Mutation

COS 326

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Princeton University

C structures are *mutable*, ML structures are immutable

C program

```
struct foo {int x; int y} *p;
int a,b,u;

a = p->x;
u = f(p);
b = p->x;

/* does a==b?  maybe */
```

OCaml program

```
let fst(x:int,y:int) = x
let p: int*int = ... in

let a = fst p in
let u = f p in
let b = fst p in

  (* does a==b?  Yes! *)
```

Reasoning about Mutable State is Hard

mutable set

```
insert i s1;
f x;
member i s1
```

immutable set

```
let s1 = insert i s0 in
f x;
member i s1
```

Is member i s1 == true? …

- When s1 is mutable, one must look at f to determine if it modifies s1.
- Worse, one must often solve the *aliasing problem*.
- Worse, in a concurrent setting, one must look at *every other function* that *any other thread may be executing* to see if it modifies s1.

Thus far…

We have considered the (almost) purely functional subset of OCaml.

- We've had a few side effects: printing & raising exceptions.

Two reasons for this emphasis:

- *Reasoning about functional code is easier*.
  - Both formal reasoning
    - equationally, using the substitution model
    - and informal reasoning
  - Data structures are *persistent*.
    - They don't change – we build new ones and let the garbage collector reclaim the unused old ones.
  - *Hence, any invariant you prove true stays true*.
    - e.g., 3 is a member of set S.
- *To convince you that you don't need side effects for many things where you previously thought you did*.
  - Programming with *basic immutable data like ints, pairs, lists is easy*.
    - types do a lot of testing for you!
    - do not fear recursion!
  - You can implement *expressive*, *highly reusable functional* data structures like polymorphic 2-3 trees or dictionaries or stacks or queues or sets or expressions or programming languages with reasonable space and time.

But alas…

Purely functional code is pointless.

- The whole reason we write code is to have some effect on the world.
- For example, the OCaml top-level loop prints out your result.
  - Without that printing (a side effect), how would you know that your functions computed the right thing?

Some algorithms or data structures need mutable state.

- Hash-tables have (essentially) constant-time access and update.
  - The best functional dictionaries have either:
    - logarithmic access & logarithmic update
    - constant access & linear update
    - constant update & linear access
  - Don’t forget that we give up something for this:
    - we can’t go back and look at previous versions of the dictionary. We *can* do that in a functional setting.
- Robinson’s unification algorithm
  - A critical part of the OCaml type-inference engine.
  - Also used in other kinds of program analyses.
- Depth-first search, union-find, more ...

However, ~~purely~~ mostly functional code is amazingly productive

The value of a classics degree

Inventor (1960s) of algorithms now fundamental to computational logical reasoning (about software, hardware, and other things…)

John Alan Robinson
1928 – 2016
PhD Princeton 1956 (philosophy)

"Robinson was born in Yorkshire, England in 1930 and left for the United States in 1952 with a classics degree from Cambridge University. He studied philosophy at the University of Oregon before moving to Princeton University where he received his PhD in philosophy in 1956. He then worked at Du Pont as an operations research analyst, where he learned programming and taught himself mathematics. He moved to Rice University in 1961, spending his summers as a visiting researcher at the Argonne National Laboratory's Applied Mathematics Division. He moved to Syracuse University as Distinguished Professor of Logic and Computer Science in 1967 and became professor emeritus in 1993."
--Wikipedia

OCAML MUTABLE REFERENCES

References

- New type: `t ref`
  - Think of it as a pointer to a *box* that holds a t value.
  - The contents of the box can be read or written.

