

# **COS 326**

## **Functional Programming**

Andrew Appel  
Princeton University



Alonzo Church, 1903-1995  
Princeton Professor, 1929-1967

In 1936, Alonzo Church invented the lambda calculus. He called it a logic, but it was a language of pure functions -- the world's first programming language.

He said:

*"There may, indeed, be other applications of the system than its use as a logic."*

**Indeed!**



Alonzo Church  
1934 -- developed lambda calculus



*Programming Languages*



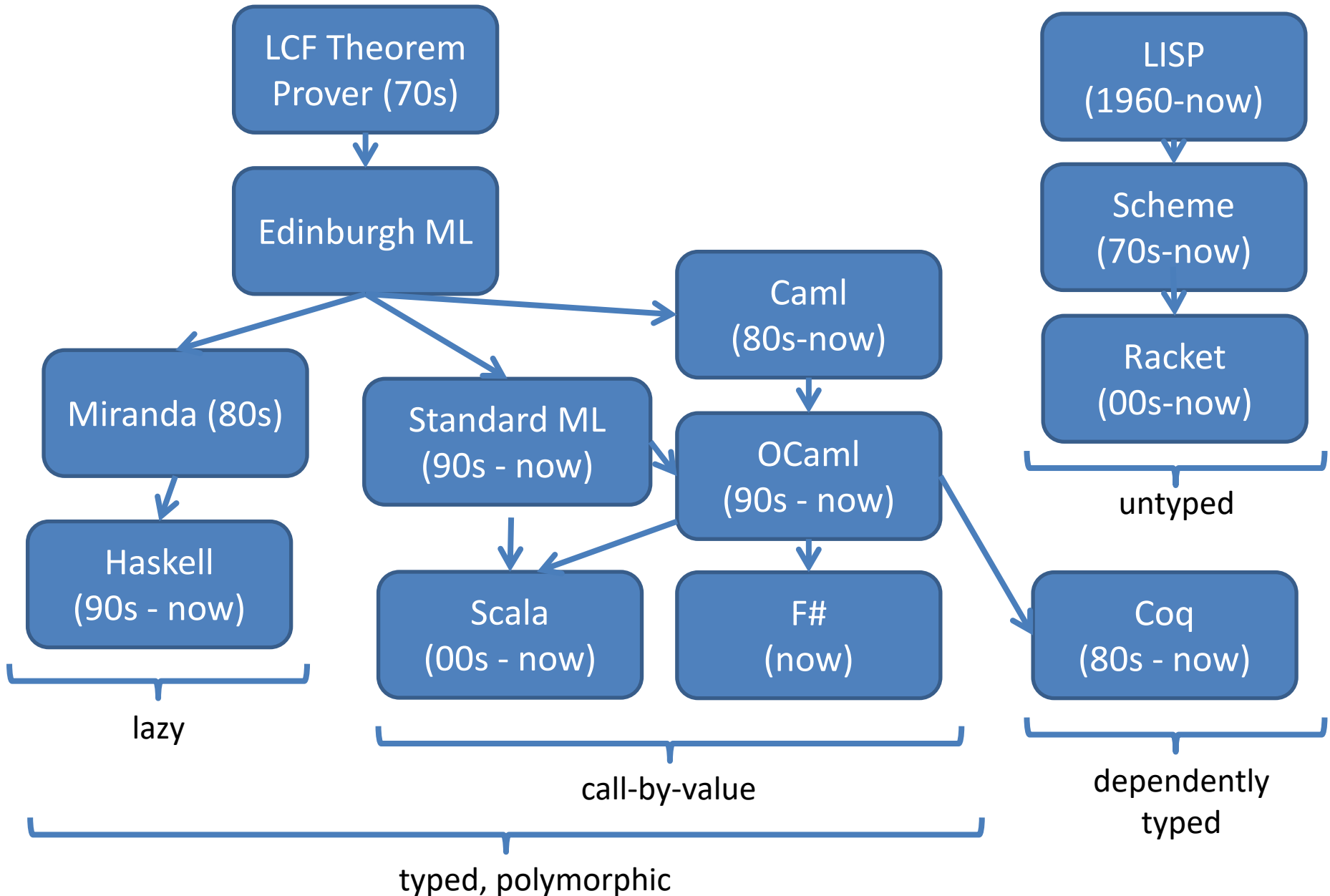
Alan Turing (PhD Princeton 1938)  
1936 -- developed Turing machines



