Modules and Abstract Data Types

COS 326

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motherhood and apple pie



Turn Tives

photo: pexels.com

photo: en.wiktionary.org

Phrase

- 1. (US, symbolically) Wholesomeness and traditional (American) values.
- 2. (*politics*, *US*) Something that cannot be questioned because it appeals to universally held beliefs or values.

(from wictionary.org)

modules, interfaces, data abstraction

Phrase

- 1. Wholesomeness and traditional (software engineering) values.
- 2. Something that cannot be questioned because it appeals to universally held beliefs or values.

The Reality of Development

We rarely know the *right* algorithms or the *right* data structures when we start a design project.

— When implementing a search engine, what data structures and algorithms should you use to build the index? To build the query evaluator?

Reality is that we often have to go back and change our code, once we've built a prototype.

- Often, we don't even know what the user wants (requirements)
 until they see a prototype.
- Often, we don't know where the performance problems are until we can run the software on realistic test cases.
- Sometimes we just want to change the design -- come up with simpler algorithms, architecture later in the design process

Engineering for Change

Given that we know the software will change, how can we write the code so that doing the changes will be easier?

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The primary trick: use data and algorithm abstraction.

Engineering for Change

Given that we know the software will change, how can we write the code so that doing the changes will be easier?

The primary trick: use data and algorithm abstraction.

- Don't code in terms of concrete representations that the language provides.
- Do code with high-level abstractions in mind that fit the problem domain.
- Implement the abstractions using a well-defined interface.
- Swap in different implementations for the abstractions.
- Parallelize the development process.

Goal: Implement a query engine.

Requirements: Need a scalable *dictionary* (a.k.a. index)

- maps words to set of URLs for the pages on which words appear.
- want the index so that we can efficiently satisfy queries
 - e.g., all links to pages that contain "Dave" and "Jill".

Wrong way to think about this:

- Aha! A list of pairs of a word and a list of URLs.
- We can look up "Dave" and "Jill" in the list to get back a list of URLs.

```
type query =
 Word of string
| And of query * query
| Or of query * query
type index = (string * (url list)) list
let rec eval(q:query) (h:index) : url list =
 match q with
  | Word x ->
      let ( ,urls) = (try List.find (fun (w,urls) -> w=x) h
                      with Not found -> [])
       in urls
  | And (q1, q2) ->
       intersect lists (eval q1 h) (eval q2 h)
  | Or (q1,q2) ->
       (eval q1 h) @ (eval q2 h)
```

```
type query =
 Word of string
| And of query * query
| Or of query * query
type index = (string * (url list)) list
let rec eval(q:query) (h:index)
                                     intersect expects
 match q with
                                      to be passed
  | Word x →
                                      sorted lists.
      let ( ,urls) = List.find
                                                      = x) in
      urls
  | And (q1,q2) ->
       intersect lists (eval q1 h) (eval q2 h)
  | Or (q1,q2) ->
       (eval q1 h) @ (eval q2 h)
```

```
type query =
 Word of string
| And of query * query
| Or of query * query
type index = (string * (url list)) list
let rec eval(q:query) (h:index)
                                     intersect expects
 match q with
                                      to be passed
  | Word x →
                                       sorted lists
      let (_,urls) = List.find
      urls
                                                  Oops!
  | And (q1, q2) ->
       intersect lists (eval q1
  | Or (q1,q2) ->
       (eval q1 h) @ (eval q2 h)
```

```
type query =
  Word of string
| And of query * query
| Or of query * query
type index = string (url list) hashtable
let rec eval(q:query)(h:index) : url list =
 match q with
  | Word x →
       let i = hash string x in
       let l = Array.get h [i] in
       let urls = assoc list find l x in
       urls
  | And (q1,q2) \rightarrow ...
  | \text{ Or } (q1,q2) -> \dots
```

I find out there's a better hash-table implementation

```
type query =
 Word of string
| And of query * query
| Or of query * query
type index = string url set dictionary
let rec eval(q:query)(d:index) : url set =
 match q with
  | Word x -> Dict.lookup d x
  | And (q1,q2) -> Set.intersect (eval q1 h) (eval q2 h)
  | Or (q1,q2) -> Set.union (eval q1 h) (eval q2 h)
```

```
talked about an
type query =
                                               abstract type of
 Word of string
                                             dictionaries and sets of
| And of query * query
                                                   URLs.
| Or of query * query
type index = string url set dictionary
let rec eval(q:query)(d:index) : url set =
 match q with
  | Word x -> Dict.lookup d x
  | And (q1,q2) -> Set.intersect (eval q1 h) (eval q2 h)
  | Or (q1,q2) -> Set.union (eval q1 h) (eval q2 h)
```

The problem domain

```
type query =
 Word of string
| And of query * query
| Or of query * query
type index = string url set dictionar
let rec eval(q:query)(d:index) : url
 match q with
  | Word x -> Dict.lookup d x
  | And (q1,q2) -> Set.intersect (eval q1 h) (eval q2 h)
```

The problem domain talked about an abstract type of <u>dictionaries</u> and <u>sets</u> of URLS.

Once we've written the client, we know what operations we need on these abstract types.

```
type query =
 Word of string
| And of query * query
| Or of query * query
type index = string url set dictionar
let rec eval(q:query)(d:index) : url
 match q with
  | Word x -> Dict.lookup d x
  | And (q1,q2) -> Set.intersect (eval q1 h) (eval q2 h)
  | Or (q1,q2) -> Set.union (eval q1 h) (eval q2 h)
```

The problem domain talked about an abstract type of dictionaries and sets of URLS.

Once we've written the client, we know what operations we need on these abstract types.

So we can define an Later on, when we find interface, and send a pal out linked lists aren't so off to implement the good for sets, we can abstract types dictionary replace them with and set. balanced trees.

Abstract Data Types



Barbara Liskov Assistant Professor, MIT 1973

Invented CLU language that enforced data abstraction



Barbara Liskov Professor, MIT Turing Award 2008

"For contributions to practical and theoretical foundations of programming language and system design, especially related to data abstraction, fault tolerance, and distributed computing."

Building Abstract Types in OCaml

OCaml has mechanisms for building new abstract data types:

- module type (or "signature"): an interface.
 - specifies the abstract type(s) without specifying their implementation
 - specifies the set of operations on the abstract types
- module (or "structure"): an implementation.
 - a collection of type and value definitions
 - notion of an implementation matching or satisfying an interface
 - gives rise to a notion of subtyping
- parameterized module (or "functor"):
 - really, a function from modules to modules
 - allows us to factor out and re-use modules

The Abstraction Barrier

Rule of thumb: Use the language to enforce the abstraction barrier.

