

Algorithms

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<https://algs4.cs.princeton.edu>

3.1 SYMBOL TABLES

- ▶ *API*
- ▶ *elementary implementations*
- ▶ *ordered operations*



3.1 SYMBOL TABLES

- ▶ **API**
- ▶ *elementary implementations*
- ▶ *ordered operations*

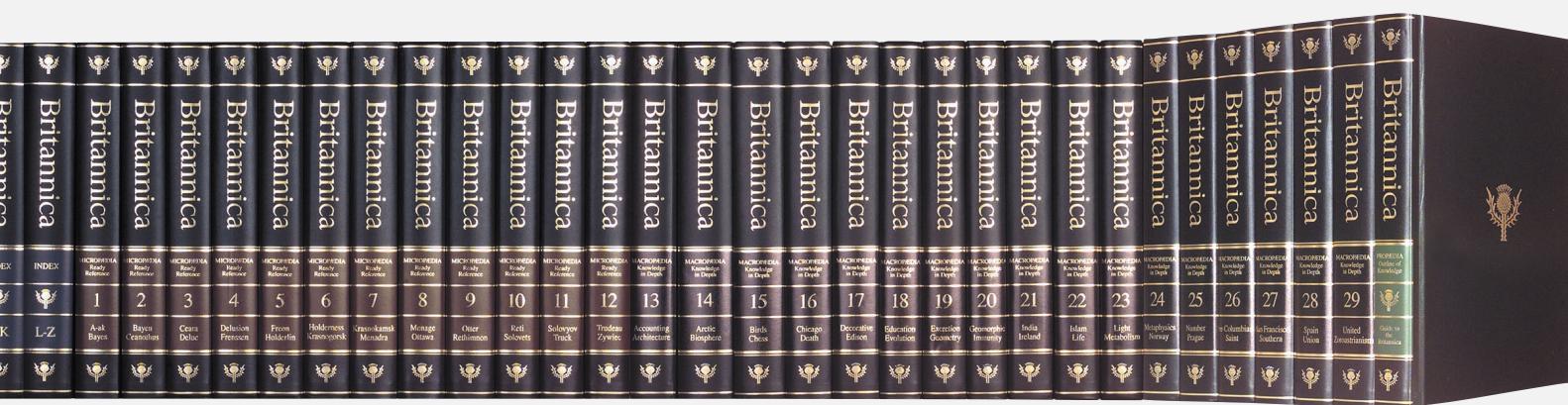
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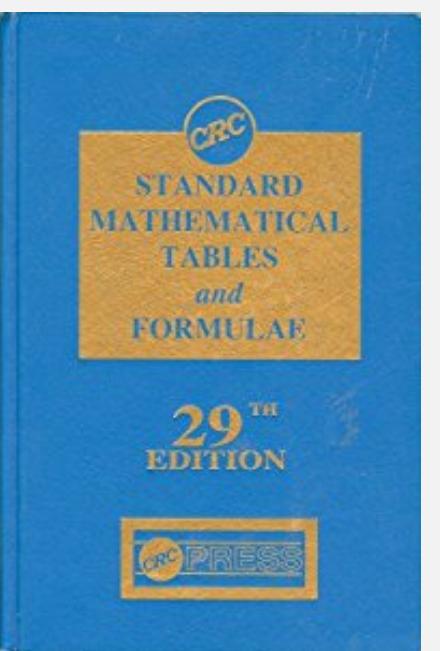
Why are telephone books obsolete?

Unsupported operations.

