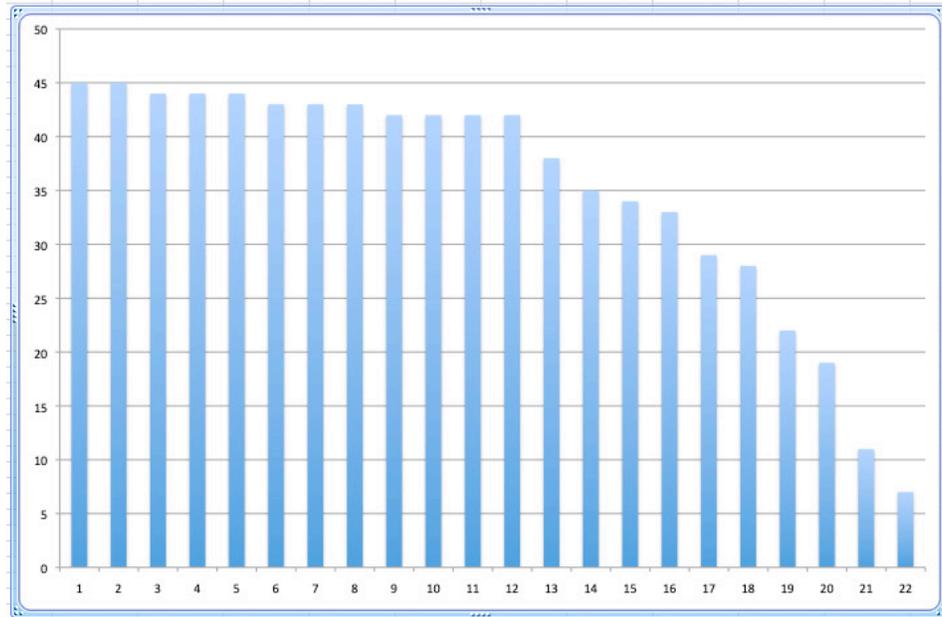


Solutions for COS 109 Problem Set 4

The further to the right you are on this curve, the more you should be paying careful attention to where you went wrong.



Problem 1: 18 pts (6 x 3)

Quite a few people were confused by how area relates to length or width or radius. It's a quadratic relationship, most easily seen with the area of a circle: πr^2 . That's the square of the radius: if you double the radius, the area goes up by a factor of four. And there's no need to do any more complicated arithmetic than that; the π and the actual values of the radii don't matter. That answers parts (d) through (f).

The first three parts are just arithmetic with powers of 10, but again you have to keep the dimensionality right: there are $10^4 \times 10^4$ square microns in a square centimeter, for example.

(a) Area of focal plane in square meters?

0.32 (approx). 10 microns is 10^{-5} meters, so the number of 10 micron x 10 micron squares is $3.2 \times 10^9 \times 10^{-5} \times 10^{-5}$.

(b) MB of memory for the pixels?

$3.2 \times 10^9 \times 3 \text{ bytes} = 9.6 \times 10^9 \text{ B} / 10^6 = 9,600 \text{ MB}$

(c) How many transistors in 1 cm x 1 cm?

10¹⁰ or 10 billion. There are 100 in a square 1 micron by 1 micron, and there are 10⁴ microns in a cm, so 10⁴ x 10⁴ square microns in a square cm, so 10⁴ x 10⁴ * 10².

(d) How many chips on a 15-inch wafer?

900. The radius goes up by 1.5, so the area goes up by 2.25.

(e) How many 1-inch patties?

48. A one-inch patty is 1/4 of the area of a 2 inch.

(f) How many 4-inch patties?

3. Same thing: area is proportional to the square of the radius.

Problem 2: (12 pts, 4 x 3)

(a) Total number of handshakes?

81. 9 x 9. A fair number didn't read the question carefully: it's one team shaking hands with another, not with itself.

(b) How does the number of handshakes grow in proportion to n?

n² (quadratic), yet another example of "every one does something to every one."

(c) How many comparisons to look up a name in the whole-earth phone book?

33. Just about 8 billion people, which is just under 2³³. Some confusion here about searching vs sorting and what we were trying to do. The phone book is sorted, so binary search is the way to go.

(d) How many eons to play Tower of Hanoi with 30 disks?

1024. Each new disk doubles the time of the previous case, so 10 doublings. That's the point of the story, and we saw it demoed (virtually) in class.

Problem 3: (15 pts, 5 x 3)

A fair amount of confusion here if the interval wasn't instantly clear, and too much precision. The Rule of 72 is itself an approximation, so quoting its results to three significant figures is ungood.

(a) What year will processors be twice as fast as they are today?

24 years, or 2044. $72 / 3\%/year$

(b) What is the annual rate of tuition increase?

6% (roughly). Quadruples in 25 years, so it doubles in about 12 years. 5.76% is too precise.

(c) What will the endowment be worth in 25 years?

About \$100B (same numbers as the previous part).

(d) How fast is power consumption growing each month?

about 1%/month 6 years = 72 months.

(e) In what year will servers consume 10 terawatts?

Going from 10 GW to 10 TW is 10 doublings, each 6 years long, so

60 years (2019+60 = 2079).