- Reveal little information about how something is implemented
- Provide maximum flexibility for change moving forward.
- Murphy's Law: What is not enforced, will be broken

But rules are meant to broken: Exercise judgement.

- may want to reveal more information for debugging purposes
 - eg: conversion to string so you can print things out

ML gives you precise control over how much of the type is left abstract

- different amounts of information can be revealed in different contexts
- type checker helps you detect violations of the abstraction barrier

Recall assigment #2:

query.ml

```
type movie = { ... }
let sort_by_studio = ...
let sort_by_year = ...
```

main.ml

```
open lo
open Query

let main () = ... sort_by_studio ...

let _ = main ()
```

Recall assigment #2:

query.ml

```
type movie = { ... }
let sort_by_studio = ...
let sort_by_year = ...
```

main.ml

```
open lo
open Query
let main () = ... sort_by_studio ...
let _ = main ()
```

Each .ml file actually defines an ML module.

Convention: the file foo.ml or Foo.ml defines the module named Foo.

Recall assigment #2:

open gives direct access to module components

query.ml

```
type movie = { ... }

let sort_by_studio = ...

let sort_by_year = ...
```

```
main.ml

open lo
open Query

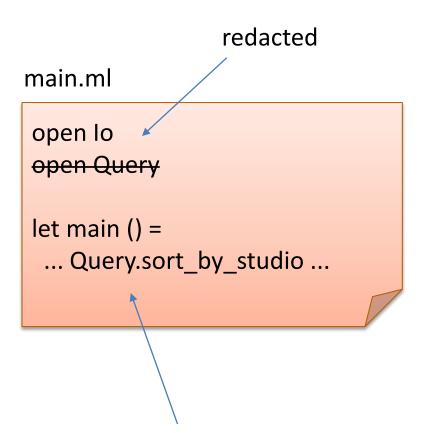
let main () = ... sort_by_studio ...

let _ = main ()
```

Recall assigment #2:

query.ml

```
type movie = { ... }
let sort_by_studio = ...
let sort_by_year = ...
```



Can refer to module components using dot notation

query.ml

```
type movie = { ... }
let sort_by_studio = ...
let sort_by_year = ...
```

main.ml

```
open lo
open Query

let main () =
... Query.sort_by_studio ...
```

query.mli

```
type movie
```

```
val sort_by_studio : movie list -> movie list
val sort_by_year : movie list -> movie list
```

You can add interface files (.mli) (also called *signatures* in ML)

These interfaces can hide module components or render types abstract.

query.ml

```
type movie = { ... }
let sort_by_studio = ...
let sort_by_year = ...
```

main.ml

```
open lo
open Query

let main () =
... Query.sort_by_studio ...
```

query.mli

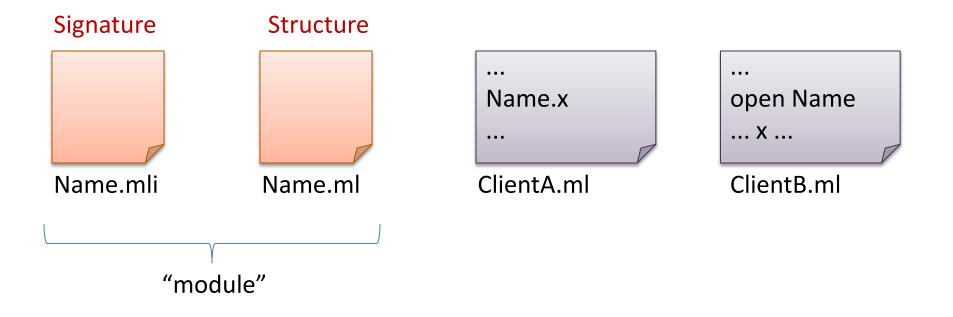
```
type movie
```

```
val sort_by_studio : movie list -> movie list
val sort_by_year : movie list -> movie list
```

If you have no signature file, then the default signature is used: all components are fully visible to clients.

Simple summary:

- file Name.ml is a structure implementing a module named Name
- file Name.mli is a signature for the module named Name
 - if there is no file Name.mli, OCaml infers the default signature



At first glance: OCaml modules = C modules?

C has:

- .h files (signatures) similar to .mli files?
- .c files (structures) similar to .ml files?

But ML also has:

- tighter control over type abstraction
 - define abstract, transparent or translucent types in signatures
 - i.e.: give none, all or some of the type information to clients
- more structure
 - modules can be defined within modules
 - i.e.: signatures and structures can be defined inside files
- more reuse
 - multiple modules can satisfy the same interface
 - the same module can satisfy multiple interfaces
 - modules take other modules as arguments (functors)
- fancy features: dynamic, first class modules

```
module type INT STACK =
  sig
    type stack
    val empty : unit -> stack
    val push : int -> stack -> stack
    val is empty : stack -> bool
    val pop : stack -> stack
    val top : stack -> int option
  end
```

```
empty and push
module type INT STACK =
                                         are abstract
                                         constructors:
  sig
                                       functions that build
    type stack
                                       our abstract type.
    val empty : unit -> stack
    val push : int -> stack -> stack
    val is empty : stack -> bool
    val pop : stack -> stack option
    val top: stack -> int option
  end
```

```
module type INT STACK =
  sig
    type stack
    val empty : unit -> stack
    val push : int -> stack -> stack
    val is empty : stack -> bool
    val pop : stack -> stack
    val top : stack
                                is empty is an
                               observer – useful
  end
                               for determining
                               properties of the
                                   ADT.
```

```
module type INT STACK =
  sig
    type stack
    val empty : unit -> stack
    val push : int -> stack -> stack
    val is empty : stack -> bool
    val pop : stack -> stack
    val top . stack -> int option
  end
                      pop is sometimes
                      called a mutator
                      (though it doesn't
                      really change the
                         input)
```

```
module type INT STACK =
  sig
     type stack
     val empty : unit -> stack
    val push : int -> stack -> stack
    val is empty : stack -> bool
     val pop : stack -> stack
    val top: stack -> int option
  end
                                      top is also an
                                     observer, in this
                                     functional setting
                                      since it doesn't
                                     change the stack.
```

Put comments in your signature!

```
module type INT STACK =
  sig
   type stack
    (* create an empty stack *)
    val empty : unit -> stack
    (* push an element on the top of the stack *)
    val push : int -> stack -> stack
    (* returns true iff the stack is empty *)
    val is empty : stack -> bool
    (* pops top element off the stack;
       returns empty stack if the stack is empty *)
    val pop : stack -> stack
    (* returns the top element of the stack; returns
       None if the stack is empty *)
    val top : stack -> int option
  end
```