References

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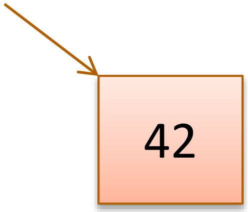
  - Think of it as a pointer to a *box* that holds a t value.
  - The contents of the box can be read or written.
- To create a fresh box: `ref 42`
  - allocates a new box, initializes its contents to 42, and returns a pointer:



  - `ref 42 : int ref`

References

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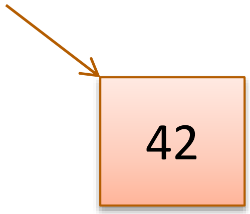


  - `ref 42 : int ref`
- To read the contents: `!r`
  - if `r` points to a box containing 42, then return 42.
  - if `r : t ref` then `!r : t`
- To write the contents: `r := 5`
  - updates the box that r points to so that it contains 5.
  - if `r : t ref` then `r := 5 : unit`

Example

```
let c = ref 0 in

let x = !c in      (* x will be 0 *)

c := 42;

let y = !c in   (* y will be 42.
                   x will still be 0! *)
```

Another Example

```
let c = ref 0 ;;


let next() =
  let v = !c in
  (c := v+1 ; v)
```

Another Example

```
let c = ref 0

let next() =
  let v = !c in
  (c := v+1 ; v)
```

If `e1 : unit`
and `e2 : t` then
`(e1 ; e2) : t`

You can also write it like this:

```
let c = ref 0

let next() =
  let v = !c in
  let _ = c := v+1 in
  v
```

Another Idiom

Global Mutable Reference

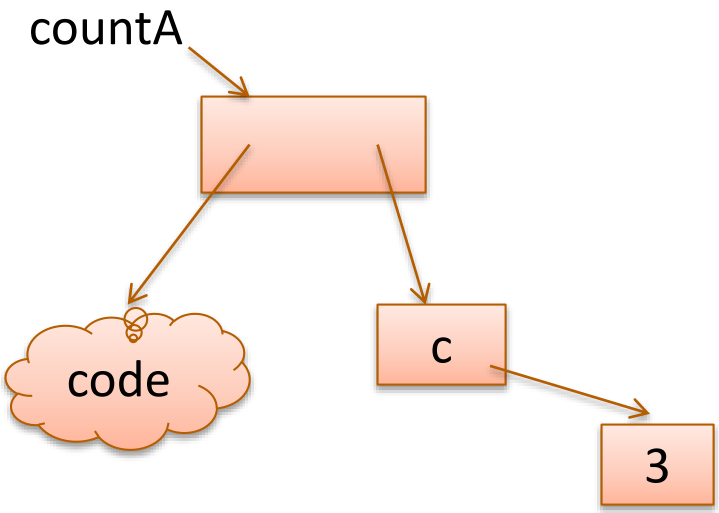
```
let c = ref 0

let next () : int =
  let v = !c in
  (c := v+1 ; v)
```

Mutable Reference Captured in Closure

```
let counter () =
    let c = ref 0 in
   fun () ->
     let v = !c in
     (c := v+1 ; v)

let countA = counter() in
let countB = counter() in
countA() ; (* 0 *)
countA() ; (* 1 *)
countB() ; (* 0 *)
countB() ; (* 1 *)
countA() ; (* 2 *)
```



Imperative loops

```
(* sum of 0 .. n *)

let sum (n:int) =
  let       s = ref 0 in
  let current = ref n in
  while !current > 0 do
    s := !s + !current;
    current := !current - 1
  done;
  !s
```

```
(* print n .. 0 *)
let count_down (n:int) =
  for i = n downto 0 do
    print_int i;
    print_newline()
  done


(* print 0 .. n *)
let count_up (n:int) =
  for i = 0 to n do
    print_int i;
    print_newline()
  done
```

Imperative loops?

```
(* print n .. 0 *)

let count_down (n:int) =
  for i = n downto 0 do
    print_int i;
    print_newline()
  done
```

```
(* for i=n downto 0 do f i *)

let rec for_down
         (n : int)
         (f : int -> unit)
            : unit =
  if n >= 0 then
   (f n; for_down (n-1) f)
  else
   ()


let count_down (n:int) =
  for_down n (fun i ->
    print_int i;
    print_newline()
  )
```

REFS AND MODULES

Types and References

Concrete, first-order type tells you a lot about a data structure:

- int ==> immutable
- int ref ==> mutable
- int * int ==> immutable
- int * (int ref) ==> 1st component immutable, 2nd mutable
- ... etc

What about higher-order types?