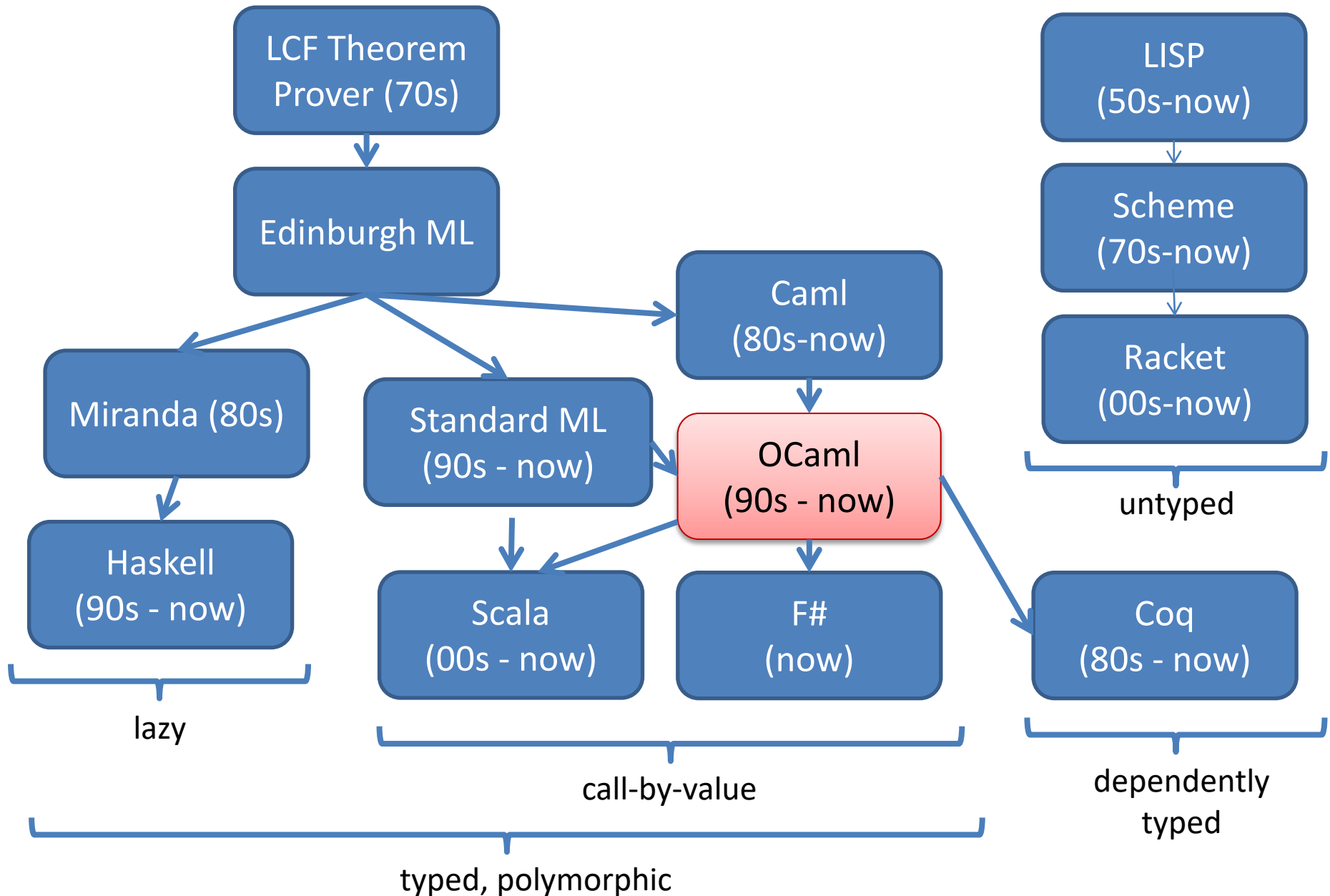
*Computers*

Optional reading: ***The Birth of Computer Science at Princeton in the 1930s***  
by Andrew W. Appel, 2012. <http://press.princeton.edu/chapters/s9780.pdf>

# Vastly Abbreviated FP Genealogy



# Vastly Abbreviated FP Genealogy



# Functional Languages: Who's using them?

6

map-reduce in their data centers



Scala for  
correctness, maintainability, flexibility



Erlang for concurrency,  
Haskell for managing PHP,  
OCaml for bug-finding



F# in Visual Studio

mathematicians

Coq (re)proof of  
4-color theorem



Haskell to  
synthesize hardware



Haskell  
for specifying  
equity derivatives

[www.artima.com/scalazine/articles/twitter\\_on\\_scala.html](http://www.artima.com/scalazine/articles/twitter_on_scala.html)

[www.infoq.com/presentations/haskell-barclays](http://www.infoq.com/presentations/haskell-barclays)

[www.janestreet.com/technology/index.html#work-functionally](http://www.janestreet.com/technology/index.html#work-functionally)

[msdn.microsoft.com/en-us/fsharp/cc742182](http://msdn.microsoft.com/en-us/fsharp/cc742182)

[research.google.com/archive/mapreduce-osdi04.pdf](http://research.google.com/archive/mapreduce-osdi04.pdf)

[www.lightbend.com/case-studies/how-apache-spark-scala-and-functional-programming-made-hard-problems-easy-at-barclays](http://www.lightbend.com/case-studies/how-apache-spark-scala-and-functional-programming-made-hard-problems-easy-at-barclays)

[www.haskell.org/haskellwiki/Haskell\\_in\\_industry](http://www.haskell.org/haskellwiki/Haskell_in_industry)

# **COURSE LOGISTICS**

# Course Staff

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[Home](#) [Info](#) [Schedule](#) [Assignments](#) [Resources](#)

# COS 326

## Functional Programming

### Resources

[Topics Notes](#)[Installing and Editors](#)[Program Style Guide](#)[Standard Library](#)[Standard Modules](#)[Ocamlbuild Docs](#)[OCaml Manual](#)[OCaml Books](#)

### Other Useful Stuff

[Course Info](#)

## Welcome

Welcome to COS 326: Functional Programming. In this course, you will learn about the joy of functional programming: From functions to futures, map-reduce to monads, interfaces to invariants, and types to tail calls.

## Getting Started

- Go to our page on [installing OCaml and VSCode](#).
- [Create a github account](#) if you don't already have one.
- Visit the [Assignments Dashboard](#) to link your PU netid to your github account.
- Go to the [Course Info](#) page and take a look at the course policies on collaboration, late

# Collaboration Policy

The COS 326 collaboration policy can be found here:

<http://www.cs.princeton.edu/~cos326/info.php#collab>

Read it in full prior to beginning the first assignment.

Please ask questions whenever anything is unclear, at any time during the course.

