Homework 3 Solutions

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Section 3.6 #36; Section 5.8 #21,25,27,30,37

3.6 #36 If there are k types of objects, each with infinite repetition number, then the number of r-combinations is $\binom{r+k-1}{r}$.

We have two types of r-combinations:

- Containing r objects from types $2, ..., k$: $\binom{r+(k-1)-1}{r}$ combinations.
- Containing $r-1$ objects from types $2, ..., k$ and item a_1: $\binom{(r-1)+(k-1)-1}{r-1}$ combinations.

The solution is the sum of the above.

5.8 #21 If $k > 0$ then

$$\begin{aligned}\binom{-r}{k} &= \frac{-r(-r-1)\cdots(-r-k+1)}{k!} \\ &= \frac{(-1)^k r(r+1)\cdots(r+k-1)}{k!} \\ &= (-1)^k\binom{r+k-1}{k}.\end{aligned}$$

If $k = 0$, both are defined to be 1. If $k < 0$ both are defined to be 0.

5.8 #25 Suppose we want the number of ways to select n items from two groups, with m_1 and m_2 items respectively. There are $\binom{m_1+m_2}{n}$ combinations.

To select n items, we can choose 0 from group 1 and n from group 2, etc., or i items from group 1 and $n-i$ items from group 2, for $0 \leq i \leq n$. For each i, there are $\binom{m_1}{i}\binom{m_2}{n-i}$ combinations. This combinatorial argument shows our claim.

Equation (5.11) follows by letting $m_1 = m_2$.

5.8 #27 Clearly, the clutter that is the 2-combination of S contains 6 items. Consider clutters of other sizes.

- 1-combinations: Wlog, suppose the clutter contains the item 1. All the other combinations are combinations of $\{2, 3, 4\}$. By Theorem 5.4.3, that leaves at most $\binom{3}{\lfloor 3/2 \rfloor}$ or 3 combinations.
- 4-combinations: Obviously, if the clutter contains S then that is the only set in the clutter.
- 3-combinations: We cannot have a 1- or 4-combination. If the clutter contained a 3-combination, say $\{1, 2, 3\}$ wlog, then other candidate sets in the clutter are: $\{1, 4\}$, $\{2, 4\}$, $\{3, 4\}$, $\{1, 2, 4\}$, $\{1, 3, 4\}$, $\{2, 3, 4\}$. The presence of each of the two-combinations rules out two of the 3-combinations. This prevents us from getting a clutter of size 6.

5.8 #30 10 symmetric chains:
$\emptyset \subset \{1\} \subset \{1, 2\} \subset \{1, 2, 3\} \subset \{1, 2, 3, 4\} \subset \{1, 2, 3, 4, 5\}$
$\{5\} \subset \{1, 5\} \subset \{1, 2, 5\} \subset \{1, 2, 3, 5\}$
$\{4\} \subset \{1, 4\} \subset \{1, 2, 4\} \subset \{1, 2, 4, 5\}$
$\{4, 5\} \subset \{1, 4, 5\}$
$\{2\} \subset \{2, 3\} \subset \{2, 3, 4\} \subset \{2, 3, 4, 5\}$
$\{2, 5\} \subset \{2, 3.5\}$
$\{2, 4\} \subset \{2, 4, 5\}$
$\{3\} \subset \{1, 3\} \subset \{1, 3, 4\} \subset \{1, 3, 4, 5\}$
$\{3, 5\} \subset \{1, 3, 5\}$
$\{3, 4\} \subset \{3, 4, 5\}$

5.8 #37 Coefficient of $x_1^3 x_2 x_3^4 x_4^2$ in $(x_1 - x_2 + 2x_3 - 2x_4)$ is

$$\binom{9}{3;3;1;2}(1)^3(-1)^3(2)^1(-2)^2 = \frac{9!}{3!3!1!2!} \cdot -8 = -40320$$