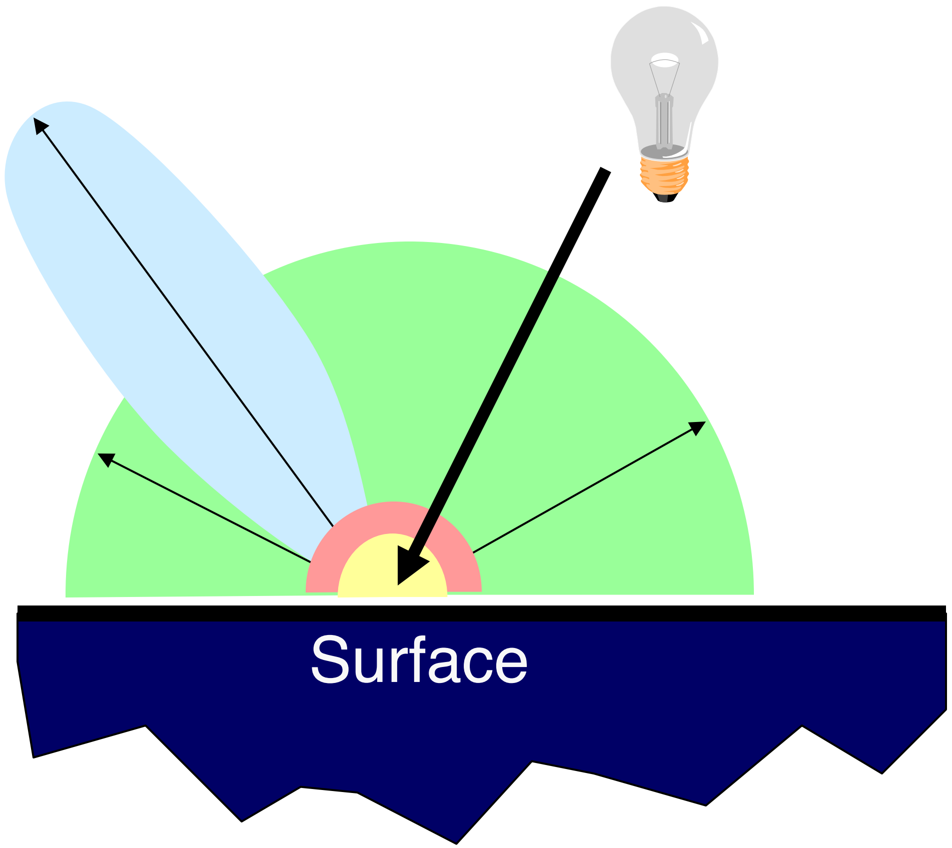
OpenGL Reflectance Model

- Simple analytic model:
  - diffuse reflection +
  - specular reflection +
  - emission +



OpenGL Reflectance Model

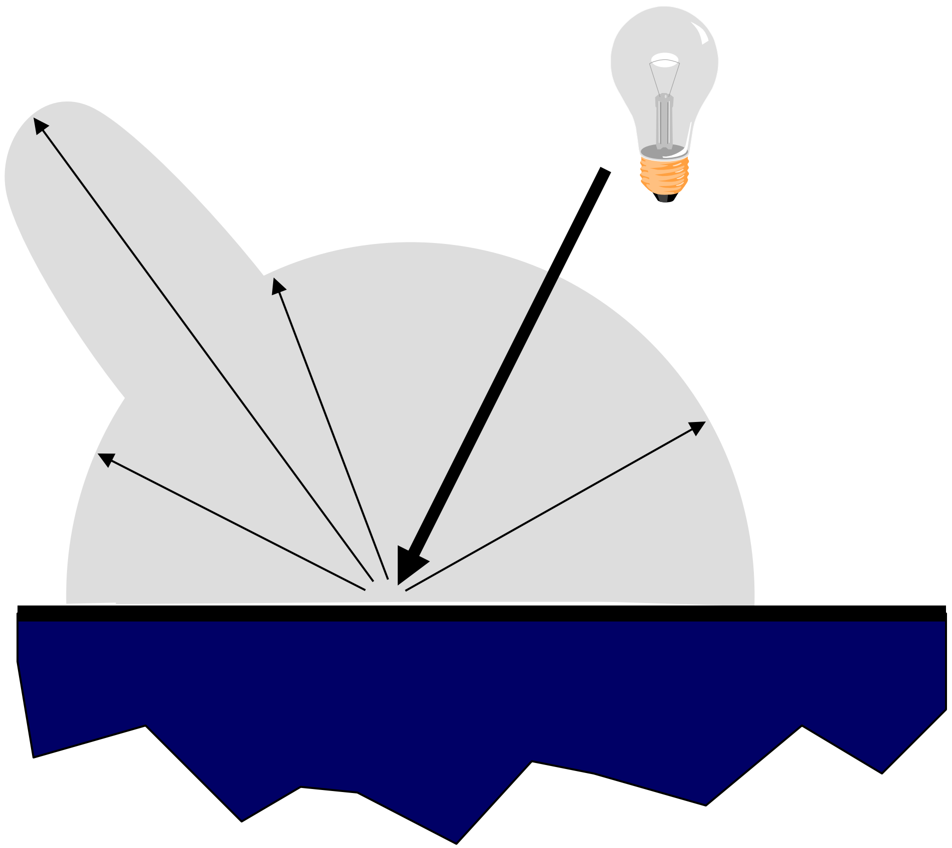
- Simple analytic model:
  - diffuse reflection +
  - specular reflection +
  - emission +
  - “ambient”



OpenGL Reflectance Model

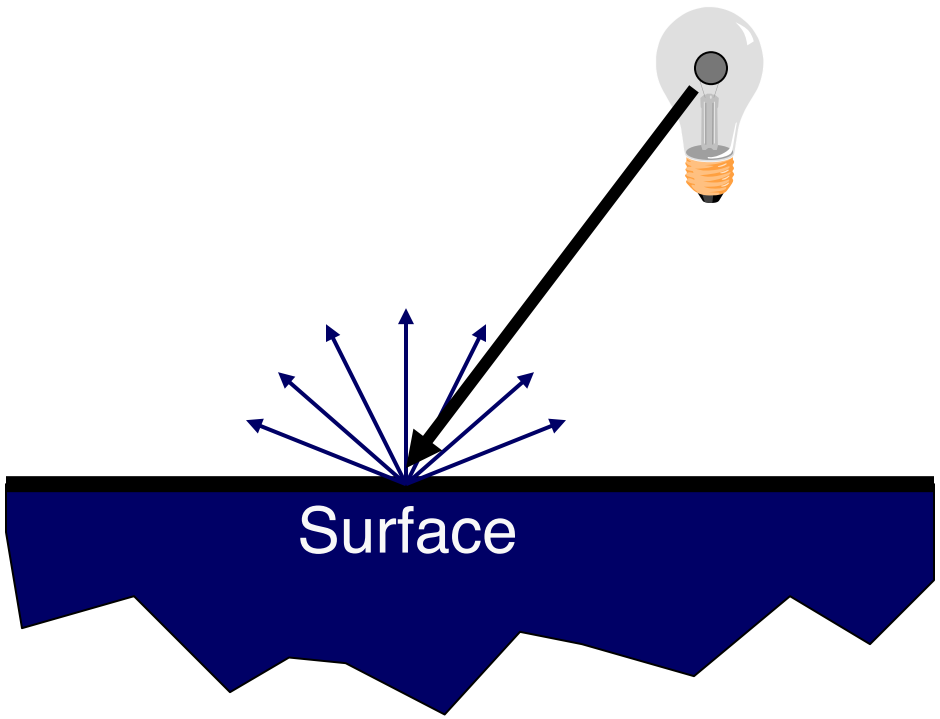
- Simple analytic model:
  - diffuse reflection +
  - specular reflection +
  - emission +
  - “ambient”