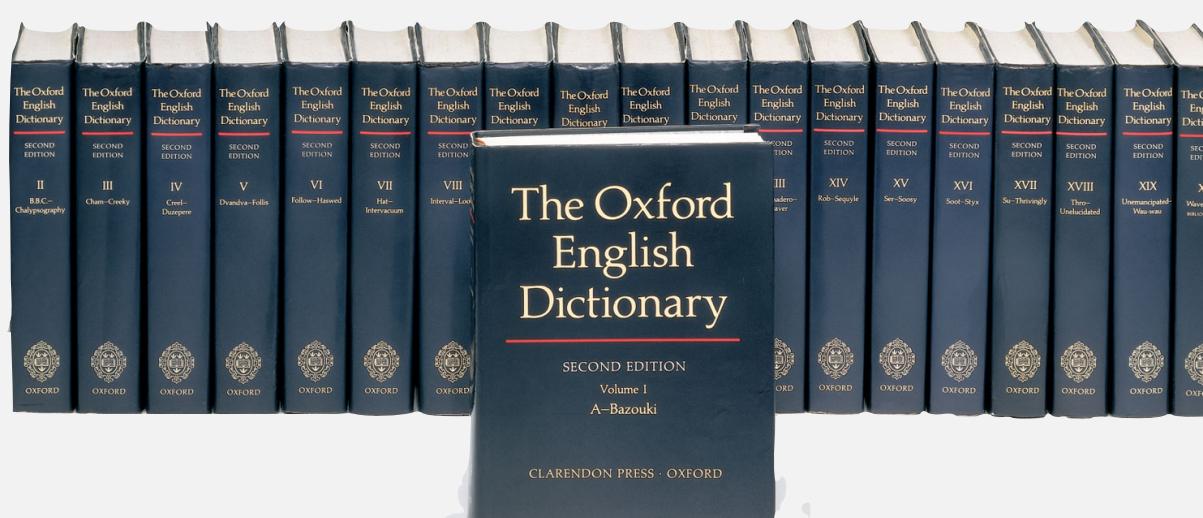
- Add a new name and associated number.
- Remove a given name and associated number.
- Change the number associated with a given name.