Signature Comments

Signature comments are for clients of the module

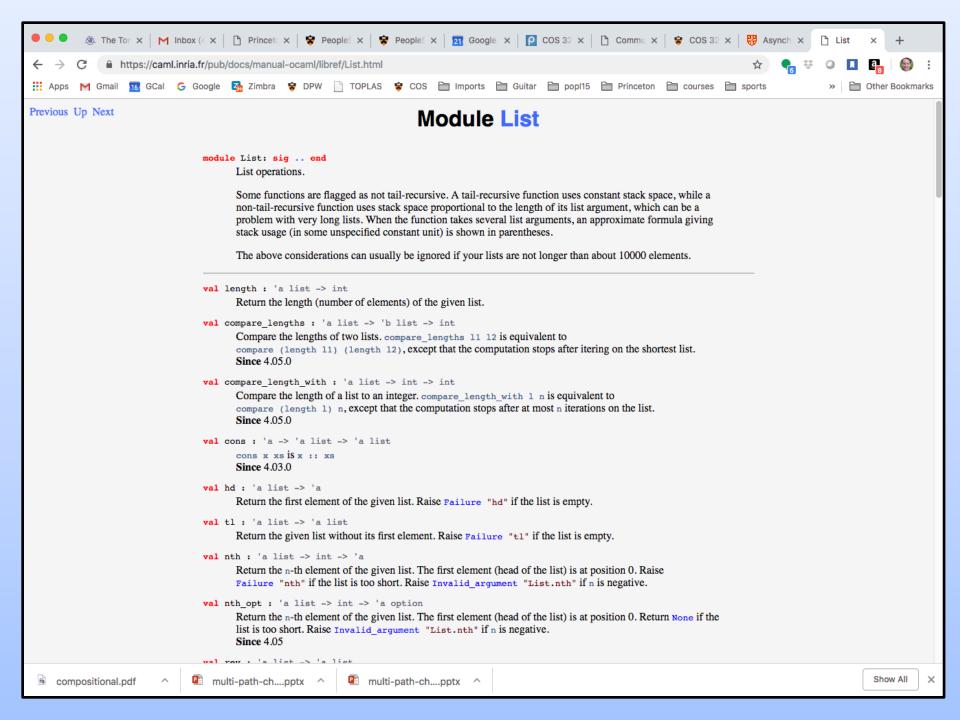
- explain what each function should do
 - how it manipulates abstract values (stacks)
- not how it manipulates concrete values
- don't reveal implementation details that should be hidden behind the abstraction

Don't copy signature comments into your structures

- your comments will get out of date in one place or the other
- an extension of the general rule: don't copy code

Place implementation comments inside your structure

- comments about implementation invariants hidden from client
- comments about helper functions



Example Structure Inside a File

```
module ListIntStack : INT STACK =
  struct
    type stack = int list
    let empty () : stack = []
    let push (i:int) (s:stack) : stack = i::s
    let is empty (s:stack) =
      match s with
       | [] -> true
       | :: -> false
    let pop (s:stack) : stack =
      match s with
       | [] -> []
       | ::t -> t
    let top (s:stack) : int option =
      match s with
       | [] -> None
       | h:: -> Some h
  end
```

Example Structure Inside a File

```
module ListIntStack : INT STACK =
  struct
    type stack = int list
                                          Inside the module,
    let empty () : stack = []
                                           we know the
    let push (i:int) (s:stack) = i:
                                          concrete type used
    let is empty (s:stack) =
                                           to implement the
      match s with
                                            abstract type.
         [] -> true
       | :: -> false
    let pop (s:stack) : stack =
      match s with
       | [] -> []
        | ::t -> t
    let top (s:stack) : int option =
      match s with
        | [] -> None
       | h:: -> Some h
  end
```

Example Structure Inside a File

```
module ListIntStack : INT STACK =
  struct
    type stack = int list
    let empty () : stack = []
    let push (i:int) (s:stack)
    let is empty (s:stack) =
      match s with
         [] -> true
       | :: -> false
    let pop (s:stack) : stack =
      match s with
       | [] -> []
       | ::t -> t
    let top (s:stack) : int option =
      match s with
       | [] -> None
       | h:: -> Some h
  end
```

But by giving the module the INT_STACK interface, which does not reveal how stacks are being represented, we prevent code outside the module from knowing stacks are lists.

```
module ListIntStack : INT STACK =
  struct
  end
let s0 = ListIntStack.empty ()
let s1 = ListIntStack.push 3 s0
let s2 = ListIntStack.push 4 s1
let x = ListIntStack.top s2
```

```
module ListIntStack : INT STACK =
  struct
  end
let s0 = ListIntStack.empty ()
let s1 = ListIntStack.push 3 s0
let s2 = ListIntStack.push 4 s1
let x = ListIntStack.top s2
                            s0 : ListIntStack.stack
                            s1 : ListIntStack.stack
                            s2 : ListIntStack.stack
```

```
module ListIntStack : INT STACK =
  struct
  end
let s0 = ListIntStack.empty ()
let s1 = ListIntStack.push 3 s0
let s2 = ListIntStack.push 4 s1
let x = ListIntStack.top s2
x : option int = Some 4
```

```
module ListIntStack : INT STACK =
  struct
  end
let s0 = ListIntStack.empty ()
let s1 = ListIntStack.push 3 s0
let s2 = ListIntStack.push 4 s1
let x = ListIntStack.top s2
x : option int = Some 4
let x = ListIntStack.top (ListIntStack.pop s2)
x : option int = Some 3
```

```
module ListIntStack : INT STACK =
  struct
  end
let s0 = ListIntStack.empty ()
let s1 = ListIntStack.push 3 s0
let s2 = ListIntStack.push 4 s1
let x = ListIntStack.top s2
x : option int = Some 4
let x = ListIntStack.top (ListIntStack.pop s2)
x : option int = Some 3
open ListIntStack
```

```
module ListIntStack : INT STACK =
  struct
  end
let s0 = ListIntStack.empty ()
let s1 = ListIntStack.push 3 s0
let s2 = ListIntStack.push 4 s1
let x = ListIntStack.top s2
x : option int = Some 4
let x = ListIntStack.top (ListIntStack.pop s2)
x : option int = Some 3
open ListIntStack
let x = top (pop (pop s2))
x : option int = None
```

```
module type INT STACK =
  sig
    type stack
    val push : int -> stack -> stack
  end
                                               Notice that the
module ListIntStack : INT STACK
                                                client is not
                                              allowed to know
let s2 = ListIntStack.push
                                              that the stack is a
                                                   list.
List.rev s2
Error: This expression has type stack but an
expression was expected of type 'a list.
```

Example Structure

```
module ListIntStack (* : INT STACK *) =
  struct
    type stack = int list
    let empty () : stack = []
    let push (i:int) (s:stack) = i::s
    let is empty (s:stack) =
      match s with
       | [ ] -> true
       | :: -> false
    exception EmptyStack
    let pop (s:stack) =
      match s with
       | [] -> []
       | ::t -> t
    let top (s:stack) =
      match s with
       | [] -> None
       | h:: -> Some h
  end
```

Note that when you are debugging, you may want to comment out the signature ascription so that you can access the contents of the module.

