# COS 484: Natural Language Processing

# Recurrent Neural Networks

How to model sequences using neural networks?

Fall 2019

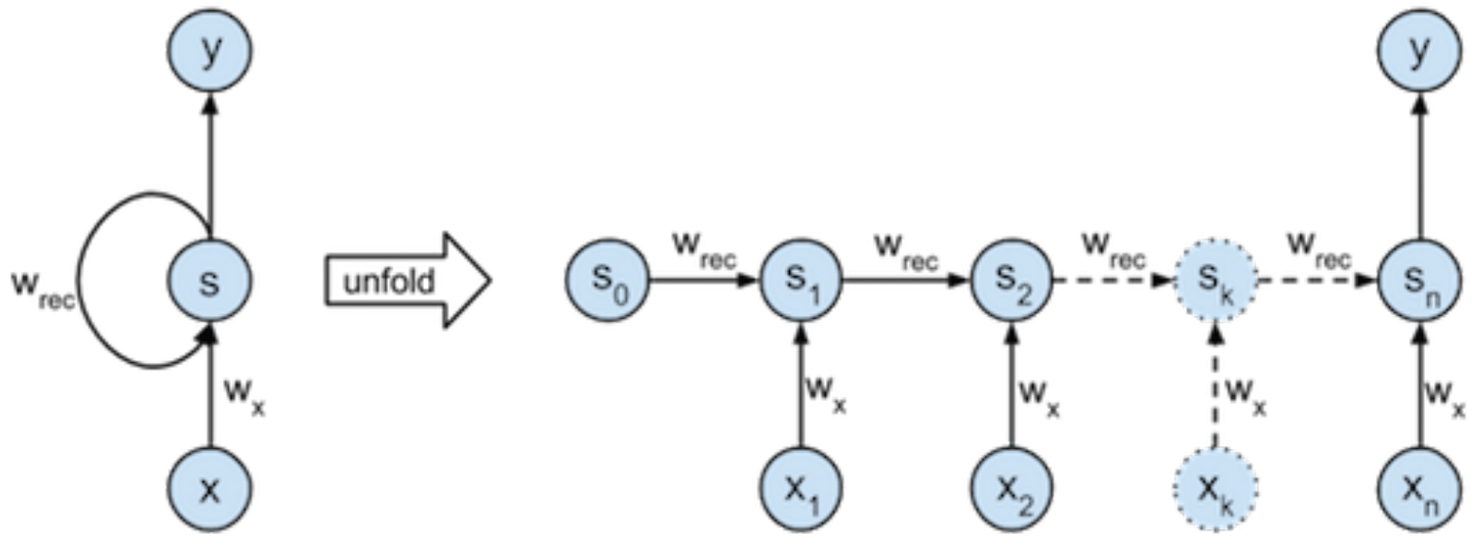
(Some slides adapted from Chris Manning, Abigail See, Andrej Karpathy)

# Overview

- What is a recurrent neural network (RNN)?
- Simple RNNs
- Backpropagation through time
- Long short-term memory networks (LSTMs)
- Applications
- Variants: Stacked RNNs, Bidirectional RNNs

# Recurrent neural networks (RNNs)

A class of neural networks allowing to handle **variable length inputs**



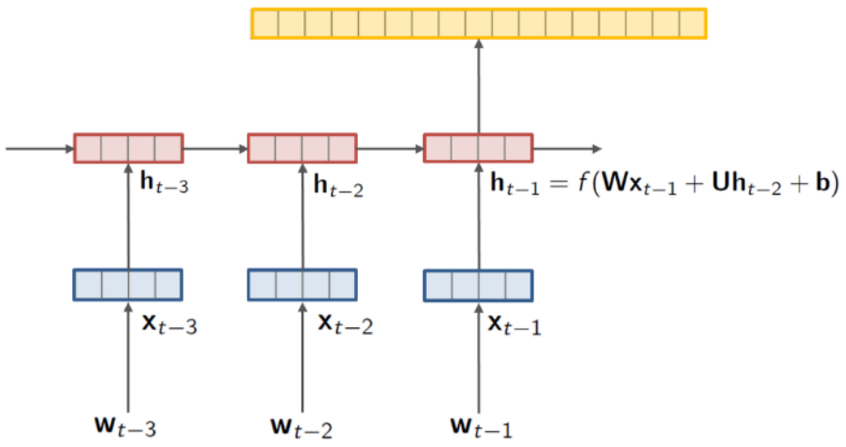

A function: $y = \text{RNN}(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n) \in \mathbb{R}^d$

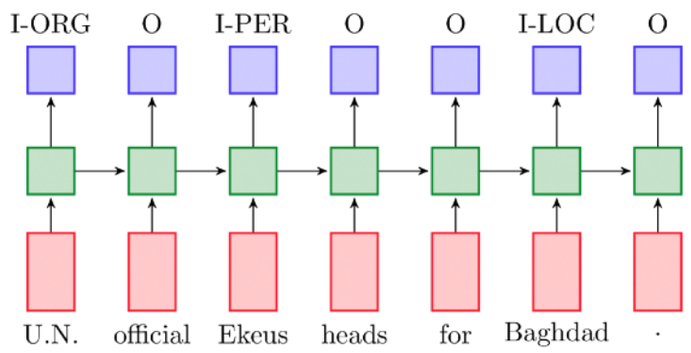
where $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \mathbb{R}^{d_{in}}$