# A Typical Week

## Monday

- Lecture

## Tuesday

- Assignment from last week due (7 assignments total)
- Your first assignment is due Tuesday Sept 13 at 11:59pm

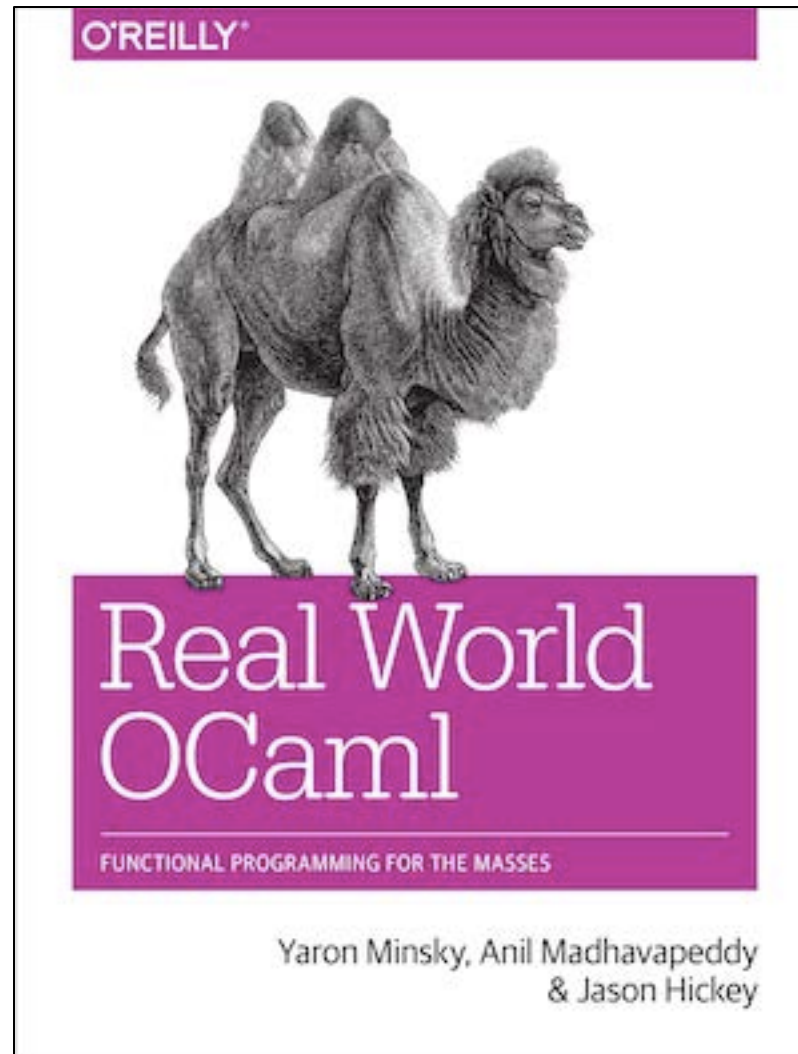
## Wednesday

- Lecture
- Next assignment is available
- *start assignment* with material from lecture

## Thursday/Friday

- mandatory precept reinforces lecture content
- you may have questions for your preceptor about the assignment

# Course Textbook



<http://realworldocaml.org/>

# Exams

## Midterm

- in class during midterm week

## Final

- during exam period in December
- make your travel plans accordingly
- I have *no control at all* over when the exam occurs, the Registrar schedules exams.
- The final is **not** “cumulative” over the whole semester, it covers just “equational reasoning”

# Special Calendar Note

## DECEMBER 2022

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
				1	2	3
4	5 Lecture	6	7 Lecture	8 Precept	9 Precept	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Friday precepts ***will meet*** on December 9<sup>th</sup>, even though it may officially be Reading Period.

# Assignment 0

Install opam, ocaml, VS Code  
[and if you use Windows: WSL2]  
on your machine by the time precept begins tomorrow.

Resources Page:

<http://www.cs.princeton.edu/~cos326/resources.php>

**Hint:**

ocaml.org

# Public Service Announcement

## **The Pen is Mightier than the Keyboard: Advantages of Longhand Over Laptop Note Taking**

Pam Mueller (Princeton University)

Daniel Oppenheimer (UCLA)

Journal of Psychological Science, June 2014, vol 25, no 6

<http://pss.sagepub.com/content/25/6/1159.fullkeytype=ref&siteid=sppss&ijkey=CjRAwmrlURGNw>

<https://www.scientificamerican.com/article/a-learning-secret-don-t-take-notes-with-a-laptop/>

- You learn conceptual topics better by taking notes by hand.
- Instagram and Fortnite distract your classmates.



# Functional Programming

# Thinking Functionally

pure, functional code:

```
let (x,y) = pair in  
(y,x)
```

you *analyze* existing data (like pair)  
and you *produce* new data (y,x)

imperative code:

```
temp = pair.x;  
pair.x = pair.y;  
pair.y = temp;
```

commands *modify* or *change* an  
existing data structure (like pair)

# Thinking Functionally

pure, functional code:

```
let (x,y) = pair in  
(y,x)
```

- *outputs are everything!*
- *output is function of input*
- *data properties are stable*
- *repeatable*
- *parallelism apparent*
- *easier to test*
- *easier to compose*

imperative code:

```
temp = pair.x;  
pair.x = pair.y;  
pair.y = temp;
```

- *outputs are irrelevant!*
- *output is not function of input*
- *data properties change*
- *unrepeatable*
- *parallelism hidden*
- *harder to test*
- *harder to compose*

This simple switch in perspective can change the way you  
*think*  
about programming and problem solving.

# Why OCaml?

Small, orthogonal core based on the *lambda calculus*.

- Control is based on (recursive) functions.
- Instead of for-loops, while-loops, do-loops, iterators, etc.
  - can be defined as library functions.
- Makes it easy to define semantics



Supports *first-class, lexically scoped, higher-order* procedures

- a.k.a. first-class functions or closures or lambdas.
- *first-class*: functions are data values like any other data value
  - like numbers, they can be stored, defined anonymously, ...
- *lexically scoped*: meaning of variables determined statically.
- *higher-order*: functions as arguments and results
  - programs passed to programs; generated from programs

These features also found in Scheme, Haskell, Scala, F#, Clojure, ....

# Why OCaml?

**Statically typed:** debugging and testing aid

- compiler catches many silly errors before you can run the code.
  - A type is worth a thousand tests
- Java is also strongly, statically typed.
- Scheme, Python, Javascript, etc. are all strongly, *dynamically typed* – type errors are discovered while the code is running.

