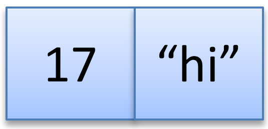
Thinking Inductively

COS 326
Andrew Appel
Princeton University

Options

Often, we either have a thing …. or we don't:

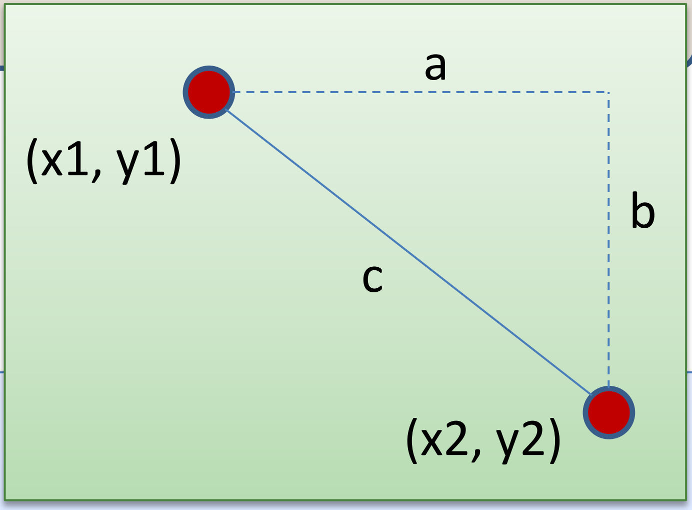


Option types are used in this situation: t option

There's *one way* to build a pair, but *two ways* to build an optional value:

- None -- when we've got nothing
- Some v -- when we've got a value v of type t

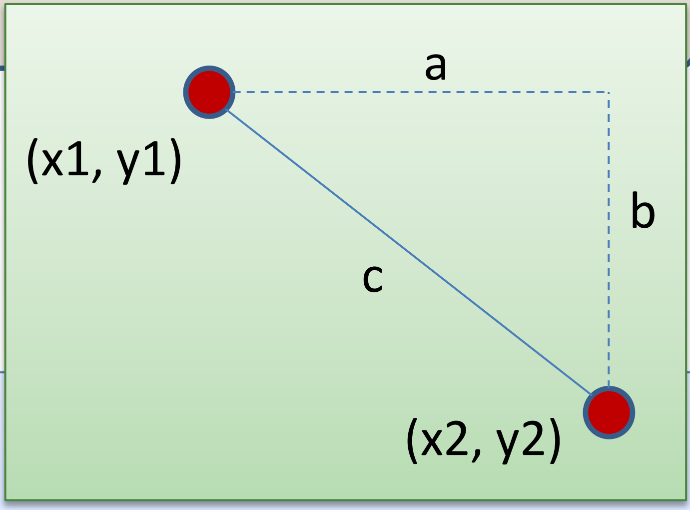
Slope between two points




```
type point = float * float

let slope (p1:point) (p2:point) : float =
```

Slope between two points

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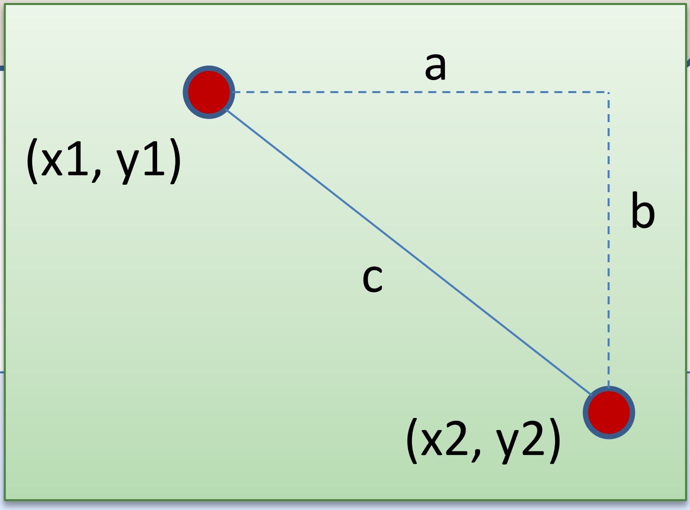
type point = float * float