The Client without the Signature

```
module ListIntStack (* : INT STACK *) =
  struct
  end
let s = ListIntStack.empty()
let s1 = ListIntStack.push 3 s
                                             If we don't seal
let s2 = ListIntStack.push 4 s1
                                            the module with
                                             a signature, the
                                             client can know
                                             that stacks are
let x = List.rev s2
                                                 lists.
x : int list = [3; 4]
```

Example Structure

```
module ListIntStack : INT STACK =
  struct
    type stack = int list
    let empty () : stack = []
    let push (i:int) (s:stack) =
    let is empty (s:stack) =
      match s with
       | [ ] -> true
       | :: -> false
    exception EmptyStack
    let pop (s:stack) =
      match s with
       | [] -> []
       | ::t -> t
    let top (s:stack) =
      match s with
       | [] -> None
       | h:: -> Some h
  end
```

When you put the signature on here, you are restricting client access to the information in the signature (which does not reveal that stack = int list.) So clients can only use the stack operations on a stack value (not list operations.)

Example Structure

```
module type INT STACK =
  siq
    type stack
    val inspect : stack -> int list
    val run unit tests : unit -> unit
                                                 Another technique:
  end
                                              Add testing components to
                                                   your signature.
module ListIntStack : INT STACK =
  struct
                                               Or have 2 signatures, one
    type stack = int list
                                               for testing and one for the
                                                   rest of the code)
    let inspect (s:stack) : int list = s
    let run_unit_tests () : unit = ...
  end
```

DESIGN CHOICES FOR CORNER CASES

Interface design

```
module type INT STACK =
  sig
    type stack
    (* create an empty stack *)
    val empty : unit -> stack
    (* push an element on the t
                                          stack *)
    val push : int ->
                           Is this a good
                              idea?
    (* returns true if
    val is empty : stac}
    (* pops top element off the stack;
       returns empty stack if the stack is empty *)
    val pop : stack -> stack
  end
```

```
type stack
  (* pops top element;
    returns empty if empty
    *)
  val pop : stack -> stack
end
```

```
type stack
  (* pops top element;
    returns arbitrary stack
    if empty *)
  val pop : stack -> stack
end
```

```
type stack
  (* pops top element;
    returns option *)
  val pop:
    stack -> stack option
end
```

```
type stack
exception EmptyStack
  (* pops top element;
    raises EmptyStack if empty
    *)
val pop: stack -> stack
end
```

For some functions, there are some input values outside the *domain* of the function & the domain is not easily described by a simple type.

```
type stack
  (* pops top element;
    returns arbitrary stack
    if empty *)
  val pop : stack -> stack
end
```

Say the function returns an arbitrary result on those inputs.

When proving things about the program, there's an extra proof obligation: Prove that the input is in the domain of the function.

For some functions, there are some input values outside the *domain* of the function & the domain is not easily described by a simple type.

```
sig
type stack
  (* pops top element;
    returns arbitrary stack
    if empty *)
    val pop : stack -> stack
end
```

Say the function returns an arbitrary result on those inputs.

When proving things about the program, there's an extra proof obligation: Prove that the input is in the domain of the function.

But when a programmer forgets to do this proof (or makes a mistake), such silent errors can be hard to track down.

For some functions, there are some input values outside the *domain* of the function.

```
type stack
  (* pops top element;
     crashes the program
     if empty *)
  val pop : stack -> stack
end
```

This is not *completely* crazy. One might still be able to guarantee that the input is always in the domain of the function.

It's what the C language does, for example.

For some functions, there are some input values outside the *domain* of the function.

```
type stack
  (* pops top element;
     crashes the program
     if empty *)
  val pop : stack -> stack
end
```

This is not *completely* crazy. One might still be able to guarantee that the input is always in the domain of the function.

It's what the C language does, for example.

But it's *almost completely* crazy. This is the biggest source of security vulnerabilities ever. It's why the hackers can drive your car, steal your money, read your e-mails, ...

```
type stack
  (* pops top element;
    returns empty if empty
    *)
  val pop : stack -> stack
end
```

```
type stack
  (* pops top element;
    returns arbitrary stack
    if empty *)
    val pop : stack -> stack
end
```

It's also reasonable to say the function returns a *specified, convenient,* result on those inputs. This is pretty much the same thing, in practice.

Consider: If supplying an empty stack to pop is probably a mistake on the part of the caller, it is better to stop the program right away (by raising an exception) than to let the error silently slip by. In the long run, finding the real error is tougher.

For some functions, there are some input values outside the *domain* of the function.

That's what exceptions are for!
Raise an exception for values
not in the domain.

```
type stack
exception EmptyStack
(* pops top element;
    raises EmptyStack if empty
    *)
val pop : stack -> stack
end
```

Finally, you can just use option types in the obvious way.

Using an option has the advantage of forcing the caller to consider what to do on the "error" condition every time the function is called. They can't *forget* to handle this situation.

```
sig
  type stack
  (* pops top element;
    returns option *)
  val pop:
      stack -> stack option
end
```

```
type stack
  (* pops top element;
    returns empty if empty
    *)
  val pop : stack -> stack
end
```

```
type stack
  (* pops top element;
    returns arbitrary stack
    if empty *)
    val pop : stack -> stack
```

All of these are reasonable design choices!

```
type stack
  (* pops top element;
    returns option *)
  val pop:
    stack -> stack option
end
```

```
type stack
exception EmptyStack
  (* pops top element;
    raises EmptyStack if empty
    *)
  val pop : stack -> stack
end
```

```
sig
                 But use these two with extreme care
 type stack
                                    type stack
 (* pops top element;
                                    (* pops top element;
    returns empty if empty
                                       returns arbitrary stack
  *)
                                     if empty *)
                                     val pop : stack -> stack
  val pop : stack -> stack
end
                      All of these are reasonable
                          design choices!
sig
                                    type stack
 type stack
                                    exception EmptyStack
 (* pops top element;
                                    (* pops top element;
    returns option *)
                                       raises EmptyStack if empty
  val pop:
                                      *)
       stack -> stack option
                                     val pop : stack -> stack
end
                                   end
```

The bottom two are more common. Options are the "safest." They force consideration of the error condition every time.

