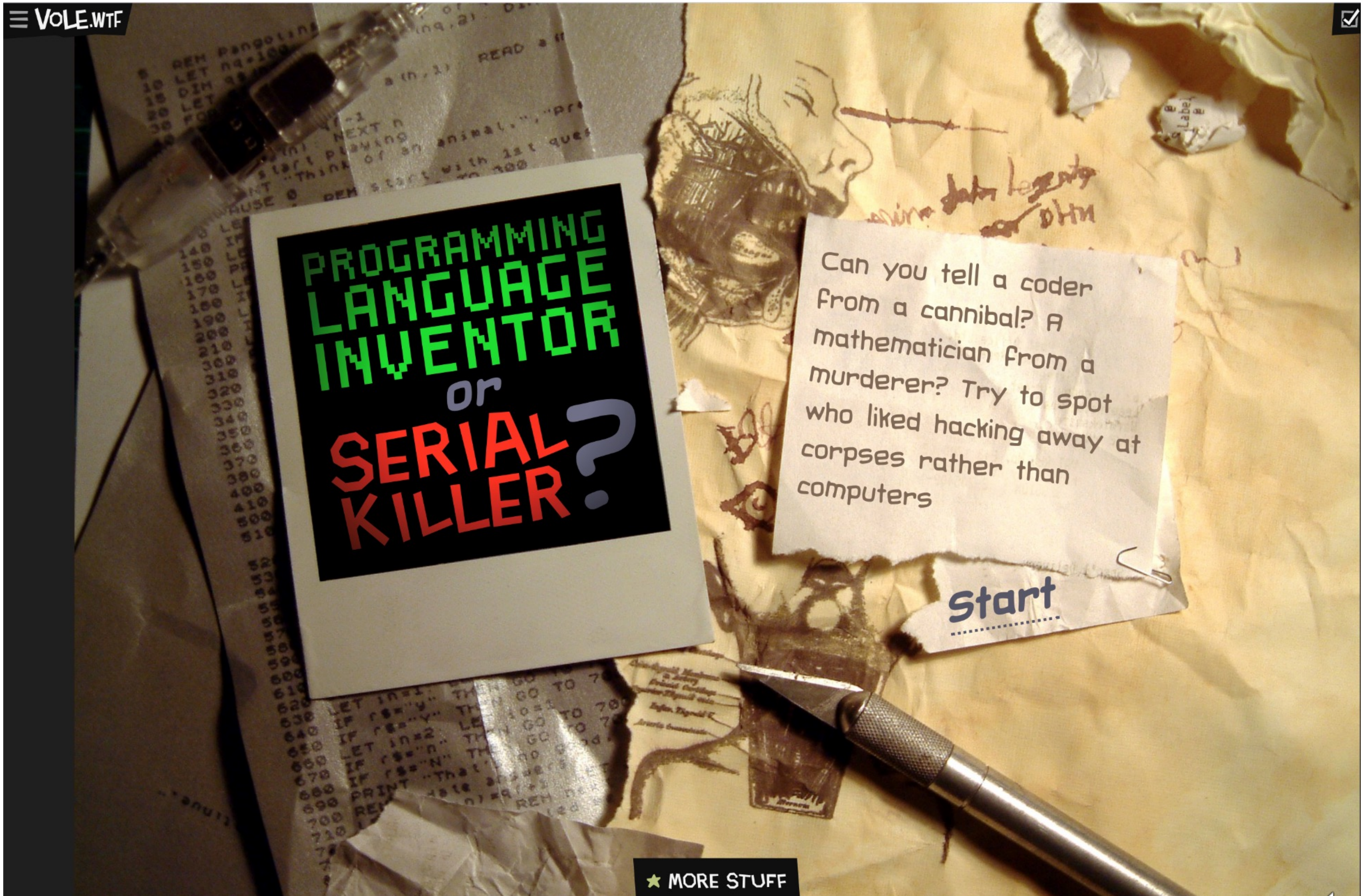
A Functional Space Model

Speaker: David Walker
COS 326
Princeton University

Interlude



https://vole.wtf/coder-serial-killer-quiz/

Space

Understanding the space complexity of functional programs

- At least two interesting components:
  - the amount of *live space* at any instant in time
  - the *rate of allocation*
    - a function call may not change the amount of live space by much but may allocate at a substantial rate
    - because functional programs act by generating new data structures and discarding old ones, they often allocate a lot
      - » OCaml garbage collector is optimized with this in mind
      - » interesting fact: at the assembly level, the number of writes by a functional program is roughly the same as the number of writes by an imperative program
- *What takes up space*?
  - conventional first-order data: tuples, lists, strings, datatypes
  - function representations (closures)
  - the call stack

CONVENTIONAL DATA

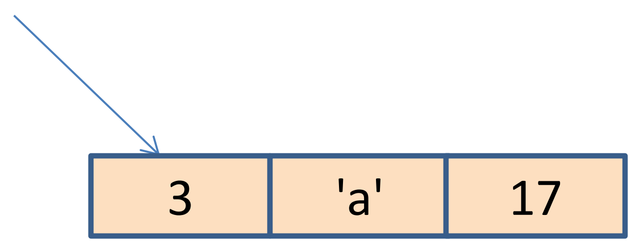
OCaml Representations for Data Structures

Type:

```
type triple = int * char * int
```

Representation:

(3, 'a', 17)



OCaml Representations for Data Structures

Type:

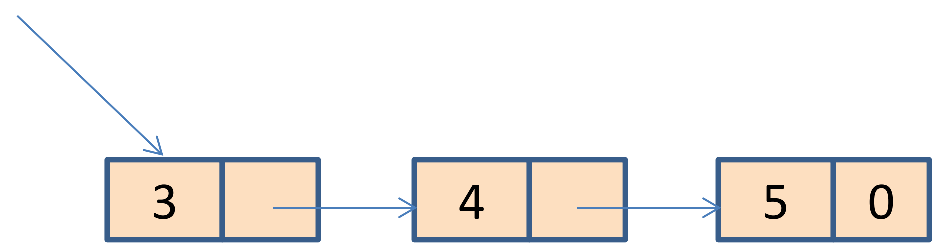
```
type mylist = int list
```

Representation:

[ ]

[3; 4; 5]

0



Space Model

Type:

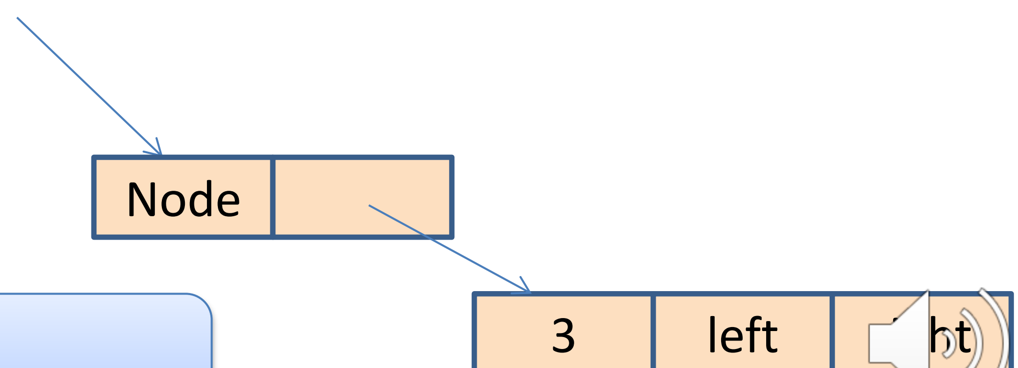
```
type tree = Leaf | Node of int * tree * tree
```

Representation:

Leaf

0

Node(3, left, right)



Actually like this in Ocaml:

Allocating space

In C, you allocate when you call "malloc"

In Java, you allocate when you call "new"

What about ML?

