Consistency Models

Strict Serializability

Linearizability

Sequential

Causal+

Eventual

Stronger

Weaker
Consistency Models

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Strict Serializability

- **Transactions**: operations that span multiple objects (e.g., keys in KV store) *atomically* commit (or abort).
- **Total order**: There exists some legal total ordering of transactions.
  - Legal (intuitively defined for strict serializability): in the total ordering, read operations “see” the latest write operation.
- **Preserves real-time commit order**: if txn A commits before txn B begins, then txn A occurs before txn B in the total order.
  - Write ops in a committed txn are visible to all future txns’ read ops.
  - Intuition: once a read “sees” a txn and commits, all future reads must also “see” that txn.

**Pros**: applications can easily reason about correctness of transactions.

**Cons**: strict serializability imposes high read and write latencies on system.
## Strict Serializability Example

<table>
<thead>
<tr>
<th></th>
<th>Strictly Serializable?</th>
<th>P1:</th>
<th>P2:</th>
<th>P3:</th>
<th>P4:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Example 1</strong></td>
<td>Yes</td>
<td>{W(x)b, W(y)b}</td>
<td>{W(x)a}</td>
<td>{R(x)a}</td>
<td>{R(x)b}</td>
</tr>
<tr>
<td><strong>Example 2</strong></td>
<td>No</td>
<td>{W(x)b, W(y)b}</td>
<td>{W(x)a}</td>
<td>{R(x)b}</td>
<td>{R(x)a}</td>
</tr>
</tbody>
</table>
Consistency Models

Strict Serializability → Linearizability → Causal+ → Eventual

- Stronger
- Sequential
- Weaker
Linearizability

- **Total order:** There exists some legal total order of operations (not txns).
- **Difference from strict serializability?**
  - Single-object operations! No transactions!
- **Preserves real-time ordering:** if an operation $A$ completes before operation $B$ begins, then op $A$ occurs before op $B$ in the total order.
  - A completed write op is visible to all future read ops.
  - Intuition: once a read “sees” a new write, all future reads must also “see” that write.

**Pros:** Easy to reason about correctness

**Cons:** High read and write latencies
## Linearizability Example

<table>
<thead>
<tr>
<th>Linearizable?</th>
<th>No</th>
<th>Linearizable?</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1:</td>
<td>W(x)a</td>
<td>P1:</td>
<td>W(x)a</td>
</tr>
<tr>
<td>P2:</td>
<td>W(x)b</td>
<td>P2:</td>
<td>W(x)b</td>
</tr>
<tr>
<td>P3:</td>
<td>R(x)b, R(x)a</td>
<td>P3:</td>
<td>R(x)a, R(x)b</td>
</tr>
<tr>
<td>P4:</td>
<td>R(x)b, R(x)a</td>
<td>P4:</td>
<td>R(x)a, R(x)b</td>
</tr>
</tbody>
</table>
Consistency Models

Strict Serializability  Linearizability  Causal+  Sequential  Eventual

Stronger  Weaker
Sequential Consistency

- **Total order**: there exists some legal total order of operations.
- **Preserves process ordering**: total order respects order of each process’s operations.
- **Difference from linearizability?**
  - Order of ops across processes not determined by real-time

**Pros**: Can allow more orderings than linearizability → better performance.

**Cons**: Many possible sequential executions → increased application complexity
**Sequential Consistency Example**

<table>
<thead>
<tr>
<th>Operation</th>
<th>P1:</th>
<th>P2:</th>
<th>P3:</th>
<th>P4:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W(x)a</td>
<td>W(x)b</td>
<td>R(x)b, R(x)a</td>
<td>R(x)b, R(x)a</td>
</tr>
<tr>
<td><strong>Sequentially Consistent?</strong></td>
<td><strong>Yes</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operation</th>
<th>P1:</th>
<th>P2:</th>
<th>P3:</th>
<th>P4:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W(x)a</td>
<td>W(x)b</td>
<td>R(x)b, R(x)a</td>
<td>R(x)a, R(x)b</td>
</tr>
<tr>
<td><strong>Sequentially Consistent?</strong></td>
<td><strong>No</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Consistency Models

- Strict Serializability
- Linearizability
- Sequential
- Causal+
- Eventual

- Stronger
- Weaker
Causal+ Consistency

- **Partial order**: order causally related ops the same way across all processes.
- **+**: replicas’ total order eventually converge.
- **Difference from sequential consistency?**
  - Only causally related ops need to be ordered: no guaranteed total order.
  - Concurrent ops may be ordered differently across different processes.