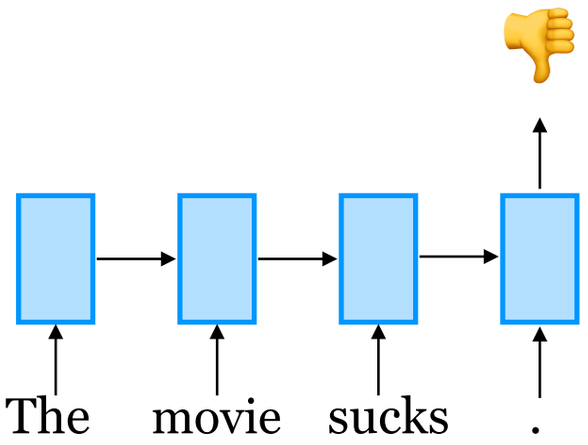
# Recurrent neural networks (RNNs)

Proven to be an highly effective approach to language modeling, sequence tagging as well as text classification tasks:

| Language modeling | Sequence tagging | Text classification |
|---|---|---|

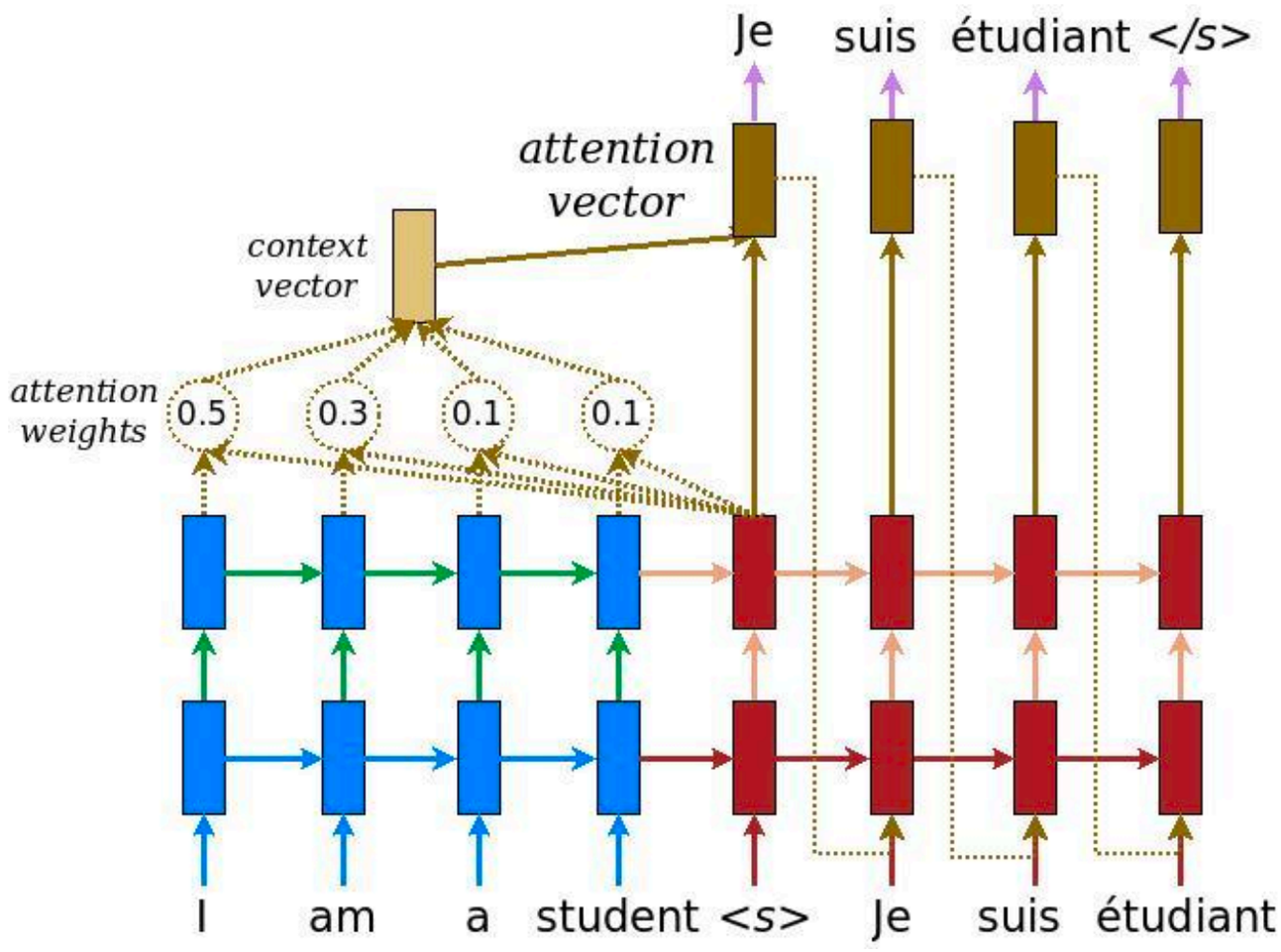


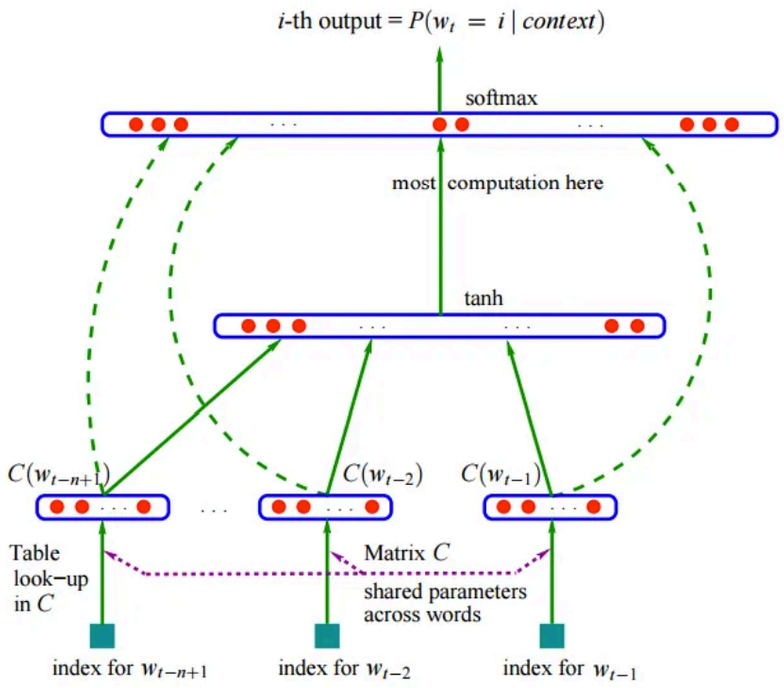
# Recurrent neural networks (RNNs)

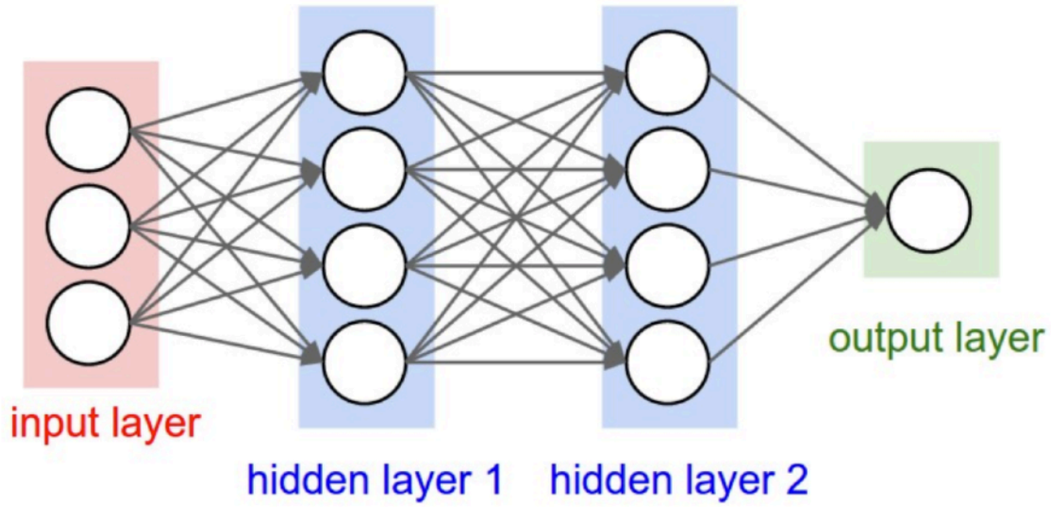
Form the basis for the modern approaches to machine translation, question answering and dialogue:



# Why variable-length?

Recall the feedfoward neural LMs we learned:




The dogs are barking

$$\mathbf{x} = [\mathbf{e}_{\text{the}}, \mathbf{e}_{\text{dogs}}, \mathbf{e}_{\text{are}}] \in \mathbb{R}^{3d}$$

(fixed-window size = 3)

the dogs in the neighborhood are ____

# Simple RNNs

$\mathbf{h}_0 \in \mathbb{R}^d$ is an initial state

$$\boxed{\mathbf{h}_t = f(\mathbf{h}_{t-1}, \mathbf{x}_t) \in \mathbb{R}^d}$$

$\mathbf{h}_t$ : hidden states which store information from $\mathbf{x}_1$ to $\mathbf{x}_t$

**Simple RNNs**:

$$\boxed{\mathbf{h}_t = g(\mathbf{W}\mathbf{h}_{t-1} + \mathbf{U}\mathbf{x}_t + \mathbf{b}) \in \mathbb{R}^d}$$