**Strongly typed:** compiler enforces type abstraction.

- cannot cast an integer to a record, function, string, etc.
  - so we can utilize *types as capabilities*; crucial for local reasoning
- C/C++ are *weakly typed* (statically typed) languages. The compiler will happily let you do something smart (*more often stupid*).

**Type inference:** compiler fills in types for you



Integer Functor Ord Char  
 Either Monad  
 Bool Enum  
 Int [...] → Eq  
 Num Read  
 Bounded (→, →)  
 Integral () IO Show  
 Maybe String Ratio Float

# Installing, Running OCaml

- OCaml comes with compilers:
  - "ocamlc" – fast bytecode compiler
  - "ocamlopt" – optimizing, native code compiler
  - "dune" – a build system for OCaml
- And an interactive, top-level shell:
  - useful for trying something out.
  - "ocaml" or "utop" at the prompt.
  - *but use the compiler (via dune) most of the time*
- See the course web pages for installation pointers
  - also OCaml.org

# Editing OCaml Programs

- Many options:
  - We recommend VS Code, with its OCaml mode

But you can use other text editors if you want, such as:

- Emacs
  - what your professors tend to use
  - good but not great support for OCaml.
- Sublime, atom
  - Many CS326 students have used these



# **AN INTRODUCTORY EXAMPLE (OR TWO)**

# A First OCaml Program

hello.ml:

```
print_string "Hello COS 326!!\n"
```

EXPLORER

▼ LEC1 [WSL: UBUNTU-20....] [New] [Folder] [Refresh] [Copy]

- assert.ml
- broken.ml
- countn.ml
- hello.ml
- hello2.ml
- Makefile
- sum\_test.ml
- sum.ml

> OUTLINE

> TIMELINE

Get Started X

# Visual Studio Code

## Editing evolved

### Start

- New File...
- Open File...
- Open Folder...
- Clone Git Repository...

### Recent

test [WSL: Ubuntu-20.04] /home/a...

### Walkthroughs

- Get Started with VS Code**  
Discover the best customizations to make VS Code yours.
- Learn the Fundamentals**  
Jump right into VS Code and get an overview of the must-have features.
- Boost your Productivity**

[More...](#)

☒ Show welcome page on startup



1

hello.ml

ml  
mli

home &gt; appel &gt; lec1 &gt; hello.ml

1

2

3

```
print_string "Hello COS 326!!\n"
```



8



- File >
- Edit >
- Selection >
- View >
- Go >
- Run >
- Terminal >**
- Help >

New Terminal Ctrl+Shift+`  
Split Terminal Ctrl+Shift+5

Run Task...  
Run Build Task... Ctrl+Shift+B  
Run Active File  
Run Selected Text

Show Running Tasks...  
Restart Running Task...  
Terminate Task...

Configure Tasks...  
Configure Default Build Task...



1

hello.ml

ml  
mli

...

home &gt; appel &gt; lec1 &gt; hello.ml

1

2

3

print\_string "Hello COS 326!!\n"



8

PROBLEMS

OUTPUT

TERMINAL

...



bash



...

ubuntu\$





1

hello.ml

ml  
mli

...

home &gt; appel &gt; lec1 &gt; hello.ml

1

2

3

print\_string "Hello COS 326!!\n"



8

PROBLEMS

OUTPUT

TERMINAL

...



bash



...

ubuntu\$ ocaml hello.ml



hello.ml

ml  
mli

home &gt; appel &gt; lec1 &gt; hello.ml

```
1  
2 print_string "Hello COS 326!!\n"  
3
```

PROBLEMS

OUTPUT

TERMINAL

...

bash - lec1

+ v

[]

🗑

^

✕

```
• ubuntu$ ocaml hello.ml  
Hello COS 326!!  
○ ubuntu$
```



# A First OCaml Program

hello.ml:

```
print_string "Hello COS 326!!\n"
```

# A First OCaml Program

hello.ml:

```
print_string "Hello COS 326!!\n"
```

a function

its string argument  
enclosed in " . . . "

no parens. normally call a function *f* like this:

*f* *arg*

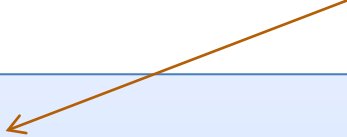
a program  
can be nothing  
more than  
just a single  
expression  
(but that is  
uncommon)

(parens are used for grouping, precedence  
only when necessary)

# A Second OCaml Program

sumTo8.ml:

a comment  
(\* ... \*)



```
(* sum the numbers from 0 to n
   precondition: n must be a natural number
   *)
let rec sumTo (n:int) : int =
  match n with
    0 -> 0
  | n -> n + sumTo (n-1)

let _ =
  print_int (sumTo 8);
  print_newline()
```

# A Second OCaml Program

the name of the function being defined

sumTo8.ml:

```
(* sum the numbers from 0 to n
   precondition: n must be a natural number
   *)
let rec sumTo (n:int) : int =
  match n with
    0 -> 0
  | n -> n + sumTo (n-1)

let _ =
  print_int (sumTo 8);
  print_newline();
```

the keyword “let” begins a definition; keyword “rec” indicates recursion

# A Second OCaml Program

sumTo8.ml:

```
(* sum the numbers from 0 to n
   precondition: n must be a natural number
   *)
let rec sumTo (n:int) : int =
  match n with
    0 -> 0
  | n -> n + sumTo (n-1)

let _ =
  print_int (sumTo 8);
  print_newline();
```

result type int

argument  
named n  
with type int

# A Second OCaml Program

deconstruct the value  $n$   
using pattern matching

sumTo8.ml:

```
(* sum the numbers from 0 to n
   precondition: n must be a natural number
*)
let rec sumTo (n:int) : int =
  match n with
    0 -> 0
  | n' -> n' + sumTo (n-1)

let _ =
  print_int (sumTo 8);
  print_newline();
```

data to be  
deconstructed  
appears  
between  
key words  
“match” and  
“with”