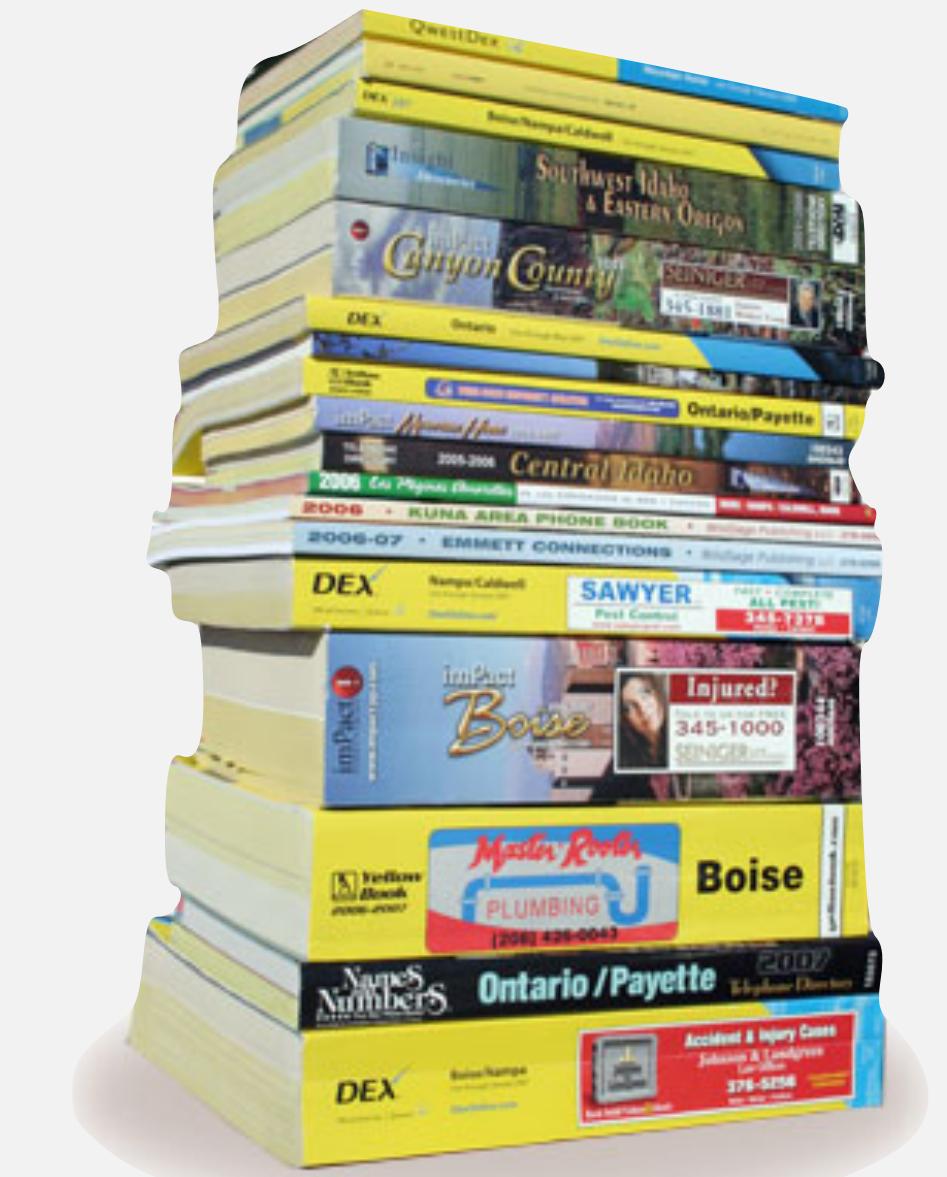
key = term
value = article



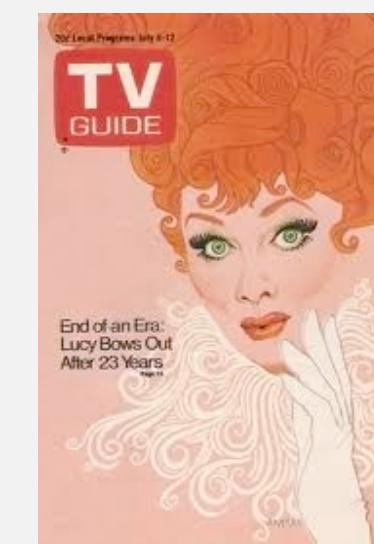
key = math function and input
value = function output



key = word
value = definition



key = name
value = phone number



key = time and channel
value = TV show

Symbol tables

Key–value pair abstraction.

- **Insert** a value with specified key.
- Given a key, **search** for the corresponding value.

Ex. DNS lookup.

- Insert domain name with specified IP address.
- Given domain name, find corresponding IP address.

domain name	IP address
www.cs.princeton.edu	128.112.136.61
goprinctontigers.com	67.192.28.17
wikipedia.com	208.80.153.232
google.com	172.217.11.46

↑
key

↑
value

Symbol table applications

application	purpose of search	key	value
dictionary	find definition	word	definition
book index	find relevant pages	term	list of page numbers
file share	find song to download	name of song	computer ID
financial account	process transactions	account number	transaction details
web search	find relevant web pages	keyword	list of page names
compiler	find properties of variables	variable name	type and value
routing table	route Internet packets	destination	best route
DNS	find IP address	domain name	IP address
reverse DNS	find domain name	IP address	domain name
genomics	find markers	DNA string	known positions
file system	find file on disk	filename	location on disk

Symbol tables: context

Also known as: maps, dictionaries, associative arrays.

Generalizes arrays. Keys need not be integers between 0 and $n - 1$.

Language support.

- External libraries: C, VisualBasic, Standard ML, bash, ...
- Built-in libraries: Java, C#, C++, Scala, ...
- Built-in to language: Awk, Perl, PHP, Tcl, JavaScript, Python, Ruby, Lua.

```
has_nice_syntax_for_dictionaries['Python'] = True  
has_nice_syntax_for_dictionaries['Java'] = False
```

legal Python code

Basic symbol table API

Associative array abstraction. Associate key–value pairs.

two generic type parameters	
public class ST<Key extends Comparable<Key>, Value>	
ST()	<i>create an empty symbol table</i>
void put(Key key, Value val)	<i>insert key–value pair</i> ← $a[key] = val;$
Value get(Key key)	<i>value paired with key</i> ← $a[key]$
boolean contains(Key key)	<i>is there a value paired with key?</i>
Iterable<Key> keys()	<i>all the keys in the symbol table</i>
void delete(Key key)	<i>remove key (and associated value)</i>
boolean isEmpty()	<i>is the symbol table empty?</i>
int size()	<i>number of key–value pairs</i>

Conventions

- Method `put()` overwrites old value with new value.
- Method `get()` returns `null` if key not present.
- Values are not `null`. ← `java.util.Map` allows `null` values

“ Careless use of `null` can cause a staggering variety of bugs. Studying the Google code base, we found that something like 95% of collections weren’t supposed to have any `null` values in them, and having those fail fast rather than silently accept `null` would have been helpful to developers. ”



<https://code.google.com/p/guava-libraries/wiki/UsingAndAvoidingNullExplained>

Key and value types

Value type. Any generic type.