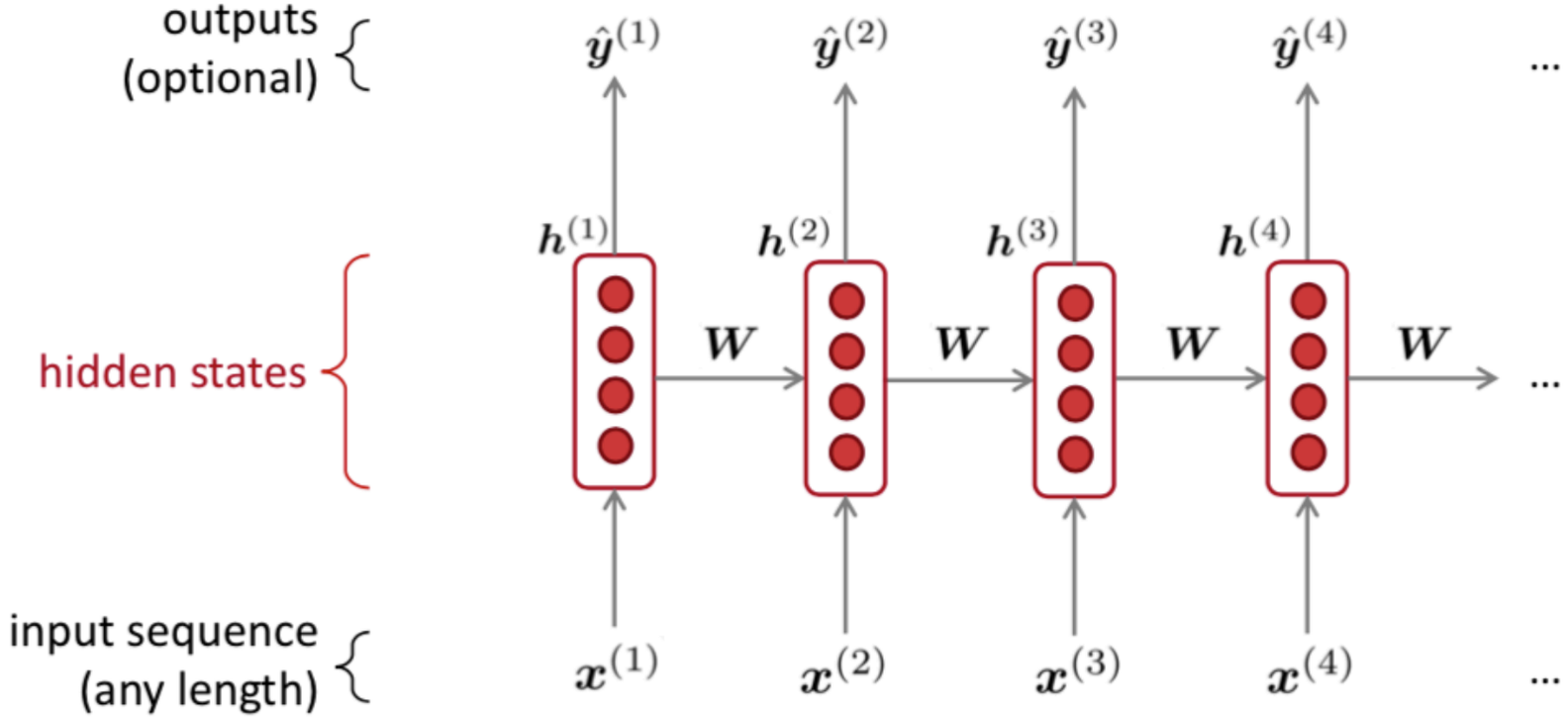
$g$: nonlinearity (e.g. tanh),

$\mathbf{W} \in \mathbb{R}^{d \times d}, \mathbf{U} \in \mathbb{R}^{d \times d_{in}}, \mathbf{b} \in \mathbb{R}^d$

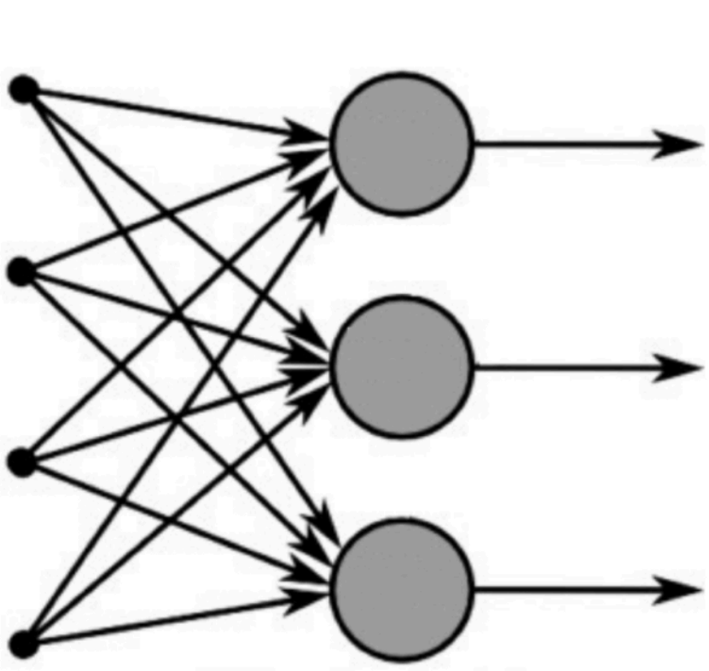
# Simple RNNs

$$\mathbf{h}_t = g(\mathbf{W}\mathbf{h}_{t-1} + \mathbf{U}\mathbf{x}_t + \mathbf{b}) \in \mathbb{R}^d$$