let slope (p1:point) (p2:point) : float =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
```

deconstruct tuple

Slope between two points

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let slope (p1:point) (p2:point) : float =
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
    (y2 -. y1) /. xd
  else
    ???
```
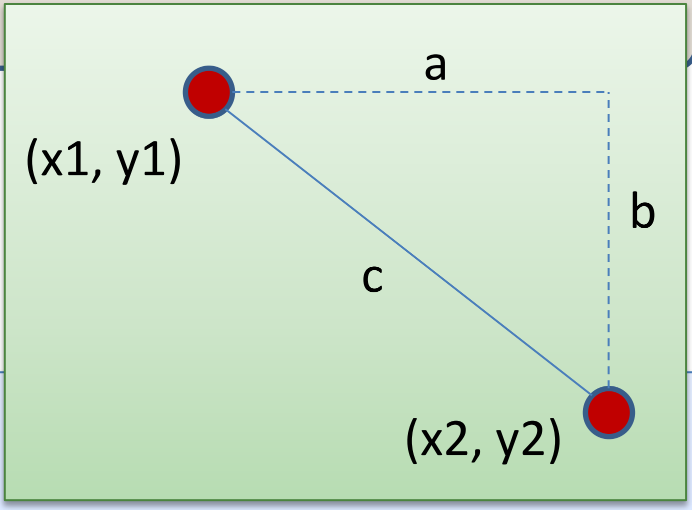

avoid divide by zero

what can we return?

Slope between two points

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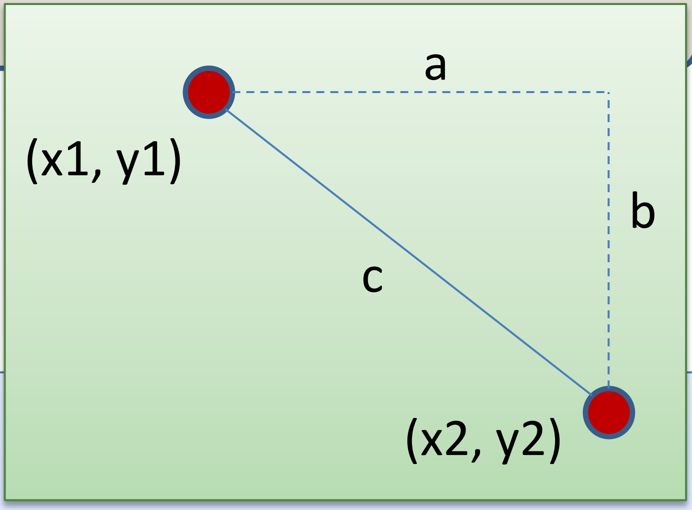
  else
    ???
```

we need an option type as the result type

Slope between two points

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  let (x2,y2) = p2 in
  let xd = x2 -. x1 in
  if xd != 0.0 then
    Some ((y2 -. y1) /. xd)
  else
    None
```
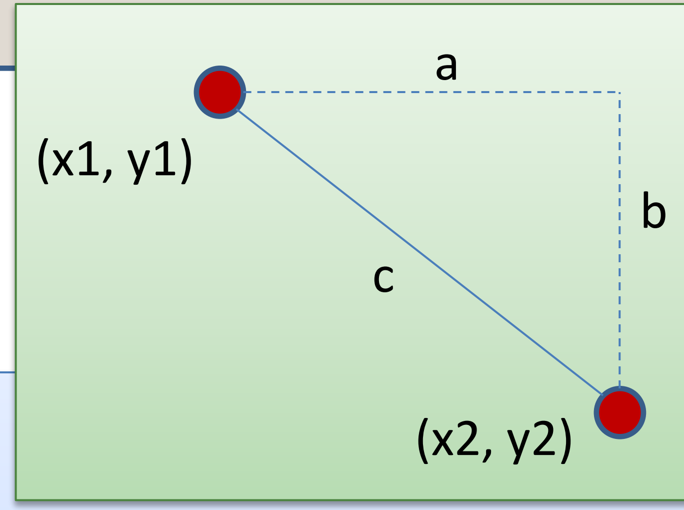