Allocating space

Whenever you *use a constructor*, space is allocated:

```
let rec insert (t:tree) (i:int) =
  match t with
    Leaf -> Node (i, Leaf, Leaf)
  | Node (j, left, right) ->
      if i <= j then
        Node (j, insert left i, right)
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        Node (j, left, insert right i)
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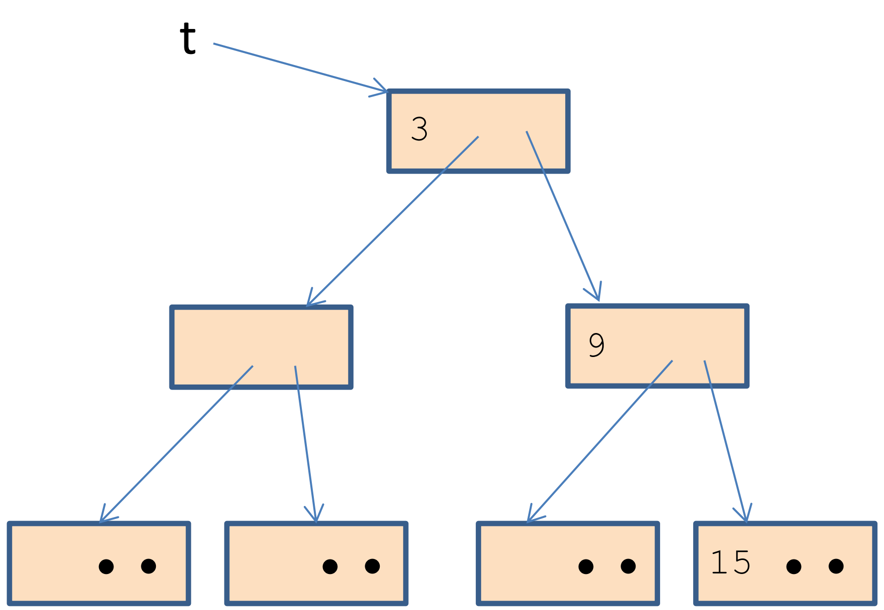
Consider:

insert t 21



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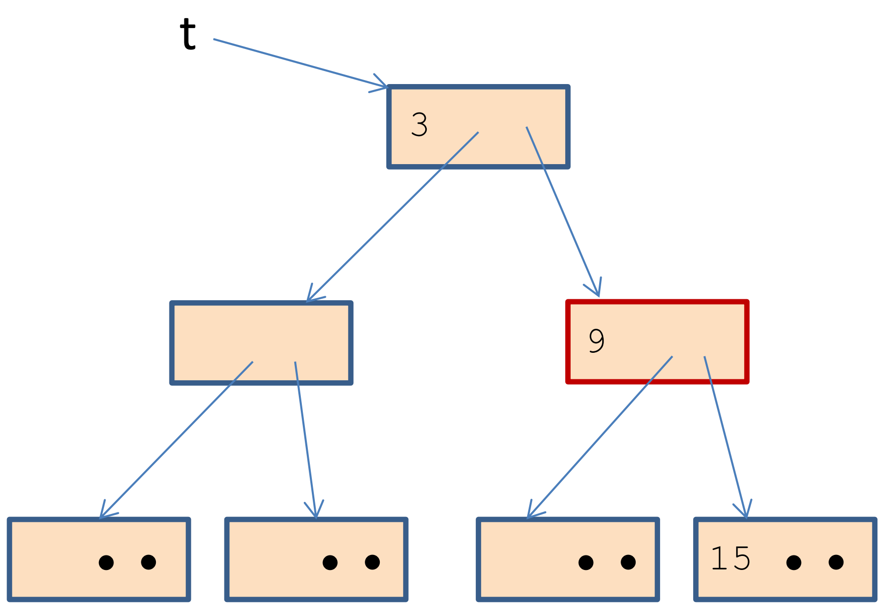

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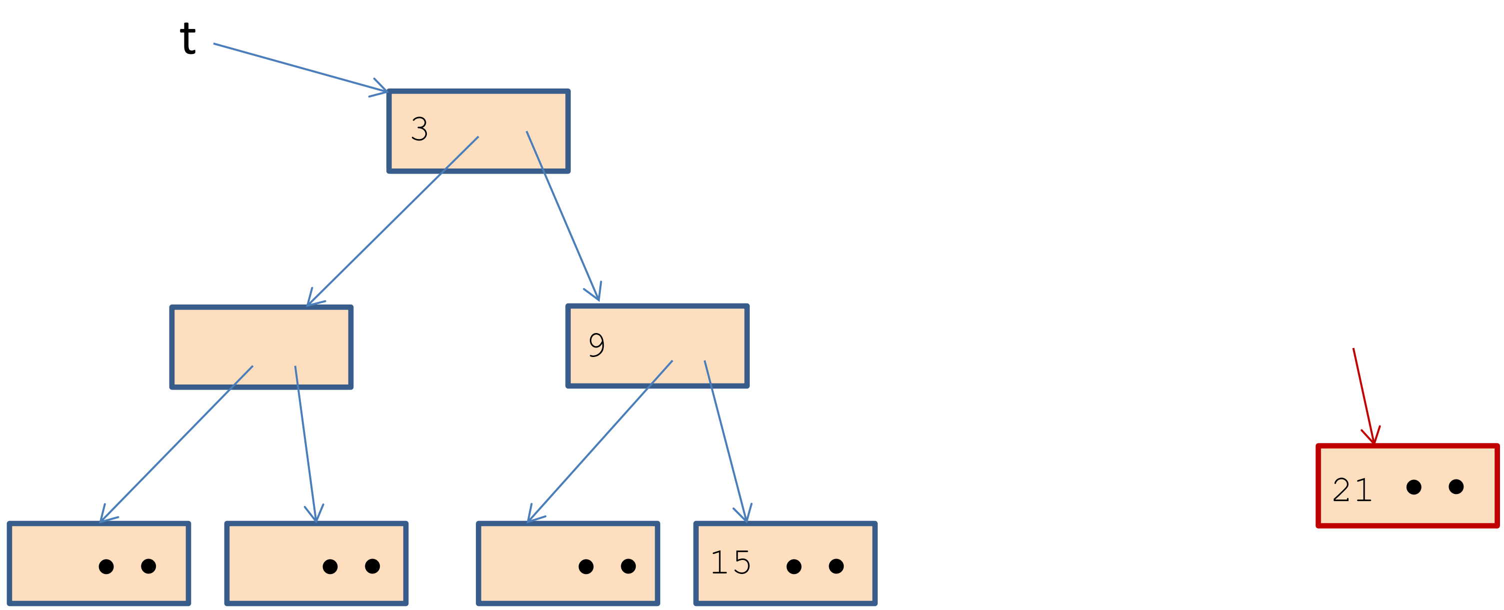

