COS 341, October 10, 2001
Due: October 17, 2001

Homework Set 4

Reading Assignments Start reading Chapter 7.

Written Assignments

Do exercises 18 and 20 in Section 5.8.

Do the following problems:

Special Problem 1 (counted as 1 exercise) Evaluate $f(n) = \sum_{1 \le k \le n} k7^k \binom{n}{k}$. Your answer should be a closed-form expression as a function of n.

Special Problem 2 (counted as 1 exercise) Show that

$$\sum_{0 \le k \le n} \frac{(2n)!}{(k!)^2((n-k)!)^2} = \binom{2n}{n}^2.$$

Special Problem 3 (counted as 1 exercise) Evaluate

$$g(n) = \sum_{\substack{0 \le k \le n \\ k \bmod 3 = 0}} \binom{n}{k}.$$

Your answer should be a closed-form expression as a function of n. (You may want to use the fact that $1 + u + u^2 = 0$ where $u = e^{i2\pi/3}$.)

Special Problem 4 (counted as 2 exercises) Consider a sequence of $2n$ people in a line at a cashier. Suppose n of the people pay \$1 and n of the people get \$1. A *paying pattern* is a binary sequence $\sigma = a_1 a_2 a_3 \cdots a_{2n}$ with exactly n $1's$ and n $0's$; the interpretation is that $a_j = 1$ if person j pays \$1, and $a_j = 0$ otherwise. Note that there are exactly $\binom{2n}{n}$ paying patterns. Prove that the number of paying patterns in which the cashier never goes in debt (ie, at every stage at least as many people have paid in \$1 as were paid out \$1) is equal to $\binom{2n}{n} - \binom{2n}{n+1}$. (**Hint:** Show a one-to-one correspondence between paying patterns where at some stage the cashier goes at least \$1 in debt and all binary sequences of length $2n$ with exactly $n+1$ $1's$.)