Slope between two points

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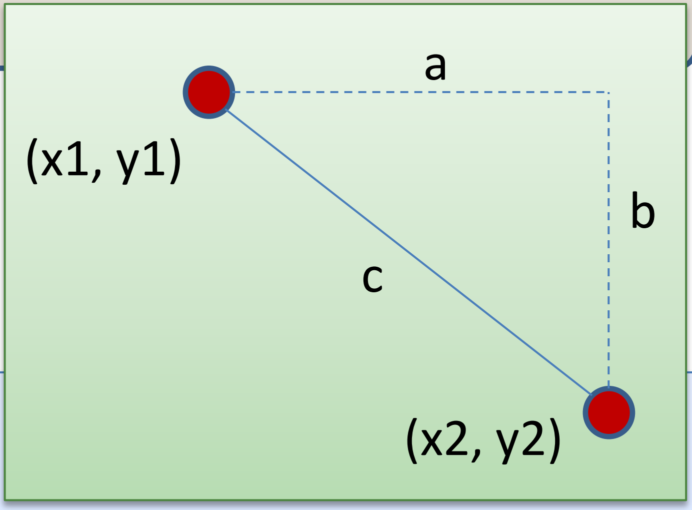
Has type float

Can have type float option

Slope between two points

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  if xd != 0.0 then
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  else
    None
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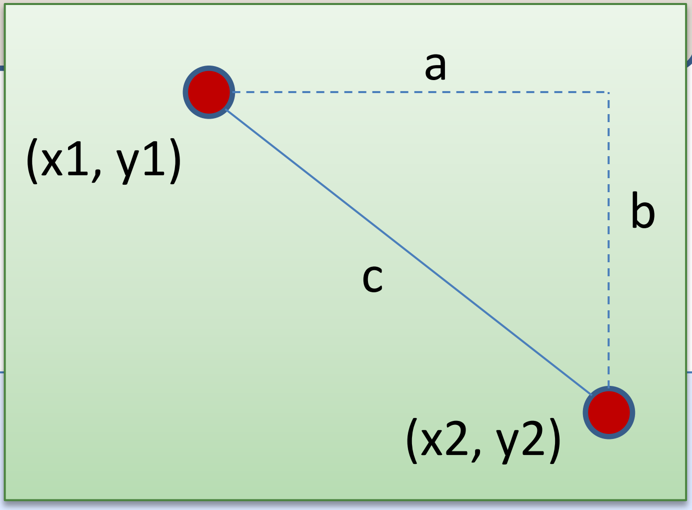
Has type float

Can have type float option

WRONG: Type mismatch

Slope between two points

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  let xd = x2 -. x1 in
  if xd != 0.0 then
    (y2 -. y1) /. xd
  else
    None
```

Has type float

doubly WRONG:
result does not
match declared result

Remember the typing rule for if

if e1 : bool
and e2 : t and e3 : t (for some type t)
then if e1 then e2 else e3 : t

Returning an optional value from an if statement:

```
if … then
   None          : t option
else
   Some ( … )    : t option
```

How do we use an option?

```
slope : point -> point -> float option
```

returns a float option

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let print_slope (p1:point) (p2:point) : unit =
```

How do we use an option?

```
slope : point -> point -> float option


let print_slope (p1:point) (p2:point) : unit =
        slope p1 p2
```

returns a float option;
to print we must discover if it is
None or Some

How do we use an option?

```
slope : point -> point -> float option


let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
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How do we use an option?

```
slope : point -> point -> float option


let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
    Some s ->
  | None ->
```

There are two possibilities

Vertical bar separates possibilities

How do we use an option?

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slope : point -> point -> float option


let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
    Some s ->
  | None ->
```

The "Some s" pattern includes the variable s

The object between | and -> is called a pattern

How do we use an option?

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slope : point -> point -> float option


let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
   | Some s ->
   | None ->
```

You can put a “|” on the first line if you want.
It is generally considered better style to do so.

How do we use an option?

```
slope : point -> point -> float option


let print_slope (p1:point) (p2:point) : unit =
  match slope p1 p2 with
    Some s ->
      print_string ("Slope: " ^ string_of_float s)
  | None ->
      print_string "Vertical line.\n"
```

Writing Functions Over Typed Data

- Steps to writing functions over typed data:
  1. Write down the function and argument names
  2. Write down argument and result types
  3. Write down some examples (in a comment)
  4. Deconstruct input data structures
  5. Build new output values
  6. Clean up by identifying repeated patterns
- For option types:

when the input has type t option, deconstruct with:

```
match … with
  | None -> …
  | Some s -> …
```

when the output has type t option, construct with:

```
Some (…)
```

```
None
```

MORE PATTERN MATCHING

Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

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```

(x2, y2) is an example of a pattern – a pattern for tuples.

So let declarations can contain patterns just like match statements

The difference is that a match allows you to consider multiple different data shapes

Recall the Distance Function

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match p1 with
  | (x1,y1) ->
       let (x2,y2) = p2 in
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```

There is only 1 possibility when matching a pair

Recall the Distance Function

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let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match p1 with
  | (x1,y1) ->
     match p2 with
     | (x2,y2) ->
        sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

We can nest one match expression inside another.
(We can nest any expression inside any other, if the expressions have the right types)

Better Style: Complex Patterns

we built a pair of pairs

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match (p1, p2) with
  | ((x1,y1), (x2, y2)) ->
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

Pattern for a pair of pairs: ((variable, variable), (variable, variable))
All the variable names in the pattern must be different.

Better Style: Complex Patterns

we built a pair of pairs

```
type point = float * float

let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  match (p1, p2) with
  | (p3, p4) ->
    let (x1, y1) = p3 in
    let (x2, y2) = p4 in
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

A pattern must be consistent with the type of the expression
in between match … with
We use (p3, p4) here instead of ((x1, y1), (x2, y2))

Pattern-matching in function parameters

```
type point = float * float

let distance ((x1,y1):point) ((x2,y2):point) : float =
  let square x = x *. x in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

Function parameters are patterns too!

What's the best style?

```
let distance (p1:point) (p2:point) : float =
  let square x = x *. x in
  let (x1,y1) = p1 in
  let (x2,y2) = p2 in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

```
let distance ((x1,y1):point) ((x2,y2):point) : float =
  let square x = x *. x in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

Either of these is reasonably clear and compact.
Code with unnecessary nested matches/lets is particularly ugly to read.
You'll be judged on code style in this class.

What's the best style?

```
let distance (x1,y1) (x2,y2) =
  let square x = x *. x in
  sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

This is how I'd do it ... the types for tuples + the tuple patterns are a little ugly/verbose ... but for now in class, use the explicit type annotations. We will loosen things up later in the semester.

Combining patterns

```
type point = float * float

(* returns a nearby point in the graph if one exists *)
nearby : graph -> point -> point option

let printer (g:graph) (p:point) : unit =
  match nearby g p with
  | None -> print_string "could not find one\n"
  | Some (x,y) ->
      print_float x;
      print_string ", ";
      print_float y;
      print_newline();
```

Other Patterns

Constant values can be used as patterns

```
let small_prime (n:int) : bool =
  match n with
  | 2 -> true
  | 3 -> true
  | 5 -> true
  | _ -> false
```

```
let iffy (b:bool) : int =
  match b with
  | true -> 0
  | false -> 1
```

the underscore pattern
matches anything
it is the "don't care" pattern

INDUCTIVE THINKING

Inductive Programming

An *inductive data type* T is a data type defined by:

- base cases
  - don’t refer to T
- inductive cases
  - build new data of type T from pre-existing data of type T
  - the pre-existing data is guaranteed to be *smaller* than the new values

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- inductive cases
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  - the pre-existing data is guaranteed to be *smaller* than the new values

Example: a tree

- base case:
  - the leaf of the tree
- inductive case:
  - the internal nodes of the tree
  - the left- and right- subtrees are the “smaller” data

Inductive Programming

To *program* a function over inductive data:

- think: what does my function need to do to be correct?
- solve the programming problem for the base cases
  - solve them one-by-one
- solve the programming problem for inductive cases:
  - solve them one-by-one
  - *assume your function already works correctly on smaller data values*
  - *call your function, when necessary, on smaller data values*

Inductive Proving

To *prove* a function over inductive data is correct:

- think: what is the correctness theorem for this function?
- prove the function correct for the base cases
  - prove them one-by-one
- prove the function correct for the inductive cases:
  - prove them one-by-one
  - *assume your function already works correctly on smaller data values*
  - *use this assumption to reason about calls over smaller data values*
  - this assumption is called the *induction hypothesis* of your proof

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To be a good programmer, you also need to be a good prover.

LISTS: AN INDUCTIVE DATA TYPE

Lists are Inductive Data

In OCaml, a list value is:

- [ ] (the empty list)
- v :: vs (a value v followed by a shorter list of values vs)

Inductive Case

Base Case

Lists are Inductive Data

In OCaml, a list value is:

[ ]   (the empty list)

v :: vs   (a value v followed by a shorter list of values vs)

An example:

- 2 :: 3 :: 5 :: [ ] has type int list
- is the same as: 2 :: (3 :: (5 :: [ ]))
- "::" is called "cons"

An alternative syntax ("syntactic sugar" for lists):

- [2; 3; 5]
- But this is just a shorthand for 2 :: 3 :: 5 :: []. If you ever get confused fall back on the 2 basic *constructors*, :: and []

Typing Lists

Typing rules for lists:

(1) [ ] may have any list type, t list

(2) if e1 : t and e2 : t list
then (e1 :: e2) : t list

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More examples:

(1 + 2) :: (3 + 4) :: [ ] : ??

