Algorithms

Robert Sedgewick | Kevin Wayne

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5.5 Data Compression

- *introduction*
- *run-length encoding*
- *Huffman compression*
- *LZW compression*

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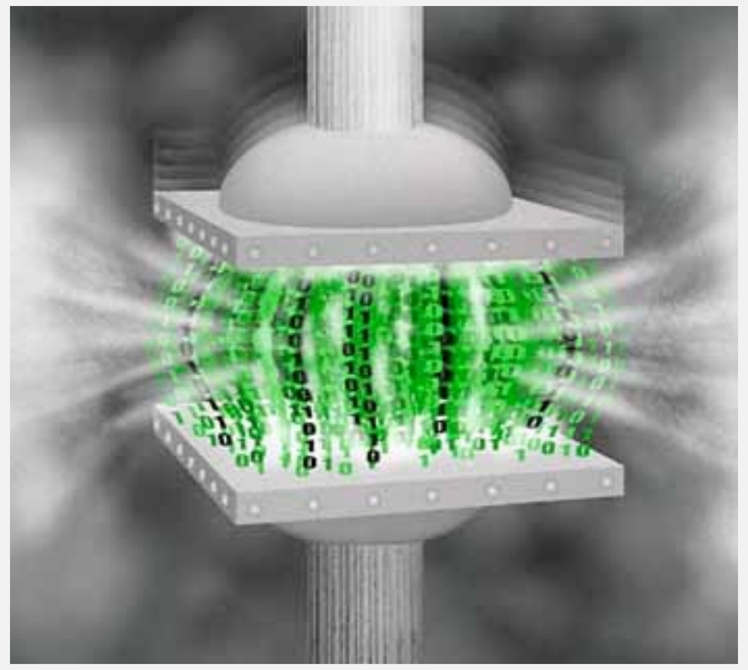
- *LZW compression*

Data compression

Compression reduces the size of a file:

- To save space when storing it.
- To save time when transmitting it.
- Most files have lots of redundancy.



Who needs compression?

- Moore's law: # transistors on a chip doubles every 18–24 months.
- Parkinson's law: data expands to fill space available.
- Text, images, sound, video, sensors, …

Every day, we create:

- 500 million Tweets.
- 300 billion emails.
- 350 million Facebook photos.
- 500,000 hours YouTube video.

Basic concepts ancient (1950s), best technology recently developed.

Applications

Generic file compression.

- Files: Gzip, bzip2, 7z, PKZIP, ...
- File systems: ZFS, HFS+, ReFS, GFS, APFS, ...

Multimedia.

- Images: GIF, JPEG, PNG, RAW, ...
- Sound: MP3, AAC, Ogg Vorbis, ...
- Video: MPEG, HDTV, H.264, HEVC, ...

Communication. Fax, Skype, WeChat, Zoom, ...

Databases. SQL, Google, Facebook, NSA, ...

Smart sensors. Phone, watch, car, health, ...

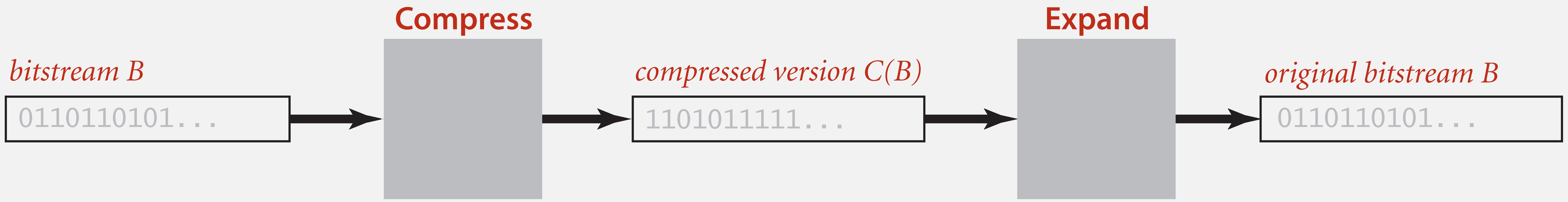
Lossless compression and expansion

Message. Bitstream B we want to compress.

Compress. Generates a “compressed” representation $C(B)$.

Expand. Reconstructs original bitstream B.

uses fewer bits (you hope)



Compression ratio. Bits in $C(B)$ / bits in B.

Ex. 50–75% or better compression ratio for English language.

Data representation: genomic code

Genome. String over the alphabet `{ A, T, C, G }`.

Goal. Encode an n-character genome: `A T A G A T G C A T A G . . .`

Standard ASCII encoding.

- 8 bits per char.
- $8\,n$ bits.

char	hex	binary
'A'	41	01000001
'T'	54	01010100
'C'	43	01000011
'G'	47	01000111

Two-bit encoding.

- 2 bits per char.
- $2\,n$ bits. ← compression ratio = 25% (compared to ASCII)

char	binary
'A'	00
'T'	01
'C'	10
'G'	11

Fixed-length code. k-bit code supports alphabet of size 2^k.

Amazing but true. Some genomic databases in 1990s used ASCII.

Binary representation

Q. How to examine the contents of a bitstream (e.g., when debugging)?

standard character stream

```
~> more abra.txt
ABRACADABRA!
```

bitstream represented with hex digits

```
~> java HexDump < abra.txt
41 42 52 41 43 41 44 41 42 52 41 21
96 bits
```

lots of unprintable characters
(don't `System.out.print()` binary data)

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2	SP	!	"	#	$	%	&	'	(	)	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[	\	]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{	|	}	~	DEL

hexadecimal-to-ASCII conversion table

bitstream represented as 0 and 1 characters

```
~> java BinaryDump < abra.txt
0100000101000010
0101001001000001
0100001101000001
0100010001000001
0100001001010010
0100000100100001
96 bits
```

bitstream of binary file represented with hex digits

```
~> java HexDump < us.gif
47 49 46 38 39 61 8e 01 01 01 d5 00 00 94 18 29
06 02 03 84 29 4a 6b 18 4a 73 29 6b 6e 5d 6e 4a
45 4a f7 ef f7 42 00 52 1f 00 37 42 10 6b 31 00
63 31 08 5a 7e 70 8d 36 08 6b 4a 21 7b 5b 36 86
...
b7 de 7b f3 dd b7 df 7f 03 1e 38 cc 41 00 00 3b
99200 bits
```

Reading and writing binary data

Binary standard input. Read bits from standard input.

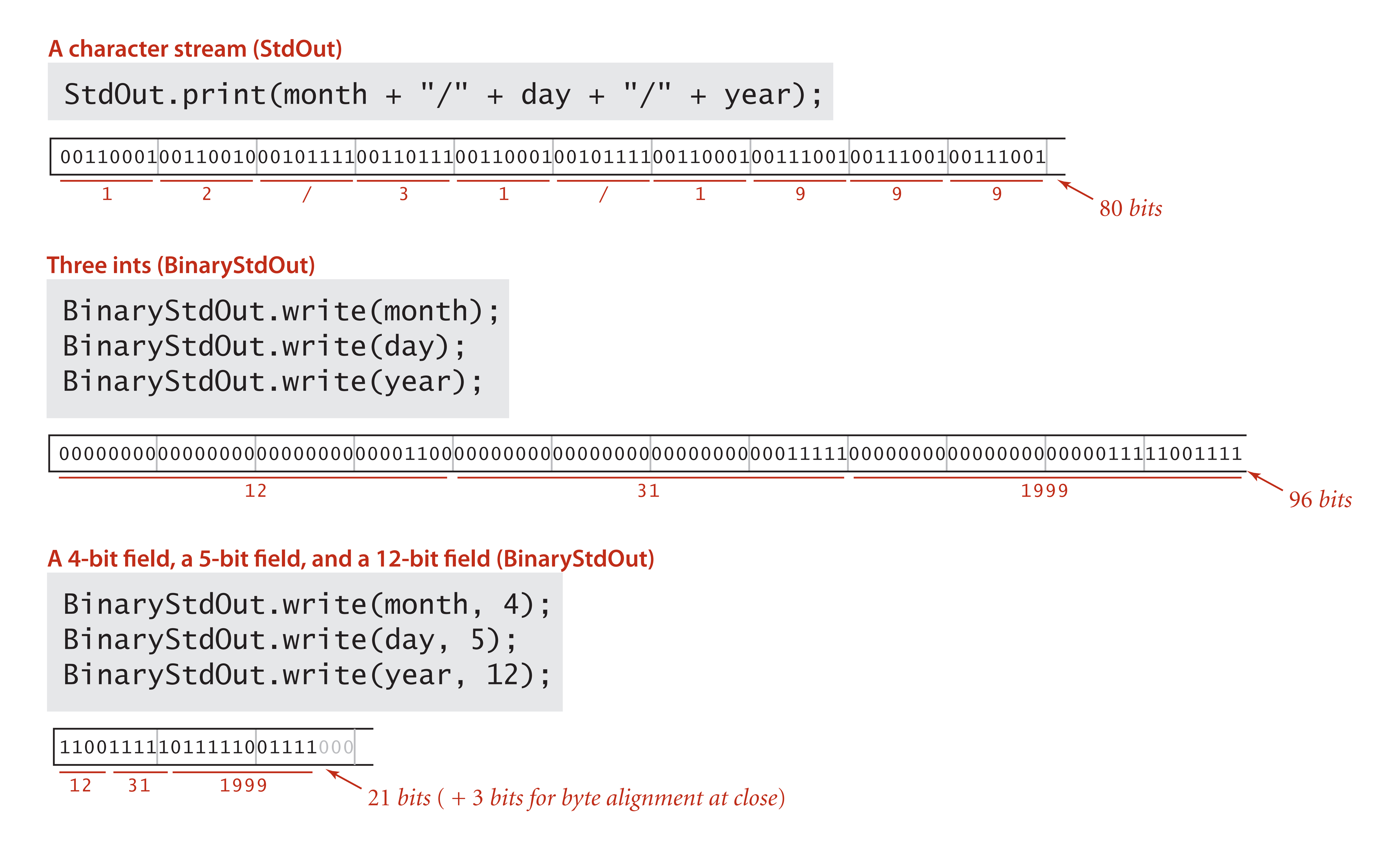
public class BinaryStdIn		
`boolean`	`readBoolean()`	*read 1 bit of data and return as a* `boolean` *value*
`char`	`readChar()`	*read 8 bits of data and return as a* `char` *value*
`char`	`readChar(int r)`	*read* `r` *bits of data and return as a* `char` *value*
[similar methods for `byte` *(8 bits);* `short` *(16 bits);* `int` *(32 bits);* `long` *and* `double` *(64 bits)]*		
`boolean`	`isEmpty()`	*is the bitstream empty?*
`void`	`close()`	*close the bitstream*

