More on Vectors and Matrices

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Perspectives on Vectors and Matrices

• Physicist vs. Mathematician vs. Computer Scientist

• Things we do in Computer Science and Machine Learning that make Physicists and Mathematicians go hmm…
Vectors with Different Units Per Dimension

- Housing

\[
\begin{bmatrix}
2600 \text{ ft}^2 \\
$300,000
\end{bmatrix}
\]
Vectors with Different Units Per Dimension

- Web page ranking

\[
\begin{bmatrix}
3 \text{ keyword mentions} \\
42 \text{ incoming links}
\end{bmatrix}
\]
Vectors with Different Units Per Dimension

- Health monitoring

\[
\begin{bmatrix}
72 \text{ beats/min} \\
123 \text{ mm Hg}
\end{bmatrix}
\]
Linear Operations “Accommodate” Units

- Components with different “units” add

\[
\begin{bmatrix}
3 \text{ keyword mentions} \\
42 \text{ incoming links}
\end{bmatrix} + \begin{bmatrix}
5 \text{ keyword mentions} \\
17 \text{ incoming links}
\end{bmatrix} = \begin{bmatrix}
8 \text{ keyword mentions} \\
59 \text{ incoming links}
\end{bmatrix}
\]

Web page #1  Web page #2  Web page collection
One way of thinking about dot product is “weighting” dimensions or “probing” individual components, while accommodating units.

\[
\begin{bmatrix}
72 \text{ beats/min} \\
123 \text{ mm Hg}
\end{bmatrix} \cdot \begin{bmatrix}
1 \text{ unit risk} / (\text{beat/min}) \\
0.5 \text{ units risk} / (\text{mm Hg})
\end{bmatrix} = 133.5 \text{ units risk}
\]

\[
\begin{bmatrix}
72 \text{ beats/min} \\
123 \text{ mm Hg}
\end{bmatrix} \cdot \begin{bmatrix}
1 \\
0
\end{bmatrix} = 72 \text{ beats/min}
\]
• We will often go back and forth between arrays of numbers, matrices, and “unrolled” vectors
Linear Algebra on Images

- Digital images are arrays of pixels
  - Value at each pixel is intensity of light
  - For color, store intensity in Red, Green, Blue channels
    (3 channels enough because of human visual system)
- Can “unroll” an image, treat it as a vector in a (high-dimensional) vector space
  - Light is linear! *(But images are often nonlinearily mapped)*
  - Can perform usual manipulations, such as …
Average Princetonian Face

- From 2005 BSE thesis project by Clay Bavor and Jesse Levinson
Vector Spaces of Images

\[ \text{Image 1} = \text{Image 2} + \text{Image 3} \]
Detecting Princetonians

Matching response (darker = better match)

[Bavor & Levinson]