ANOTHER EXAMPLE

Polymorphic Queues

```
module type QUEUE =
  sig
    type 'a queue
    val empty : unit -> 'a queue
    val enqueue : 'a -> 'a queue -> 'a queue
    val is empty : 'a queue -> bool
    exception EmptyQueue
    val dequeue : 'a queue -> 'a queue
    val front : 'a queue -> 'a
  end
```

Polymorphic Queues

```
module type QUEUE =
  sig
    type 'a queue
    val empty : unit -> 'a queue
    val enqueue : 'a -> 'a queue -> 'a queue
    val is empty : 'a queue -> bool
    exception EmptyQueue
    val dequeue : 'a queue ->
                                  aueue
    val front : 'a queue -> 'a
  end
```

These queues are re-usable for different element types.

Here's an exception

that client code

might want to

catch

```
module AppendListQueue : QUEUE =
  struct
    type 'a queue = 'a list
    let empty() = []
    let enqueue(x:'a)(q:'a queue) : 'a queue = q @ [x]
    let is empty(q:'a queue) =
     match q with
     | [] -> true
     | :: -> false
```

end

```
module AppendListQueue : QUEUE =
  struct
    type 'a queue = 'a list
    let empty() = []
    let enqueue(x:'a)(q:'a queue) : 'a queue = q @ [x]
    let is empty(q:'a queue) = ...
    exception EmptyQueue
    let deq(q:'a queue) : ('a * 'a queue) =
      match q with
      | [] -> raise EmptyQueue
      | h::t -> (h,t)
    let dequeue(q:'a queue) : 'a queue = snd (deq q)
    let front(q:'a queue) : 'a = fst (deq q)
end
```

```
module AppendListQueue : QUEUE =
  struct
    type 'a queue = 'a list
    let empty() = []
    let enqueue(x:'a)(q:'a queue) : 'a
                                            Notice deg is a helper
    let is empty(q:'a queue) = ...
                                            function that doesn't
                                               show up in the
                                                signature.
    exception EmptyQueue
    let deq(q:'a queue) : ('a * 'a queue) =
      match q with
       | [] -> raise EmptyQueue
       | h::t -> (h,t)
                                               You can't use it
                                              outside the module.
    let dequeue (q:'a queue) : 'a que
    let front(q:'a queue) : 'a = fst
end
```

```
enqueue takes time
module AppendListQueue : QUEUE =
                                             proportional to the
                                             length of the queue
  struct
    type 'a queue = 'a list
    let empty() = []
    let enqueue(x:'a)(q:'a queue) : 'a queue = q @ [x]
    let is empty(q:'a queue) = ...
                                              Dequeue runs in
                                               constant time
    exception EmptyQueue
    let deq(q:'a queue) : ('a)
      match q with
       | [] -> raise EmptyQueue
       | h::t -> (h,t)
    let dequeue(q:'a queue) : 'a queue = snd (deq q)
    let front(q:'a queue) : 'a = fst (deq q)
end
```

An Alternative Implementation

```
module DoubleListQueue : QUEUE =
  struct
    type 'a queue = {front:'a list; rear:'a list}
end
```

In Pictures

abstraction

implementation

```
a, b, c, d, e -----
```

```
{front=[a; b];rear=[e; d; c]}
```

```
let q0 = empty
              {front=[];rear=[]}
let q1 = enqueue 3 q0 {front=[]; rear=[3]}
let q2 = enqueue 4 q1 {front=[]; rear=[4;3]}
let q3 = enqueue 5 q2 {front=[]; rear=[5;4;3]}
let q4 = dequeue q3 {front=[4;5]; rear=[]}
let q5 = dequeue q4 {front=[5]; rear=[]}
let q6 = enqueue 6 q5 {front=[5]; rear=[6]}
let q7 = enqueue 7 q6 {front=[5]; rear=[7;6]}
```

An Alternative Implementation

```
module DoubleListQueue : QUEUE =
  struct
    type 'a queue = {front:'a list;
                                               enqueue runs in
                       rear: 'a list}
                                                constant time
    let empty() = {front=[]; rear=[]}
    let enqueue x q = {front=q.front; rear=x::q.rear}
    let is empty q =
      match q.front, q.rear with
       | [], [] -> true
       \mid , \rightarrow false
end
```

An Alternative Implementation

```
module DoubleListQueue : QUEUE =
  struct
                                              dequeue runs in
    type 'a queue = {front:'a list;
                                                amortized
                      rear: 'a list}
                                              constant time
    exception EmptyQueue
    let deq (q:'a queue) : 'a * 'a queue =
      match q.front with
       | h::t -> (h, {front=t; rear=q.rear})
       | [] -> match List.rev q.rear with
               | h::t -> (h, {front=t; rear=[]})
               | [] -> raise EmptyQueue
    let dequeue (q:'a queue) : 'a queue = snd(deq q)
    let front (q:'a queue) : 'a = fst(deq q)
  end
```

How would we design an abstraction?

Think:

- what data do you want?
 - define some types for your data
- what operations on that data do you want?
 - define some types for your operations

Write some test cases:

example data, operations

From this, we can derive a signature

- list the types
- list the operations with their types
- don't forget to provide enough operations that you can debug!

Then we can build an implementation

- when prototyping, build the simplest thing you can.
- later, we can swap in a more efficient implementation.
- (assuming we respect the abstraction barrier.)

Common Interfaces

The stack and queue interfaces are quite similar:

```
module type STACK =
  sig
    type 'a stack
    val empty : unit -> 'a stack
    val push : int -> 'a stack -> 'a stack
    val is empty : 'a stack -> bool
    exception EmptyStack
    val pop module type QUEUE =
    val top sig
  end
                type 'a queue
                val empty : unit -> 'a queue
                val enqueue : 'a -> 'a queue -> 'a queue
                val is empty : 'a queue -> bool
                exception EmptyQueue
                val dequeue : 'a queue -> 'a queue
                val front : 'a queue -> 'a
              end
```

It's a good idea to factor out patterns

Stacks and Queues share common features.