Binary standard output. Write bits to standard output.

public class BinaryStdOut		
`void`	`write(boolean b)`	*write the specified bit*
`void`	`write(char c)`	*write the specified 8-bit* `char`
`void`	`write(char c, int r)`	*write the* `r` *least significant bits of the specified* `char`
[similar methods for `byte` *(8 bits);* `short` *(16 bits);* `int` *(32 bits);* `long` *and* `double` *(64 bits)]*		
`void`	`close()`	*close the bitstream*

Writing binary data

Date representation. Three different ways to represent 12/31/1999.



A character stream (StdOut)

```
StdOut.print(month + "/" + day + "/" + year);
```

```
00110001|00110010|00101111|00110111|00110001|00101111|00110001|00111001|00111001|00111001
    1        2        /        3        1        /        1        9        9        9
```

80 bits

Three ints (BinaryStdOut)

```
BinaryStdOut.write(month);
BinaryStdOut.write(day);
BinaryStdOut.write(year);
```

```
00000000|00000000|00000000|00001100|00000000|00000000|00000000|00011111|00000000|00000000|00000111|11001111
                  12                                  31                                 1999
```

96 bits

A 4-bit field, a 5-bit field, and a 12-bit field (BinaryStdOut)

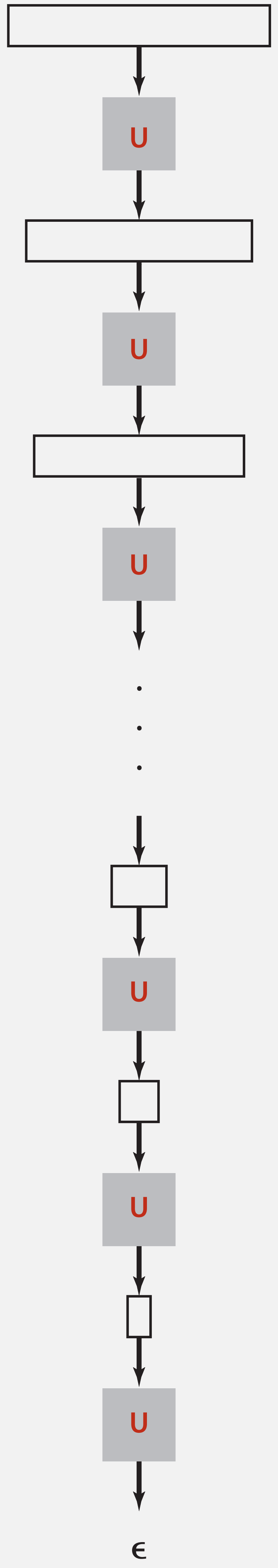
```
BinaryStdOut.write(month, 4);
BinaryStdOut.write(day, 5);
BinaryStdOut.write(year, 12);
```

```
11001111|10111110|01111000
 12  31     1999
```

21 bits (+ 3 bits for byte alignment at close)

Universal data compression

Proposition. No algorithm can compress every bitstring.



Pf 1. [by contradiction]

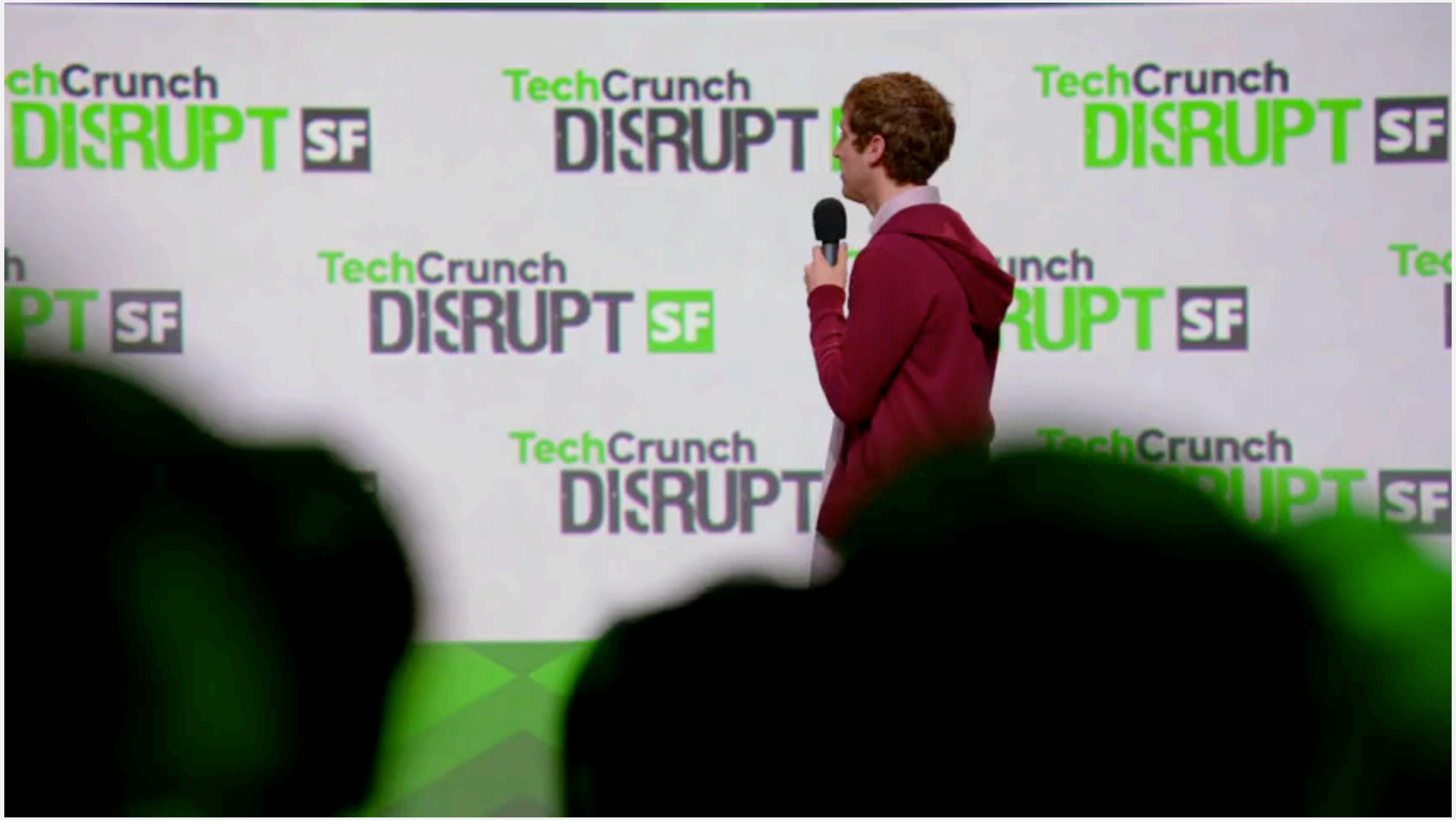
ASOK, TAKE THESE PROJECT SUMMARIES AND SUMMARIZE THEM INTO ONE SUMMARY.
scottadams@aol.com
www.dilbert.com
AND WHEN YOU'RE DONE, TAKE THAT SUMMARY AND SUMMARIZE IT.
© 2002 United Feature Syndicate, Inc.
12/04/02
WHAT IS A SHORTER WORD FOR DOOMED?