# A Second OCaml Program

vertical bar "|" separates the alternative patterns

sumTo8.ml :

```
(* sum the numbers from 0 to n
   precondition: n must be a natural number
   *)
let rec sumTo (n:int) : int =
  match n with
  | 0 -> 0
  | n -> n + sumTo (n-1)
let _ =
  print_int (sumTo 8);
  print_newline()
—
```

deconstructed data matches one of 2 cases:

(i) the data matches the pattern 0, or (ii) the data matches the variable pattern n

# A Second OCaml Program

Each branch of the match statement constructs a result

sumTo8.ml:

```
(* sum the numbers from 0 to n
   precondition: n must be a natural number
   *)
let rec sumTo (n:int) : int =
  match n with
    0 -> 0
  | n -> n + sumTo (n-1)

let _ =
  print_int (sumTo 8);
  print_newline();
```

construct  
the result 0

construct  
a result  
using a  
recursive  
call to sumTo



# A Second OCaml Program

sumTo8.ml:

```
(* sum the numbers from 0 to n
   precondition: n must be a natural number
   *)
let rec sumTo (n:int) : int =
  match n with
    0 -> 0
  | n -> n + sumTo (n-1)

let _ =
  print_int (sumTo 8);
  print_newline()
```

print the  
result of  
calling  
sumTo on 8

print a  
new line



2

hello.ml



sum.ml

ml  
mli

home &gt; appel &gt; lec1 &gt; sum.ml &gt; ...

```
1  (* sum the numbers from 0 to n
2      sumTo 0 = 0
3      sumTo 3 = 6
4      sumTo (-1) = 0
5  *)
```

int -&gt; int

```
6  let rec sumTo (n:int) : int =
7      if n <= 0 then
8          0
9      else
10         n + sumTo (n-1)
```

PROBLEMS

TERMINAL



bash - lec1



Hello COS 326!!

ubuntu\$



hello.ml

sum.ml

ml  
mli

home &gt; appel &gt; lec1 &gt; sum.ml &gt; ...

```
1  (* sum the numbers from 0 + ...
2      sumTo 0 = 0
3      sumTo 3 = 6
4      sumTo (-1) = 0
5  *)
   int -> int
6  let rec sumTo (n:int) : int =
7      if n <= 0 then
8          0
9      else
10         n + sumTo (n-1)
```

This is not part of the program, it's just VS Code reminding you the type of the "sumTo" function

PROBLEMS

TERMINAL

&gt; bash - lec1

Hello COS 326!!

ubuntu\$

hello.ml

sum.ml

ml  
mli

home &gt; appel &gt; lec1 &gt; sum.ml &gt; ...

```
1  (* sum the numbers from 0 to n *)
2      sumTo 0 = 0
3      sumTo 3 = 6
4      sumTo (-1) = 0
5  *)
   int -> int
6  let rec sumTo (n:int) : int =
7      if n <= 0 then
8          0
9      else
10         n + sumTo (n-1)
```

Good program style: *before* each function definition, write a comment saying what it's supposed to do, perhaps with examples

PROBLEMS

TERMINAL

&gt; bash - lec1

Hello COS 326!!

ubuntu\$



hello.ml



sum.ml

ml  
mli

home &gt; appel &gt; lec1 &gt; sum.ml &gt; ...

```
6 let rec sumTo (n:int) : int =
7   if n <= 0 then
8     0
9   else
10    n + sumTo (n-1)
11
12 let _ =
13   Printf.printf "The sum of the numbers from 0
to 8 is %d\n" (sumTo 8)
```

PROBLEMS

TERMINAL

...



bash - lec1



- ubuntu\$ `ocaml hello.ml`  
Hello COS 326!!
- ubuntu\$





hello.ml

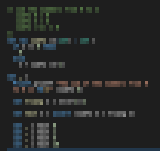
sum.ml

ml  
mli

home &gt; appel &gt; lec1 &gt; sum.ml &gt; ...

ml -&gt; ml

```
6 let rec sumTo (n:int) : int =
7   if n <= 0 then
8     0
9   else
10    n + sumTo (n-1)
11
12 let _ =
13   Printf.printf "The sum of the numbers from 0
14   to 8 is %d\n" (sumTo 8)
```



PROBLEMS

TERMINAL



bash - lec1



• ubuntu\$ ocaml sum.ml

The sum of the numbers from 0 to 8 is 36

• ubuntu\$





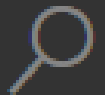
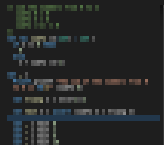
hello.ml

sum.ml

ml  
mli

home &gt; appel &gt; lec1 &gt; sum.ml &gt; ...

```
15   let triang n = n*(n+1)/2
16
17   int -> unit
18   let test n = assert (sumTo n = triang n)
19
20   let _ = test 0
21   let _ = test 1
22   let _ = test 5
23   let _ = test 10
```



PROBLEMS

TERMINAL



bash - lec1



• ubuntu\$ ocaml sum.ml

The sum of the numbers from 0 to 8 is 36

• ubuntu\$



hello.ml

sum.ml

ml / mli [Icons: Split View, More]

home &gt; appel &gt; lec1 &gt; sum.ml &gt; ...

15 | let triang n = n\*(n+1)/2

PROBLEMS

TERMINAL

...

bash - lec1

+ v

[Icon: Split View]

[Icon: Clear]

^

X

ubuntu\$ utop





hello.ml



sum.ml



ml  
mli



home > appel > lec1 > sum.ml > ...

15 | let triang n = n\*(n+1)/2

PROBLEMS

TERMINAL



ocamlrun - lec1



ubuntu\$ utop

elcome to utop version 2.9.1 (using OCaml version 4.13.1)

Type #utop\_help for help about using utop.