Key type: different assumptions.

- This lecture: keys are Comparable; use compareTo().
- Hashing lecture: keys are any generic type;
use equals() to test equality and hashCode() to scramble key.

specify Comparable in API



Best practices. Use immutable types for symbol-table keys.

- Immutable in Java: String, Integer, Double, Color, ...
- Mutable in Java: StringBuilder, Stack, URL, arrays, ...

ST test client for analysis

Frequency counter. Read a sequence of strings from standard input;
print one that occurs most often.

```
% more tinyTale.txt
it was the best of times
it was the worst of times
it was the age of wisdom
it was the age of foolishness
it was the epoch of belief
it was the epoch of incredulity
it was the season of light
it was the season of darkness
it was the spring of hope
it was the winter of despair
```

```
% java FrequencyCounter 3 < tinyTale.txt
the 10
```

```
% java FrequencyCounter 8 < tale.txt
business 122
```

```
% java FrequencyCounter 10 < leipzig1M.txt
government 24763
```

← tiny example
(60 words, 20 distinct)

← real example
(135,635 words, 10,769 distinct)

← real example
(21,191,455 words, 534,580 distinct)

Frequency counter implementation

```
public class FrequencyCounter
{
    public static void main(String[] args)
    {
        int minLength = Integer.parseInt(args[0]);

        ST<String, Integer> st = new ST<String, Integer>();
        while (!StdIn.isEmpty())
        {
            String word = StdIn.readString();
            if (word.length() < minLength) continue;
            if (!st.contains(word)) st.put(word, 1);
            else st.put(word, st.get(word) + 1);
        }

        String max = "";
        st.put(max, 0);
        for (String word : st.keys()) ← iterate over key-value pairs
            if (st.get(word) > st.get(max))
                max = word;
        StdOut.println(max + " " + st.get(max));
    }
}
```

compute frequencies

create ST

read string and
update frequency

print a string with max frequency



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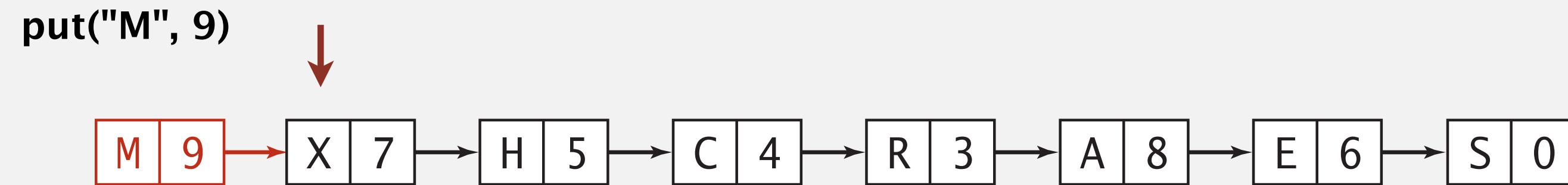
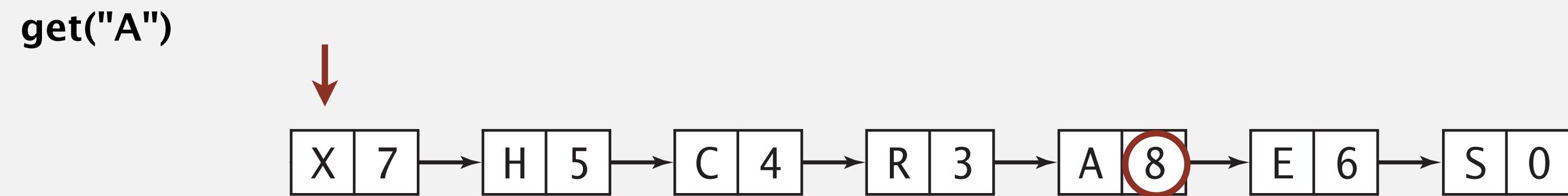
- ▶ API
- ▶ *elementary implementations*
- ▶ *ordered operations*

Sequential search in a linked list

Data structure. Maintain an (unordered) linked list of key–value pairs.