Pf 2. [by counting]

- Suppose your algorithm that can compress all $1{,}000$-bit strings.
- 2^{1000} possible bitstrings with $1{,}000$ bits.
- Only $1 + 2 + 4 + \ldots + 2^{998} + 2^{999} \;=\; 2^{1000} - 1$ can be encoded with ≤ 999 bits.
- Similarly, only 1 in 2^{499} bitstrings can be encoded with ≤ 500 bits!

Universal data compression

Pied Piper. Claims 1:3.8 lossless compression of arbitrary data.



Data compression: quiz 1

Did Pied Piper achieve a lossless compression ratio of 1:3.8 for arbitrary files?

A. Yes.

B. No.

Rdenudcany in Enlgsih lnagugae

Q. How much redundancy in the English language?

A. Quite a bit.

> “ *… randomising letters in the middle of words [has] little or no effect on the ability of skilled readers to understand the text. This is easy to denmtrasote. In a pubiltacion of New Scnieitst you could ramdinose all the letetrs, keipeng the first two and last two the same, and reibadailty would hadrly be aftcfeed. My ansaylis did not come to much beucase the thoery at the time was for shape and senqeuce retigcionon. Saberi’s work sugsegts we may have some pofrweul palrlael prsooscers at work. The resaon for this is suerly that idnetiyfing coentnt by paarllel prseocsing speeds up regnicoiton. We only need the first and last two letetrs to spot chganes in meniang.*” — Graham Rawlinson

The gaol of data cmperisoson is to inetdify rdenudcany and epxloit it.

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- *LZW compression*

Run-length encoding (RLE)

Simple type of redundancy in a bitstream. Long runs of repeated bits.

$$\underbrace{0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0}_{\text{run of length 15}}\,\underbrace{1\,1\,1\,1\,1\,1\,1}_{\text{run of length 7}}\,0\,0\,0\,0\,0\,0\,0\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1\,1 \longleftarrow \text{40 bits}$$

Representation. 4-bit counts to represent alternating runs of 0s and 1s: 15 0s, then 7 1s, then 7 0s, then 11 1s.

$$\underbrace{1\,1\,1\,1}_{15}\,\underbrace{0\,1\,1\,1}_{7}\,\underbrace{0\,1\,1\,1}_{7}\,\underbrace{1\,0\,1\,1}_{11} \longleftarrow \text{16 bits (instead of 40)}$$

Q. How many bits r to store each run length?

A. Typically 8 bits (but 4 on this slide for brevity).

Q. What to do when run length exceeds max count $2^r - 1$?

A. Intersperse runs of length 0.

Applications. JPEG, TIFF, BMP, ITU-T T4 Group 3 Fax, ...

Run-length encoding: Java implementation

```
public class RunLength
{

   public static void compress(int r)
   {  /* see textbook */  }

   public static void expand(int r)
   {
      boolean bit = false;
      while (!BinaryStdIn.isEmpty())
      {
         int run = BinaryStdIn.readInt(r);
         for (int i = 0; i < run; i++)
            BinaryStdOut.write(bit);
         bit = !bit;
      }
      BinaryStdOut.close();
   }

}
```

number of bits per count

initial run is of 0s

read count from standard input (8 bits)

write run of 0s or 1s to standard output

flip bit (for next run)

pad last byte with 0s (if needed)
and close output stream

Data compression: quiz 2

What is the best compression ratio achievable from run-length encoding when using 8-bit counts?

A. 1 / 256

B. 1 / 16

C. 8 / 255

D. 1 / 8

E. 16 / 255

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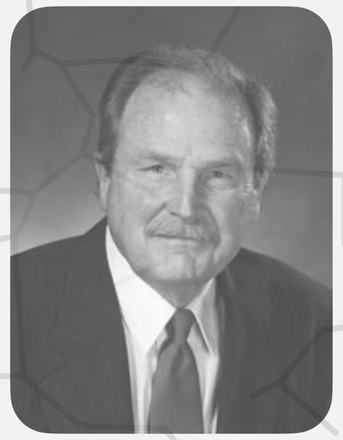


David Huffman

Variable-length codes

Key idea. Use different number of bits to encode different characters.

Ex. Morse code: ● ● ● — — — ● ● ●

Issue. Ambiguity.

SOS ?

VZE ?

EEJIE ?

EEWNI ?

A	● ▬	N	▬ ●
B	▬ ● ● ●	O	▬ ▬ ▬
C	▬ ● ▬ ●	P	● ▬ ▬ ●
D	▬ ● ●	Q	▬ ▬ ● ▬
E	●	R	● ▬ ●
F	● ● ▬ ●	S	● ● ●
G	▬ ▬ ●	T	▬
H	● ● ● ●	U	● ● ▬
I	● ●	V	● ● ● ▬
J	● ▬ ▬ ▬	W	● ▬ ▬
K	▬ ● ▬	X	▬ ● ● ▬
L	● ▬ ● ●	Y	▬ ● ▬ ▬
M	▬ ▬	Z	▬ ▬ ● ●

codeword for S is a prefix of codeword for V

In practice. Use a short gap to separate characters.

Variable-length codes

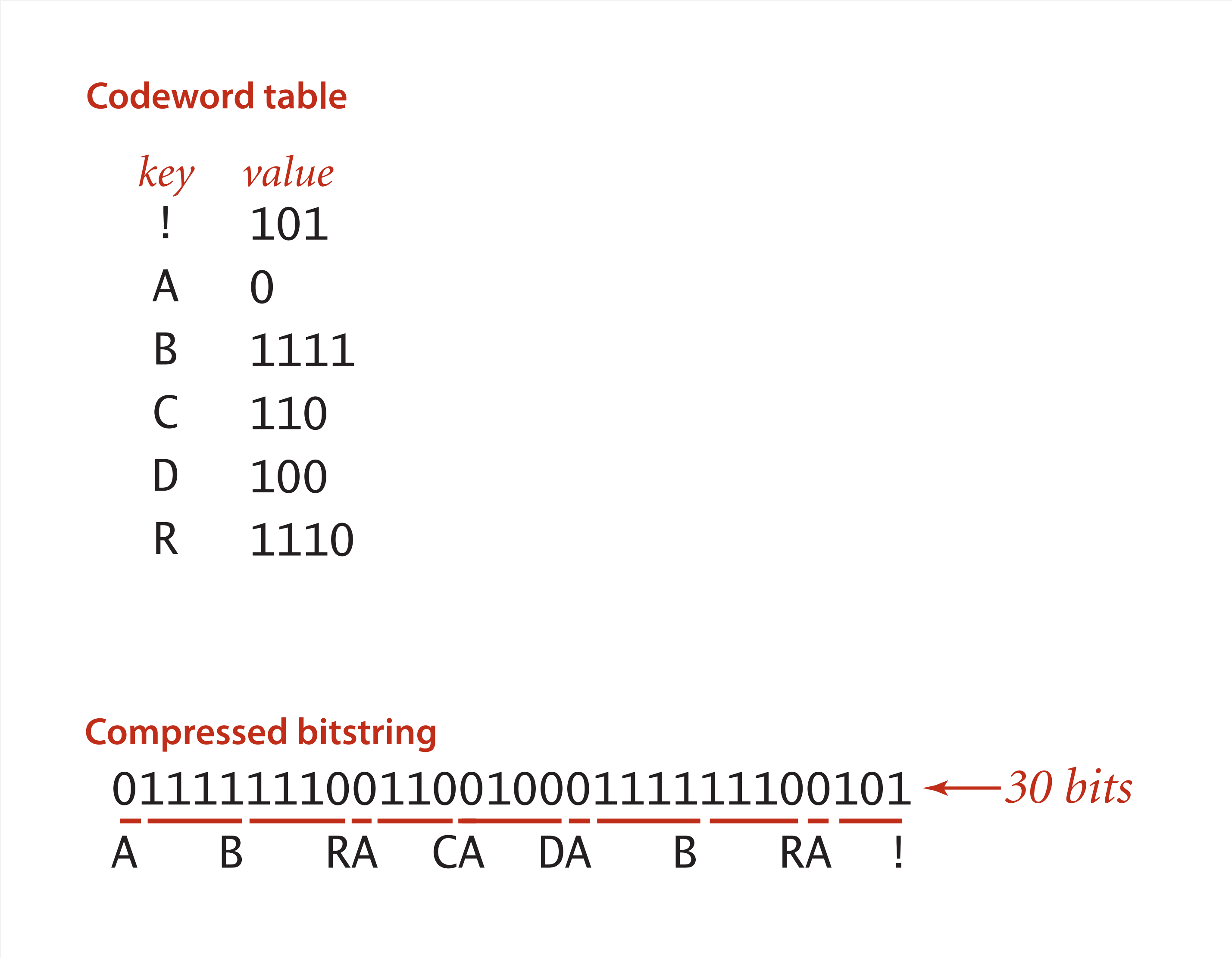
Q. How do we avoid ambiguity?