- int -> int ==> the function can't be changed
  ==> what happens when we run it?

What about abstract types?

- stack, queue? stack * queue?

Functional Stacks

```
module type STACK =
  sig
    type ‘a stack
    val empty : unit -> ‘a stack
    val push : ‘a -> ‘a stack -> ‘a stack
    val peek : ‘a stack -> ‘a option
     ...
  end
```

Functional Stacks

```
module type STACK =
  sig
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    val peek : 'a stack -> 'a option
     ...
  end
```

A functional interface takes in arguments, analyzes them, and produces new results

Imperative Stacks

```
module type IMP_STACK =
  sig
    type ‘a stack
    val empty : unit -> ‘a stack
    val push : ‘a -> ‘a stack -> unit
    val peek : ‘a stack -> ‘a option
     ...
  end
```

Imperative Stacks

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module type IMP_STACK =
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    val push : ‘a -> ‘a stack -> unit
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     ...
  end
```

When you see “unit” as the return type, you know the function is being executed for its side effects. (Like void in C/C++/Java.)

Imperative Stacks

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  sig
    type ‘a stack
    val empty : unit -> ‘a stack
    val push : ‘a -> ‘a stack -> unit
    val peek : ‘a stack -> ‘a option
    val pop : ‘a stack -> ‘a option
  end
```

Unfortunately, we can’t always tell from the type that there are side-effects going on. It’s a good idea to document them explicitly if the user can perceive them.

Imperative Stacks

```
module type IMP_STACK =
sig
  type ‘a stack
  val empty : unit -> ‘a stack
  val push: ‘a -> ‘a stack -> unit
  val pop : ‘a stack -> ‘a option
end
```

```
module ImpStack : IMP_STACK =
struct
  type ‘a stack = (‘a list) ref

  let empty() : ‘a stack = ref []

  let push(x:’a)(s:’a stack) : unit =
       s := x::(!s)

  let pop(s:’a stack) : ‘a option =
    match !s with
    | [] -> None
    | h::t -> (s := t ; Some h)
end
```

Imperative Stacks

```
module type IMP_STACK =
sig
  type ‘a stack
  val empty : unit -> ‘a stack
  val push: ‘a -> ‘a stack -> unit
  val pop : ‘a stack -> ‘a option
end
```

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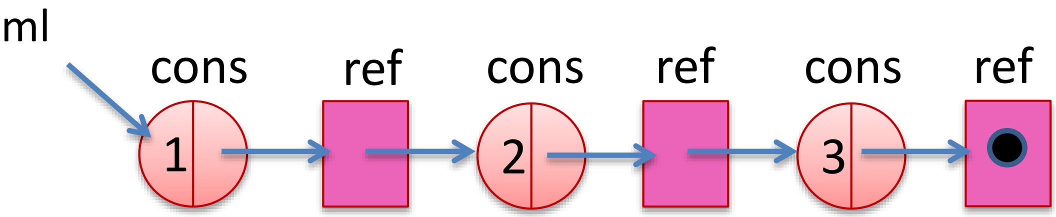
    match !s with
    | [] -> None
    | h::t -> (s := t ; Some h)
end
```

Note: We don't have to make *everything* mutable. The list is an immutable data structure stored in a single mutable cell.

Fully Mutable Lists

```
type ‘a mlist =
  Nil | Cons of ‘a * (‘a mlist ref)


let ml = Cons(1, ref (Cons(2, ref
   (Cons(3, ref Nil)))))
```



Fraught with Peril

