1. **Initialization.** Don’t forget to do this.

2. **Memory.**

   (a) `isEmpty()`, `addFront()`, `removeFront()`, `addBack()`
   
   To implement `size()`, `removeBack()`, and `sample()`, you would have to traverse the singly linked list, from first to last. The challenge with implementing `removeBack()` efficiently is updating the last pointer.

   (b) $\sim 32n$
   
   Each `Node` object uses 32 bytes of memory and there are $n$ nodes.
   - 16 bytes of object overhead
   - 8 bytes for `Node` reference
   - 8 bytes for `double` item

3. **Five sorting algorithms.**

   (3.1) insertion sort after 16 iterations
   (3.2) heap sort after heap construction phase and putting 6 keys into place
   (3.3) selection sort after 12 iterations
   (3.4) mergesort just before the last call to `merge()`
   (3.5) quicksort after first partitioning step

4. **Analysis of algorithms.**

   (4.1) $\sim 2n^2$
   Selection sort always makes $\sim \frac{1}{2} m^2$ compares to sort an array of length $m$. Here $m = 2n$.

   (4.2) $\sim n^2$
   In each of the first $n$ iterations (except the first), there is one compare (and no exchange). In each of the last $n$ iterations (except the last), there are $n$ compares and $n$ exchanges.

   (4.3) $\sim n \log_2 n$
   Mergesort requires $\frac{1}{2} n \log_2 n$ compares to sort a sorted array of length $n$. Thus, mergesort makes $\frac{1}{2} n \log_2 n$ compares to sort the left subarray (of length $n$) and $\frac{1}{2} n \log_2 n$ compares to sort the right subarray (of length $n$). Finally, it makes $n$ compares to merge the two subarrays together.

   (4.4) Timsort
   Timsort is optimized for situations when an array has a small number of non-increasing (or strictly decreasing) runs. In this case, there are only two runs (the first $n$ elements containing the value $n$, and the last $n$ elements containing the integers 1 to $n$). So, Timsort will run in linear time on staircase arrays.
(4.5) $O(n^3), O(n^4), \Theta(n^3)$

Big $O$ and big Theta notations discard both lower-order terms and the leading coefficient. The main difference is that big $O$ notation includes functions that grow more slowly. So, $O(n^4)$ includes not only functions like $2n^4$ and $\frac{1}{2}n^4$, but also $3n^3$ and $5n^2$.

5. Level-order traversal.

B F H J L M A

```java
public Iterable<Key> levelOrder() {
    Queue<Key> keys = new Queue<Key>();
    Queue<Node> queue = new Queue<Node>();
    queue.enqueue(root);
    while (!queue.isEmpty()) {
        Node x = queue.dequeue();
        if (x != null) {
            keys.enqueue(x.key);
            queue.enqueue(x.left);
            queue.enqueue(x.right);
        }
    }
    return keys;
}
```

6. Hash tables.

(6.1) B D F
(6.2) B E
(6.3) D G
(6.4) C E
7. Data structures.

(7.1) could not arise

The height of the tree is 4. However, the height of any weighted quick-union tree on \( n \) elements is at most \( \log_2 n \). Note that \( \log_2 10 < \log_2 16 = 4 \), so the height must be strictly less than 4.

(7.2) could not arise

The corresponding binary tree is not heap-ordered because 55 is greater than 66.

(7.3) could arise

Here is the BST with the given level-order traversal.

(7.4) could not arise

Perfect black balance is not satisfied. The path from the root to the right null link of 8 has only 2 black links (including the null link) but all other paths from the root to null links have 3 black links.
(7.5) could arise

It’s a valid kd-tree. It could have arisen by inserting the points in a variety of orders, including level order: (6, 7), (1, 4), (8, 5), (4, 2), (2, 8), (0, 9), (3, 6).

8. Problem identification.

8.1 Possible

This can be done with mergesort, as discussed in lecture.

8.2 Possible

This can be done with 3-way quicksort. The number of 3-way partitioning steps equals the number of distinct keys. Each partitioning step makes at most \( n \) compares.

8.3 Impossible

This would violate the sorting lower bound. We could insert the \( n \) keys; then delete-max the \( n \) keys to get them in sorted order. This would give us a compare-based sorting algorithm that makes \( \Theta(n \log \log n) \) compares in the worst case.

8.4 Possible

You could use binary search directly. Or you could compose an algorithm by combining operations that we’ve seen in the course. For example, if \( k \) is not in the array, then the predecessor is the floor (which we saw how to compute using binary search). If \( k \) is in the array, then you could search for the first occurrence of \( k \) and return the previous key (which you did on the Autocomplete assignment using binary search).

8.5 Impossible

This would violate the sorting lower bound. We could insert the \( n \) keys into a BST; then we could perform an inorder traversal to get them in sorted order. Since performing an inorder traversal doesn't require any key compares, this would give us a compare-based sorting algorithm that makes \( \Theta(n) \) compares in the worst case.

8.6 Impossible

There may be \( \Theta(n^2) \) pairs that intersect, so it will take \( \Theta(n^2) \) time to collect them in a list.

9.1 true

9.2 true

9.3 The main idea is to use binary search to find the adjacent inversion, maintaining a subarray $a[lo..hi]$ for which $(lo, hi)$ is an inversion: $lo < hi$ and $a[lo] > a[hi]$.

- Initialize $lo \leftarrow p$ and $hi \leftarrow q$
- Terminate the loop when $hi = lo + 1$, in which case $(lo, hi)$ is an adjacent inversion.
- Otherwise,
  - Set $mid = (lo + hi)/2$.
  - If $a[mid] > a[hi]$, then update $lo \leftarrow mid$. This guarantees $a[lo] > a[hi]$.
  - If $a[mid] \leq a[hi]$, then update $hi \leftarrow mid$. This guarantees $a[lo] > a[hi]$ because $a[lo]$ stays the same and $a[hi]$ does not increase.

Here’s the corresponding Java code.

```java
int lo = p, hi = q;
while (hi > lo + 1) {
    int mid = lo + (hi - lo) / 2;
    if (a[mid] > a[hi]) lo = mid;
    else hi = mid;
}
```

Here’s a symmetric version that compares $a[mid]$ to $a[lo]$.

```java
int lo = p, hi = q;
while (hi > lo + 1) {
    int mid = lo + (hi - lo) / 2;
    if (a[lo] > a[mid]) hi = mid;
    else lo = mid;
}
```

Here’s another version that does two compares per iteration of the while loop. The second compare is unnecessary because, if the first compare fails, then it must be the case that $a[mid] \geq a[lo] > a[hi]$.

```java
int lo = p, hi = q;
while (hi > lo + 1) {
    int mid = lo + (hi - lo) / 2;
    if (a[lo] > a[mid]) hi = mid;
    else if (a[mid] > a[hi]) lo = mid;
```