-( 11:03:04 )-< command 0 >-----{ counter: 0 }-

utop #

Arg

Arith\_status

Array

ArrayLabels

Assert\_failure

Atomi





hello.ml

sum.ml

ml  
mli

home &gt; appel &gt; lec1 &gt; sum.ml &gt; ...

15 | let triang n = n\*(n+1)/2

PROBLEMS

TERMINAL

...



ocamlrun - lec1



---

elcome to utop version 2.9.1 (using OCaml version 4.13.1)

---

Type #utop\_help for help about using utop.

-( 11:03:04 )-&lt; command 0 &gt;-----{ counter: 0 }-

utop # let x = 5;;

val x : int = 5

-( 11:03:04 )-&lt; command 1 &gt;-----{ counter: 0 }-

utop #

Arg

Arith\_status

Array

ArrayLabels

Assert\_failure

Atomi





hello.ml

sum.ml

ml  
mli

home &gt; appel &gt; lec1 &gt; sum.ml &gt; ...

15 let triang n = n\*(n+1)/2

PROBLEMS

TERMINAL

...

ocamlrun - lec1



-( 11:03:04 )-&lt; command 0 &gt;-----{ counter: 0 }-

utop # let x = 5;;

val x : int = 5

-( 11:03:04 )-&lt; command 1 &gt;-----{ counter: 0 }-

utop # #use "sum.ml";;

val sumTo : int -&gt; int = &lt;fun&gt;

The sum of the numbers from 0 to 8 is 36

- : unit = ()

-( 11:03:35 )-&lt; command 2 &gt;-----{ counter: 0 }-

utop #

Arg

Arith\_status

Array

ArrayLabels

Assert\_failure

Atomi





hello.ml

sum.ml

ml  
mli

home &gt; appel &gt; lec1 &gt; sum.ml &gt; ...

15 | let triang n = n\*(n+1)/2

PROBLEMS

TERMINAL



ocamlrun - lec1



```
-( 11:03:04 )-< command 0 >-----{ counter: 0 }-
```

```
utop # let x = 5;;
```

```
val x : int = 5
```

```
-( 11:03:04 )-< command 1 >-----{ counter: 0 }-
```

```
utop # #use "sum.ml";;
```

```
val sumTo : int -> int = <fun>
```

```
The sum of the numbers from 0 to 8 is 36
```

```
- : unit = ()
```

```
-( 11:03:35 )-< command 2 >-----{ counter: 0 }-
```

```
utop # sumTo 6;;
```

Arg

Arith\_status

Array

ArrayLabels

Assert\_failure

Atomi





hello.ml



sum.ml



home &gt; appel &gt; lec1 &gt; sum.ml &gt; ...

15 | let triang n = n\*(n+1)/2

PROBLEMS

TERMINAL

...



ocamlrun - lec1



val x : int = 5

-( 11:03:04 )-&lt; command 1 &gt;-----{ counter: 0 }-

utop # #use "sum.ml";;

val sumTo : int -&gt; int = &lt;fun&gt;

The sum of the numbers from 0 to 8 is 36

- : unit = ()

-( 11:03:35 )-&lt; command 2 &gt;-----{ counter: 0 }-

utop # sumTo 6;;

- : int = 21

-( 11:04:14 )-&lt; command 3 &gt;-----{ counter: 0 }-

utop #

Arg

Arith\_status

Array

ArrayLabels

Assert\_failure

Atomi



# **OCAML BASICS:**

## **EXPRESSIONS, VALUES, SIMPLE TYPES**

# Terminology: Expressions, Values, Types

**Expressions** are computations

- $2 + 3$  is a computation

**Values** (a subset of the expressions) are the results of computations

- 5 is a value

**Types** describe collections of values and the computations that generate those values

- int is a type
- values of type int include
  - 0, 1, 2, 3, ..., max\_int
  - -1, -2, ..., min\_int

# Some simple types, values, expressions

<u>Type:</u>	<u>Values:</u>	<u>Expressions:</u>
int	-2, 0, 42	42 * (13 + 1)
float	3.14, -1., 2e12	(3.14 +. 12.0) *. 10e6
char	'a', 'b', '&'	int_of_char 'a'
string	"moo", "cow"	"moo" ^ "cow"
bool	true, false	if true then 3 else 4
unit	()	print_int 3

For more primitive types and functions over them,  
see the OCaml Reference Manual here:

<https://ocaml.org/api/Stdlib.html>




# Evaluation

42 \* (13 + 1)

# Evaluation

42 \* (13 + 1)  $\text{-->}$ \* 588




Read like this: “the expression 42 \* (13 + 1) **evaluates to** the value 588”

The “**\***” is there to say that it does so in 0 or more small steps

# Evaluation

42 \* (13 + 1)  $\text{-->}^*$  588



Read like this: “the expression 42 \* (13 + 1) **evaluates to** the value 588”

The “**\***” is there to say that it does so in 0 or more small steps

Here I’m telling you how to execute an OCaml expression --- i.e., I’m telling you something about the *operational semantics* of OCaml

More on semantics later.

# Evaluation

<code>42 * (13 + 1)</code>	<code>--&gt;*</code>	<code>588</code>
<code>(3.14 +. 12.0) *. 10e6</code>	<code>--&gt;*</code>	<code>151400000.</code>
<code>int_of_char 'a'</code>	<code>--&gt;*</code>	<code>97</code>
<code>"moo" ^ "cow"</code>	<code>--&gt;*</code>	<code>"moocow"</code>
<code>if true then 3 else 4</code>	<code>--&gt;*</code>	<code>3</code>
<code>print_int 3</code>	<code>--&gt;*</code>	<code>( )</code>

# Evaluation

1 + "hello" -->\* ???

# Evaluation

1 + "hello" -->\* ???

“+” processes integers  
“hello” is not an integer  
evaluation is undefined!

Don't worry! This expression doesn't type check.