```
type ‘a mlist =
  Nil | Cons of ‘a * (‘a mlist ref)

let rec mlength(m:’a mlist) : int =
  match m with
  | Nil -> 0
  | Cons(h,t) -> 1 + length(!t)

let r = ref Nil ;;
let m = Cons(3,r) ;;
r := m ;;
mlength m ;;
```

Fraught with Peril

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type ‘a mlist =
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let rec mlength(m:’a mlist) : int =
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let r = ref Nil in
let m = Cons(3,r) in
r := m ;
mlength m
```

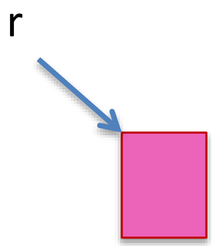

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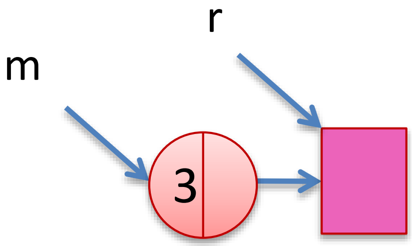
let r = ref Nil in
let m = Cons(3,r) in
r := m ;
mlength m
```



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r := m ;
mlength m
```
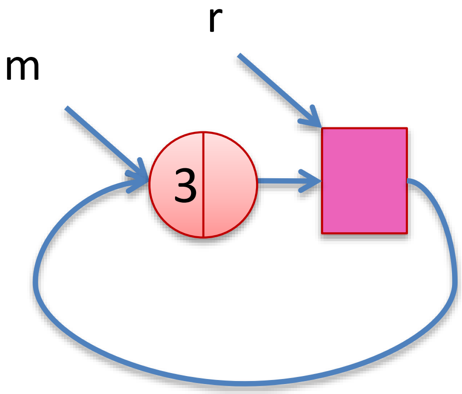
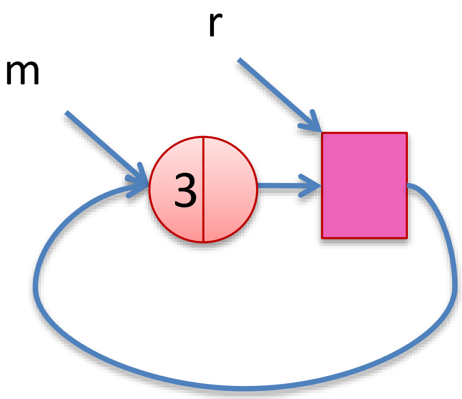



Fraught with Peril

Can’t use induction! No base case!

```
type ‘a mlist =
  Nil | Cons of ‘a * (‘a mlist ref)

let rec mlength(m:’a mlist) : int =
  match m with
  | Nil -> 0
  | Cons(h,t) -> 1 + mlength(!t)

let r = ref Nil in
let m = Cons(3,r) in
r := m ;
mlength m
```



Add mutability judiciously

Two types:

```
type 'a very_mutable_list =
  Nil
| Cons of 'a * ('a very_mutable_list ref)
```

```
type 'a less_mutable_list = 'a list ref
```

The first makes cyclic lists possible, the second doesn't

- the second preemptively avoids certain kinds of errors.
- often called a *correct-by-construction design*

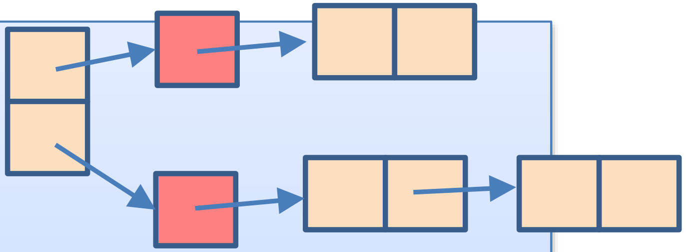
```
type 'a extremely_mutable_list =
  Nil
| Cons of 'a ref * ('a very_mutable_list ref)
```

MUTABLE RECORDS AND ARRAYS

Records with Mutable Fields

OCaml records with mutable fields:

```
type 'a queue1 =
  {front : 'a list ref;
   back  : 'a list ref }
```