(2 :: [ ]) :: (5 :: 6 :: [ ]) :: [ ] : ??

[[2]; [5; 6]] : ??

Typing Lists

Typing rules for lists:

(1) [ ] may have any list type t list

(2) if e1 : t and e2 : t list
then (e1 :: e2) : t list

More examples:

(1 + 2) :: (3 + 4) :: [ ] : int list

(2 :: [ ]) :: (5 :: 6 :: [ ]) :: [ ] : int list list

[[2]; [5; 6]] : int list list

(Remember that the 3rd example is an abbreviation for the 2nd)

Another Example

What type does this have?

[2] :: [3]

Another Example

What type does this have?

$$\underbrace{[\,2\,]}_{\text{int list}} :: \underbrace{[\,3\,]}_{\text{int list}}$$

```
# [2] :: [3];;
Error: This expression has type int but an
         expression was expected of type
         int list
#
```

Another Example

What type does this have?

$$\underbrace{[\ 2\ ]}_{\text{int list}} :: \underbrace{[\ 3\ ]}_{\text{int list}}$$

Give me a simple fix that makes the expression type check?

Another Example

What type does this have?

$$\underbrace{[\ 2\ ]}_{\text{int list}} :: \underbrace{[\ 3\ ]}_{\text{int list}}$$

Give me a simple fix that makes the expression type check?

Either: 2 :: [3] : int list

Or: [2] :: [[3]] : int list list

Analyzing Lists

Just like options, there are two possibilities when deconstructing lists. Hence we use a match with two branches

```
(* return Some v, if v is the first list element;
   return None, if the list is empty *)

let head (xs : int list) : int option =
```

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let head (xs : int list) : int option =
  match xs with
  | [] ->
  | hd :: _ ->
```

we don't care about the contents of the tail of the list so we use the underscore

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(* return Some v, if v is the first list element;
   return None, if the list is empty *)

let head (xs : int list) : int option =
  match xs with
  | [] -> None
  | hd :: _ -> Some hd
```

This function isn't recursive -- we only extracted a small , fixed amount of information from the list -- the first element

A more interesting example

```
(* Given a list of pairs of integers,
   produce the list of products of the pairs

   prods [(2,3); (4,7); (5,2)] == [6; 28; 10]
*)
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let rec prods (xs : (int * int) list) : int list =
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let rec prods (xs : (int * int) list) : int list =
  match xs with
  | [] -> []
  | (x,y) :: tl -> ?? :: ??
```

the result type is int list, so we can speculate that we should create a list

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let rec prods (xs : (int * int) list) : int list =
  match xs with
  | [] -> []
  | (x,y) :: tl -> (x * y) :: ??
```

the first element is the product

A more interesting example

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   prods [(2,3); (4,7); (5,2)] == [6; 28; 10]
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let rec prods (xs : (int * int) list) : int list =
  match xs with
  | [] -> []
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```

to complete the job, we must compute
the products for the rest of the list

A more interesting example

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   produce the list of products of the pairs

   prods [(2,3); (4,7); (5,2)] == [6; 28; 10]
*)

let rec prods (xs : (int * int) list) : int list =
  match xs with
  | [] -> []
  | (x,y) :: tl -> (x * y) :: prods tl
```

Three Parts to Constructing a Function

(1) Think about how to *break down* the input into cases:

```
let rec prods (xs : (int*int) list) : int list =
  match xs with

  | [] ->  ...

  | (x,y) :: tl ->   ...
```

(2) *Assume* the recursive call on smaller data is correct.

(3) Use the result of the recursive call to *build* correct answer.

```
let rec prods (xs : (int*int) list) : int list =
  ...
  | (x,y) :: tl ->   ...  prods tl ...
```

Another example: zip

```
(* Given two lists of integers,
   return None if the lists are different lengths
   otherwise stitch the lists together to create
     Some of a list of pairs

   zip [2; 3] [4; 5] == Some [(2,4); (3,5)]
   zip [5; 3] [4] == None
   zip [4; 5; 6] [8; 9; 10; 11; 12] == None
*)
```

(Give it a try.)