Search. Scan through all keys until find a match.

Insert. Scan through all keys until find a match; if no match add to front.



Elementary ST implementations: summary

implementation	guarantee		average case		operations on keys
	search	insert	search hit	insert	
sequential search (unordered list)	n	n	n	n	<code>equals()</code>

Challenge. Efficient implementations of both search and insert.

Binary search in a sorted array

Data structure. Maintain parallel arrays for keys and values, **sorted** by key.

Search. Use **binary search** to find key.

Proposition. At most $1 + \log_2 n$ compares to search a sorted array of length n .

`get("P")`

keys[]									
0	1	2	3	4	5	6	7	8	9
A	C	E	H	L	M	P	R	S	Z

vals[]									
0	1	2	3	4	5	6	7	8	9
8	4	2	5	11	9	10	3	0	7

Binary search in a sorted array

Data structure. Maintain parallel arrays for keys and values, sorted by key.

Search. Use **binary search** to find key.

```
public Value get(Key key)
{
    int lo = 0, hi = n-1;
    while (lo <= hi)
    {
        int mid = lo + (hi - lo) / 2;
        int cmp = key.compareTo(keys[mid]);
        if      (cmp < 0) hi = mid - 1;
        else if (cmp > 0) lo = mid + 1;
        else return vals[mid];
    }
    return null; ← no matching key
}
```

Binary search: insert

Data structure. Maintain a sorted array of key-value pairs.

Insert. Use binary search to find place to insert; shift all larger keys over.

Proposition. Takes $\Theta(n)$ time in the worst case.

`put("P", 10)`

keys[]										vals[]									
0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9
A	C	E	H	M	R	S	X	-	-	8	4	6	5	9	3	0	7	-	-

Elementary ST implementations: summary

implementation	guarantee		average case		operations on keys
	search	insert	search hit	insert	
sequential search (unordered list)	n	n	n	n	<code>equals()</code>
binary search (sorted array)	$\log n$	n^{\dagger}	$\log n$	n^{\dagger}	<code>compareTo()</code>

† can do with $\Theta(\log n)$ compares, but still requires $\Theta(n)$ array accesses

Challenge. Efficient implementations of both search and insert.



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Examples of ordered symbol table API

	<i>keys</i>	<i>values</i>
min() →	9:00:00	Chicago
	9:00:03	Phoenix
	9:00:13	Houston
	9:00:59	Chicago
	9:01:10	Houston
floor(9:05:00) →	9:03:13	Chicago
	9:10:11	Seattle
select(7) →	9:10:25	Seattle
rank(9:10:25) = 7	9:14:25	Phoenix
	9:19:32	Chicago
	9:19:46	Chicago
	9:21:05	Chicago
	9:22:43	Seattle
	9:22:54	Seattle
	9:25:52	Chicago
ceiling(9:30:00) →	9:35:21	Chicago
	9:36:14	Seattle
max() →	9:37:44	Phoenix

Ordered symbol table API

```
public class ST<Key extends Comparable<Key>, Value>  
{  
    :  
  
    Key min()                               smallest key  
    Key max()                               largest key  
    Key floor(Key key)                     largest key less than or equal to key  
    Key ceiling(Key key)                   smallest key greater than or equal to key  
    int rank(Key key)                      number of keys less than key  
    Key select(int k)                      key of rank k  
    :  
}
```

RANK IN A SORTED ARRAY



Problem. Given a sorted array of n distinct keys,
find the number of keys strictly less than a given query key.

Ordered symbol table operations: summary

	sequential search	binary search
search	n	$\log n$
insert	n	n
min / max	n	1
floor / ceiling	n	$\log n$
rank	n	$\log n$
select	n	1

order of growth of the running time for ordered symbol table operations

Challenge. Efficient implementations of all operations.