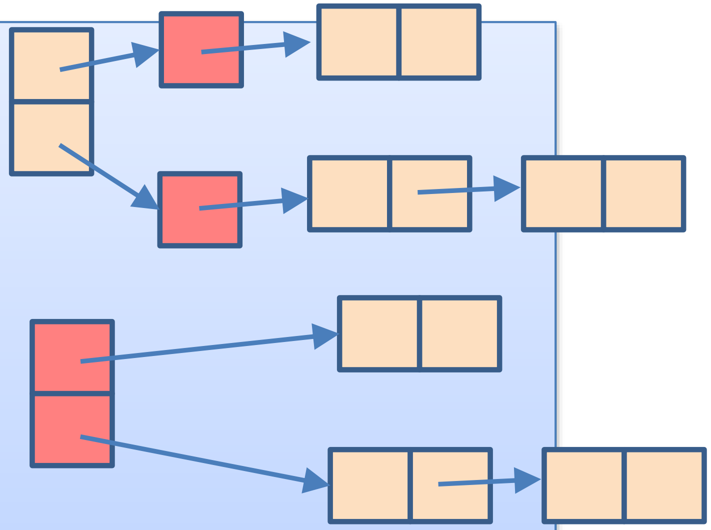


In fact: `type 'a ref = {mutable contents : 'a}`

Records with Mutable Fields

OCaml records with mutable fields:

```
type 'a queue1 =
  {front : 'a list ref;
   back  : 'a list ref }

type 'a queue2 =
  {mutable front : 'a list;
   mutable back : 'a list}

let q1 = {front = [1]; back = [2]} in
let q2 = {front = [1]; back = [2]} in

let x = q2.front @ q2.back in

q2.front <- [3]
```



In fact: `type 'a ref = {mutable contents : 'a}`

Mutable Arrays

For arrays, we have:

`A.(i)`

- to read the ith element of the array `A`

`A.(i) <- 42`

- to write the ith element of the array `A`

`Array.make : int -> ‘a -> ‘a array`

- `Array.make 42 ‘x’` creates an array of length 42 with all elements initialized to the character `‘x’`.

See the reference manual for more operations.

www.caml.inria.fr/pub/docs/manual-ocaml/libref/Array.html

Is it possible to avoid all state?

Yes! (in single-threaded programs)

- Pass in old values to functions; return new values from functions ... but this isn't necessarily the most efficient thing to do

Example: Depth-First Search

A “graph” is a mapping from node-number to list-of-node-number.

“Mark each node” using a mapping from node-number to bool.

Implement these mappings as “dictionaries”,

implemented by 2-3 trees:

```
module type DICT =
sig
  type 'a dict
  val empty : 'a -> 'a dict
  val lookup : 'a dict -> int -> 'a
  val insert : 'a dict -> int -> 'a -> 'a dict
end

module Dict : DICT =
struct  . . .  end
```

default value

Example: Depth-First Search

Pass the “marks dictionary” around from function-call to function-call:

```
type node = int
type graph = node list dict

let rec dfs (g: graph) (marks: bool dict) (n: int) : bool dict =
  if lookup marks n
  then marks
  else List.fold_left (dfs g) (insert marks n true) (lookup g n)
```

Or, if that fold_left is too concise for you,

```
let rec dfs (g: graph) (marks: bool dict) (n: int) : bool dict =
  if lookup marks n
  then marks
  else let rec f m es =
                match es with
                | [] -> m
         | e::es' -> let m' = dfs g m e
                                  in f m' es'
      in f marks (lookup g n)
```

Warning:
I haven't
tested
this code!

Asymptotic time complexity

This implementation of DFS runs in O(N log N) time.

But you know that DFS is a linear-time algorithm.

Extra cost comes from logN cost for dictionary lookup and insert,
whereas array subscript takes constant time.

You can implement this in ML with mutable arrays,
(pretty much like you'd do it in C or Java)
and it will be linear time, O(N).