Both can be considered "containers"

Create a reuseable container interface!

```
module type CONTAINER =
  sig
    type 'a t
    val empty : unit -> 'a t
    val insert : 'a -> 'a t -> 'a t
    val is empty : 'a t -> bool
    exception Empty
    val remove : 'a t -> 'a t
    val first : 'a t -> 'a
  end
```

It's a good idea to factor out patterns

```
module type CONTAINER = sig ... end
module Queue : CONTAINER = struct ... end
module Stack : CONTAINER = struct ... end
```

```
module DepthFirstSearch : SEARCHER =
   struct
   type to_do : Graph.node Stack.t
   end
```

```
module BreadthFirstSearch : SEARCHER =
   struct
   type to_do : Graph.node Queue.t
   end
```

Still repeated code!

Breadth-first and depth-first search code is the same!

Just use different containers!

Need parameterized modules!



David MacQueen
Bell Laboratories 1983-2001
U. of Chicago 2001-2012

Designer of ML module system, functors, sharing constraints, etc.

FUNCTORS

Matrices

Suppose I ask you to write a generic package for matrices.

e.g., matrix addition, matrix multiplication

The package should be *parameterized* by the element type.

- Matrix elements may be ints or floats or complex ...
- And the elements still have a collection of operations on them:
 - addition, multiplication, zero element, etc.

What we'll see:

- RING: a signature for matrix elements
- MATRIX: a signature for operations on matrices
- DenseMatrix: a functor that will generate a MATRIX with a specific RING as an element type

Ring Signature

```
module type RING =
    sig
    type t
    val zero : t
    val one : t
    val add : t -> t -> t
    val mul : t -> t -> t
    end
```

Some Rings

```
module IntRing =
  struct
  type t = int
  let zero = 0
  let one = 1
  let add x y= x + y
  let mul x y = x * y
  end
```

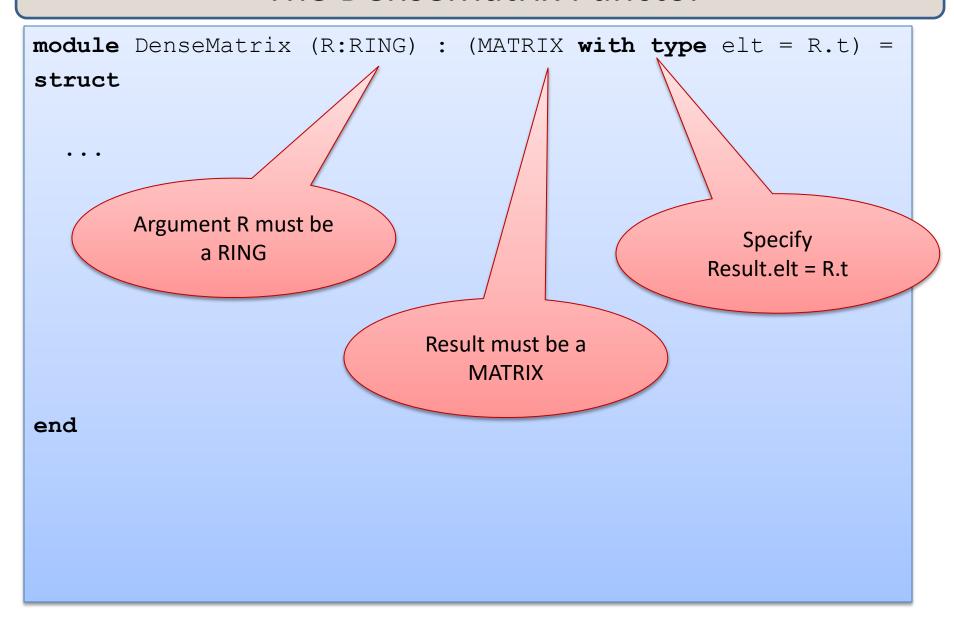
```
module BoolRing =
  struct
  type t = bool
  let zero = false
  let one = true
  let add x y= x || y
  let mul x y = x && y
  end
```

```
module FloatRing =
  struct
  type t = float
  let zero = 0.0
  let one = 1.0
  let add = (+.)
  let mul = ( *. )
  end
```

Matrix Signature

```
module type MATRIX =
    sig
        type elt
        type matrix
        val matrix_of_list : elt list list -> matrix
        val add : matrix -> matrix -> matrix
        val mul : matrix -> matrix -> matrix
    end
```

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct
end
```



```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct
                      Use DenseMatrix like
                      it is a function from
                      modules to modules
end
module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct ... end
module IntMatrix = DenseMatrix(IntRing)
       What is the signature
          of IntMatrix?
```

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct ... end
module IntMatrix = DenseMatrix(IntRing)
        What is the signature
           of IntMatrix?
                                               It depends on both
                                               the signatures of
                                              DenseMatrix and of
                                              it's argument IntRing
```

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct ... end
module IntMatrix = DenseMatrix(IntRing)
module type MATRIX =
  sig
   type elt
   type matrix
   val matrix of list : elt list list -> matrix
                                                     type elt = R.t
   val add : matrix -> matrix -> matrix
   val mul : matrix -> matrix -> matrix
   end
```

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct ... end

module IntMatrix = DenseMatrix(IntRing)

module type MATRIX =
```

```
module type MATRIX =
sig
    type elt
    type matrix
    val matrix_of_list : elt list list -> matrix
    val add : matrix -> matrix -> matrix
    val mul : matrix -> matrix -> matrix
    end
```

Recall:

```
module IntRing =
  struct
  type t = int
  let zero = 0
  ...
end
```

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct ... end
module IntMatrix = DenseMatrix(IntRing)
```

```
module type MATRIX =
  siq
    type elt
    type matrix
    val matrix of list : elt list list -> matrix
    val add : matrix -> matrix -> matrix
    val mul : matrix -> matrix -> matrix
   end
```

type elt = R.t

module type MATRIX = sig type elt = int type matrix end

module IntRing = struct type t = int end

```
module DenseMatrix (R:RING) : (MATRIX with type alt = R
struct
                                                  redacted
end
module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

```
module DenseMatrix (R:RING) : (MATRIX with type alt = R t)
struct
                                                      redacted
                            module type MATRIX =
                                                         abstract =
                              sig
                                                         unknown!
                                type elt <
                                type matrix
                                val matrix of list:
                                      elt list list -> matrix
                                val add : matrix -> matrix -> matrix
                                val mul : matrix -> matrix -> matrix
end
                               end
module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix (BoolRing)
```

```
module DenseMatrix (R:RING) : (MATRIX with type alt = R t)
struct
                                                          redacted
    If the "with" clause is
       redacted then
                              module type MATRIX =
   IntMatrix.elt is abstract
                                                             abstract =
                                 sig
   -- we could never build
                                                             unknown!
                                   type elt <
    a matrix because we
                                  type matrix
    could never generate
                                  val matrix of list:
          an elt
                                         elt list list -> matrix
                                  val add : matrix -> matrix -> matrix
                                  val mul : matrix -> matrix -> matrix
end
                                  end
module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix (BoolRing)
```