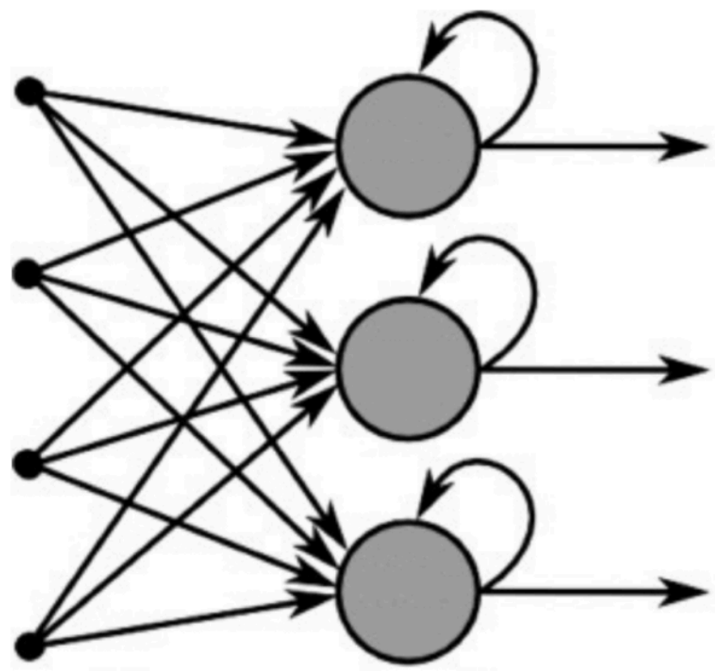
Key idea: apply the same weights $\mathbf{W}$ repeatedly



# RNNs vs Feedforward NNs



Feed-Forward Neural Network



Recurrent Neural Network

# Recurrent Neural Langhage Models (RNNLMs)

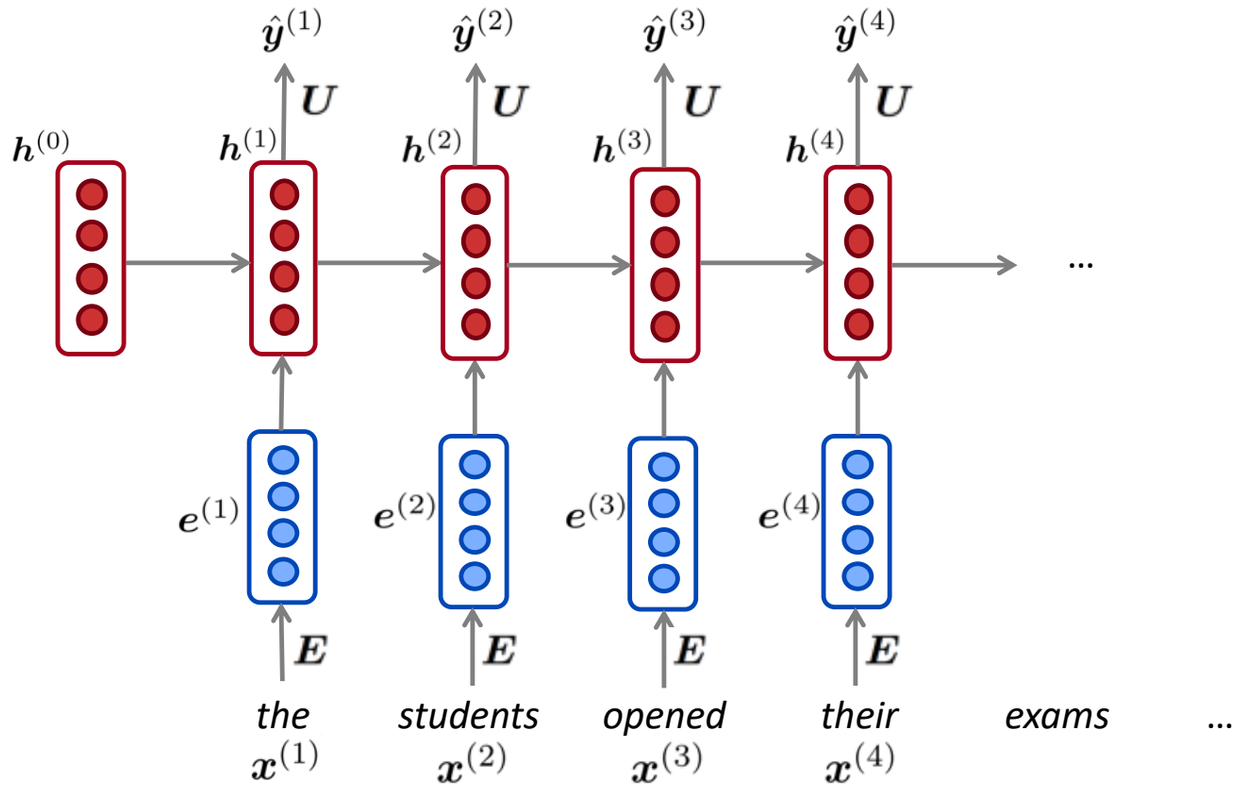
$$P(w_1, w_2, \ldots, w_n) = P(w_1) \times P(w_2 \mid w_1) \times P(w_3 \mid w_1, w_2) \times \ldots \times P(w_n \mid w_1, w_2, \ldots, w_{n-1})$$