Based on model proposed by Phong



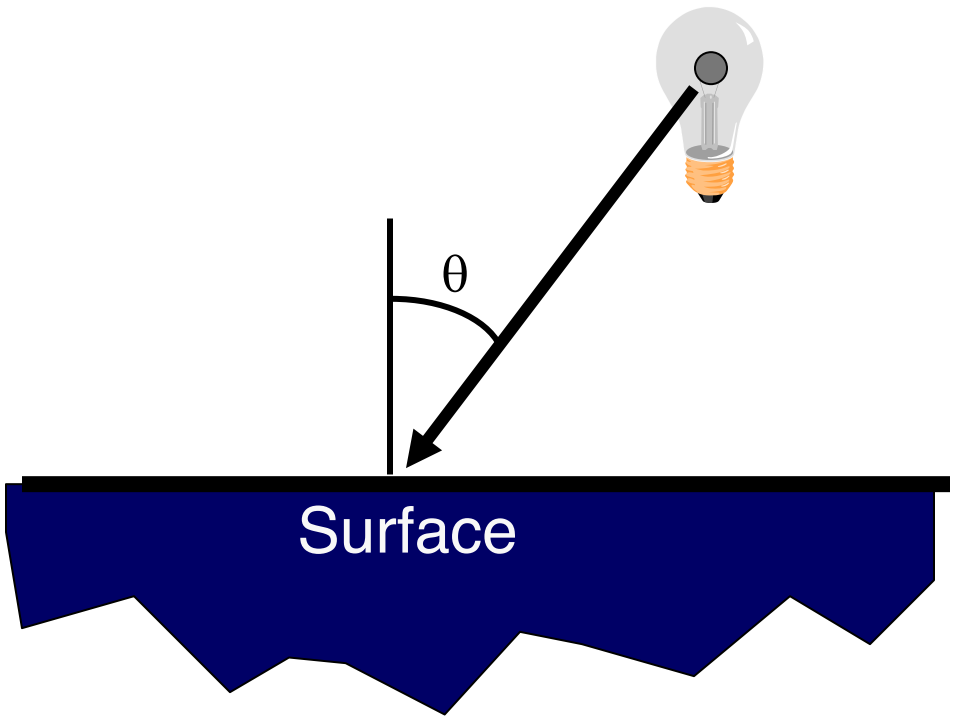
Diffuse Reflection

- Assume surface reflects equally in all directions
  - Examples: chalk, clay



Diffuse Reflection

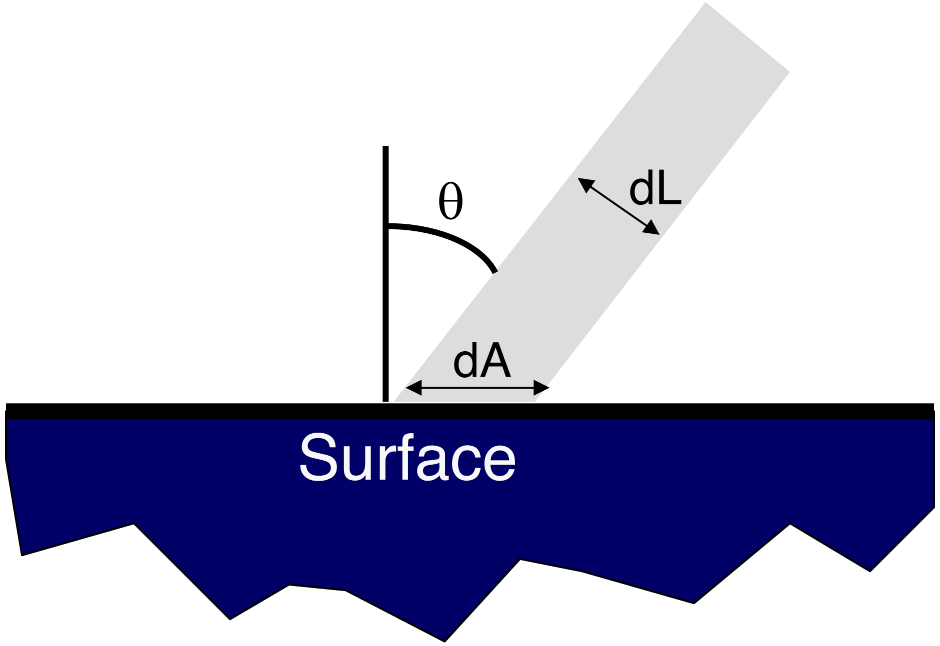
- What is brightness of surface?
  - Depends on angle of incident light



Diffuse Reflection

- What is brightness of surface?
  - Depends on angle of incident light

$$dL = dA \cos \Theta$$



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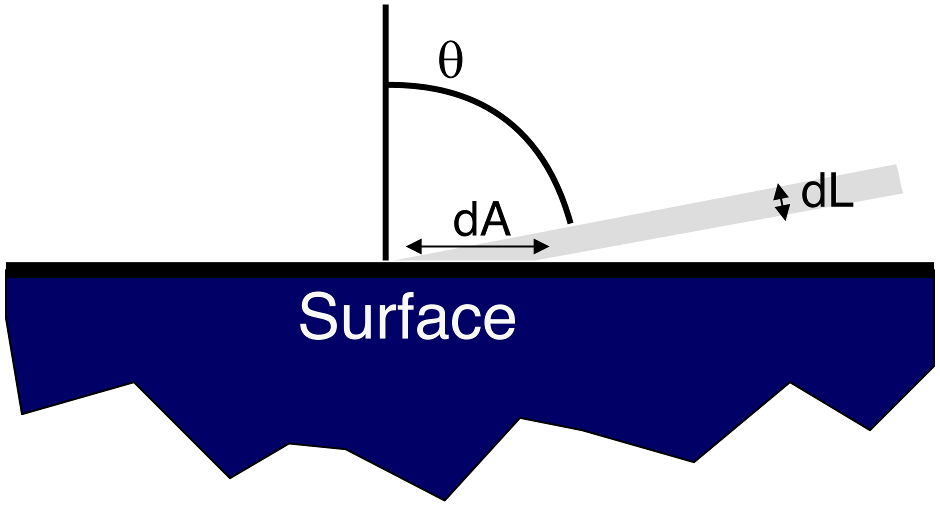
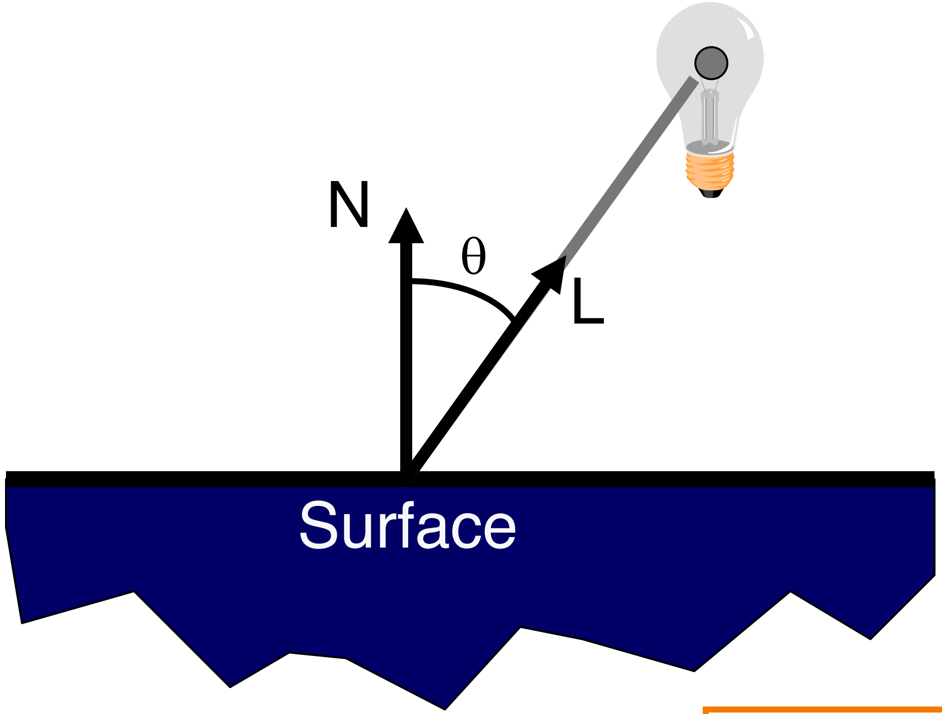


Diffuse Reflection

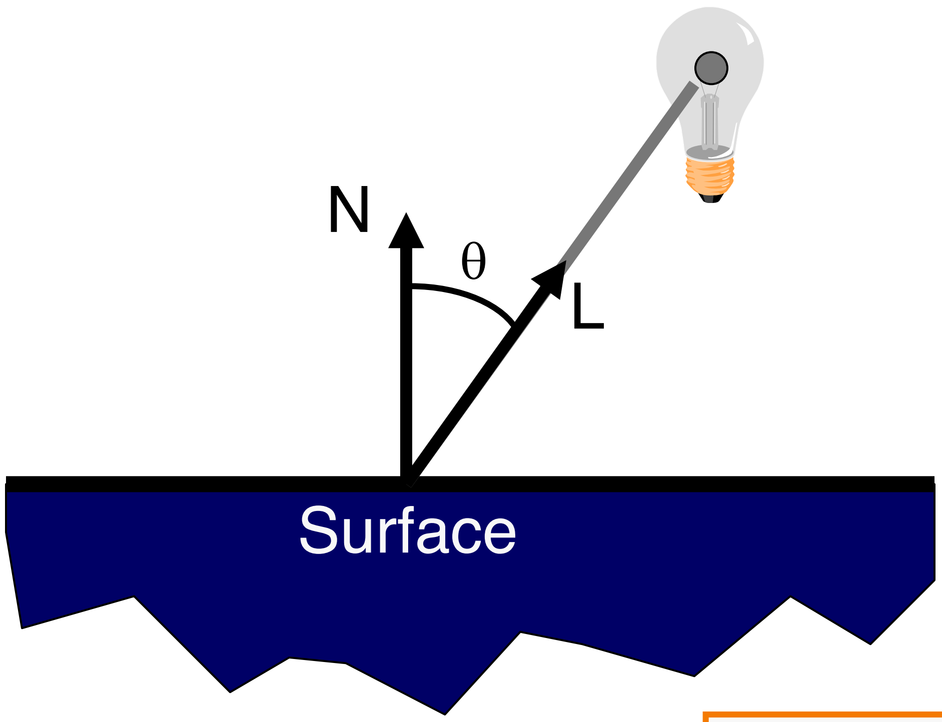
- Lambertian model
  - cosine law (dot product)



$$I_D = (N \cdot L) I_L$$

Diffuse Reflection

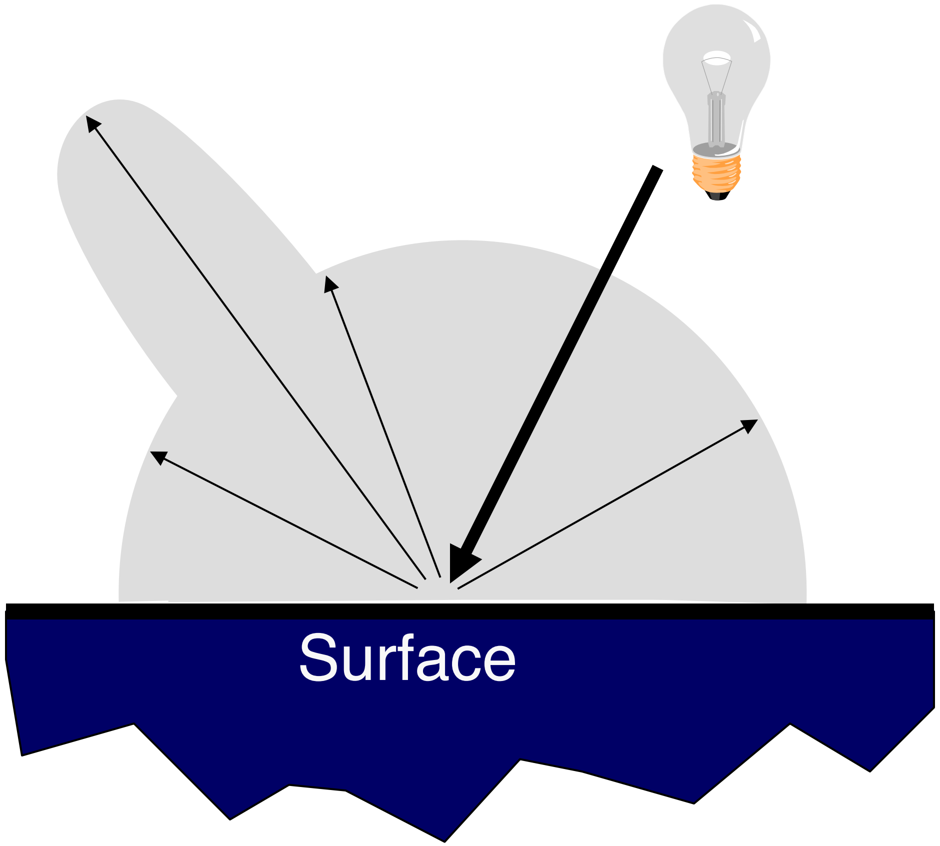
- Lambertian model
  - cosine law (dot product)



