COS 341, November 14, 2001
Due: November 28, 2001

Homework Set 7

Written Assignments

Do exercises 2, 6, 10 and 11 in Section 9.5.

Do the following special problem:

Special Problem (Counted as 2 exercises)

The purpose is to give an alternative way to analyze the algorithm in Handout 9. For each $1 \leq j \leq n$, let Y_j denote the random variable such that, for any $u = (i_1, i_2, \cdots, i_n) \in U$, $Y_j(u) = 1$ if $i_j = \max\{i_1, i_2, \cdots, i_j\}$ and $Y_j(u) = 0$ otherwise. That is, $Y_j = 1$ if and only if we have a left-to-right maximum at location j. Clearly,

$$Y = \sum_{1 \leq j \leq n} Y_j. \tag{1}$$

(a) Prove that $E(Y_j) = 1/j$ for all $1 \leq j \leq n$, and use equation (1) to prove $E(Y) = H_n$.
(b) Prove that $E(Y_k Y_j) = 1/(kj)$ for all $1 \leq k < j \leq n$.
(c) Use parts (a), (b) and equation (1) to prove that $E(Y^2) = H_n + (H_n)^2 - H_n^{(2)}$.
(d) Prove that $Var(Y) = H_n - H_n^{(2)}$.