Fully encapsulated state

We can't always tell from the type that there are side-effects going on. It's a good idea to document them explicitly **if the user can perceive them.**

This is a terrific way to use references in ML. Look for these opportunities

Sometimes, one uses references inside a module but the data structures have functional (persistent) semantics

Factoring!

```
let factor n =
  let s = int_of_float (sqrt (float_of_int n)) in
  let rec f i =
    if i<=s then
      if n mod i = 0 then
        Some i
      else
        f (i+1)
    else
      None
   in f 2
```

Factoring!

```
let factor n =
  let s = int_of_float (sqrt (float_of_int n)) in
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        Some i
      else
        f (i+1)
    else
      None
   in f 2
```

```
factor 77 = Some 7
```

```
factor 97 = None
```

Caveats

```
let factor n =
  let s = int_of_float (sqrt (float_of_int n)) in
  let rec f i =
    if i<=s then
      if n mod i = 0 then
        Some i
      else
        f (i+1)
```

Caveat 1:
Many applications of prime numbers are for many-bit (500-bit, 2000-bit) numbers; OCaml ints are 31-bit or 63-bit, so you'd want a version of this for the bignums

Caveat 2:
This primitive factoring algorithm, already obsolete 2000 years ago, is not what you'd really use. Modern algorithms based on fancy number theory are much faster.

Caveat 3:
Even the fancy number-theory algs take superpolynomial time (as function of the number of bits in n)

Memoized factoring

```
let table = Hashtbl.create 1000

let memofactor n =
  try Hashtbl.find table n
  with Not_found ->
       let p = factor n
       in Hashtbl.add table n p; p
```

```
memofactor 77 = Some 7
```

```
memofactor 97 = None
```

Encapsulating the side effects

```
struct
  let table = Hashtbl.create 1000

  let memofactor n =
    try Hashtbl.find table n
    with Not_found ->
         let p = factor n
         in Hashtbl.add table n p; p

  let factor n = memofactor n
end
```

```
sig
  val factor : int -> int
end
```

The table is hidden inside the function closure.

There's no way for the client to access it, or know it’s there.

We can pretend memofactor is a pure function.

OCaml Objects

```
class point =
  object
    val mutable x = 0
    method get_x = x
    method move d = x <- x + d
  end;;
```

```
let p = new point in
let x = p#get in

p#move 4;

x + p#get  (* 0 + 4 *)
```

http://caml.inria.fr/pub/docs/manual-ocaml-4.00/manual005.html

Xavier Leroy (OCaml inventor):

- No one ever uses objects in OCaml!
- Adding objects to OCaml was one of the best decisions I ever made!

SUMMARY

Summary: How/when to use state?

- A complicated question!
- In general, I try to write the functional version first.
  - e.g., prototype
  - don’t have to worry about sharing and updates
  - don’t have to worry about race conditions
  - reasoning is easy (the substitution model is valid!)
- Sometimes you find you can’t afford it for efficiency reasons.
  - example: routing tables need to be fast in a switch
  - constant time lookup, update (hash-table)
- When I do use state, I try to *encapsulate* it behind an interface.
  - try to reduce the number of error conditions a client can see
    - correct-by-construction design
  - module implementer must think explicitly about sharing and invariants
  - write these down, write assertions to test them
  - if encapsulated in a module, these tests can be localized
  - *most of your code should still be functional*

Summary

Mutable data structures can lead to *efficiency improvements*.

- e.g., Hash tables, memoization, depth-first search

But they are *much* harder to get right, so don't jump the gun

- *updating in one place may have an effect on other places.*
- *writing and enforcing invariants becomes more important*.
  - e.g., assertions we used in the queue example
  - why more important? because the types do less ...
- *cycles in data (other than functions) can't happen until* we introduce refs.
  - must write operations much more carefully to avoid looping
  - more cases to deal with and the compiler doesn't help you!
- we haven't even gotten to the multi-threaded part.

So use refs when you must, but try hard to avoid it.