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct
                                                        sharing constraint
                             module type MATRIX =
                                                           known to be
                               sig
                                                           int when
                                 type elt = int
                                 type matrix
                                                           R.t = int like
                                                           when R = IntRing
                                 val matrix of list:
                                     > elt list list -> matrix
                                 val add : matrix -> matrix -> matrix
                   list of list of
                                 val mul : matrix -> matrix -> matrix
                   ints
end
                                end
module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix (BoolRing)
```

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct
                                                           sharing constraint
     The "with" clause
                               module type MATRIX =
     makes IntMatrix.elt
                                                              known to be
                                 sig
    equal to int -- we can
                                                              int when
                                   type elt = int
   build a matrix from any
                                   type matrix
                                                              R.t = int like
         int list list
                                                              when R = IntRing
                                   val matrix of list :
                                       > elt list list -> matrix
                                   val add : matrix -> matrix -> matrix
                    list of list of
                                   val mul : matrix -> matrix -> matrix
                    ints
end
                                  end
module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix (BoolRing)
```

Matrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct
  type elt = ...
  type matrix = ...
  let matrix of list = ...
  let add m1 m2 = \dots
  let mul m1 m2 = \dots
end
```

To define a functor, just write down a module as its body.

That module has to match the result signature (MATRIX).

This module may refer to the functor arguments, like R.

```
module IntMatrix = DenseMatrix(IntRing)
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

Matrix Functor

```
module DenseMatrix (R:RING) : (MATRIX with type elt = R.t) =
struct
  type elt = R.t
                                                   Satisfies the sharing
  type matrix = (elt list) list
                                                      constraint
  let matrix of list rows = rows
  let add m1 m2 =
    List.map (fun (r1,r2) \rightarrow
                  List.map (fun (e1, e2) \rightarrow R.add e1 e2))
                      (List.combine r1 r2))
       (List.combine m1 m2)
  let mul m1 m2 = (* good exercise *)
                                                    Can refer to functor
end
                                                        argument
                                                    values or argument
module IntMatrix = DenseMatrix(IntRing)
                                                         types!
module FloatMatrix = DenseMatrix(FloatRing)
module BoolMatrix = DenseMatrix(BoolRing)
```

ANONYMOUS STRUCTURES

Another Example

```
module type UNSIGNED BIGNUM =
sig
  type ubignum
  val fromInt : int -> ubignum
  val toInt : ubignum -> int
  val plus : ubignum -> ubignum -> ubignum
  val minus : ubignum -> ubignum -> ubignum
  val times : ubignum -> ubignum -> ubignum
end
```

An Implementation

```
module My UBignum 1000 : UNSIGNED BIGNUM =
struct
                                          What if we want
  let base = 1000 —
                                           to change the
                                           base? Binary?
  type ubignum = int list
                                         Hex? 2^32? 2^64?
  let toInt(b:ubiqnum):int = ...
  let plus(b1:ubiqnum)(b2:ubiqnum):ubiqnum = ...
  let minus(b1:ubiqnum)(b2:ubiqnum):ubiqnum = ...
  let times(b1:ubignum) (b2:ubignum):ubignum = ...
end
```

Another Functor Example

```
module type BASE =
sig
  val base : int
end
module UbignumGenerator (Base: BASE) : UNSIGNED BIGNUM =
struct
  type ubiqnum = int list
  let toInt(b:ubignum):int =
    List.fold left (fun a c -> c*Base.base + a) 0 b ...
end
                                                   Anonymous
                                                   structures
module Ubiqnum 10 =
  UbignumGenerator(struct let base = 10 end) ;;
module Ubignum 2 =
  UbiqnumGenerator(struct let base = 2 end) ;;
```

SIGNATURE SUBTYPING

Subtyping

A module matches any interface as long as it provides at least the definitions (of the right type) specified in the interface.

But as we saw earlier, the module can have more stuff.

e.g., the deq function in the Queue modules

Basic principle of subtyping for modules:

- wherever you are expecting a module with signature S, you can use a module with signature S', as long as all of the stuff in S appears in S'.
- That is, S' is a bigger interface.

```
module type GROUP =
  sig
  type t
   val zero : t
   val add : t -> t -> t
  end
module type RING =
  sig
  type t
   val zero : t
   val one : t
   val add : t -> t -> t
   val mul : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

```
module type GROUP =
  sig
   type t
   val zero : t
   val add : t -> t -> t
                                RING is a sub-type
                                  of GROUP.
  end
module type RING =
  sig
   type t
   val zero : t
   val one : t
   val add : t -> t -> t
   val mul : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

```
module type GROUP =
  sig
   type t
    val zero : t
    val add : t -> t -> t
  end
module type RING =
  sig
                                     There are more
   type t
                                    modules matching
    val zero : t
                                      the GROUP
                                    interface than the
    val one : t
                                       RING one.
    val add : t -> t -> t
   val mul : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

```
module type GROUP =
  sig
   type t
   val zero : t
   val add : t -> t -> t
  end
module type RING =
  siq
   type t
   val zero : t
                                        Any module
   val one : t
    val add : t -> t -> t
                                        expecting a
                                       GROUP can be
    val mul : t -> t -> t
                                       passed a RING.
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
                                    The include primitive
  end
                                     is like cutting-and-
module type RING =
                                   pasting the signature's
                                      content here.
  sig
    include GROUP
    val one : t
    val mul : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

```
module type GROUP =
  sig
    type t
    val zero : t
    val add : t -> t -> t
  end
                                That ensures we
                               will be a sub-type
module type RING =
                                of the included
  sig
                                  signature.
    include GROUP
    val one : t
    val mul : t -> t -> t
  end
module IntGroup : GROUP = IntRing
module FloatGroup : GROUP = FloatRing
module BoolGroup : GROUP = BoolRing
```

MODULE EVALUATION

A structure is a series of declarations

- How does one evaluate a type declaration? We'll ignore it (because it doesn't do anything at run time).
- How does one evaluate a let declaration?