Another example: zip

```
let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =
```

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let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =

  match (xs, ys) with
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let rec zip (xs : int list) (ys : int list)
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  match (xs, ys) with
  | ([], []) ->
  | ([], y::ys') ->
  | (x::xs', []) ->
  | (x::xs', y::ys') ->
```

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is this ok?

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let rec zip (xs : int list) (ys : int list)
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```

No! zip returns a list option, not a list!
We need to match it and decide if it is Some or None.

Another example: zip

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let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =

  match (xs, ys) with
  | ([], []) -> Some []
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  | (x::xs', []) -> None
  | (x::xs', y::ys') ->
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         None -> None
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Is this ok?

Another example: zip

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let rec zip (xs : int list) (ys : int list)
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  | (x::xs', y::ys') ->
      (match zip xs' ys' with
         None -> None
       | Some zs -> Some ((x,y) :: zs))
```

Another example: zip

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let rec zip (xs : int list) (ys : int list)
  : (int * int) list option =

  match (xs, ys) with
  | ([], []) -> Some []
  | (x::xs', y::ys') ->
      (match zip xs' ys' with
         None -> None
       | Some zs -> Some ((x,y) :: zs))
  | (_, _) -> None
```

Clean up.
Reorganize the cases.
Pattern matching proceeds in order.

A bad list example

```
let rec sum (xs : int list) : int =
  match xs with
  | hd::tl -> hd + sum tl
```

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  match xs with
  | hd::tl -> hd + sum tl
```

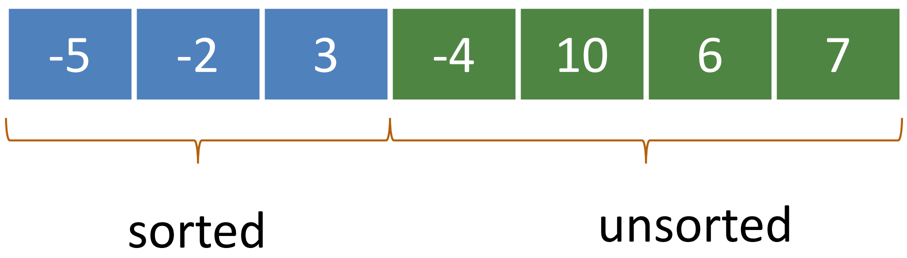
```
Warning 8: this pattern-matching is not exhaustive.
Here is an example of a value that is not matched: []
val sum : int list -> int = <fun>
```

INSERTION SORT

Recall Insertion Sort

At any point during the insertion sort:

- some initial segment of the array will be sorted
- the rest of the array will be in the same (unsorted) order as it was originally



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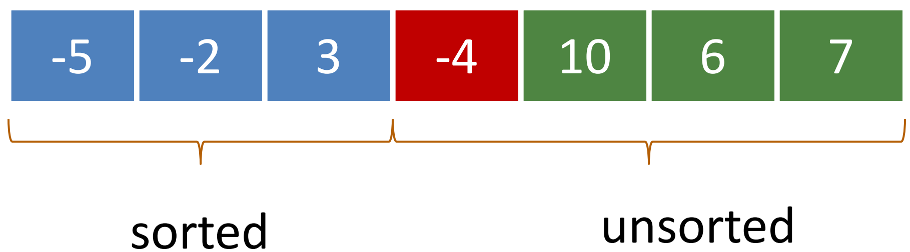
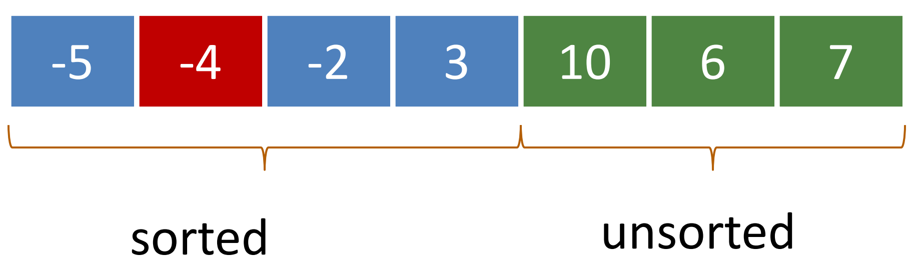
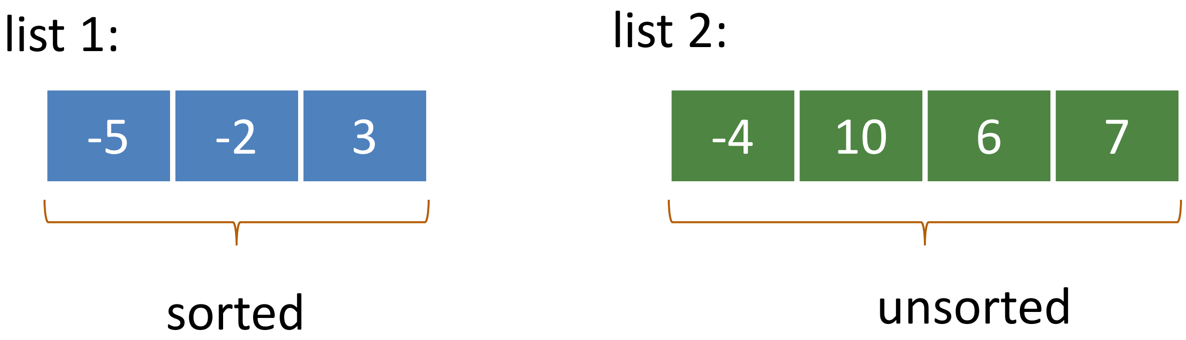