Aside: See this 4-min talk on Javascript:  
<https://www.destroyallsoftware.com/talks/wat>

# **OCAML BASICS:**

## **CORE EXPRESSION SYNTAX**

# Core Expression Syntax

The simplest OCaml expressions **e** are:

- values *numbers, strings, bools, ...*
- id *variables (x, foo, ...)*
- $e_1 \text{ op } e_2$  *operators (x+3, ...)*
- id  $e_1 e_2 \dots e_n$  *function call (foo 3 42)*
- **let** id =  $e_1$  **in**  $e_2$  *local variable decl.*
- **if**  $e_1$  **then**  $e_2$  **else**  $e_3$  *a conditional*
- (e) *a parenthesized expression*
- (e : t) *an expression with its type*



# A note on parentheses

In most languages, arguments are parenthesized & separated by commas:

```
f(x,y,z)      sum(3,4,5)
```

In OCaml, we don't write the parentheses or the commas:

```
f x y z      sum 3 4 5
```

But we do have to worry about *grouping*. For example,

```
f x y z
f x (y z)
```

The first one passes three arguments to f (x, y, and z)

The second passes two arguments to f (x, and the result of applying the function y to z.)

# **OCAML BASICS: TYPE CHECKING**

# Type Checking

Every value has a type and so does every expression

This is a concept that is familiar from Java but it becomes more important when programming in a functional language

We write ( $e : t$ ) to say that *expression e has type t*. eg:

$2 : \text{int}$

$\text{"hello"} : \text{string}$

$2 + 2 : \text{int}$

$\text{"I say " ^ "hello"} : \text{string}$

# Type Checking Rules

There are a set of **simple rules** that govern type checking

- programs that do not follow the rules will not type check and OCaml will refuse to compile them for you (the nerve!)
- at first you may find this to be a pain ...

But types are a great thing:

- help us *think* about *how to construct* our programs
- help us *find stupid programming errors*
- help us track down errors quickly when we *edit our code*
- allow us to *enforce powerful invariants* about data structures

# Type Checking Rules

Example rules:

- (1) `0 : int` (and similarly for any other integer constant `n`)
- (2) `"abc" : string` (and similarly for any other string constant `"..."`)

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- (2)  $"\text{abc}" : \text{string}$  (and similarly for any other string constant "...")
- (3) if  $e1 : \text{int}$  and  $e2 : \text{int}$   
then  $e1 + e2 : \text{int}$
- (4) if  $e1 : \text{int}$  and  $e2 : \text{int}$   
then  $e1 * e2 : \text{int}$

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then  $e1 * e2 : \text{int}$
- (5) if  $e1 : \text{string}$  and  $e2 : \text{string}$   
then  $e1 \wedge e2 : \text{string}$
- (6) if  $e : \text{int}$   
then  $\text{string\_of\_int } e : \text{string}$

Using the rules:

$2 : \text{int}$  and  $3 : \text{int}$ . (By rule 1)



# Type Checking Rules

Example rules:

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Using the rules:

$2 : \text{int}$  and  $3 : \text{int}$ . (By rule 1)  
Therefore,  $(2 + 3) : \text{int}$  (By rule 3)

# Type Checking Rules

Example rules:

- (1)  $0 : \text{int}$  (and similarly for any other integer constant  $n$ )
- (2)  $"abc" : \text{string}$  (and similarly for any other string constant "...")
- (3) if  $e1 : \text{int}$  and  $e2 : \text{int}$   
then  $e1 + e2 : \text{int}$
- (4) if  $e1 : \text{int}$  and  $e2 : \text{int}$   
then  $e1 * e2 : \text{int}$
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then  $e1 \wedge e2 : \text{string}$
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then  $\text{string\_of\_int } e : \text{string}$

Using the rules:

$2 : \text{int}$ and $3 : \text{int}$ .	(By rule 1)
Therefore, $(2 + 3) : \text{int}$	(By rule 3)
$5 : \text{int}$	(By rule 1)

# Type Checking Rules

Example rules:

- (1)  $0 : \text{int}$  (and similarly for any other integer constant  $n$ )
- (2)  $"\text{abc}" : \text{string}$  (and similarly for any other string constant  $s$ )
- (3) if  $e1 : \text{int}$  and  $e2 : \text{int}$   
then  $e1 + e2 : \text{int}$
- (5) if  $e1 : \text{string}$  and  $e2 : \text{string}$   
then  $e1 \wedge e2 : \text{string}$

FYI: This is a ***formal proof***  
that the expression is well-  
typed!

Using the rules:

$2 : \text{int}$  and  $3 : \text{int}$ . (By rule 1)  
Therefore,  $(2 + 3) : \text{int}$  (By rule 3)  
 $5 : \text{int}$  (By rule 1)  
Therefore,  $(2 + 3) * 5 : \text{int}$  (By rule 4 and our previous work)

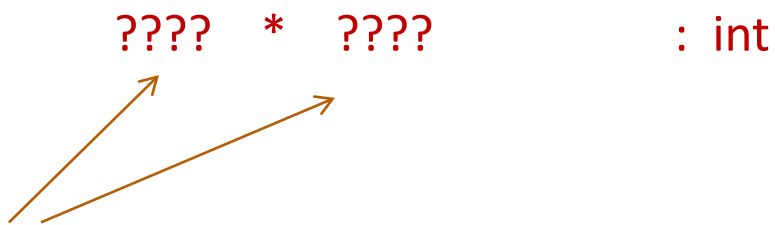
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then `string_of_int e : string`

Another perspective:

rule (4) for typing expressions  
says I can put any expression  
with type `int` in place of the `????`



`???? * ???? : int`

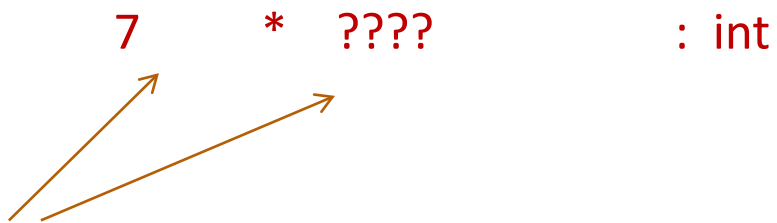
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then `string_of_int e : string`

Another perspective:

rule (4) for typing expressions  
says I can put any expression  
with type `int` in place of the `????`



The diagram shows the expression `7 * ????` followed by a colon and `int`. Two orange arrows originate from the text 'any expression with type int' and point to the `7` and the `????` in the expression, illustrating that both operands must have the type `int` for rule (4) to apply.