$$= P(w_1 \mid \mathbf{h}_0) \times P(w_2 \mid \mathbf{h}_1) \times P(w_3 \mid \mathbf{h}_2) \times \ldots \times P(w_n \mid \mathbf{h}_{n-1})$$

- Denote $\hat{\mathbf{y}}_t = softmax(\mathbf{W}_o \mathbf{h}_t)$, $\mathbf{W}_o \in \mathbb{R}^{|V| \times d}$
- Cross-entroy loss:

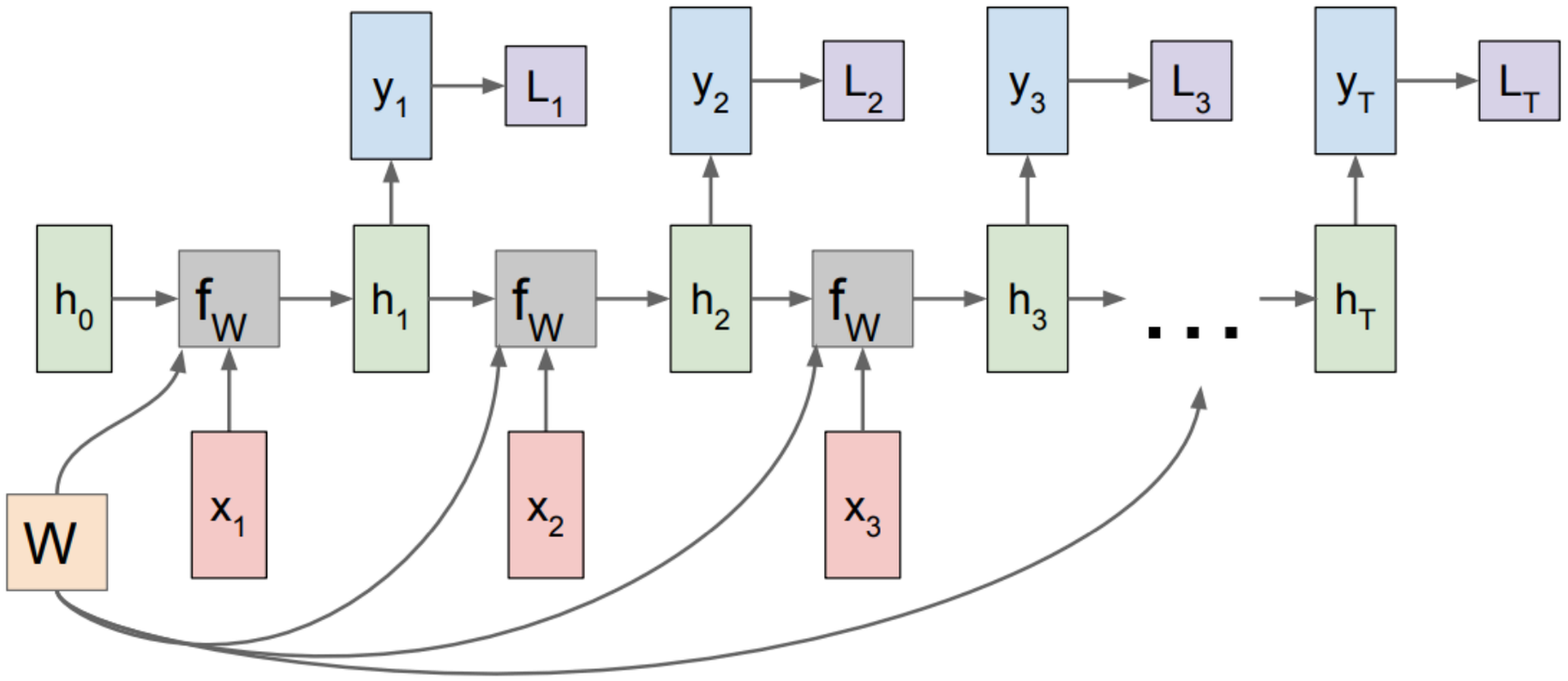
$$L(\theta) = -\frac{1}{n} \sum_{t=1}^{n} \log \hat{\mathbf{y}}_{t-1}(w_t)$$

$$\theta = \{\mathbf{W}, \mathbf{U}, \mathbf{b}, \mathbf{W}_o, \mathbf{E}\}$$



# Training RNNLMs

- Backpropagation? Yes, but not that simple!




- The algorithm is called Backpropagation Through Time (BPTT).