At each step, take the next item in the array and insert it in order into the sorted portion of the list



Insertion Sort With Lists

The algorithm is similar, except instead of *one array*, we will maintain *two lists*, a sorted list and an unsorted list



We'll factor the algorithm:

– a function to insert into a sorted list
– a sorting function that repeatedly inserts

Insert

```
(* insert x into sorted list xs *)

let rec insert (x : int) (xs : int list) : int list =
```

Insert

```
(* insert x into sorted list xs *)

let rec insert (x : int) (xs : int list) : int list =
  match xs with
  | [] ->
  | hd :: tl ->
```

a familiar pattern:
analyze the list by cases

Insert

```
(* insert x into sorted list xs *)

let rec insert (x : int) (xs : int list) : int list =
  match xs with
  | [] -> [x]
  | hd :: tl ->
```

insert x into the empty list

Insert

```
(* insert x into sorted list xs *)

let rec insert (x : int) (xs : int list) : int list =
  match xs with
  | [] -> [x]
  | hd :: tl ->
      if hd < x then
        hd :: insert x tl
```

build a new list with:

- hd at the beginning
- the result of inserting x in to the tail of the list afterwards

Insert

```
(* insert x into sorted list xs *)

let rec insert (x : int) (xs : int list) : int list =
  match xs with
  | [] -> [x]
  | hd :: tl ->
      if hd < x then
        hd :: insert x tl
      else
        x :: xs
```

put x on the front of the list,
the rest of the list follows

Insertion Sort

```
type il = int list

insert : int -> il -> il


(* insertion sort *)

let rec insert_sort(xs : il) : il =
```

Insertion Sort

```
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insert : int -> il -> il


(* insertion sort *)

let rec insert_sort(xs : il) : il =

  let rec aux (sorted : il) (unsorted : il) : il =



  in
```

Insertion Sort

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type il = int list

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  in
  aux [] xs
```

Insertion Sort

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insert : int -> il -> il


(* insertion sort *)

let rec insert_sort(xs : il) : il =

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    match unsorted with
    | [] ->
    | hd :: tl ->
  in
  aux [] xs
```

Insertion Sort

```
type il = int list

insert : int -> il -> il



(* insertion sort *)

let rec insert_sort(xs : il) : il =

  let rec aux (sorted : il) (unsorted : il) : il =
    match unsorted with
    | [] -> sorted
    | hd :: tl -> aux (insert hd sorted) tl
  in
  aux [] xs
```

Does Insertion Sort Terminate?

Recall that we said: inductive functions should call themselves recursively on *smaller data items*.

What about that loop in insertion sort?

```
let rec loop (sorted : il) (unsorted : il) : il =
    match unsorted with
    | [] -> sorted
    | hd :: tl -> loop (insert hd sorted) tl
```

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```

growing!

shrinking!