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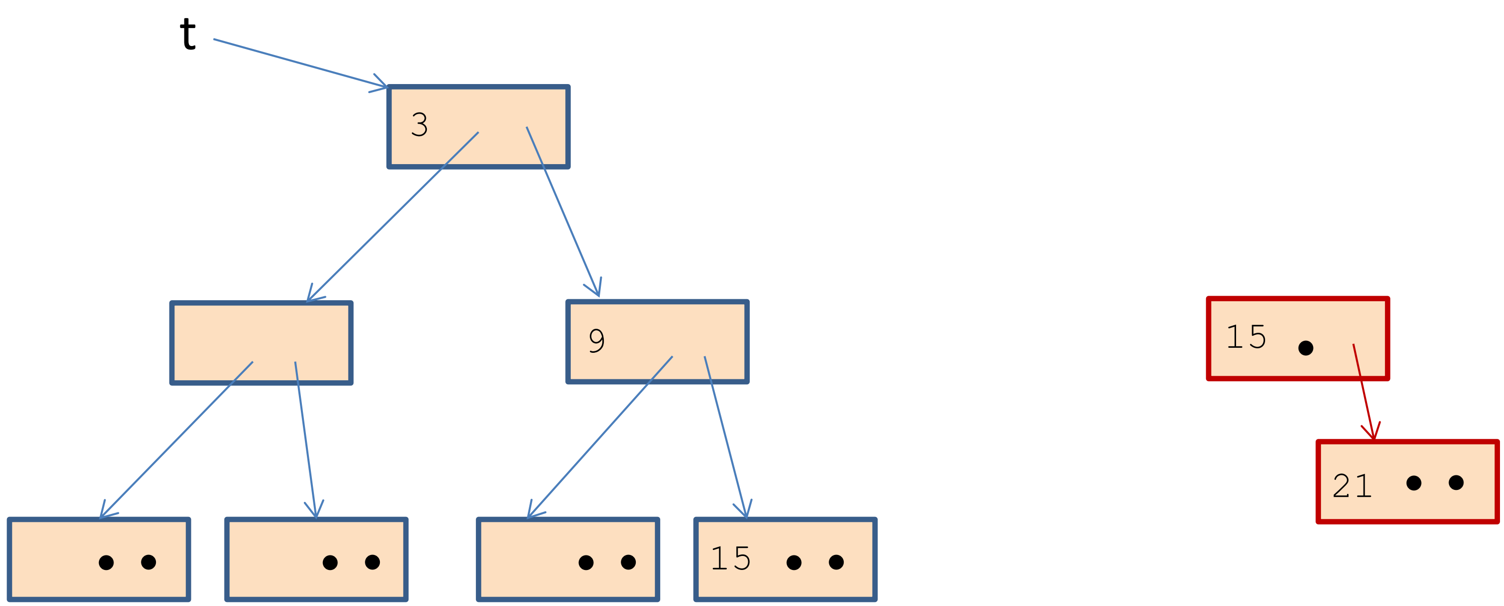

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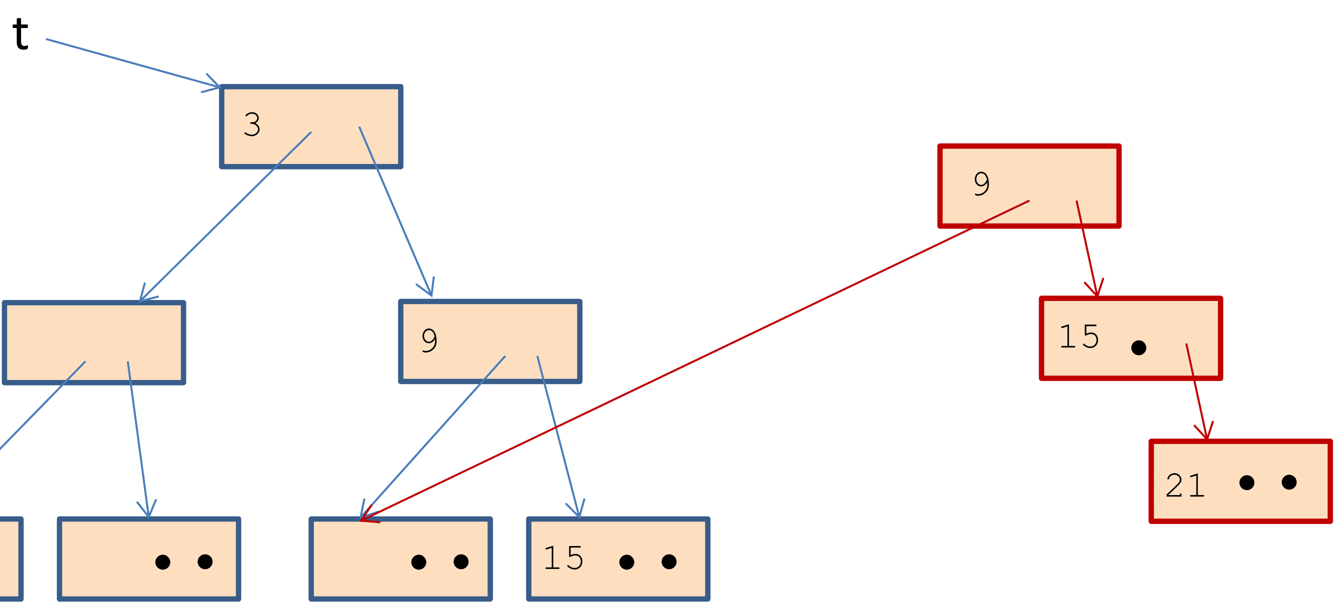

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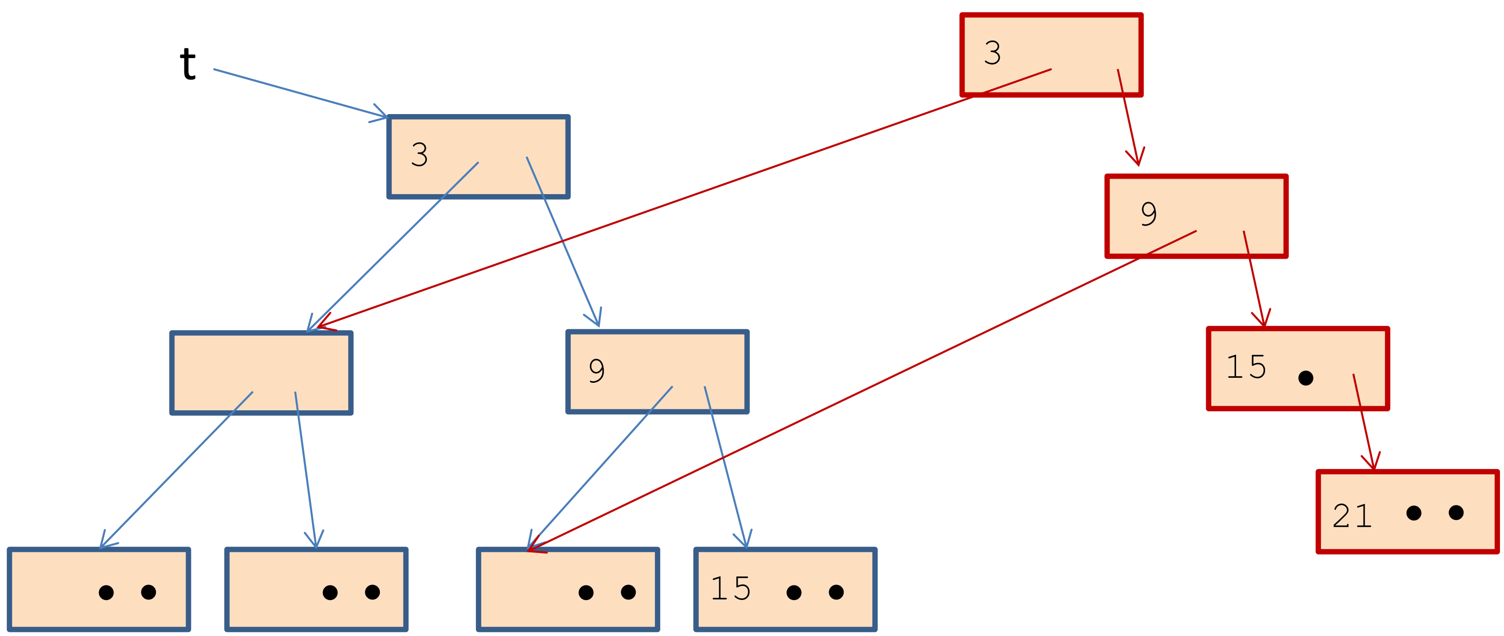

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```
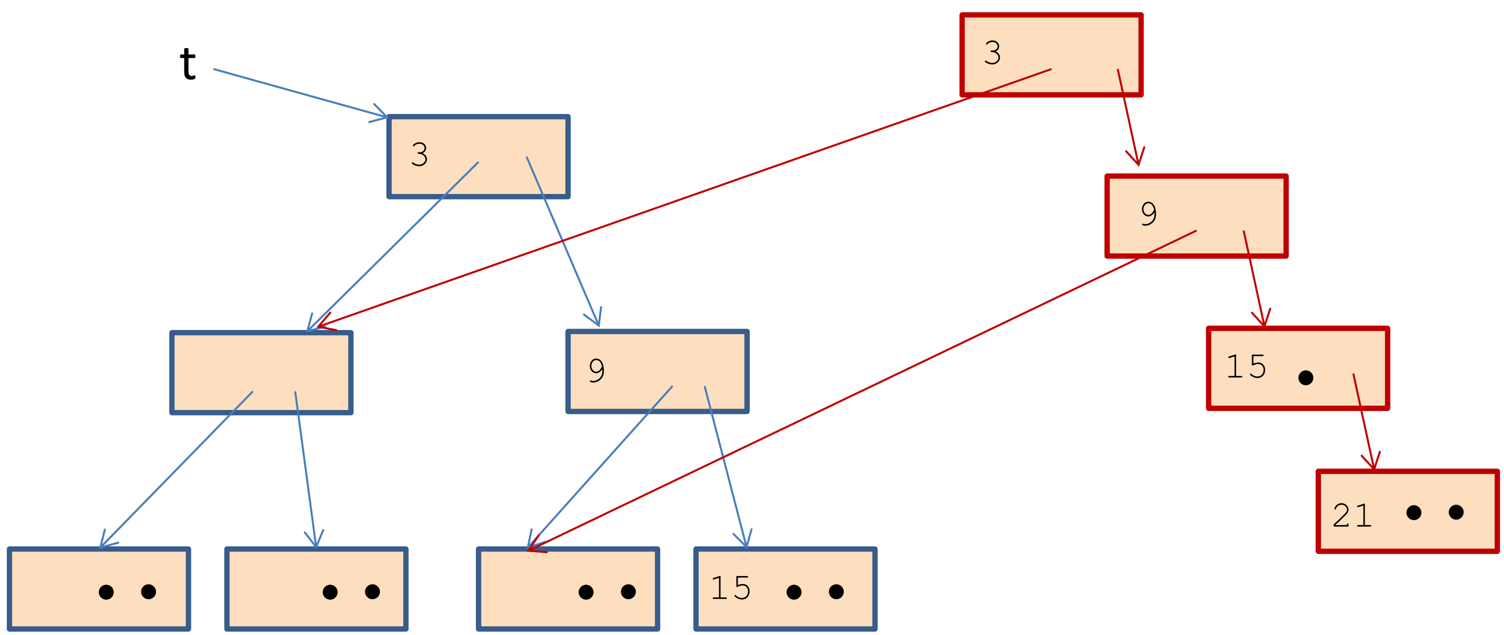

Total space allocated is proportional to the height of the tree.

~ log n, if tree with n nodes is balanced



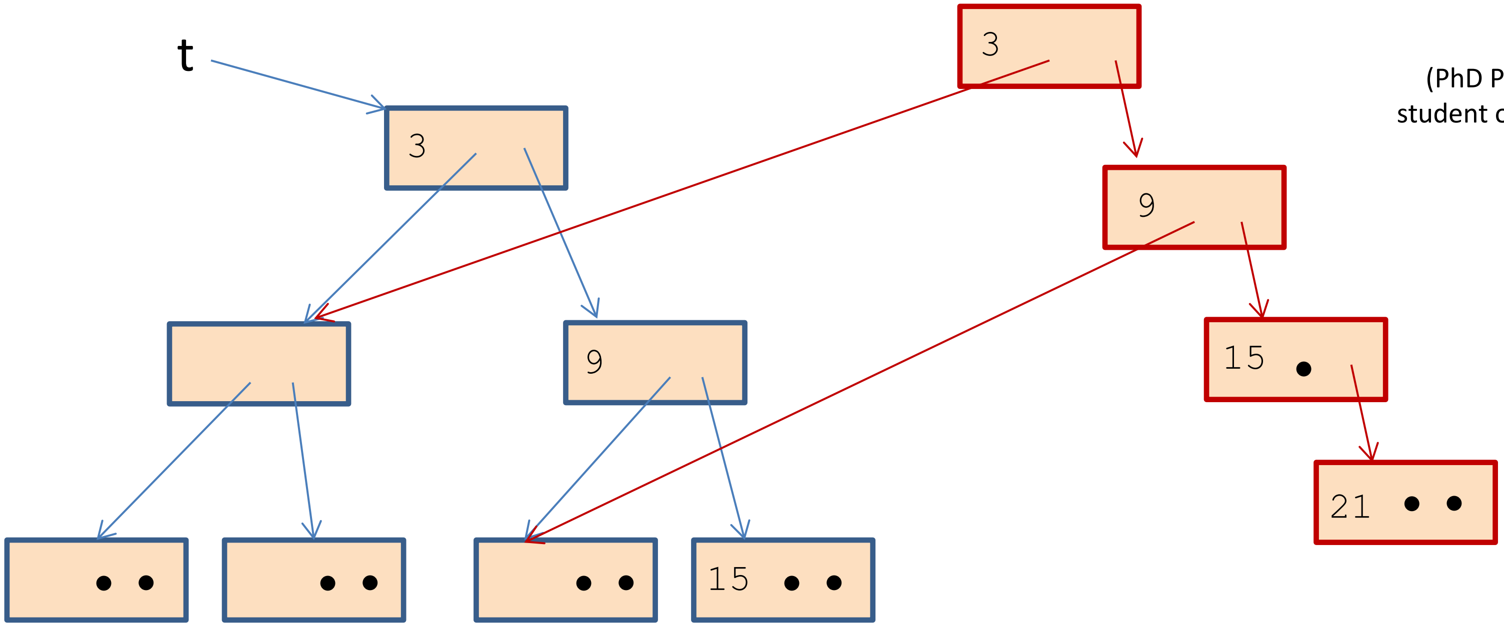
Net space allocated

The garbage collector reclaims unreachable data structures on the heap.

