General observations:

Quite a few people were confused by how area relates to length and width or radius. It’s a quadratic relationship, most easily seen with the area of a circle: pi r^2. That’s the square of the radius: if you double the radius, the area goes up by a factor of four. There’s no need to do any more arithmetic than multiplying by 4. For circles, the pi and the actual values of the radii don’t matter, just the ratio. For rectangles, the specific heights and widths would not matter. And as a distant early warning in case we ever move into 3 dimensions, the same idea applies to volume, except it’s cubic, so doubling the radius of a sphere produces 8 times the volume.

**Problem 1: [15 pts, 3 each]**

The first three parts are just arithmetic with powers of 10, but you have to keep the dimensionality right: there are 10^4 x 10^4 square microns in a square centimeter, for example.

As noted above, if you double the radius, the area goes up by a factor of four. That answers parts (d) and (e).

(a) Area of focal plane in square meters?
0.32 (approx). 10 microns is $10^{-5}$ meters, so the number of 10 micron x 10 micron squares is $3.2 \times 10^9 \times 10^{-5} \times 10^{-5}$.

Many people simply divided microns by $10^6$ to get meters, but failed to square the result. This problem is about areas. And as a sort of sanity check, suppose the focal plane was 320,000 square meters, the most popular wrong answer. That’s a square of nearly 2,000 feet on a side, which is rather larger than the main part of campus. Does that sound a bit large for a camera?

(b) MB of memory for the pixels?

$$3.2 \times 10^9 \times 3 \text{ bytes} = 9.6 \times 10^9 \text{ B} / 10^6 = 9,600 \text{ MB}.$$  Some people got confused by bits. Others approximated 9,600 by 10,000, probably intimidated by my warnings about excess precision. No penalty, but this is a place where the answer is exact.

(c) How many transistors in 1 cm x 1 cm?

**10^10 or 10 billion.** There are 100 in a square 1 micron by 1 micron, and there are $10^4$ microns in a cm, so $10^4 \times 10^4$ square microns in a square cm, so $10^4 \times 10^4 \times 10^2$. Again, quite a few people didn’t do a proper area computation and thus were off by large factors.

(d) How many 1-inch patties?

**48.** A one-inch patty is 1/4 of the area of a 2 inch, so there are 4 times as many. No need for any arithmetic beyond that.

(e) How many 4-inch patties?

**3.** Same thing: area is proportional to the square of the radius.

**Problem 2: [9 pts, 3 each]**

(a) Total number of handshakes?

36: each justice (9 of them) shakes hands with each other justice (8 of them), but divide by two so each is counted only once.

(b) How does the number of handshakes grow in proportion to n?

**$n^2$ (quadratic),** yet another example of “every one does something to every one.” I did want to see a formula that included an $n^2$, since that’s the point. Saying that every new person has to shake hands with the other $n-1$ is correct but isn’t enough.
(c) How many comparisons to look up a name in the whole-earth phone book?

**33.** About 8 billion people, which is just under $2^{33}$. You really need 33 tests; 32 would only handle 4 billion people. The phone book is sorted, so binary search is the way to go. A number of people computed $n \log n$ and/or mentioned Quicksort. This is not a sorting problem.

**Problem 3:** [12 pts, 3 each]

(a) What year will processors be twice as fast as they are today?

**24 years, or 2045.** 72 / 3%/year. The question might have been ambiguous about the starting year, so 2041 is ok too. (I originally wrote 2044, a slip of the pen; sorry.)

(b) What is the annual rate of tuition increase?

**6% (roughly).** Quadruples in 25 years, so it doubles in about 12 years. 5.76% is too precise, a strong sign of ignoring the injunction against using a calculator.

(c) What will the endowment be worth in 25 years?

I bungled the numbers here, thus making the answer not as clean as intended. At 5%, the endowment would double in about 14 years, and quadruple in 28 years, to $100B. But 25 years is less than 28, so the answer would be more like $85B (which you can also see by computing $1.05^{25} \times 25$, or by taking 15% off $100B). Any confusion here was my fault.

(d) How many years for Dartmouth to catch up to Princeton?

This time I did get the numbers clean. It’s probably easiest to say that Dartmouth doubles every 6 years, so its sequence is 6, 12, 24, 48, 96 in 24 years, while Princeton doubles every 12 years, so its sequence is 24, 48, 96 in 24 years.

But not everyone read far enough to see that I had changed the numbers to try to make it easier. Full marks if your answer was consistent with the numbers in part (c) instead.