A. Ensure that no codeword is a prefix of another.

Ex 1. Fixed-length code.

Ex 2. Append special “stop” character to each codeword.

Ex 3. General prefix-free code.

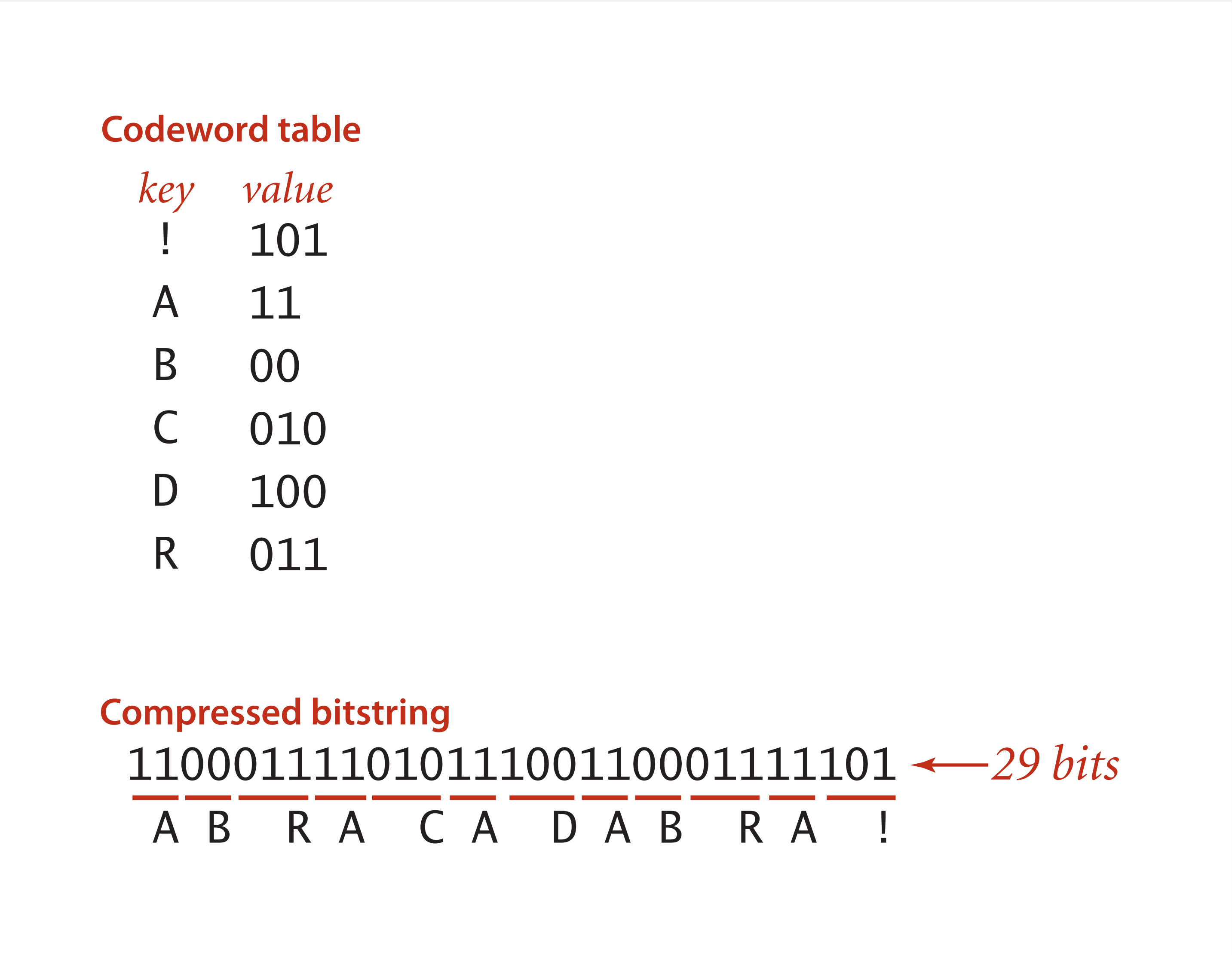


Codeword table

key	*value*
!	101
A	0
B	1111
C	110
D	100
R	1110

Compressed bitstring

```
0111111100110010001111111100101 ←30 bits
A   B    RA   CA   DA   B    RA   !
```



Codeword table

key	*value*
!	101
A	11
B	00
C	010
D	100
R	011

Compressed bitstring

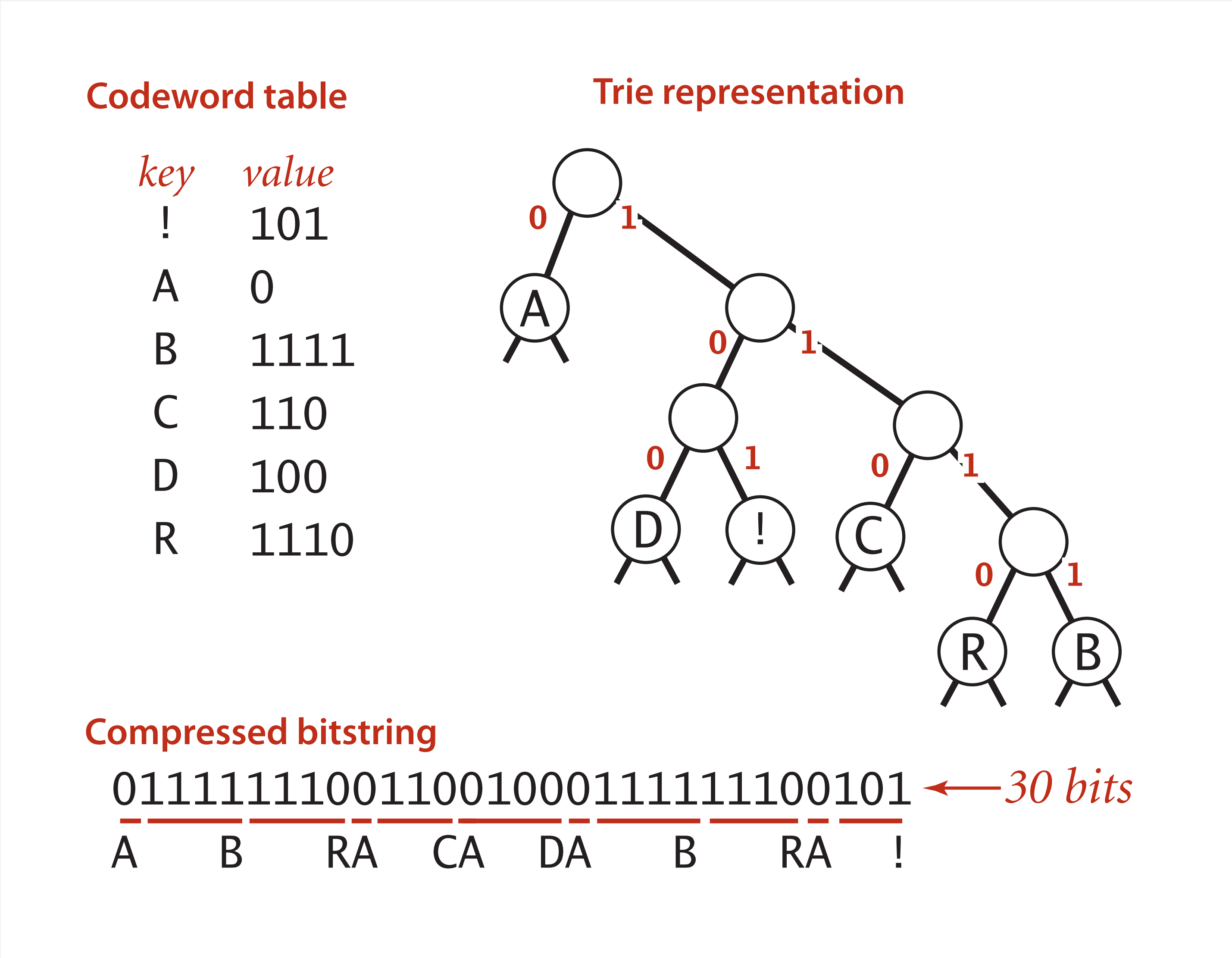
```
11000111101011100110001111101 ←29 bits
 A B  R A  C A  D A B  R A  !
```

Prefix-free codes: trie representation

Q. How to represent the prefix-free code?

A. A binary trie.

- Characters in leaves.
- Codeword is path from root to leaf.