How does one evaluate an entire structure?

evaluate each declaration in order from first to last

main.ml

```
let x = 326
let main () =
 Printf.printf "Hello COS %d\n" x
let foo =
 Printf.printf "Byeee!\n"
let _ =
  main ()
```

main.ml

```
let x = 326
let main () =
 Printf.printf "Hello COS %d\n" x
let foo =
 Printf.printf "Byeee!\n"
let =
  main ()
```

Step 1: evaluate the 1st declaration

but the RHS (326) is already a value so there's nothing to do except remember that x is bound to the integer 326

main.ml

```
let x = 326
let main () =
 Printf.printf "Hello COS %d\n" x
let foo =
 Printf.printf "Byeee!\n"
let =
  main ()
```

Step 2:

evaluate the 2nd declaration this is slightly trickier:

really declares a function. It's equivalent to:

"fun () -> ..." is already a value, like 326.
So there's nothing to do again.

main.ml

```
let x = 326
let main () =
 Printf.printf "Hello COS %d\n" x
let foo =
 Printf.printf "Byeee!\n"
let =
  main ()
```

Step 3: evaluate the 3rd declaration

let foo = ...

evaluation of this expression has an effect – it prints out "Byeee!\n" to the terminal.

the resulting value is () which is bound to foo

main.ml

```
let x = 326
let main () =
 Printf.printf "Hello COS %d\n" x
let foo =
 Printf.printf "Byeee!\n"
let _ =
  main ()
```

Step 4: evaluate the 4th declaration

let _ = ...

evaluation main () causes another effect.

"Hello ..." is printed

the resulting value is () again. the "_" indicates we don't care to bind () to any variable

main.ml

```
let x = 326
let main =
 (fun () ->
   Printf.printf "Hello COS %d\n" x)
let foo =
 Printf.printf "Byeee!\n"
let _ =
  main ()
```

This evaluates exactly the same way

We just replaced

let main () = ...

with the equivalent

let main = fun () -> ...

main.ml

```
let x = 326
let main =
 Printf.printf "Hello COS %d\n" x;
 (fun () -> ())
let foo =
 Printf.printf "Byeee!\n"
let _ =
  main ()
```

This rewrite does something different.

On the 2nd step, it prints because that's what evaluating this expression does:

```
Printf.printf "Hello COS %d\n" x; (fun () -> ())
```

The result of the expression is:

which is bound to main.
This is a pretty silly function.

main.ml

```
module C326 =
struct
 let x = 326
 let main =
  Printf.printf "Hello COS %d\n" x;
  (fun () -> ())
 let foo = Printf.printf "Byeee!\n"
 let _ = main ()
end
let _ =
 Printf.printf "Done\n"
```

Now what happens?

main.ml

```
module C326 =
struct
 let x = 326
 let main =
  Printf.printf "Hello COS %d\n" x;
  (fun () -> ())
 let foo = Printf.printf "Byeee!\n"
 let _ = main ()
end
let done =
 Printf.printf "Done\n"
```

Now what happens?

The entire file contains 2 decls:

- module C326 = ...
- let done = ...

We execute both of them in order.

main.ml

```
module C326 =
struct
 let x = 326
 let main =
  Printf.printf "Hello COS %d\n" x;
  (fun () -> ())
 let foo = Printf.printf "Byeee!\n"
 let _ = main ()
end
let done =
 Printf.printf "Done\n"
```

Now what happens?

The entire file contains 2 decls:

- module C326 = ...
- let done = ...

We execute both of them in order.

Executing the module declaration has the effect of executing every declaration within it in order.

Executing let done = ... is as before

main.ml

```
module C326 =
struct
 exception Unimplemented
 let x = raise Unimplemented
 let main =
  Printf.printf "Hello COS %d\n" x;
  (fun () -> ())
 let foo = Printf.printf "Byeee!\n"
 let _ = main ()
end
let done =
 Printf.printf "Done\n"
```

Now what happens?

main.ml

```
module C326 =
struct
 exception Unimplemented
 let x = raise Unimplemented
 let main =
  Printf.printf "Hello COS %d\n" x;
  (fun () -> ())
 let foo = Printf.printf "Byeee!\n"
 let _ = main ()
end
let done =
 Printf.printf "Done\n"
```

Now what happens?

The entire file contains 2 decls:

- module C326 = ...
- let done = ...

We execute both of them in order.

Executing the module declaration has the effect of executing every declaration within it in order.

The first declaration within it raises an exception which is not caught! That is the only result.

main.ml

```
module type S =
sig
type t = int
val x:t
end
module F (M:S) : S =
struct
 let wow = Printf.printf "%d\n" M.x
 let t = M.t
let x = M.x
end
let done = Printf.printf "Done\n"
```

Now what happens?

The entire file contains 2 decls:

- module type = ...
- module F (M:S) : S = ...
- let done = ...

main.ml

```
module type S =
sig
type t = int
 val x:t
end
module F(M:S) : S =
struct
 let wow = Printf.printf "%d\n" M.x
 let t = M.t
 let x = M.x
end
let done = Printf.printf "Done\n"
```

The signature declaration has no (run-time) effect.

The functor declaration is like declaring a function value.

The body of the functor is not executed until it is applied.

The functor is not applied here so M.x is not printed.

Only "Done\n" is printed.

main.ml

```
module type S = sig ... end
module F (M:S) : S =
struct
 let wow = Printf.printf "%d\n" M.x
 let t = M.t
let x = M.x
end
let module M1 = F (
   struct
    type t = int
    val x = 3
   end)
let done = Printf.printf "Done\n"
```

What happens now?

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```

What happens now?

When M1 is declared, F is applied to an argument.

This creates a new structure and its components are executed.

This has the effect of printing 3.

SUMMARY

Key Points

OCaml's linguistic mechanisms include:

- signatures (interfaces)
- structures (implementations)
- functors (functions from modules to modules)

We can use the module system

- provides support for name-spaces
- hiding information (types, local value definitions)
- code reuse (via functors, reuseable interfaces, reuseable modules)

Information hiding allows design in terms of abstract types and algorithms.

- think "sets" not "lists" or "arrays" or "trees"
- think "document" not "strings"
- the less you reveal, the easier it is to replace an implementation
- use linguistic mechanisms to implement information hiding
 - invariants written down as comments are easy to violate
 - use the type checker to guarantee you have strong protections in place

Wrap up and Summary

It is often tempting to break the abstraction barrier.

e.g., during development, you want to print out a set, so you
just call a convenient function you have lying around for
iterating over lists and printing them out.

But the barrier supports future change of implementations.

- e.g., moving from unsorted invariant to sorted invariant.
- or from lists to balanced trees.

Languages often allow information to leak through the barrier.

- "good" clients should not take advantage of this.
- but they always end up doing it.
- so you end up having to support these leaks when you upgrade, else you'll break the clients.

Wrap up and Summary

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