$$I_D = K_D(N \cdot L)I_L$$

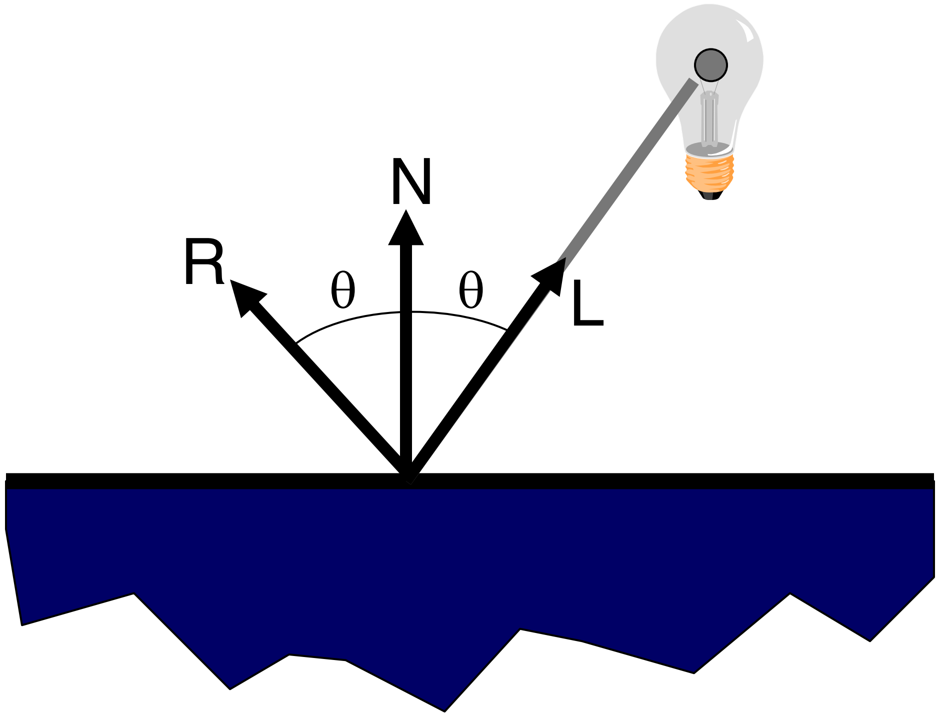
OpenGL Reflectance Model

- Simple analytic model:
  - diffuse reflection +
  - specular reflection +
  - emission +
  - “ambient”



Specular Reflection

- Reflection is strongest near mirror angle
  - Examples: mirrors, metals

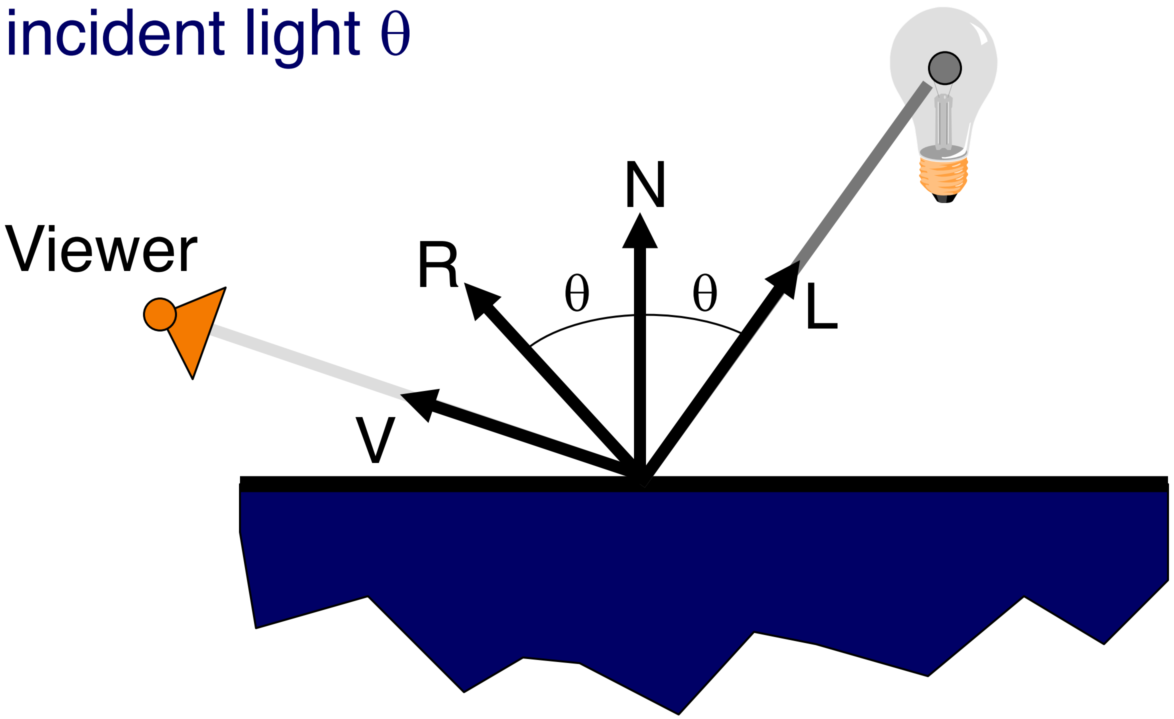


Specular Reflection

How much light is seen?

Depends on:

- angle of incident light θ

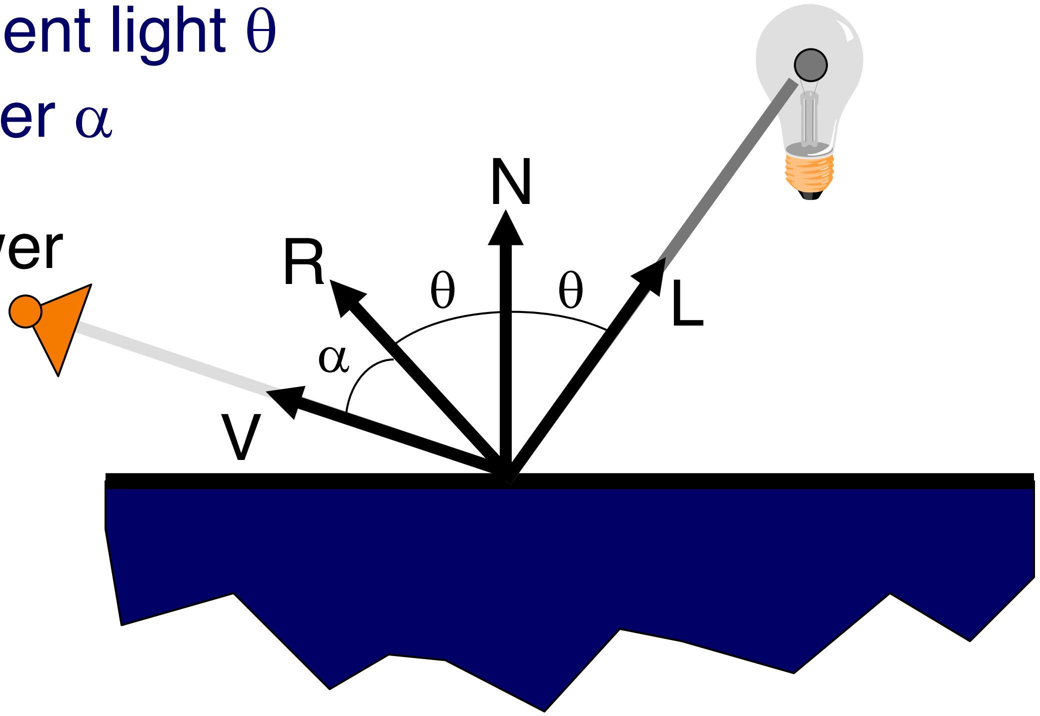


Specular Reflection

How much light is seen?

Depends on:

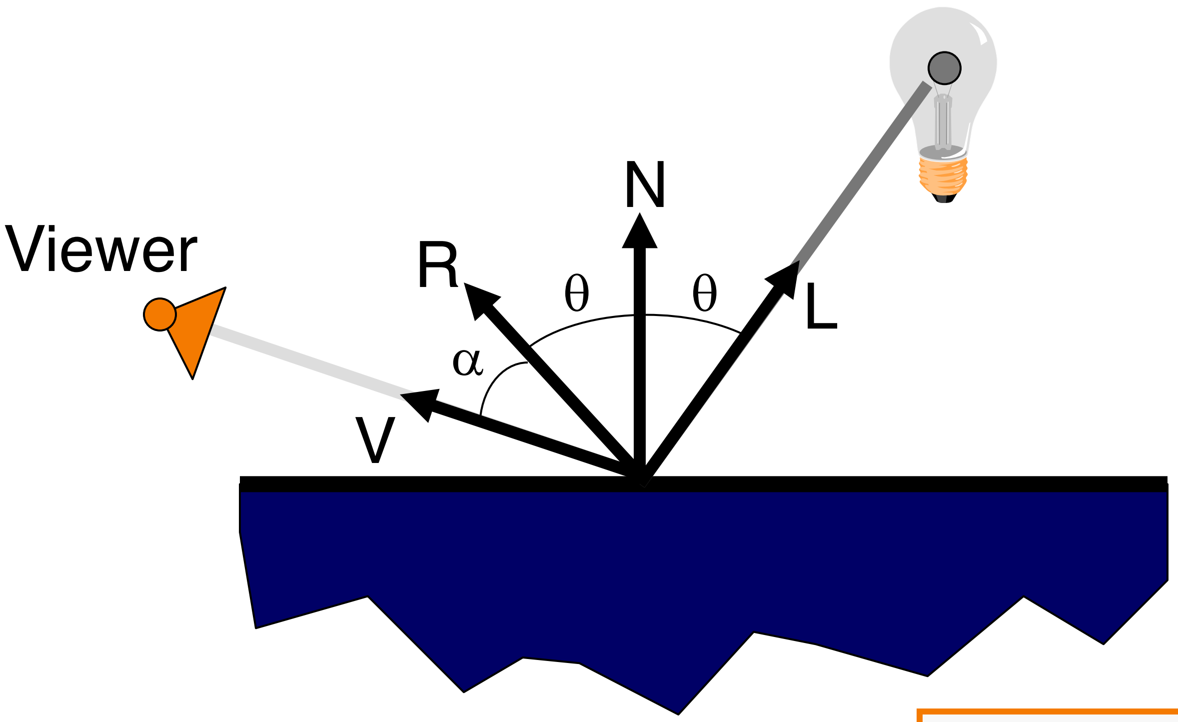
- angle of incident light θ
- angle to viewer α