Codeword table

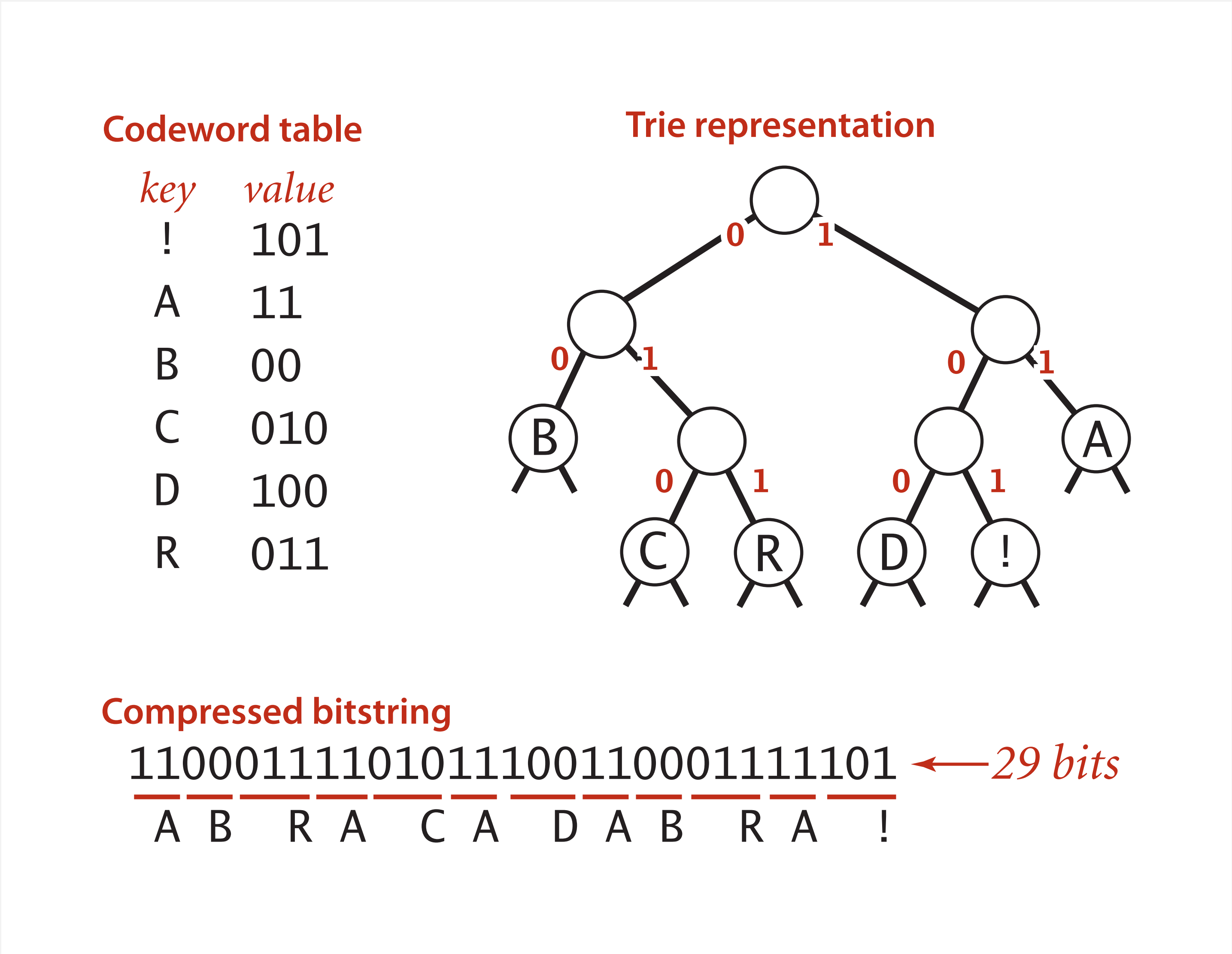
key	*value*
!	101
A	0
B	1111
C	110
D	100
R	1110

Trie representation

Compressed bitstring

011111110011001000111111100101 ← *30 bits*

A B RA CA DA B RA !



Codeword table

key	*value*
!	101
A	11
B	00
C	010
D	100
R	011

Trie representation

Compressed bitstring

11000111101011100110001111101 ← *29 bits*

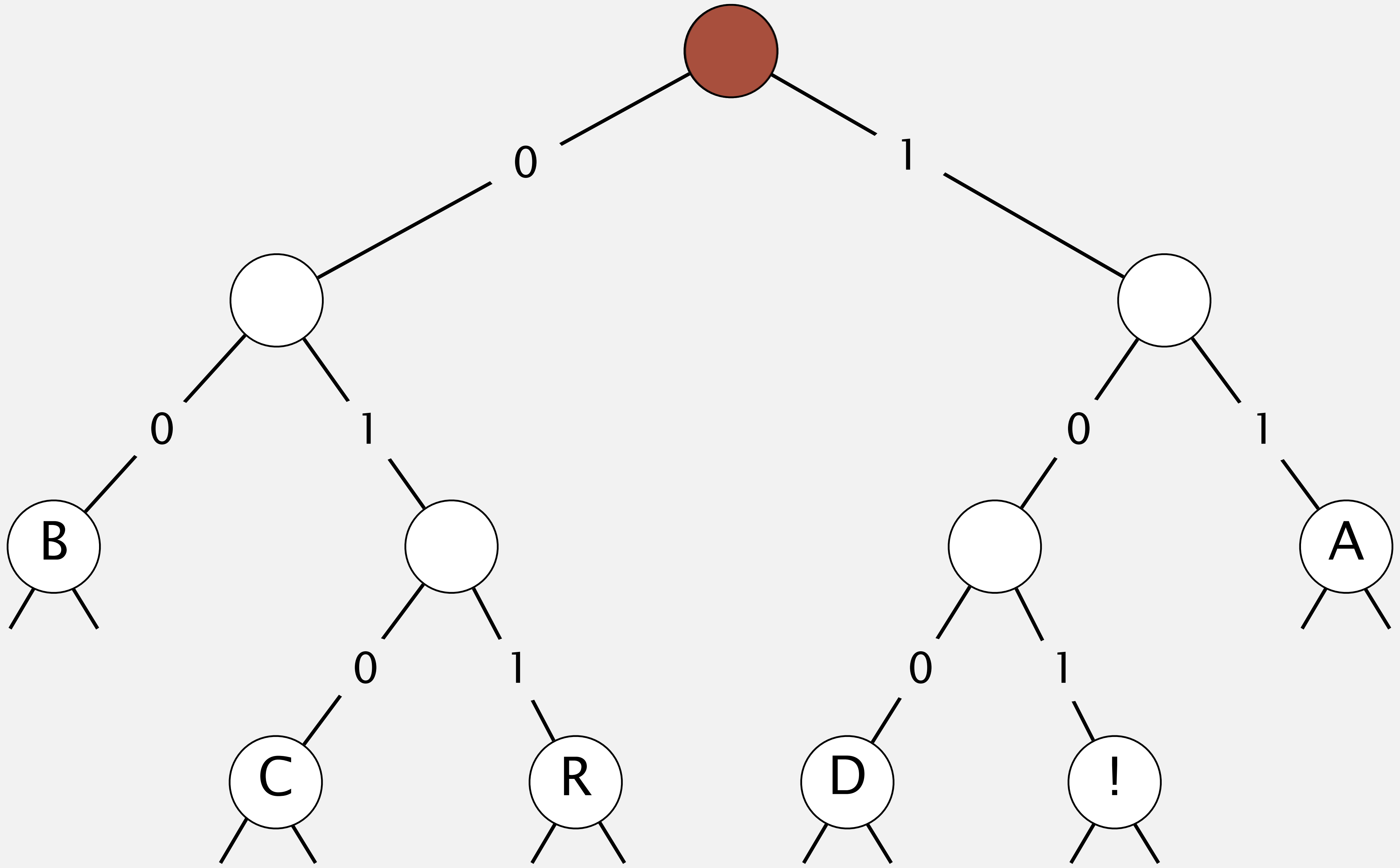
A B R A C A D A B R A !

Prefix-free codes: expansion

Expansion.

- Start at root.
- Go left if bit is 0; go right if 1.
- If leaf node, write character; return to root node; repeat.

1 1	0 0	0 1 1	1 1	0 1 0	1 1	1 0 0	1 1	0 0	0 1 1	1 1	1 0 1
A	B	R	A	C	A	D	A	B	R	A	!



Data compression: quiz 3

Consider the following trie representation of a prefix-free code.