# Backpropagation through time

$$\mathbf{h}_1 = g(\mathbf{W}\mathbf{h}_0 + \mathbf{U}\mathbf{x}_1 + \mathbf{b})$$

$$\mathbf{h}_2 = g(\mathbf{W}\mathbf{h}_1 + \mathbf{U}\mathbf{x}_2 + \mathbf{b})$$

$$\mathbf{h}_3 = g(\mathbf{W}\mathbf{h}_2 + \mathbf{U}\mathbf{x}_3 + \mathbf{b})$$

$$L_3 = -\log \hat{\mathbf{y}}_3(w_4)$$

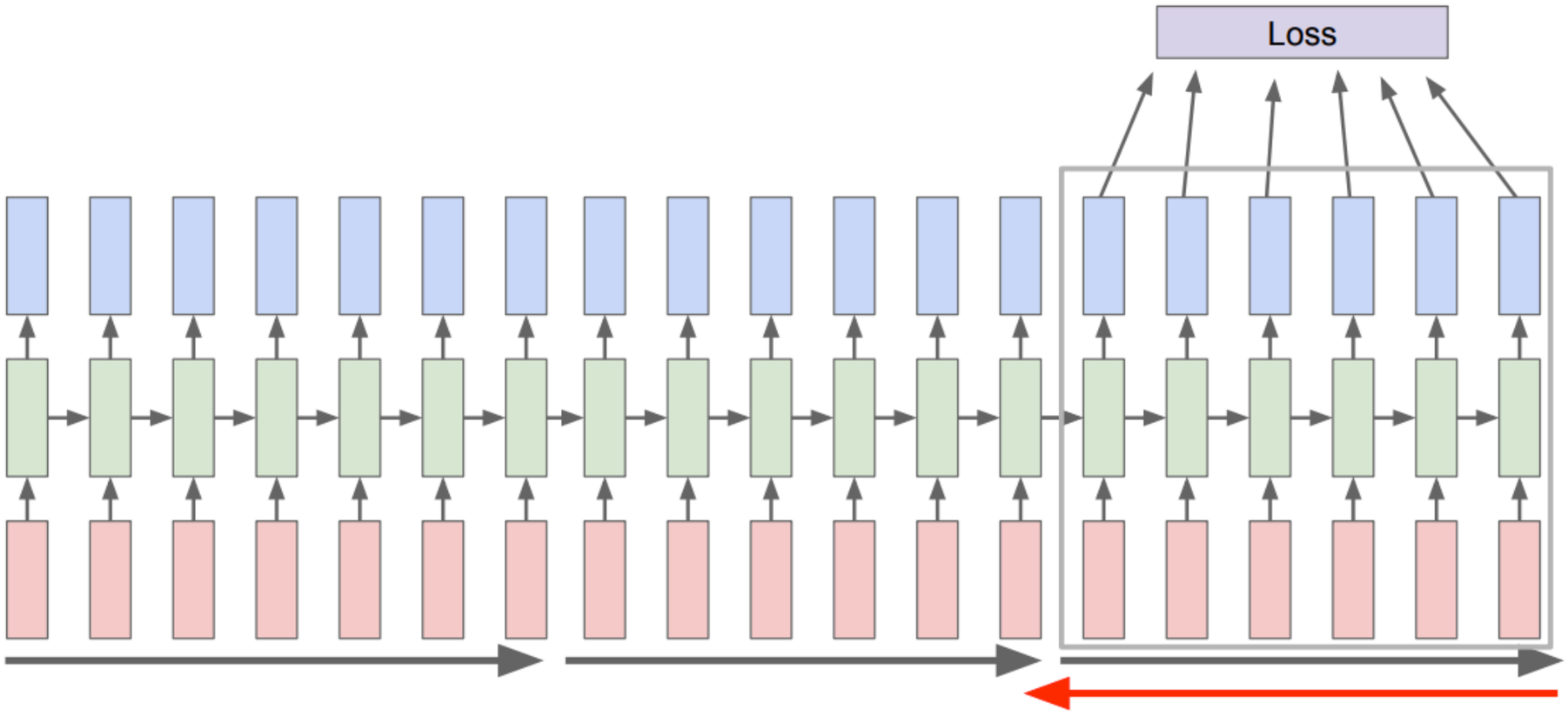
You should know how to compute: $\frac{\partial L_3}{\partial \mathbf{h}_3}$

$$\frac{\partial L_3}{\partial \mathbf{W}} = \frac{\partial L_3}{\partial \mathbf{h}_3}\frac{\partial \mathbf{h}_3}{\partial \mathbf{W}} + \frac{\partial L_3}{\partial \mathbf{h}_3}\frac{\partial \mathbf{h}_3}{\partial \mathbf{h}_2}\frac{\partial \mathbf{h}_2}{\partial \mathbf{W}} + \frac{\partial L_3}{\partial \mathbf{h}_3}\frac{\partial \mathbf{h}_3}{\partial \mathbf{h}_2}\frac{\partial \mathbf{h}_2}{\partial \mathbf{h}_1}\frac{\partial \mathbf{h}_1}{\partial \mathbf{W}}$$

$$\boxed{\frac{\partial L}{\partial \mathbf{W}} = -\frac{1}{n}\sum_{t=1}^{n}\sum_{k=1}^{t}\frac{\partial L_t}{\partial \mathbf{h}_t}\left(\prod_{j=k+1}^{t}\frac{\partial \mathbf{h}_j}{\partial \mathbf{h}_{j-1}}\right)\frac{\partial \mathbf{h}_k}{\partial \mathbf{W}}}$$