```
let fiddle (t: tree) =
  insert t 21
```

John McCarthy invented GC 1960

(PhD Princeton 1951, student of Alonzo Church)



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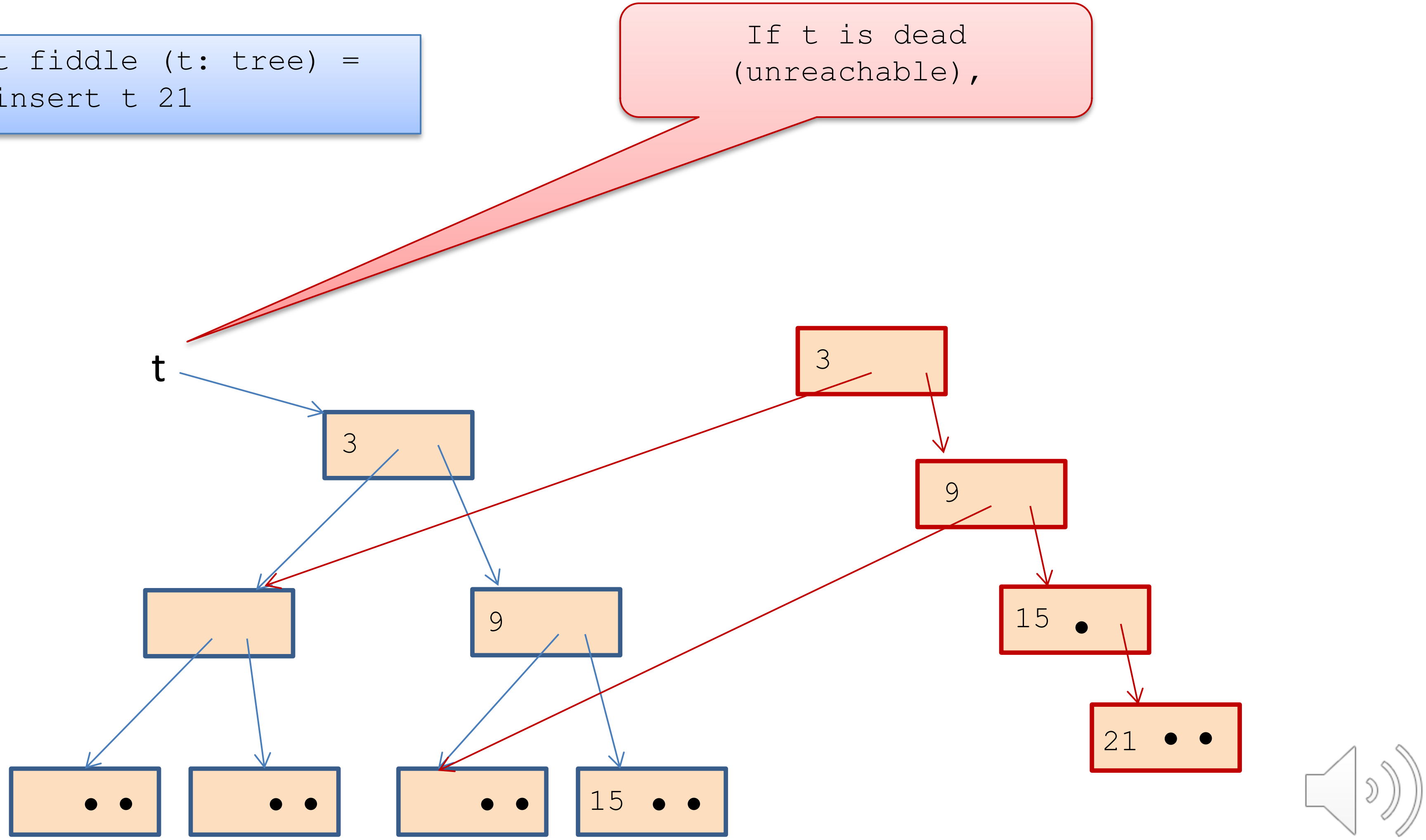

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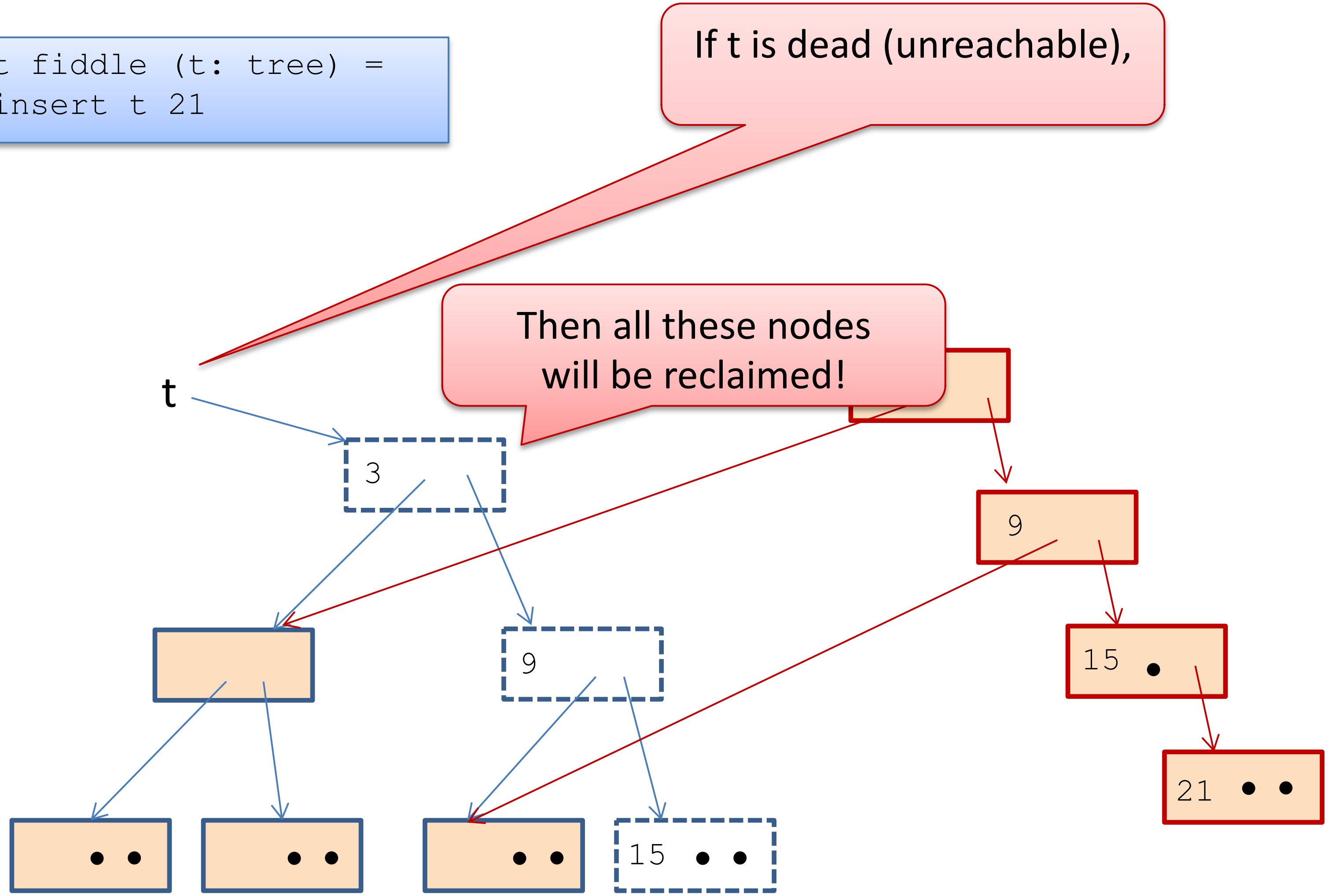

Net space allocated

The garbage collector reclaims unreachable data structures on the heap.