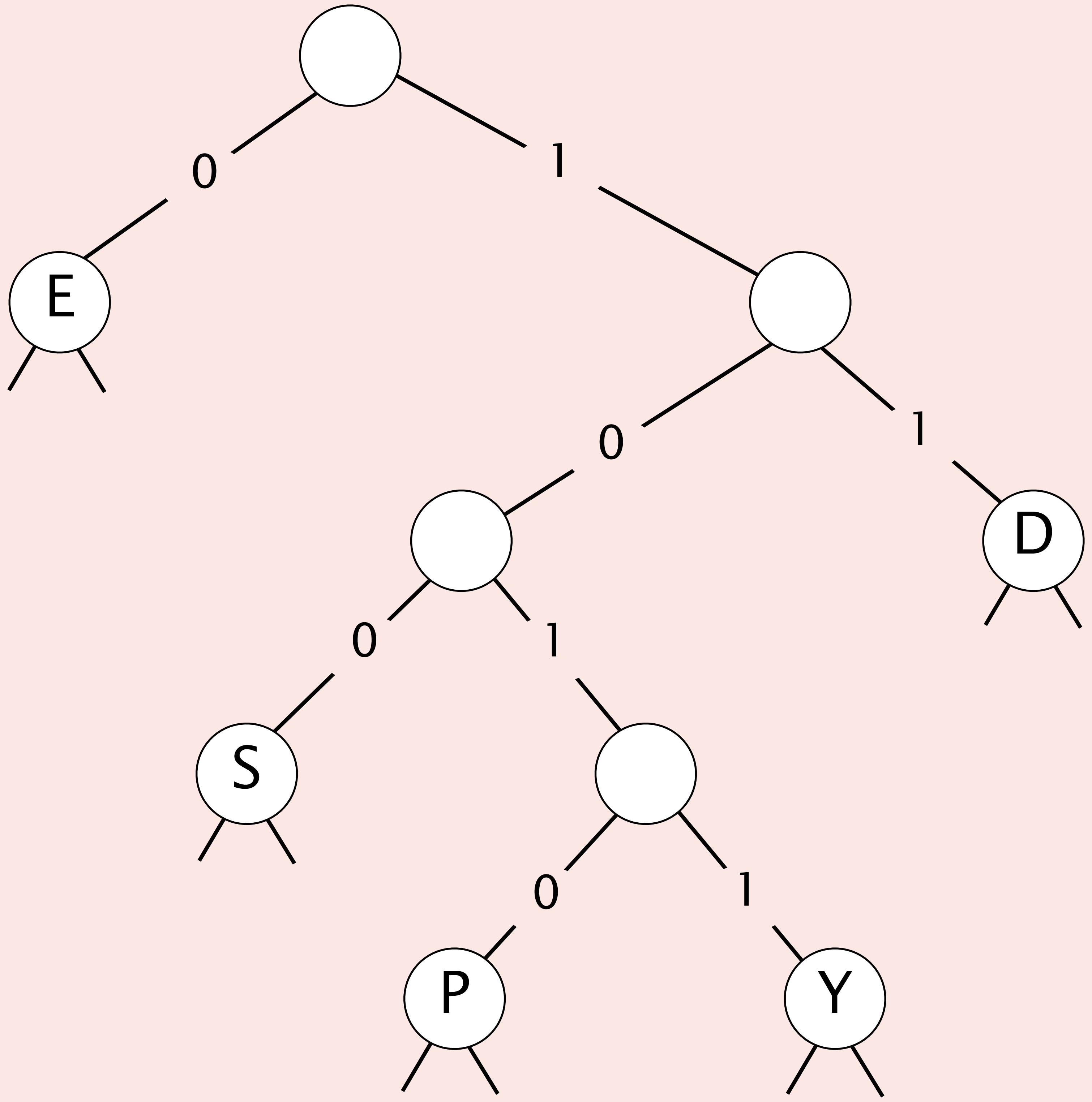
Expand the compressed bitstring 1 0 0 1 0 1 0 0 0 1 1 1 0 1 1 ?

A. PEED

B. PESDEY

C. SPED

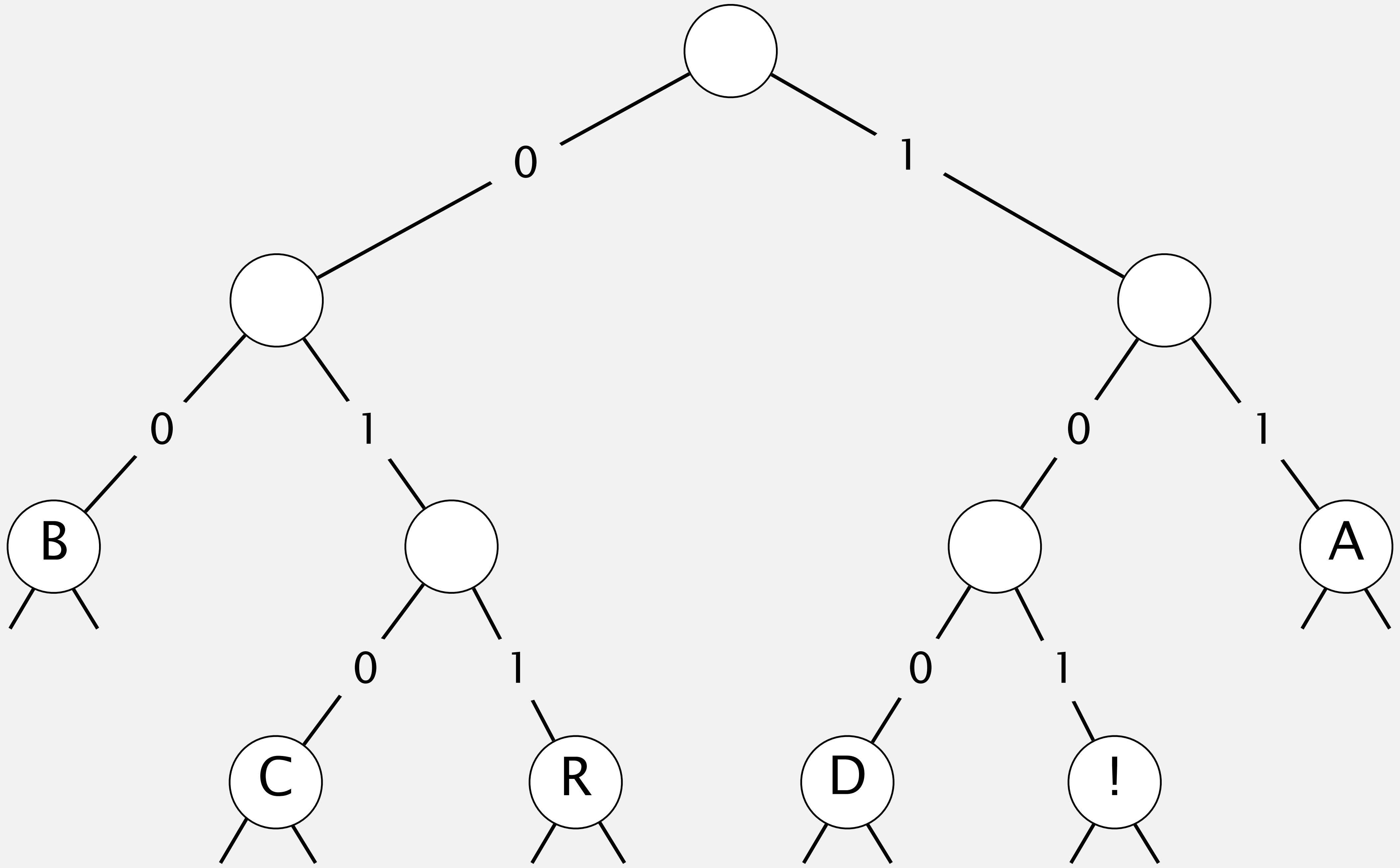
D. SPEEDY



Prefix-free codes: compression

Compression method 1. Start at leaf; follow path up to the root; print bits in reverse.

Compression method 2. Create ST of key–value pairs.



Huffman coding overview

Static model. Use the same prefix-free code for all messages.

Dynamic model. Use a custom prefix-free code for each message.

Compression.

- Read message.
- Build best prefix-free code for message. How? [ahead]
- Write prefix-free code.
- Compress message using prefix-free code.

Expansion.

- Read prefix-free code.
- Read compressed message and expand using prefix-free code.

Huffman trie node data type

```java
private static class Node implements Comparable<Node>
{
   private final char ch;     // used only for leaf nodes
   private final int freq;    // used only by compress()
   private final Node left, right;

   public Node(char ch, int freq, Node left, Node right)
   {
      this.ch    = ch;
      this.freq  = freq;
      this.left  = left;
      this.right = right;
   }

   public boolean isLeaf()
   {  return left == null && right == null; }

   public int compareTo(Node that)
   {  return this.freq - that.freq;  }

}
```

initializing constructor

is Node a leaf?

compare nodes by frequency
(stay tuned)

Prefix-free codes: expansion

```
public void expand()
{
   Node root = readTrie();
   int n = BinaryStdIn.readInt();

   for (int i = 0; i < n; i++)
   {

      Node x = root;
      while (!x.isLeaf())
      {
         if (!BinaryStdIn.readBoolean())
            x = x.left;
         else
            x = x.right;
      }
      BinaryStdOut.write(x.ch, 8);

   }
   BinaryStdOut.close();
}
```

read encoding trie

read number of chars (32 bits)

for each encoded character *i*

follow path from root to leaf
to determine character

read 0 or 1 (1 bit)

write character (8 bits)

don't forget this!