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    | [] -> sorted
    | hd :: tl -> loop (insert hd sorted) tl
```

growing!

shrinking!

Refined idea: Pick an argument up front. That argument must contain smaller data *on every recursive call*.

Exercises

- Write a function to sum the elements of a list
  - sum [1; 2; 3] ==> 6
- Write a function to append two lists
  - append [1;2;3] [4;5;6] ==> [1;2;3;4;5;6]
- Write a function to reverse a list
  - rev [1;2;3] ==> [3;2;1]
- Write a function to turn a list of pairs into a pair of lists
  - split [(1,2); (3,4); (5,6)] ==> ([1;3;5], [2;4;6])
- Write a function that returns all prefixes of a list
  - prefixes [1;2;3] ==> [[]; [1]; [1;2]; [1;2;3]]
- suffixes…

A SHORT JAVA RANT

Definition and Use of Java Pairs

```
public class Pair {

  public int x;
  public int y;

  public Pair (int a, int b) {
    x = a;
    y = b;
  }
}
```

```
public class User {

  public Pair swap (Pair p1) {
    Pair p2 =
      new Pair(p1.y, p1.x);

    return p2;
  }
}
```

What could go wrong?

A Paucity of Types

```
public class Pair {

  public int x;
  public int y;

  public Pair (int a, int b) {
    x = a;
    y = b;
  }
}
```

```
public class User {

  public Pair swap (Pair p1) {
    Pair p2 =
      new Pair(p1.y, p1.x);

    return p2;
  }
}
```

The input p1 to swap may be null and we forgot to check.

Java has no way to define a pair data structure that is *just a pair*.

How many students in the class have seen an accidental null pointer exception thrown in their Java code?

From Java Pairs to OCaml Pairs

In OCaml, if a pair may be null it is a pair option:

```
type java_pair = (int * int) option
```

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And if you write code like this:

```
let swap_java_pair (p:java_pair) : java_pair =
  let (x,y) = p in
  (y,x)
```

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And if you write code like this:

```
let swap_java_pair (p:java_pair) : java_pair =
  let (x,y) = p in
  (y,x)
```

You get a *helpful* error message like this:

```
# … Characters 91-92:
    let (x,y) = p in (y,x);;
                ^
Error: This expression has type java_pair = (int * int) option
       but an expression was expected of type 'a * 'b
```

From Java Pairs to OCaml Pairs

```
type java_pair = (int * int) option
```

And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

```
let swap_java_pair (p:java_pair) : java_pair =
  match p with
   | Some (x,y) -> Some (y,x)
```

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   | Some (x,y) -> Some (y,x)
```

OCaml to the rescue!

```
 ..match p with
      | Some (x,y) -> Some (y,x)
Warning 8: this pattern-matching is not exhaustive.
Here is an example of a value that is not matched:
None
```

From Java Pairs to OCaml Pairs

```
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And what if you were up at 3am trying to finish your COS 326 assignment and you accidentally wrote the following sleep-deprived, brain-dead statement?

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let swap_java_pair (p:java_pair) : java_pair =
  match p with
   | Some (x,y) -> Some (y,x)
```

An easy fix!

```
let swap_java_pair (p:java_pair) : java_pair =
   match p with
    | None -> None
    | Some (x,y) -> Some (y,x)
```

From Java Pairs to OCaml Pairs

Moreover, your pairs are probably almost never null!

Defensive programming & always checking for null is

AnNOyinG

From Java Pairs to OCaml Pairs

There just isn't always some "good thing" for a function to do when it receives a bad input, like a null pointer

In OCaml, all these issues disappear when you use the proper type for a pair and that type contains no "extra junk"

```
type pair = int * int
```

Once you know OCaml, it is *hard* to write swap incorrectly

Your *bullet-proof* code is much simpler than in Java.

```
let swap (p:pair) : pair =
   let (x,y) = p in (y,x)
```

Summary of Java Pair Rant

Java has a paucity of types

- There is no type to describe just the pairs
- There is no type to describe just the triples
- There is no type to describe the pairs of pairs
- There is no type …

OCaml has many more types

- use option when things may be null
- do not use option when things are not null
- OCaml types describe data structures more precisely
  - programmers have fewer cases to worry about
  - entire classes of errors just go away
  - type checking and pattern analysis help prevent programmers from ever forgetting about a case

Summary of Java Pair Rant

Java has a paucity of types

- There is no type to describe j… t the pairs
- There is n… type to describe … t…
- There is no … describ…
- There is no t…

OCam…

- use …
- d…
    - …
    - type ch… analys… help prevent programmers from ever fo… g about a case

SCORE: OCAML 1, JAVA 0

Example problems to practice

- Write a function to sum the elements of a list
  - sum [1; 2; 3] ==> 6
- Write a function to append two lists
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  - prefixes [1;2;3] ==> [[]; [1]; [1;2]; [1;2;3]]
- suffixes…