Specular Reflection

- Phong Model
  - $(\cos \alpha)^n$

This is a (vaguely physically-motivated) hack!



$$I_S = K_S (V \cdot R)^n I_L$$

OpenGL Reflectance Model

- Simple analytic model:
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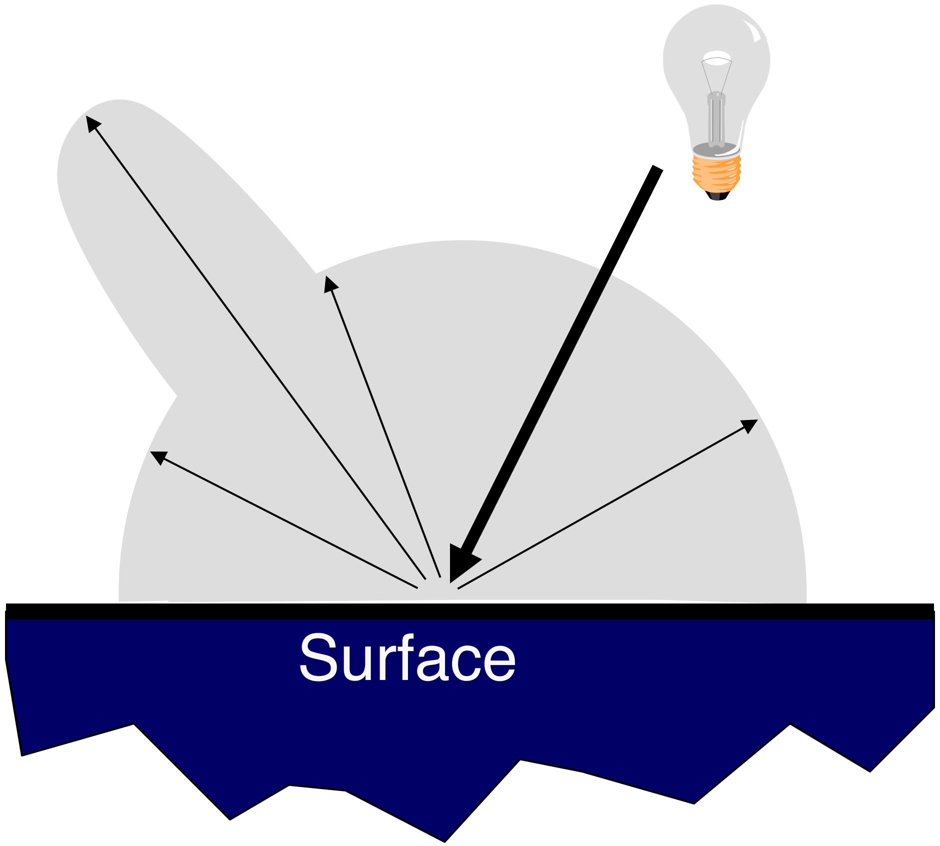
  - specular reflection +
  - emission +
  - "ambient"



Emission

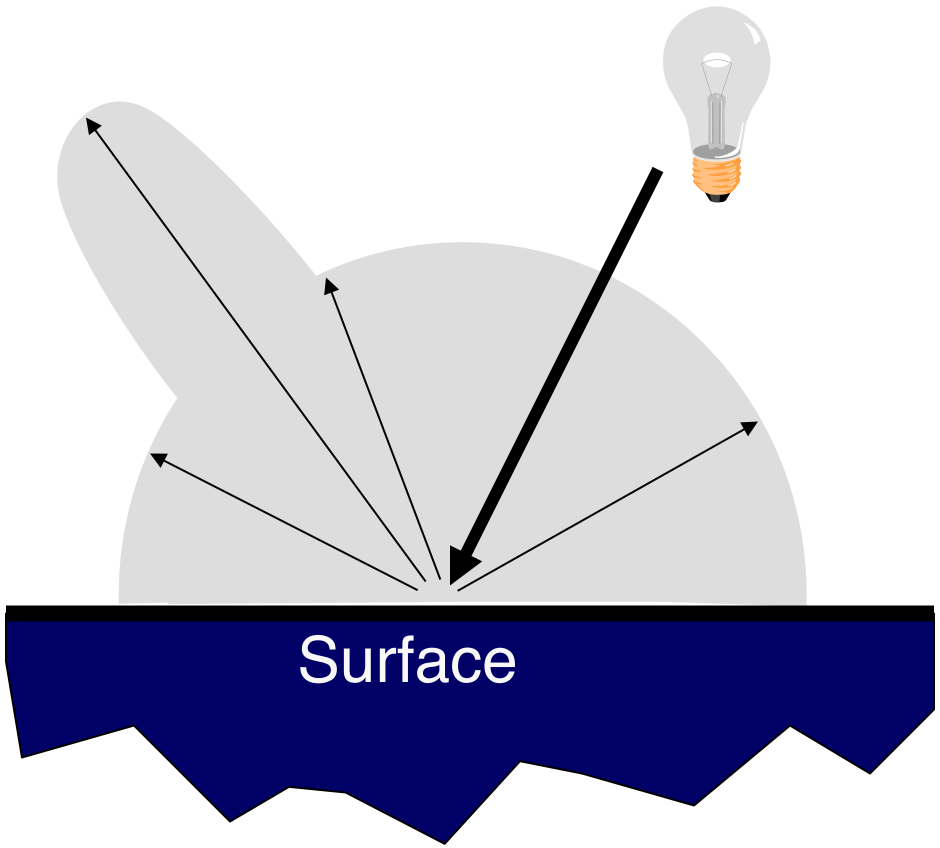
Represents light emanating directly from surface

- Note: does not "automatically" act as light source!
  Does not affect other surfaces in scene!

Emission $\neq 0$

OpenGL Reflectance Model

- Simple analytic model:
  - diffuse reflection +
  - specular reflection +
  - emission +
  - "ambient"



Ambient Term

Represents reflection of all indirect illumination

This is a hack (avoids complexity of global illumination)!

OpenGL Reflectance Model

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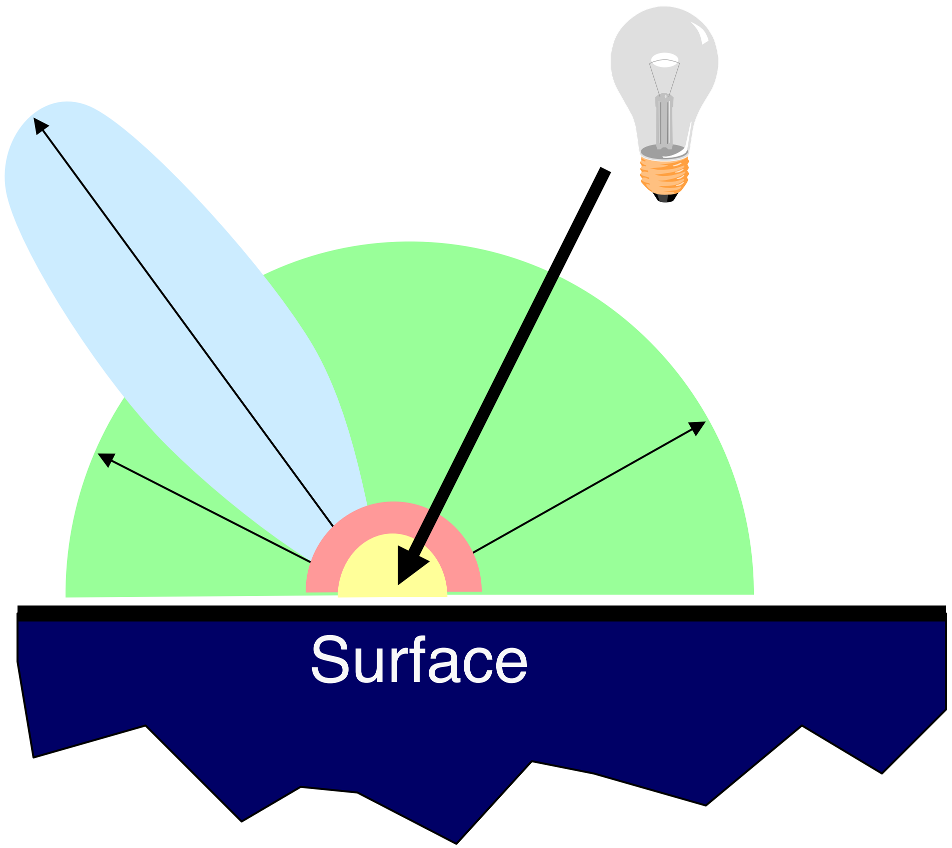
  - specular reflection +
  - emission +
  - “ambient”