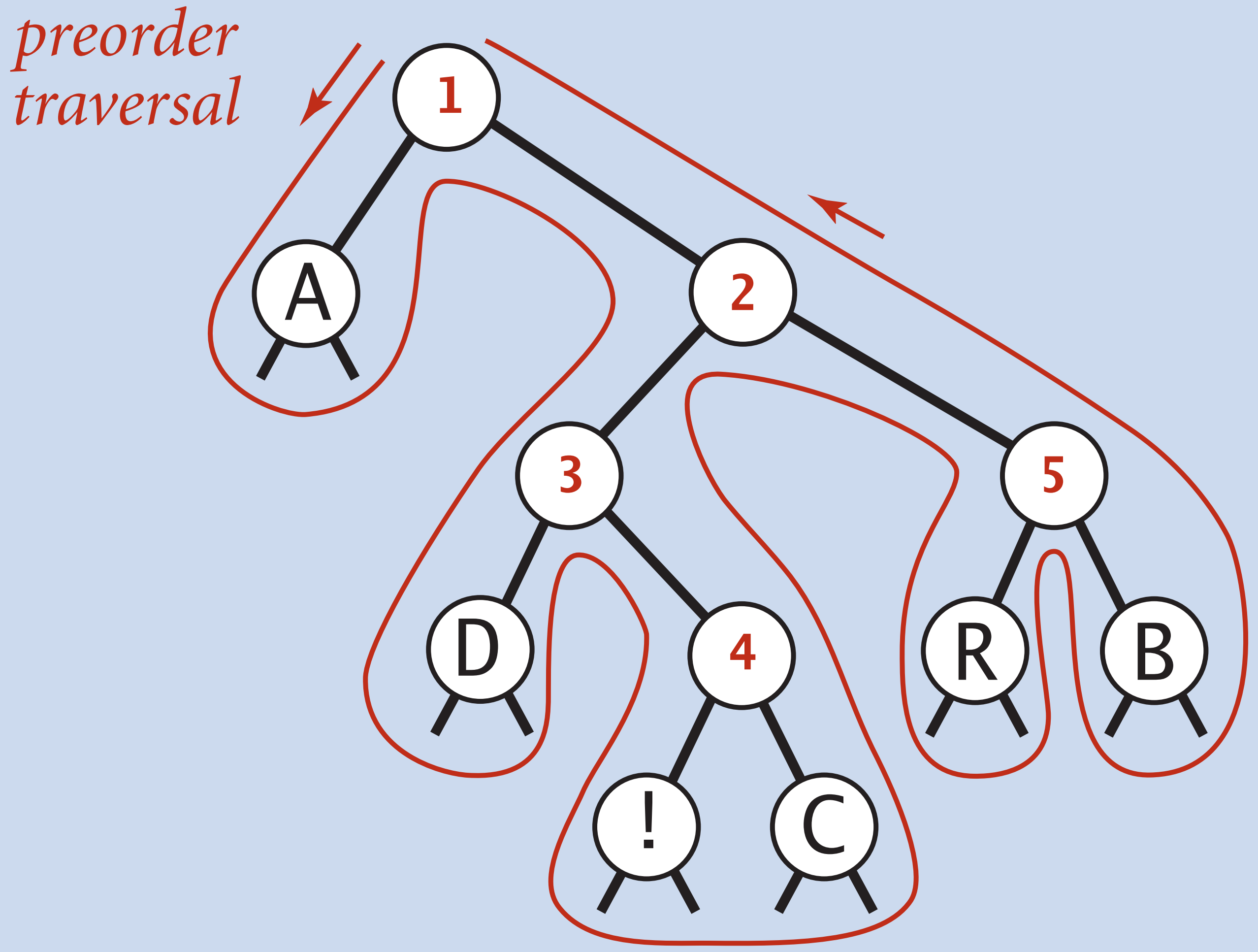
Running time. Linear in input size (number of bits).

Encoding the binary trie

Q. How to transmit the binary trie?

A. Write preorder traversal; mark leaf nodes and internal nodes with a bit.

0 for internal nodes
1 for leaf nodes



leaves

A D ! C R B

0101000001001010001000100001010101000011010101001010100010

1 2 3 4 5 ← *internal nodes*

Huffman codes

Q. How to find best prefix-free code?

Huffman algorithm:

- Count frequency `freq[i]` for each char `i` in input.
- Start with one node corresponding to each char `i` (with weight `freq[i]`).
- Repeat until single trie formed:
  - select two tries with min weight `freq[i]` and `freq[j]`
  - merge into single trie with weight `freq[i] + freq[j]`

Applications:

JPEG PDF Adobe mp3 DivX MPEG4 gzip PKZIP

Constructing a Huffman encoding trie: Java implementation

```
private static Node buildTrie(int[] freq)
{
   MinPQ<Node> pq = new MinPQ<Node>();
       for (char c = 0; c < R; c++)
          if (freq[c] > 0)
             pq.insert(new Node(c, freq[c], null, null));

   while (pq.size() > 1)
   {
      Node x = pq.delMin();
      Node y = pq.delMin();
      Node parent = new Node('\0', x.freq + y.freq, x, y);
      pq.insert(parent);
   }

   return pq.delMin();
}
```

initialize PQ with singleton tries

merge two smallest tries

not used for internal nodes

total frequency

two subtries

Huffman compression summary

Proposition. Huffman's algorithm produces an optimal prefix-free code.

Pf. See textbook.

no prefix-free code uses fewer bits

Two-pass implementation (for compression).

- Pass 1: tabulate character frequencies; build trie.
- Pass 2: encode file by traversing trie (or symbol table).

Running time (for compression). Using a binary heap ⇒ $O(n + R \log R)$.

Running time (for expansion). Using a binary trie ⇒ $\Theta(n)$.

n = input size

R = alphabet size

Q. Can we do better (in terms of compression ratio)? [stay tuned]

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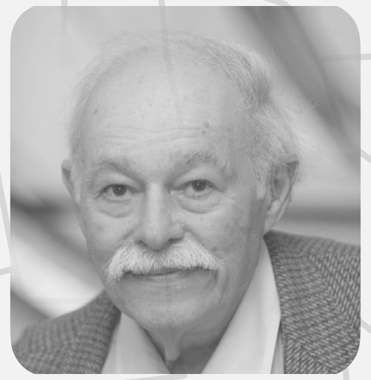
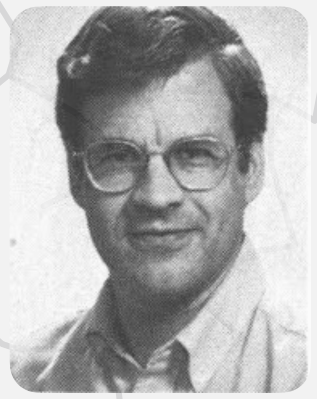
- Huffman compression
- *LZW compression*



Abraham Lempel

Jacob Ziv



Terry Welch

Statistical methods

Static model. Same model for all texts.

- Fast.
- Not optimal: different texts have different statistical properties.
- Ex: ASCII, Morse code.

Dynamic model. Generate model based on text.

- Preliminary pass needed to generate model.
- Must transmit the model.
- Ex: Huffman code.

Adaptive model. Progressively learn and update model as you read text.

- More accurate modeling can produces better compression.
- Decoding must start from beginning.
- Ex: LZW.

LZW compression demo (for 7-bit chars and 8-bit codewords)

input	A	B	R	A	C	A	D	A	B	R	A	B	R	A	B	R	A	
matches	A	B	R	A	C	A	D	A B		R A		B R		A B R			A	
value	41	42	52	41	43	41	44	81		83		82		88			41	80

LZW compression for A B R A C A D A B R A B R A B R A

key	value
⋮	⋮
A	41
B	42
C	43
D	44
⋮	⋮

key	value
AB	81
BR	82
RA	83
AC	84
CA	85
AD	86

key	value
DA	87
ABR	88
RAB	89
BRA	8A
ABRA	8B

codeword table

Input.
- 7-bit ASCII chars.
- ASCII 'A' is 41_{16}.

Codeword table.
- 8-bit codewords.
- Codewords for single chars are ASCII values.
- Use codewords 81_{16} to FF_{16} for multiple chars.
- Stop symbol = 80_{16}.