**Pros**: preserves causality while improving efficiency.

**Cons**: harder to reason about concurrency.
<table>
<thead>
<tr>
<th>Ops</th>
<th>Concurrent</th>
</tr>
</thead>
<tbody>
<tr>
<td>a,b</td>
<td>No</td>
</tr>
<tr>
<td>a,e</td>
<td>Yes</td>
</tr>
<tr>
<td>a,g</td>
<td>No</td>
</tr>
<tr>
<td>c,e</td>
<td>Yes</td>
</tr>
<tr>
<td>c,d</td>
<td>No</td>
</tr>
<tr>
<td>d,g</td>
<td>No</td>
</tr>
<tr>
<td>d,f</td>
<td>No</td>
</tr>
<tr>
<td>e,g</td>
<td>No</td>
</tr>
<tr>
<td>a,d</td>
<td>No</td>
</tr>
</tbody>
</table>

Diagram:

```
  P1  
  |   |
  | a |
  | b |
  |   |
  |   |
  | c |
  |   |
  | d |
  |   |
  | e |
  | f |
  | g |
  P2  
```
Causal+ Consistency Example

Causally+ Consistent?  Yes

P1:  W(x)a
P2:  W(x)b
P3:  R(x)b  R(x)a
P4:  R(x)a

Causally+ Consistent?  No

P1:  W(x)a
P2:  R(x)a  W(x)b
P3:  R(x)b  R(x)a
P4:  R(x)a
Consistency Models

- Strict Serializability
- Linearizability
- Sequential
- Causal
- Eventual

Stronger → Weak
Eventual Consistency

- **Eventual convergence**: If no more writes, all replicas eventually agree.
- **Difference from causal consistency?**
  - Does not preserve causal relationships
  - Is the “+” in causal+.
- Frequently used with application conflict resolution, anti-entropy

**Pros**: highly available; think Bayou.

**Cons**: no safety guarantees, need conflict resolution
In a nutshell...

**Strict Serializability**: total order + real time guarantees over *transactions*

**Linearizability**: total order + real time guarantees over *operations*

**Sequential consistency**: total order + process order

**Causal+ consistency**: causally ordered + replicas eventually converge

**Eventual consistency**: eventually, everyone should agree on state
Exercise 1:

Consistency Model:

- Strictly Serializable: Yes
- Linearizable: Yes
- Sequential: Yes
- Causal+: Yes
- Eventual: Yes

P1: \{W(x) 1, W(y) 2\} \{R(y) 4\}

P2: \{W(x) 1, R(y) 4\}

P3: \{W(x) 0, W(y) 4\}

P4: \{R(x) 0\} \{R(x) 1\}
Exercise 2:

Consistency Model:
- Linearizable: Yes
- Sequential: Yes
- Causal+: Yes
- Eventual: Yes

<table>
<thead>
<tr>
<th></th>
<th>P1: W(x) 1</th>
<th>R(y) 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>P2:</td>
<td>R(x) 1</td>
<td>R(y) 4</td>
</tr>
<tr>
<td>P3:</td>
<td>R(x) 1</td>
<td>W(y) 4</td>
</tr>
<tr>
<td>P4:</td>
<td>R(x) 1</td>
<td>R(y) 4</td>
</tr>
</tbody>
</table>
## Exercise 3:

<table>
<thead>
<tr>
<th>P1:</th>
<th>W(x) 3</th>
<th>W(y) 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>P2:</td>
<td>W(x) 1</td>
<td></td>
</tr>
<tr>
<td>P3:</td>
<td>R(x) 1</td>
<td>R(x) 3</td>
</tr>
<tr>
<td>P4:</td>
<td>R(x) 1</td>
<td>R(x) 3</td>
</tr>
<tr>
<td>P5:</td>
<td>R(x) 1</td>
<td>R(x) 3</td>
</tr>
</tbody>
</table>

### Consistency Model:
- **Linearizable**: No
- **Sequential**: Yes
- **Causal+**: Yes
- **Eventual**: Yes
### Exercise 4:

<table>
<thead>
<tr>
<th></th>
<th>W(x) 3</th>
<th>W(y) 7</th>
<th></th>
<th>R(x) 1</th>
<th>R(x) 3</th>
<th>R(y) 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td></td>
<td></td>
<td>P2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P3</td>
<td></td>
<td></td>
<td>P4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Consistency Model:

- **Linearizable**: No
- **Sequential**: No
- **Causal+**: Yes
- **Eventual**: Yes
Exercise 5:

Consistency Model:
- Linearizable: No
- Sequential: No
- Causal+: Yes
- Eventual: Yes

P1: W(x) 1
P2: W(x) 3
P3: W(x) 7
P4: R(x) 3  R(x) 7  R(x) 1
P5: R(x) 3  R(x) 1  R(x) 7
Exercise 6:

<table>
<thead>
<tr>
<th>P1:</th>
<th>W(x) 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>P2:</td>
<td>W(x) 3</td>
</tr>
<tr>
<td>P3:</td>
<td>R(x) 3  W(x) 7</td>
</tr>
<tr>
<td>P4:</td>
<td>R(x) 3  R(x) 7  R(x) 1</td>
</tr>
<tr>
<td>P5:</td>
<td>R(x) 3  R(x) 1  R(x) 7</td>
</tr>
</tbody>
</table>

Consistency Model:

- Linearizable: No
- Sequential: No
- Causal+: Yes
- Eventual: Yes
Exercise 7:

P1: \( W(x) 1 \)

P2: \( R(x) 1 \quad W(x) 3 \)

P3: \( R(x) 3 \quad W(x) 7 \)

P4: \( R(x) 3 \quad R(x) 7 \quad R(x) 1 \)

P5: \( R(x) 3 \quad R(x) 1 \quad R(x) 7 \)

Consistency Model:

- Linearizable: No
- Sequential: No
- Causal+: No
- Eventual: Yes