# Type Checking Rules

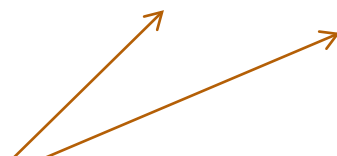
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then `string_of_int e : string`

Another perspective:

rule (4) for typing expressions  
says I can put any expression  
with type `int` in place of the `????`

`7 * (add_one 17) : int`



# Type Checking Rules

You can always start up the OCaml interpreter to find out a type of a simple expression:

```
$ ocaml
      OCaml Version 4.13.1
#
```

# Type Checking Rules

You can always start up the OCaml interpreter to find out a type of a simple expression:

or utop

```
$ ocaml
      OCaml Version 4.13.1
# 3 + 1;;
```

use “;;”  
to end  
a phrase  
in the  
top level

(“;;” can also end a top-level phrase in a file, but I’m going to avoid using it there because then some of you will confuse it with a “;” ....)

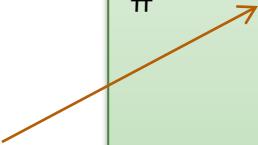


# Type Checking Rules

You can always start up the OCaml interpreter to find out a type of a simple expression:

```
$ ocaml
      OCaml Version 4.13.1
# 3 + 1;;
- : int = 4
#
```

press  
return  
and you  
find out  
the type  
and the  
value

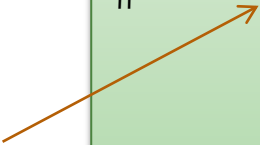


# Type Checking Rules

You can always start up the OCaml interpreter to find out a type of a simple expression:

```
$ ocaml
      OCaml Version 4.13.1
# 3 + 1;;
- : int = 4
# "hello " ^ "world";;
- : string = "hello world"
#
```

press  
return  
and you  
find out  
the type  
and the  
value



# Type Checking Rules

You can always start up the OCaml interpreter to find out a type of a simple expression:

```
$ ocaml
      OCaml Version 4.13.1
# 3 + 1;;
- : int = 4
# "hello " ^ "world";;
- : string = "hello world"
# #quit;;
$
```

# Type Checking Rules

Example rules:

- (1) `0 : int` (and similarly for any other integer constant `n`)
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then `e1 ^ e2 : string`
- (6) if `e : int`  
then `string_of_int e : string`

Violating the rules:

`"hello" : string`

`1 : int`

`1 + "hello" : ??`

(By rule 2)

(By rule 1)

(NO TYPE! Rule 3 does not apply!)

# Type Checking Rules

Violating the rules:

```
# "hello" + 1;;
```

```
Error: This expression has type string but an  
expression was expected of type int
```

The type error message tells you the type that was **expected** and the type that it **inferred** for your subexpression

By the way, this was one of the nonsensical expressions that did not evaluate to a value

It is a **good thing** that this expression does not type check!

*“Well typed programs do not go wrong”*

*Robin Milner, 1978*

# Type Checking Rules

Violating the rules:

```
# "hello" + 1;;
```

```
Error: This expression has type string but an  
expression was expected of type int
```

A possible fix:

```
# "hello" ^ (string_of_int 1);;  
- : string = "hello1"
```

*One of the keys to becoming a good ML programmer is to understand type error messages.*

# Type Checking Rules

What about this expression:

```
# 3 / 0 ;;  
Exception: Division_by_zero.
```

Why doesn't the ML type checker do us the favor of telling us the expression will raise an exception?

# Type Checking Rules

What about this expression:

```
# 3 / 0 ;;  
Exception: Division_by_zero.
```

Why doesn't the ML type checker do us the favor of telling us the expression will raise an exception?

- In general, detecting a divide-by-zero error requires we know that the divisor evaluates to 0.
- In general, deciding whether the divisor evaluates to 0 requires solving the halting problem:

```
# 3 / (if turing_machine_halts m then 0 else 1);;
```

There are type systems that will rule out divide-by-zero errors, but they require programmers supply proofs to the type checker



# Isn't that cheating?

*“Well typed programs do not go wrong”*

*Robin Milner, 1978*

(3 / 0) is well typed. Does it “go wrong?” Answer: No.

“Go wrong” is a technical term meaning, “**have no defined semantics.**” Raising an exception is perfectly well defined semantics, which we can reason about, which we can handle in ML with an exception handler.

So, it's not cheating.

*(Discussion: why do we make this distinction, anyway?)*

# Type Soundness

*“Well typed programs do not go wrong”*

Programming languages with this property have *sound* type systems. They are called *safe* languages.

Safe languages are generally *immune* to buffer overrun vulnerabilities, uninitialized pointer vulnerabilities, etc., etc.  
(but not immune to all bugs!)

Safe languages: ML, Java, Python, ...

Unsafe languages: C, C++

**OVERALL SUMMARY:  
A SHORT INTRODUCTION TO  
FUNCTIONAL PROGRAMMING**

# OCaml

OCaml is a *functional* programming language

- express control flow and iteration by defining *functions*
- *not by modifying the values of variables and data structures*

Imperative: “do this”

Functional: “be this”

OCaml is a *typed* programming language

- the *type* of an expression *correctly predicts* the kind of *value* the expression will generate when it is executed
- types help us *understand* and *write* our programs
- the type system is *sound*; the language is *safe*