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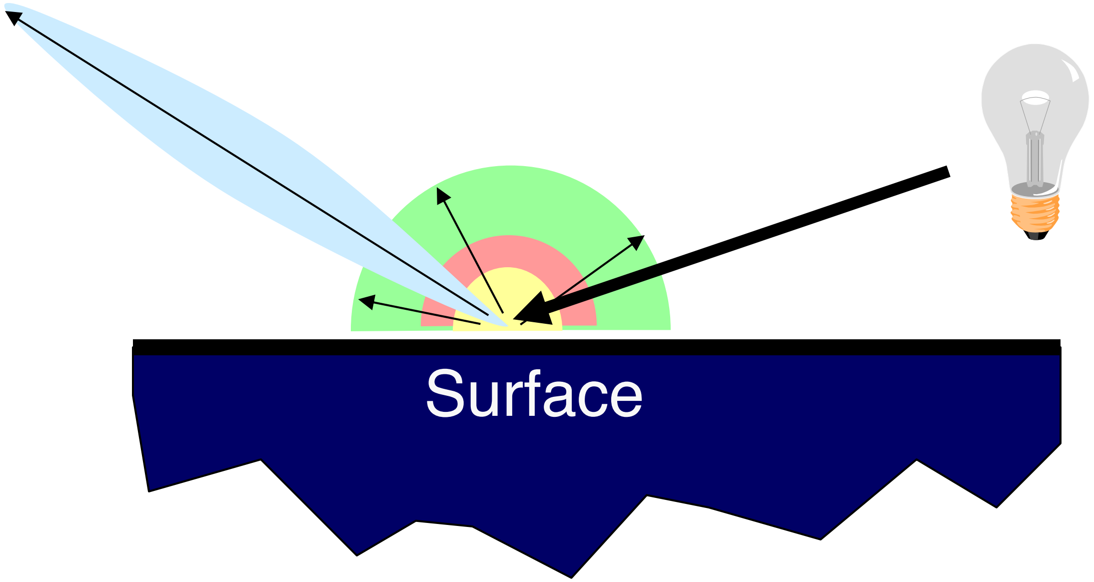
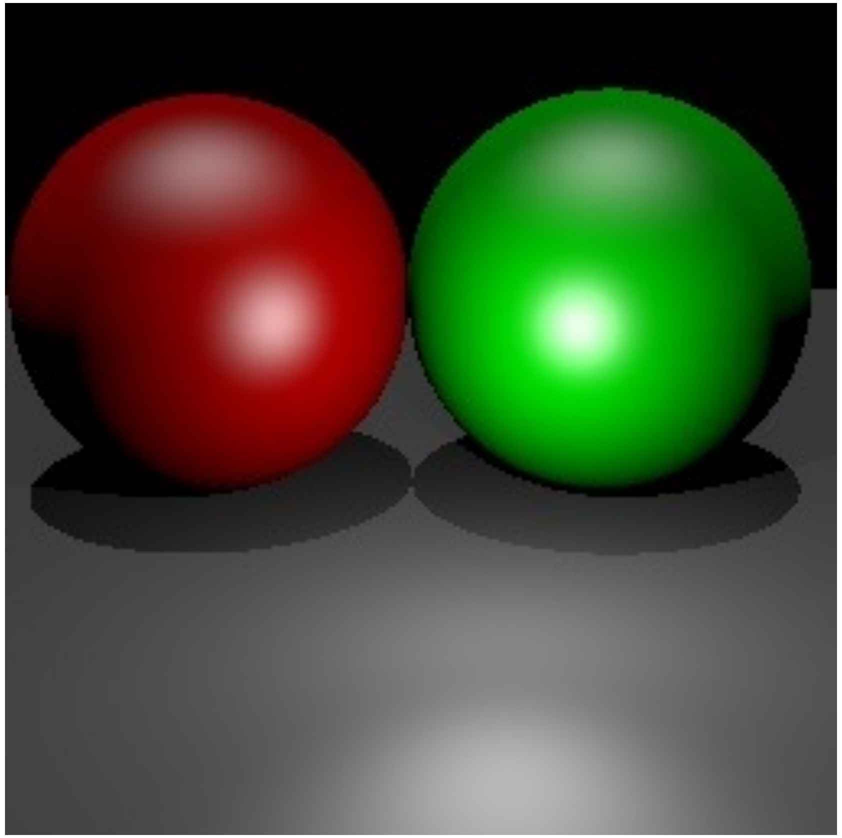


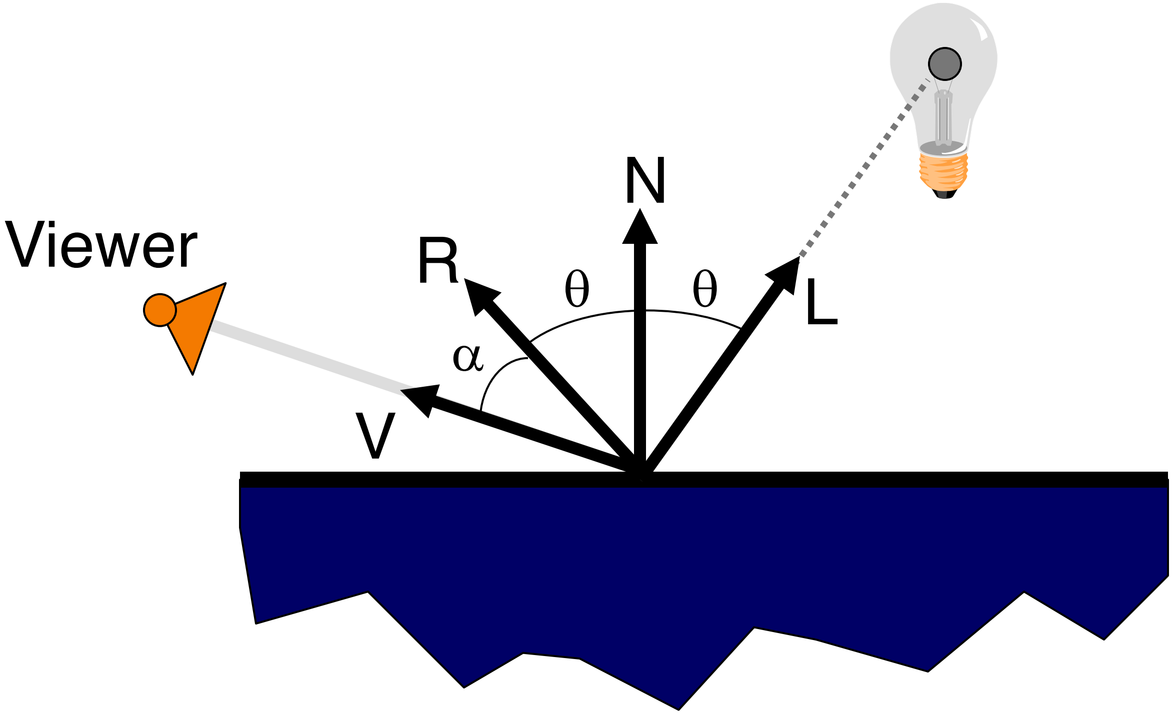
OpenGL Reflectance Model

- Good model for plastic surfaces, …



Direct Illumination Calculation

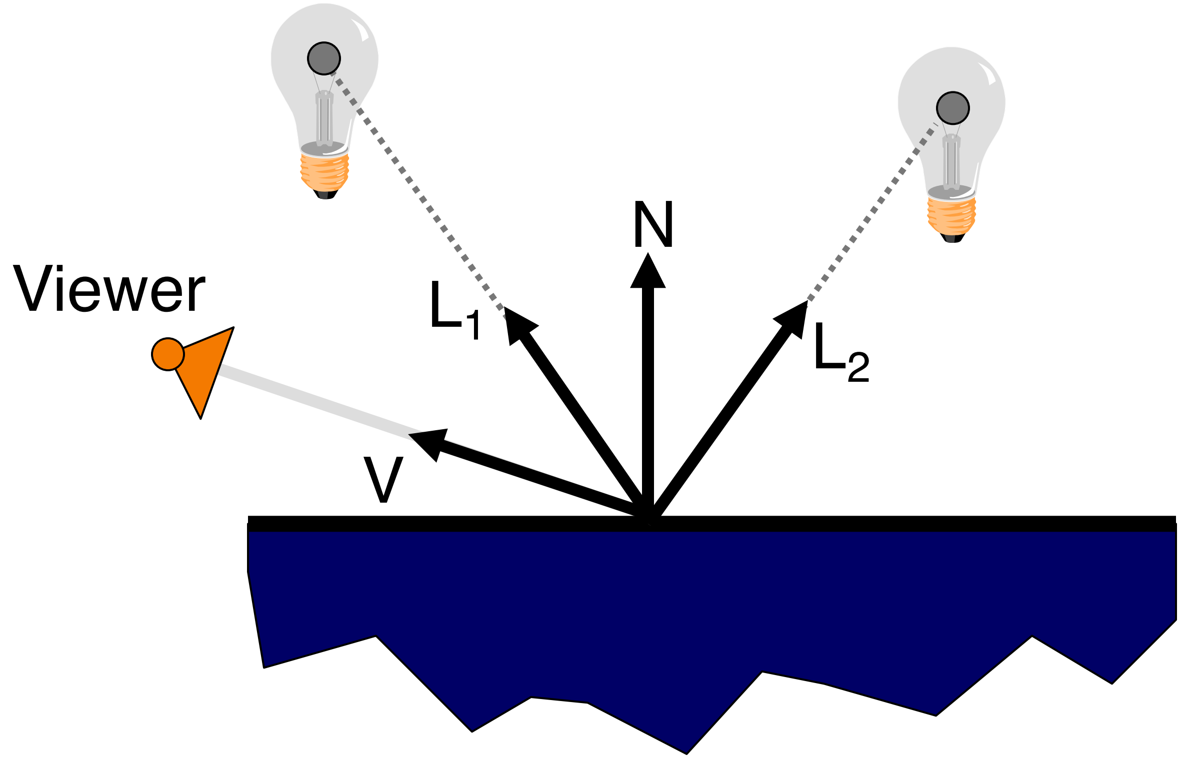
- Single light source:



$$I = I_E + K_A I_{AL} + K_D (N \cdot L) I_L + K_S (V \cdot R)^n I_L$$

Direct Illumination Calculation

- Multiple light sources:



Note: all of the K and I are RGB colors

$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) I_L$$

Overview

- Direct Illumination
  - Emission at light sources
  - Scattering at surfaces

- Global illumination
  - Shadows
  - Transmissions
  - Inter-object reflections

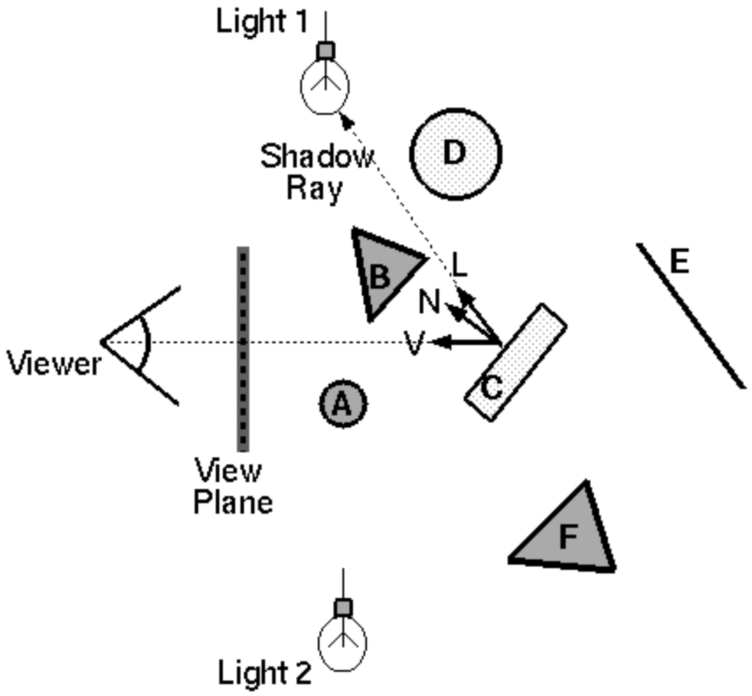
Global Illumination

Global Illumination

Greg Ward

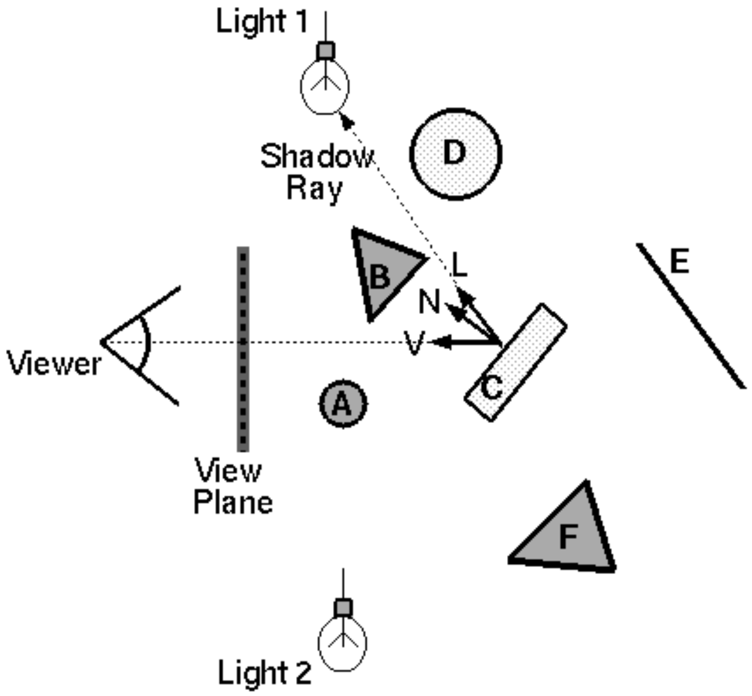
Ray Casting (last lecture)

- Trace primary rays from camera
    - Direct illumination from unblocked lights only



Ray Casting (last lecture)

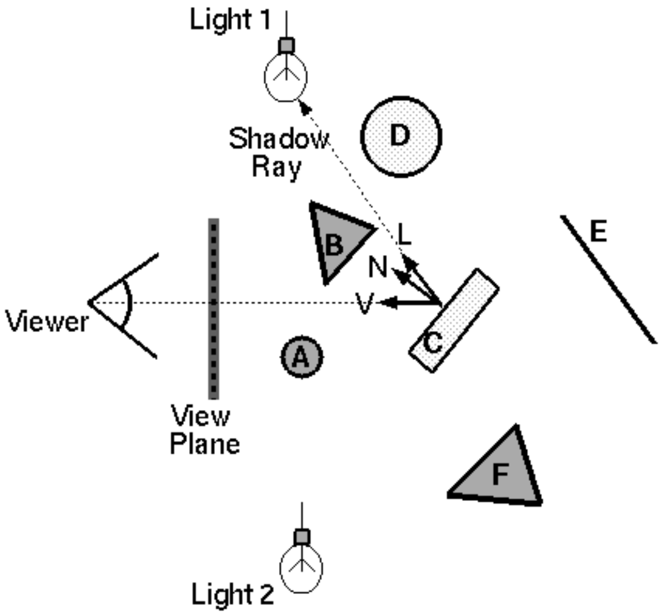
- Trace primary rays from camera
  - Direct illumination from unblocked lights only



$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) I_L$$

Shadows

- Shadow term tells if light sources are blocked
  - Cast ray towards each light source
  - $S_L = 0$ if ray is blocked, $S_L = 1$ otherwise

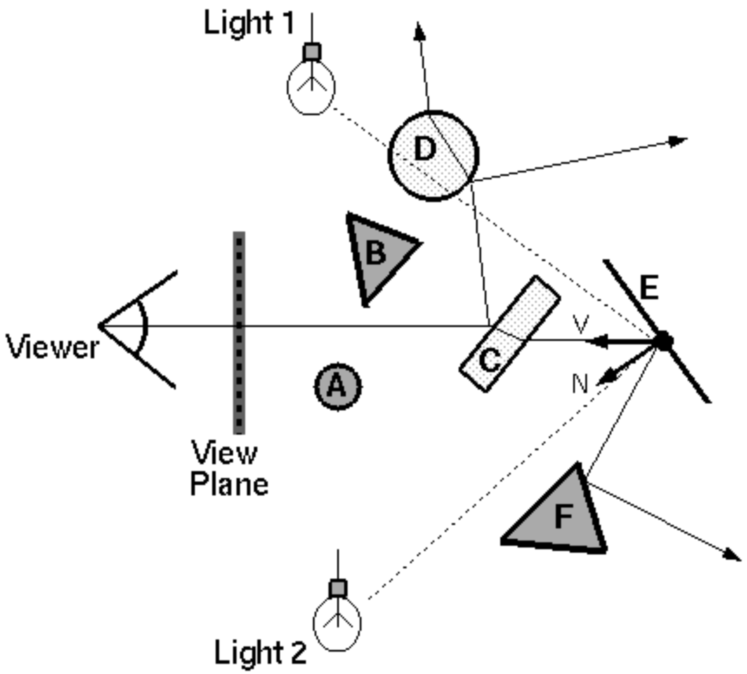


Shadow Term

$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L$$

Recursive Ray Tracing

- Also trace secondary rays from hit surfaces
  - Mirror reflection and transparency



$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L$$

Recursive Ray Tracing

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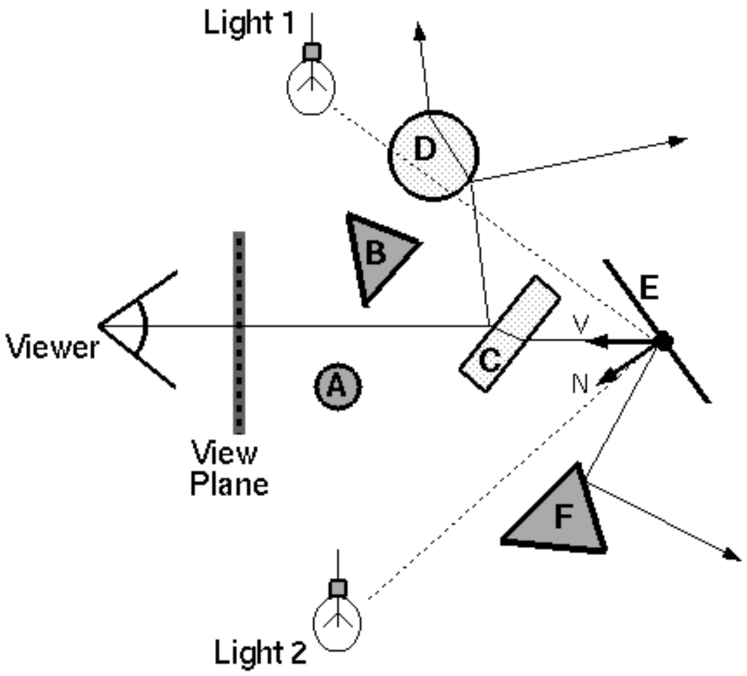
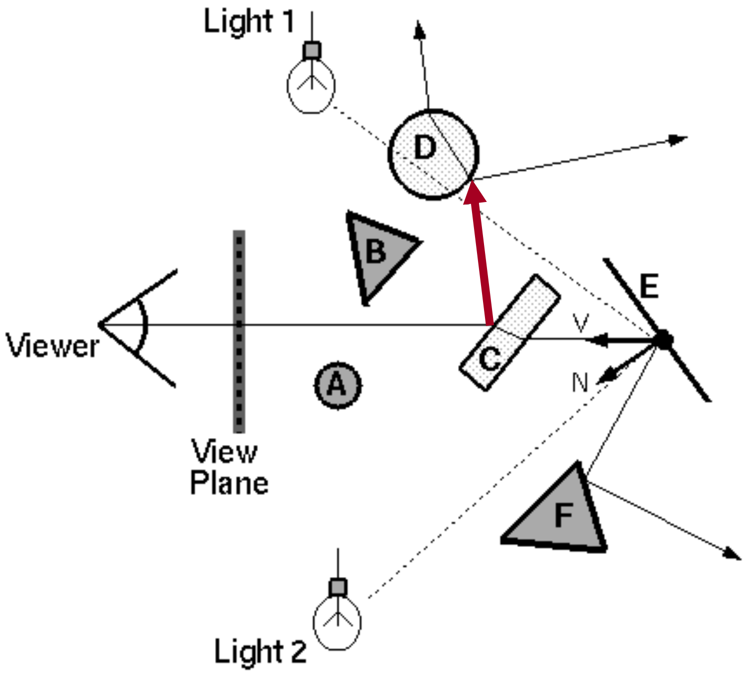


$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T I_T$$

Mirror reflections

- Trace secondary ray in mirror direction
  - Evaluate radiance along secondary ray and include it into illumination model