LZW expansion demo (for 7-bit chars and 8-bit codewords)

value	41	42	52	41	43	41	44	81	83	82	88	41	80
output	A	B	R	A	C	A	D	A B	R A	B R	A B R	A	

LZW expansion for 41 42 52 41 43 41 44 81 83 82 88 41 80

key	value
⋮	⋮
41	A
42	B
43	C
44	D
⋮	⋮

key	value
81	AB
82	BR
83	RA
84	AC
85	CA
86	AD

key	value
87	DA
88	ABR
89	RAB
8A	BRA
8B	ABRA

codeword table

Input.

- 7-bit ASCII chars.
- ASCII 'A' is 41_{16}.

Codeword table.

- 8-bit codewords.
- Codewords for single chars are ASCII values.
- Use codewords 81_{16} to FF_{16} for multiple chars.
- Stop symbol = 80_{16}.

Data compression: quiz 4

Which is the LZW compression for ABABABA ?

A. 41 42 41 42 41 42 80

B. 41 42 41 81 81 80

C. 41 42 81 81 41 80

D. 41 42 81 83 80

Data compression: quiz 5

Which is the key data structure to implement LZW compression efficiently?

A. Array

B. Red–black BST

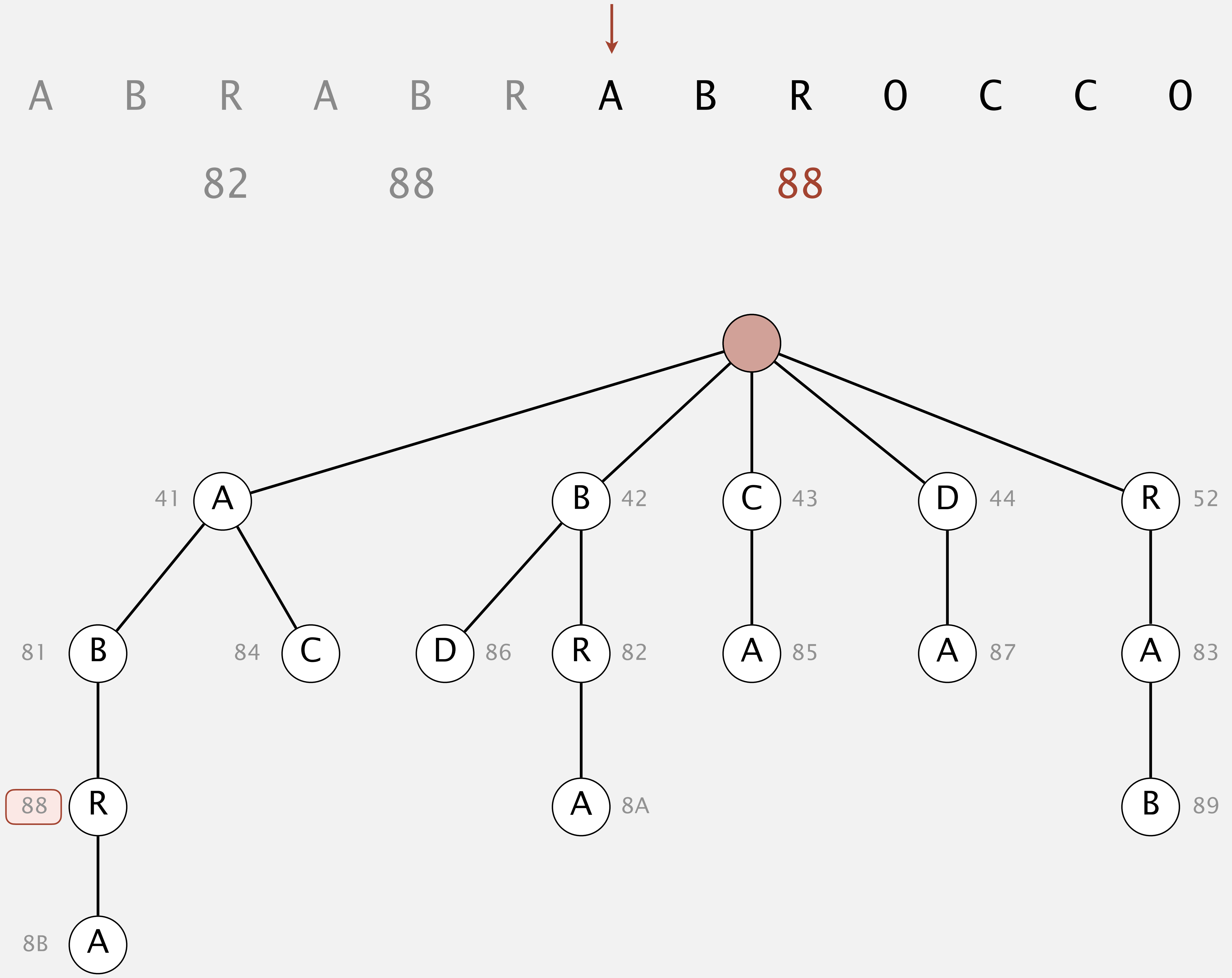
C. Hash table

D. Trie

Implementing LZW compression: longest prefix match

Find longest key in symbol table that is a prefix of query string.

input	…	R	A	B	R	A	B	R	A	B	R	O	C	C	O	L	I	…
value		89			82		88				88							



LZW tricky case: expansion

value	41	42	81	83	80
output	A	B	A B	A B A	

codeword for 83

need to know code for 83
before it is in codeword table!

we can deduce that
the code for 83 is ABx
for some character x

now, we have deduced x!

LZW expansion for 41 42 81 83 80

key	value
⋮	⋮
41	A
42	B
43	C
44	D
⋮	⋮

key	value
81	AB
82	BA
83	ABA

codeword table

LZW in the real world

Lempel–Ziv and friends.

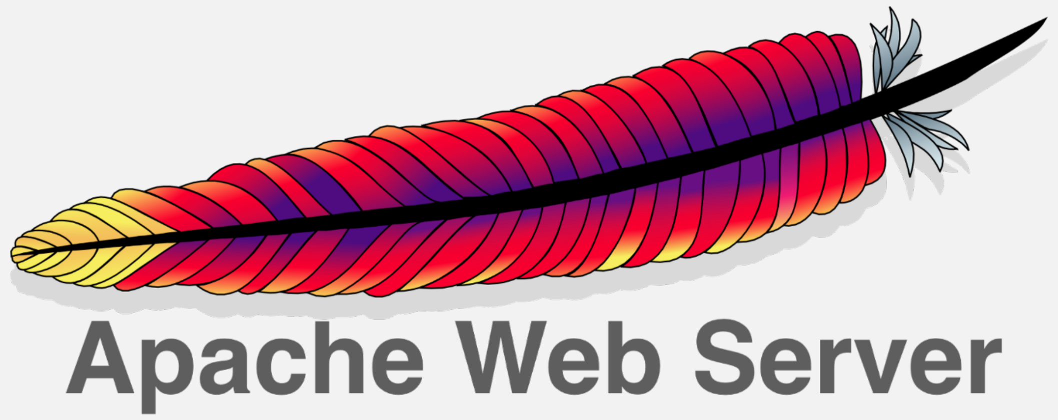
- LZ77.
- LZ78.
- LZW.
- Deflate / zlib = LZ77 variant + Huffman.

Unix compress, GIF, TIFF, V.42bis modem: LZW. ← previously under patent

zip, 7zip, gzip, jar, png, pdf: deflate / zlib.

iPhone, Wii, Apache HTTP server: deflate / zlib.

← not patented (widely used in open source)



Lossless data compression benchmarks

year	scheme	bits / char
1967	**ASCII**	7
1950	**Huffman**	4.7
1977	**LZ77**	3.94
1984	**LZMW**	3.32
1987	**LZH**	3.3
1987	**move-to-front**	3.24
1987	**LZB**	3.18
1987	**gzip**	2.71
1988	**PPMC**	2.48
1994	**SAKDC**	2.47
1994	**PPM**	2.34
1995	**Burrows-Wheeler**	**2.29**
1997	**BOA**	1.99
1999	**RK**	1.89

← next programming assignment

data compression using Calgary corpus

Data compression summary

Lossless compression.

- Represent fixed-length symbols with variable-length codes. [Huffman]
- Represent variable-length symbols with fixed-length codes. [LZW]

Lossy compression. [not covered in this course]

- JPEG, MPEG, MP3, …
- FFT/DCT, wavelets, fractals, …

$$X_k \;=\; \sum_{i=0}^{n-1} x_i \, \cos\left[\frac{\pi}{n}\left(i+\frac{1}{2}\right)k\right]$$

Theoretical limits on compression. Shannon entropy: $H(X) \;=\; -\sum_{i}^{n} p(x_i)\,\log_2 p(x_i)$

Practical compression. Exploit extra knowledge whenever possible.