```
let fiddle (t: tree) =
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```
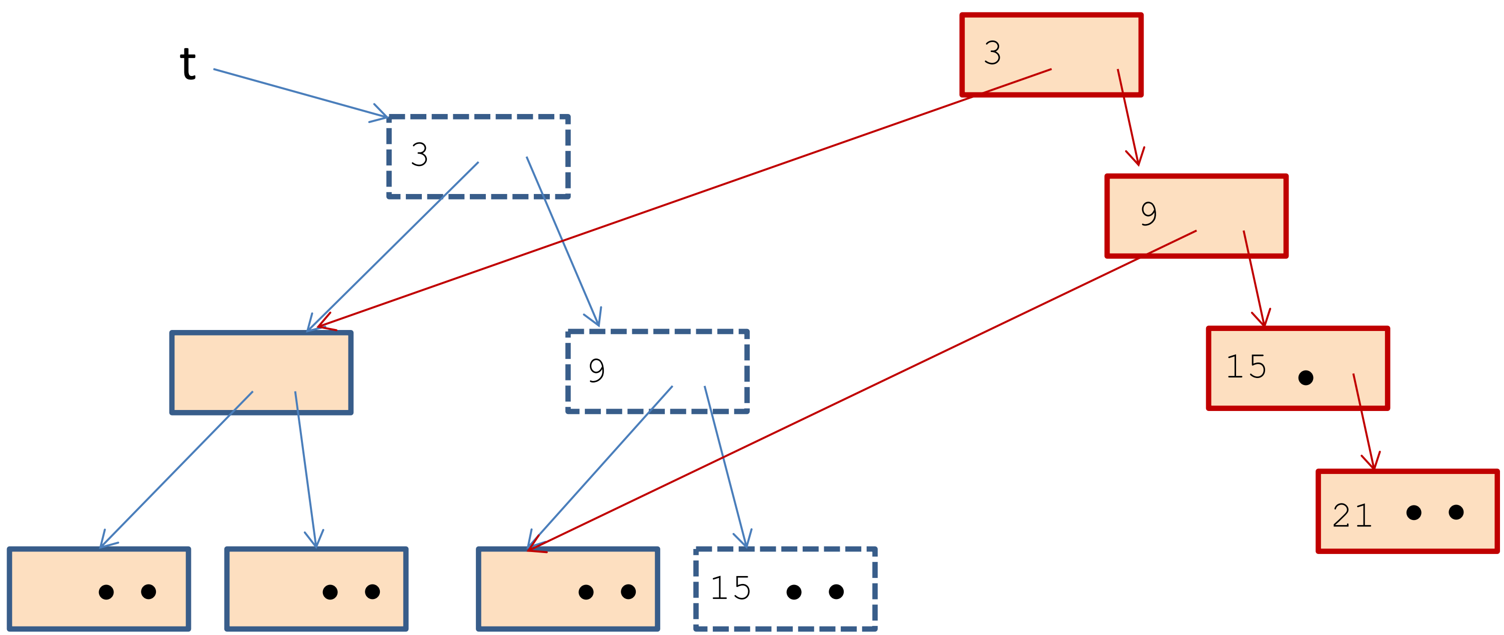