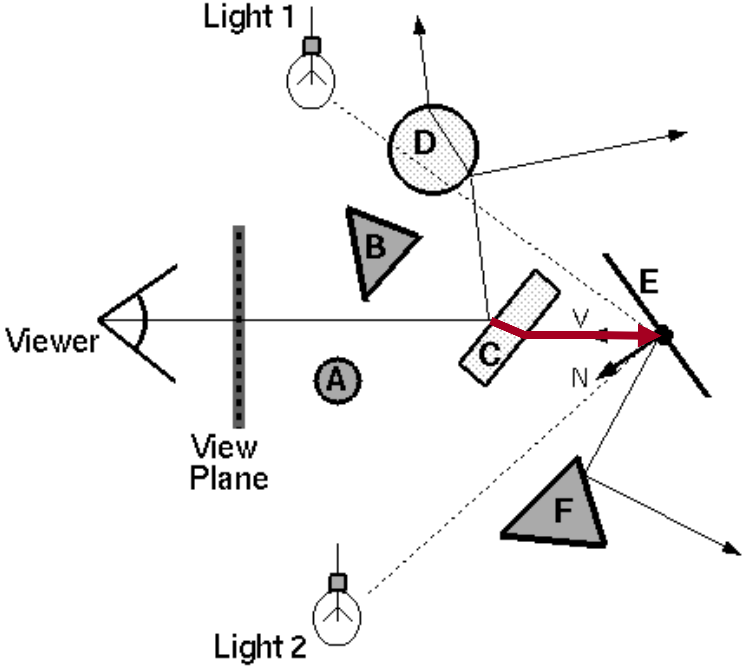


Radiance for mirror reflection ray

$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T I_T$$

Transparency

- Trace secondary ray in direction of refraction
  - Evaluate radiance along secondary ray and include it into illumination model

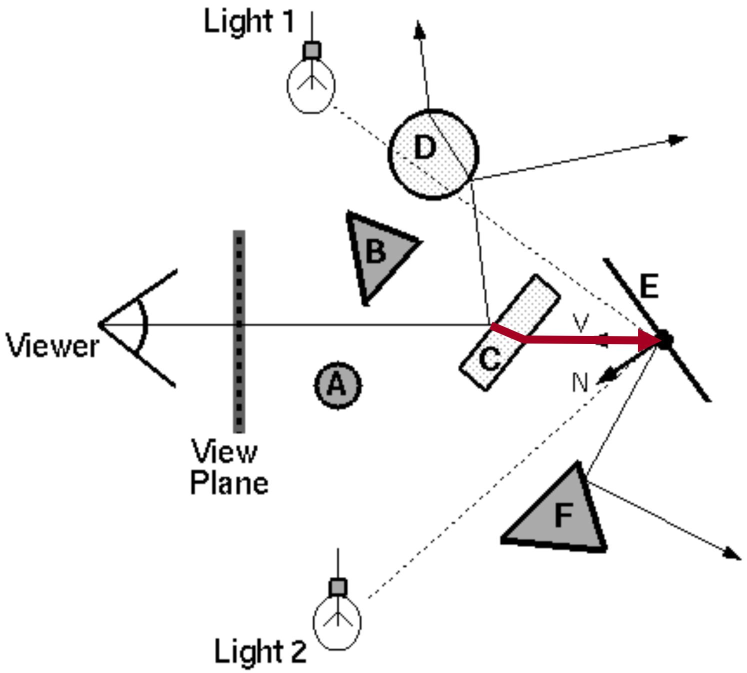


Radiance for refraction ray

$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T I_T$$

Transparency

- Transparency coefficient is fraction transmitted
  - $K_T = 1$ for translucent object, $K_T = 0$ for opaque
  - $0 < K_T < 1$ for object that is semi-translucent



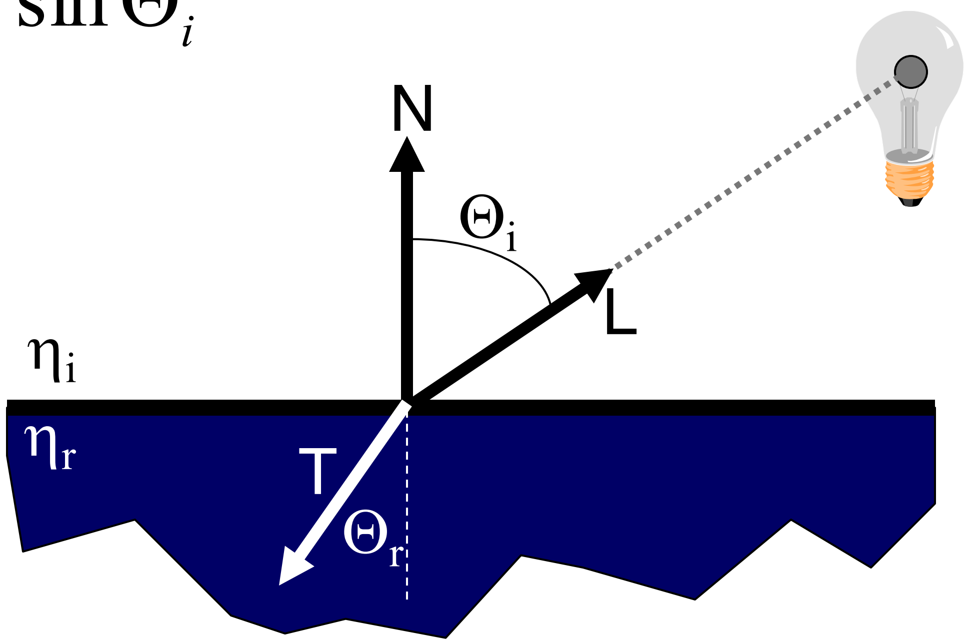
Transparency Coefficient

$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T I_T$$

Refractive Transparency

- For solid objects, apply Snell's law:

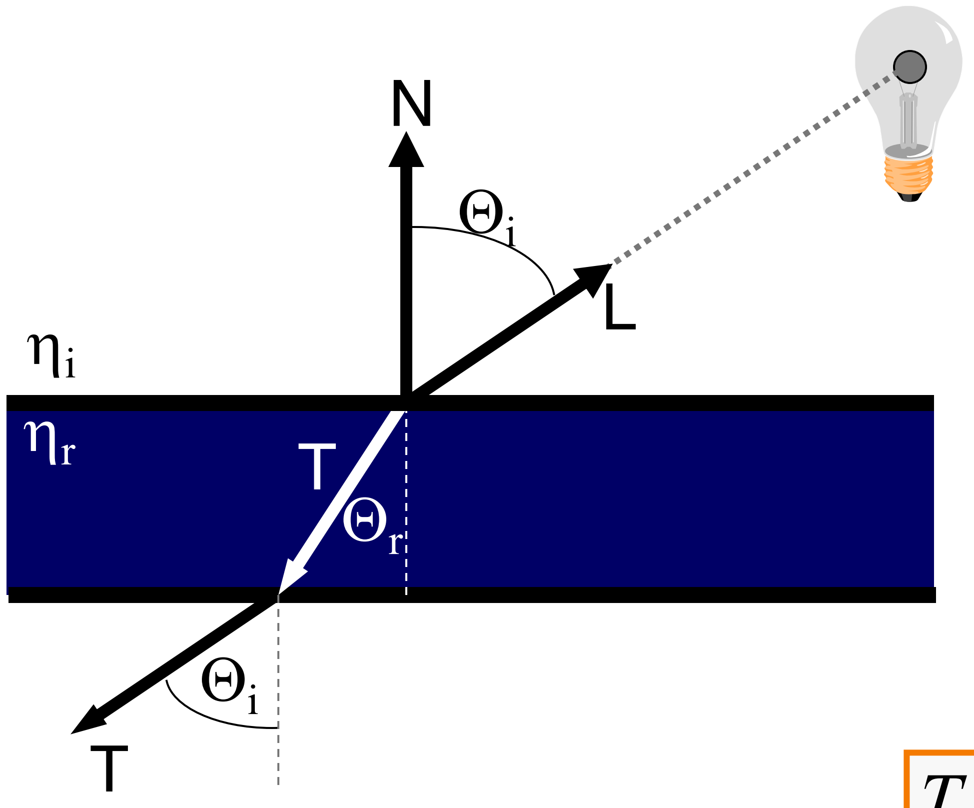
$$\eta_r \sin\Theta_r = \eta_i \sin\Theta_i$$



$$T = \left(\frac{\eta_i}{\eta_r}\cos\Theta_i - \cos\Theta_r\right)N - \frac{\eta_i}{\eta_r}L$$

Refractive Transparency

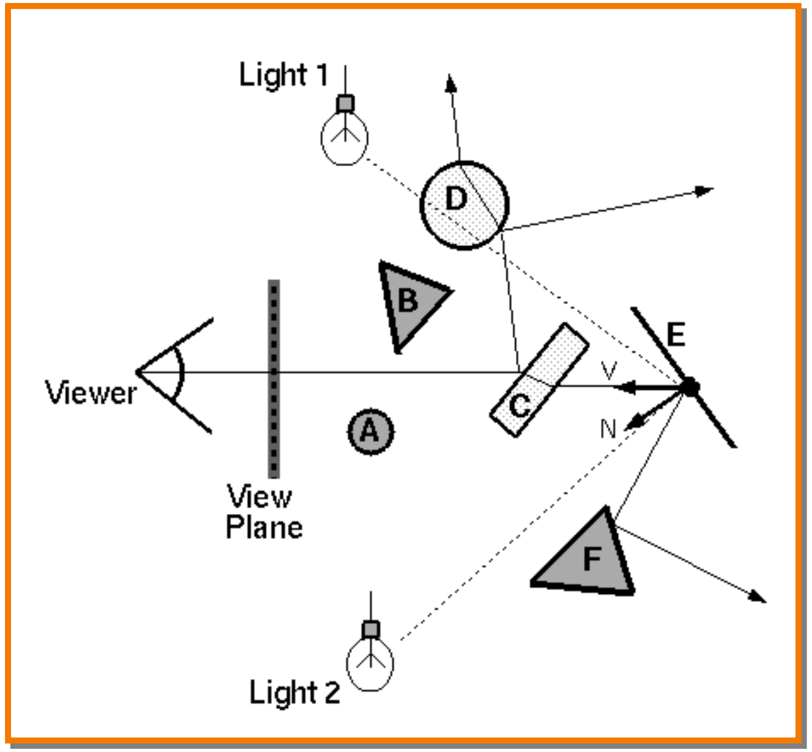
- For thin surfaces, can ignore change in direction
  - Assume light travels straight through surface



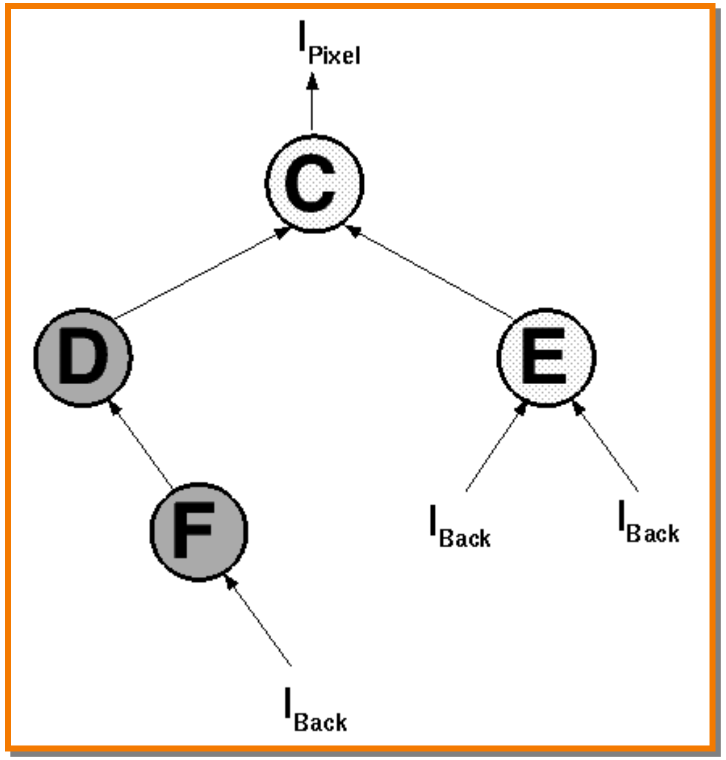
$$T \cong -L$$

Recursive Ray Tracing

- Ray tree represents illumination computation



Ray traced through scene

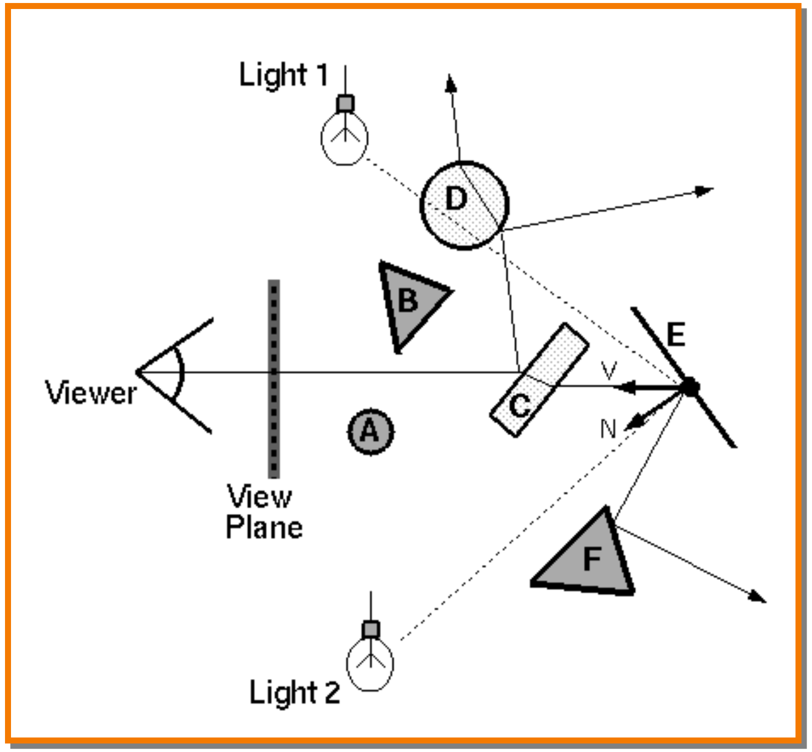


Ray tree

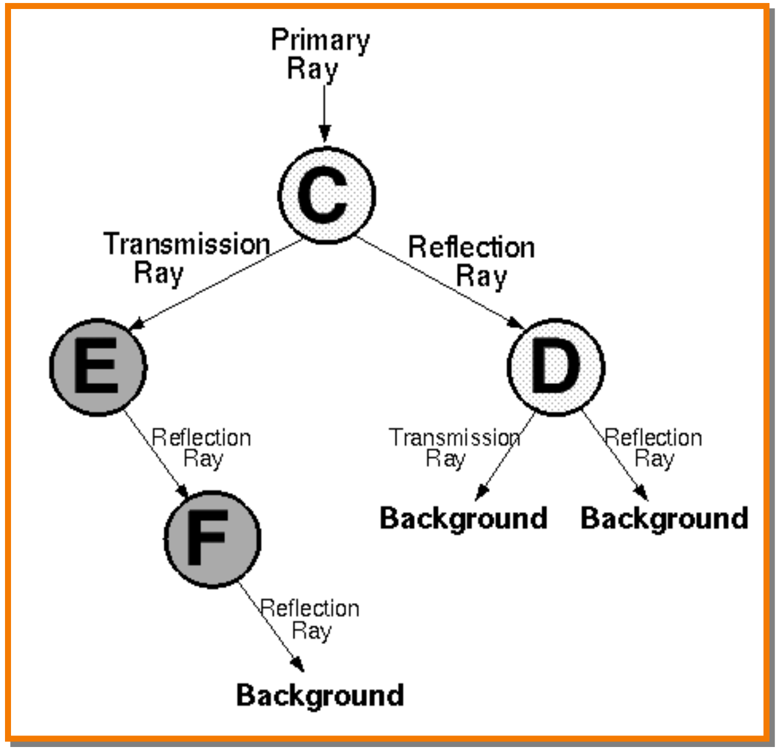
$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T I_T$$

Recursive Ray Tracing

- Ray tree represents illumination computation



Ray traced through scene



Ray tree

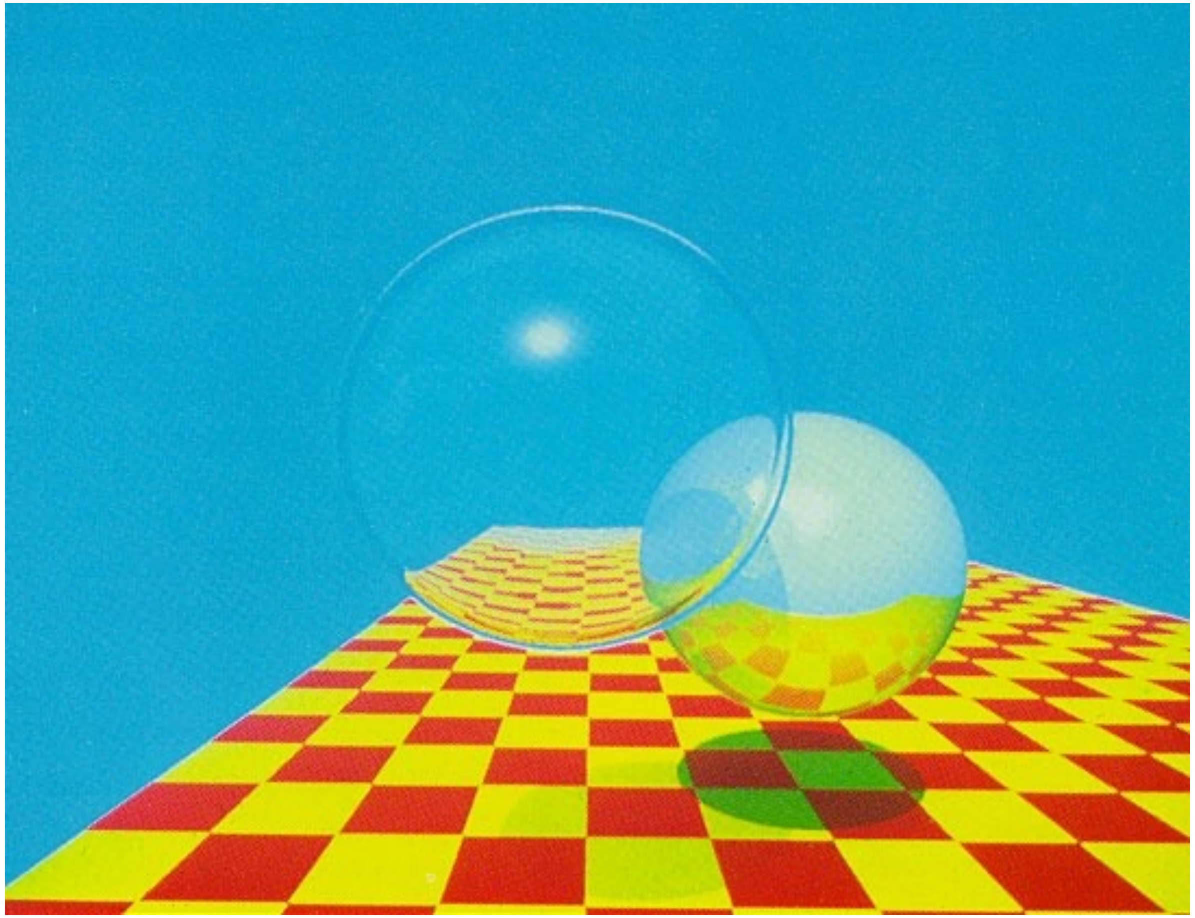
$$I = I_E + K_A I_{AL} + \sum_L \left(K_D (N \cdot L_i) + K_S (V \cdot R_i)^n \right) S_L I_L + K_S I_R + K_T I_T$$

Recursive Ray Tracing

- ComputeRadiance is called recursively

```
R3Rgb ComputeRadiance(R3Scene *scene, R3Ray *ray, R3Intersection& hit)
{
    R3Ray specular_ray = SpecularRay(ray, hit);
    R3Ray refractive_ray = RefractiveRay(ray, hit);
    R3Rgb radiance = Phong(scene, ray, hit) +
                     Ks * ComputeRadiance(scene, specular_ray) +
                     Kt * ComputeRadiance(scene, refractive_ray);
    return radiance;
}
```

Example



Turner Whitted, 1980

Summary

- Ray casting (direct Illumination)
  - Usually use simple analytic approximations for light source emission and surface reflectance
- Recursive ray tracing (global illumination)
  - Incorporate shadows, mirror reflections, and pure refractions

All of this is an approximation so that it is practical to compute

More on global illumination after next week!