Net new space allocated:
1 node

(just like "imperative" version of binary search trees)



Net space allocated

But what if you want to keep the old tree?

```
let faddle (t: tree) =
  (t, insert t 21)
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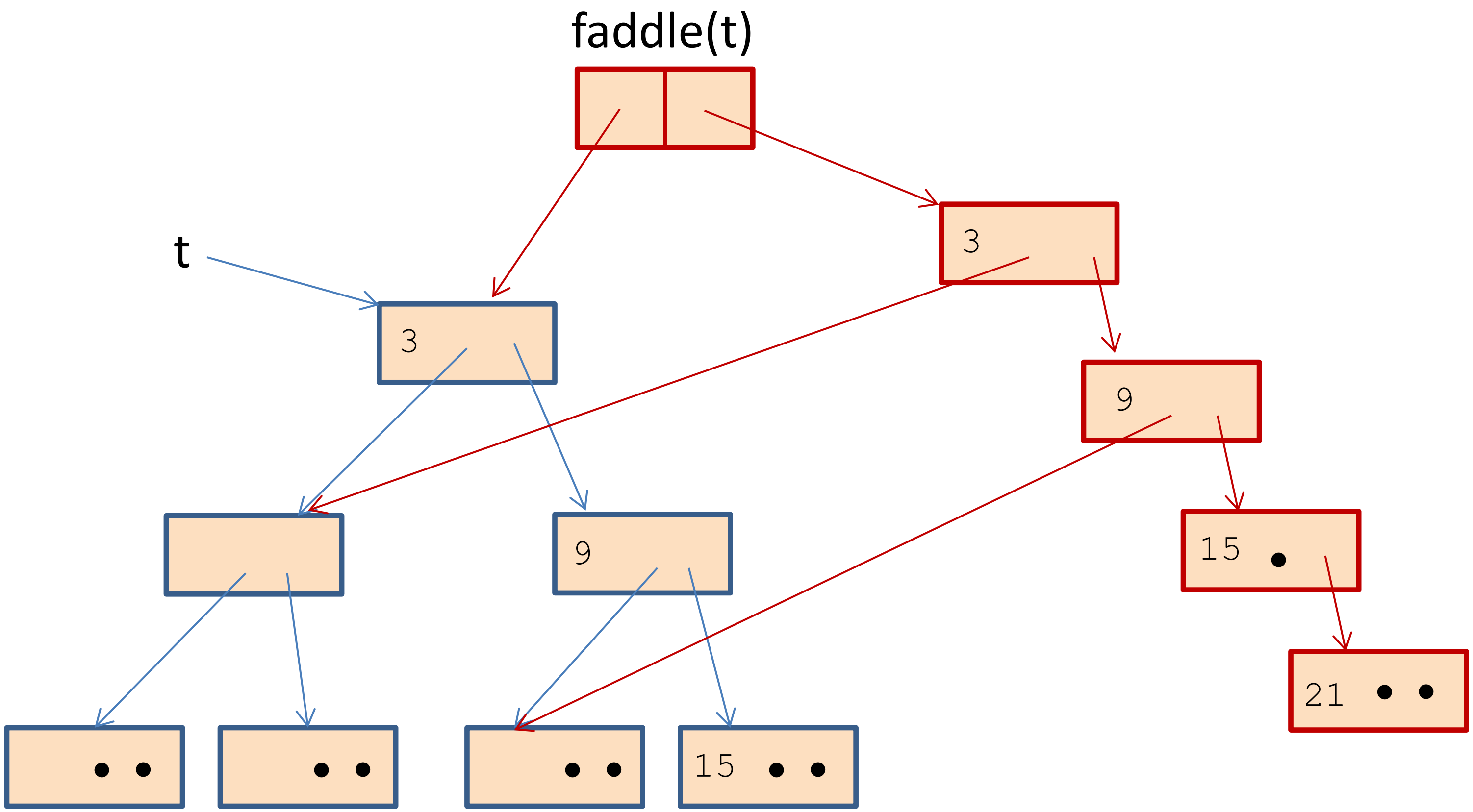

Net space allocated

But what if you want to keep the old tree?

Net new space allocated:
log(N) nodes

but note: “imperative” version would have to copy the old tree, space cost N new nodes!

```
let faddle (t: tree) =
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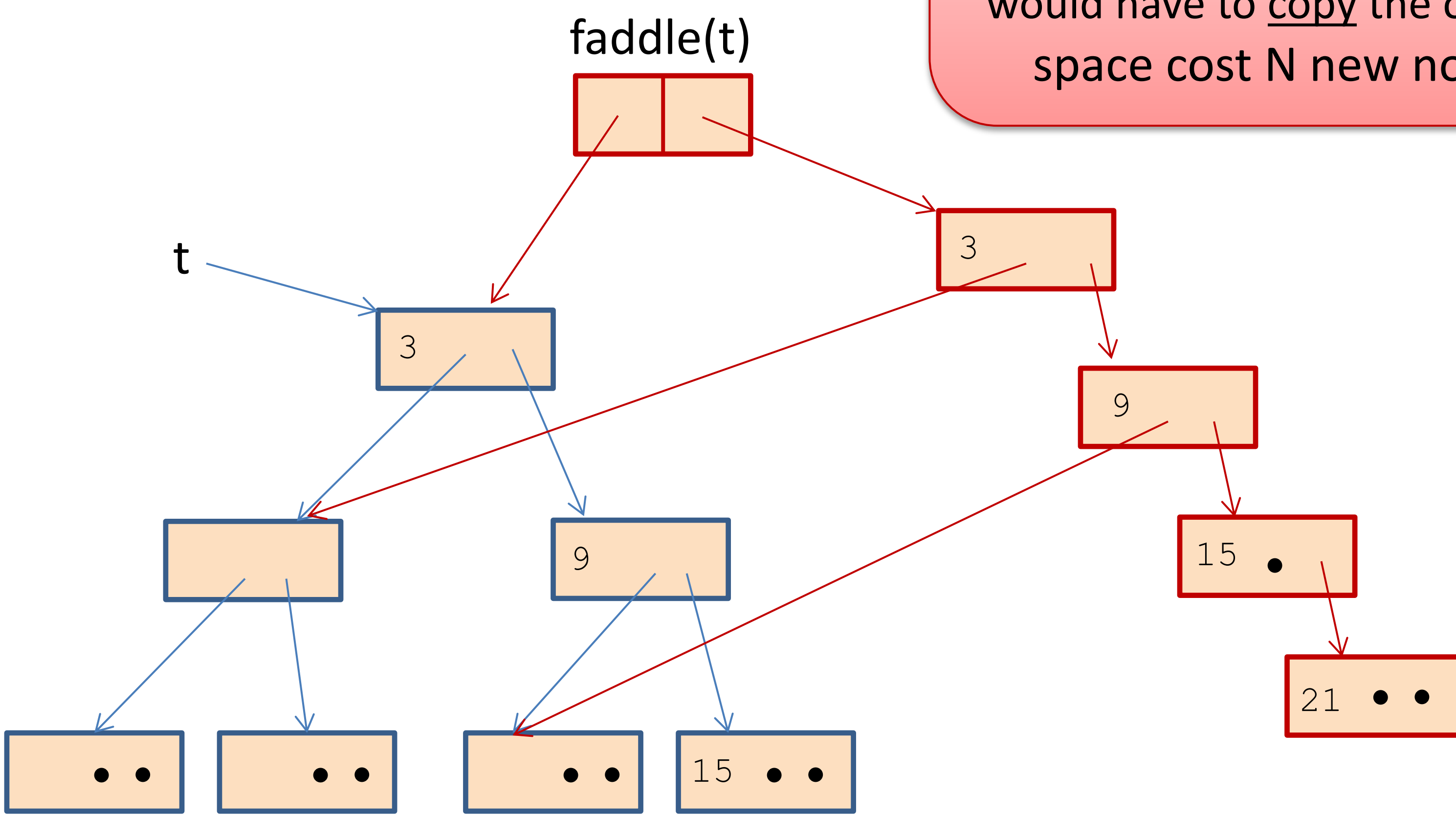

Compare


```
let check_option (o:int option) : int option =
  match o with
    Some _  -> o
  | None -> failwith "found none"
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let check_option (o:int option) : int option =
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allocates nothing when arg is Some i

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Compare


```
let cadd (c1:int*int) (c2:int*int) : int*int =
  let (x1,y1) = c1 in
  let (x2,y2) = c2 in
  (x1+x2, y1+y2)
```

```
let double (c1:int*int) : int*int =
  let c2 = c1 in
  cadd c1 c2
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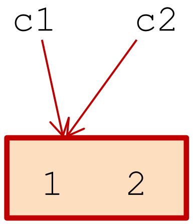

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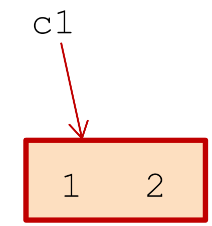

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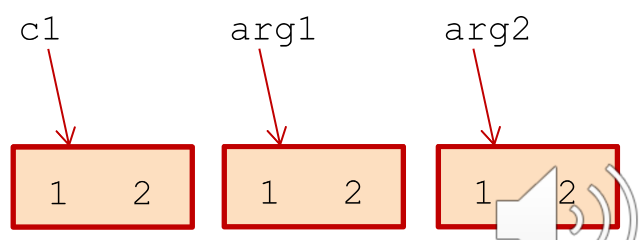

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no allocation

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let double (c1:int*int) : int*int =
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  cadd (x1,y1) (x1,y1)
```

allocates 2 pairs
(unless the compiler
happens to optimize…)

Compare


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```

```
let double (c1:int*int) : int*int =
  let (x1,y1) = c1 in
  cadd c1 c1
```

double does not allocate

extracts components: it is a read

FUNCTION CLOSURES

Closures (A reminder)

Nested functions like bar often contain free variables:

```
let foo y =
  let bar x = x + y in
  bar
```

Here's bar on its own:

```
let bar x = x + y
```

y is *free* in the definition of bar

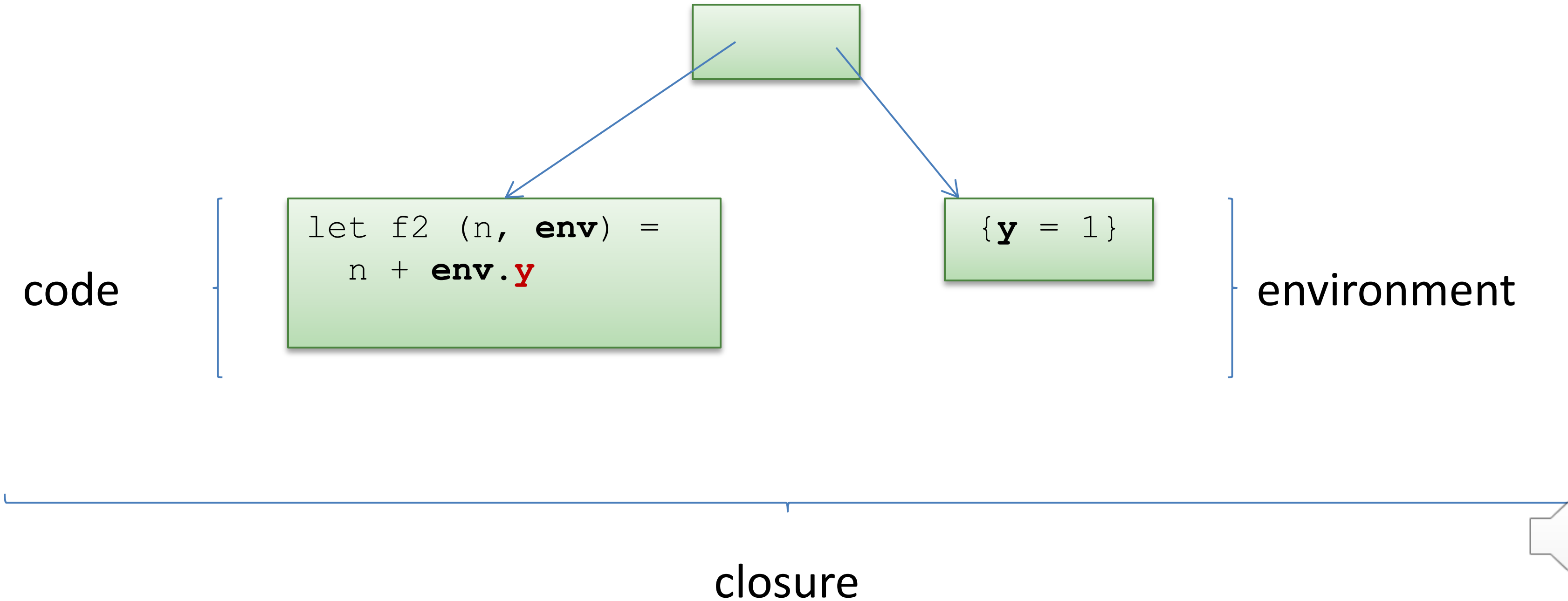
To implement bar, the compiler creates a *closure*, which is a pair of code for the function plus an environment holding the free variables.

But what about nested, higher-order functions?

bar again:

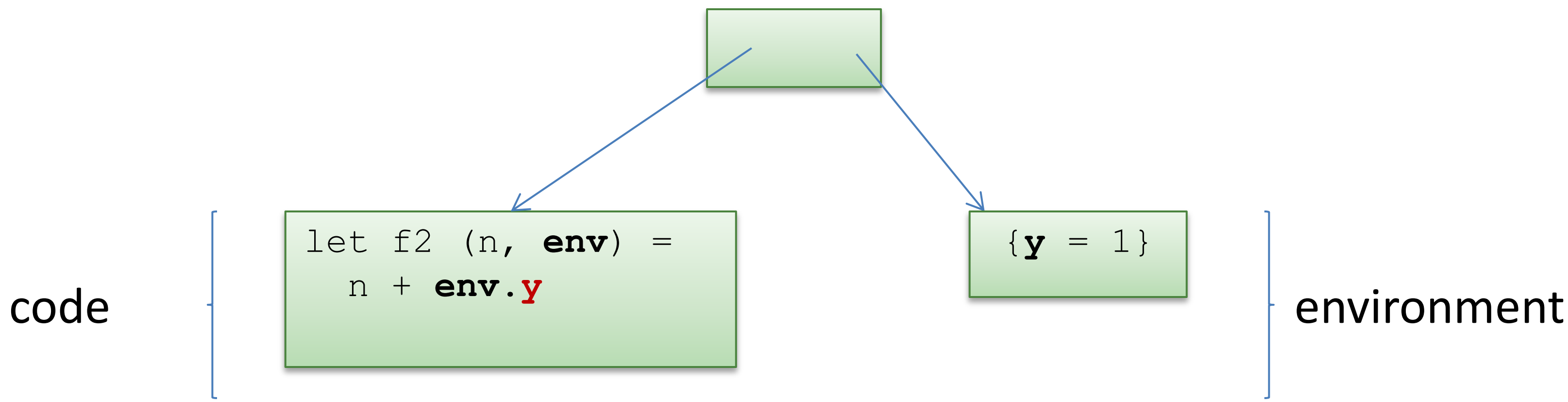
```
let bar x = x + y
```

bar's representation:



But what about nested, higher-order functions?

To estimate the (heap) space used by a program, we often need to estimate the (heap) space used by its closures.



Our estimate will include the cost of the pair:

- two pointers = 2 words (8 bytes each, or 4 bytes each on some machines)
- the cost of the environment (1 word in this case).
- but not: the cost of the code (because the same code is reused in every closure of this function)

Space Model Summary

Understanding space consumption in FP involves:

- understanding the difference between
  - live space
  - rate of allocation
- understanding where allocation occurs
  - any time a constructor is used
  - whenever closures are created
- understanding the costs of
  - data types (fairly similar to Java)
  - costs of closures (cost of a pair of pointers + environment)

Exercise

```
let rec gen n =
  if n <= 0 then
    []
  else
    n::gen (n-1)

let rec goo n =
  if n <= 0 then
    []
  else
    (fun () -> gen n)::goo (n-1)

let rec gah n =
   if n <= 0 then
     []
   else
     let l = gen n in
     (fun () -> l)::gah (n-1)
```

Assume 8-byte words. Estimate the size of the data structure generated by a call to `goo` (respectively `gah`) in terms of their arguments n. Explain your work. Discuss.