# Truncated backpropagation through time

- Backpropagation is very expensive if you handle long sequences




- Run forward and backward through chunks of the sequence instead of whole sequence
- Carry hidden states forward in time forever, but only backpropagate for some smaller number of steps

# Progress on language models

On the Penn Treebank (PTB) dataset

Metric: perplexity

KN5: Kneser-Ney 5-gram

| Model | Individual |
|---|---|
| KN5 | 141.2 |
| KN5 + cache | 125.7 |
| Feedforward NNLM | 140.2 |
| Log-bilinear NNLM | 144.5 |
| Syntactical NNLM | 131.3 |
| Recurrent NNLM | 124.7 |
| RNN-LDA LM | 113.7 |

(Mikolov and Zweig, 2012): Context dependent recurrent neural network language model

# Progress on language models

On the Penn Treebank (PTB) dataset

Metric: perplexity

| Model | #Param | Validation | Test |
|---|---|---|---|
| Mikolov & Zweig (2012) – RNN-LDA + KN-5 + cache | 9M‡ | - | 92.0 |
| Zaremba et al. (2014) – LSTM | 20M | 86.2 | 82.7 |
| Gal & Ghahramani (2016) – Variational LSTM (MC) | 20M | - | 78.6 |
| Kim et al. (2016) – CharCNN | 19M | - | 78.9 |
| Merity et al. (2016) – Pointer Sentinel-LSTM | 21M | 72.4 | 70.9 |
| Grave et al. (2016) – LSTM + continuous cache pointer† | - | - | 72.1 |
| Inan et al. (2016) – Tied Variational LSTM + augmented loss | 24M | 75.7 | 73.2 |
| Zilly et al. (2016) – Variational RHN | 23M | 67.9 | 65.4 |
| Zoph & Le (2016) – NAS Cell | 25M | - | 64.0 |
| Melis et al. (2017) – 2-layer skip connection LSTM | 24M | 60.9 | 58.3 |
| Merity et al. (2017) – AWD-LSTM w/o finetune | 24M | 60.7 | 58.8 |
| Merity et al. (2017) – AWD-LSTM | 24M | 60.0 | 57.3 |
| Ours – AWD-LSTM-MoS w/o finetune | 22M | 58.08 | 55.97 |
| Ours – AWD-LSTM-MoS | 22M | **56.54** | **54.44** |
| Merity et al. (2017) – AWD-LSTM + continuous cache pointer† | 24M | 53.9 | 52.8 |
| Krause et al. (2017) – AWD-LSTM + dynamic evaluation† | 24M | 51.6 | 51.1 |
| Ours – AWD-LSTM-MoS + dynamic evaluation† | 22M | **48.33** | **47.69** |

(Yang et al, 2018): Breaking the Softmax Bottleneck: A High-Rank RNN Language Model

(advanced)

# Vanishing/exploding gradients

- Consider the gradient of $L_t$ at step $t$, with respect to the hidden state $\mathbf{h}_k$ at some previous step $k$ ($k < t$):

$$\frac{\partial L_t}{\partial \mathbf{h}_k} = \frac{\partial L_t}{\partial \mathbf{h}_t}\left(\prod_{t \geq j > k} \frac{\partial \mathbf{h}_j}{\partial \mathbf{h}_{j-1}}\right)$$

$$= \frac{\partial L_t}{\partial \mathbf{h}_t} \times \prod_{t \geq j > k}\left(diag\left(g'(\mathbf{W}\mathbf{h}_{j-1} + \mathbf{U}\mathbf{x}_j + \mathbf{b})\right)\mathbf{W}\right)$$

- (Pascanu et al, 2013) showed that if the largest eigenvalue of $\mathbf{W}$ is less than 1 for $g = tanh$, then the gradient will shrink exponentially. This problem is called **vanishing gradients**.

- In contrast, if the gradients are getting too large, it is called **exploding gradients**.

# Why is exploding gradient a problem?

- Gradients become too big and we take a very large step in SGD.
- **Solution**: Gradient clipping — if the norm of the gradient is greater than some threshold, scale it down before applying SGD update.

**Algorithm 1** Pseudo-code for norm clipping

$\hat{\mathbf{g}} \leftarrow \frac{\partial \mathcal{E}}{\partial \theta}$
**if** $\|\hat{\mathbf{g}}\| \geq threshold$ **then**
  $\hat{\mathbf{g}} \leftarrow \frac{threshold}{\|\hat{\mathbf{g}}\|}\hat{\mathbf{g}}$
**end if**

# Why is vanishing gradient a problem?

- If the gradients becomes vanishingly small over long distances (step $k$ to step $t$), then we can't tell whether:
    - We don't need long-term dependencies
    - We have wrong parameters to capture the true dependency

the dogs in the neighborhood are ____

Still difficult to predict "barking"

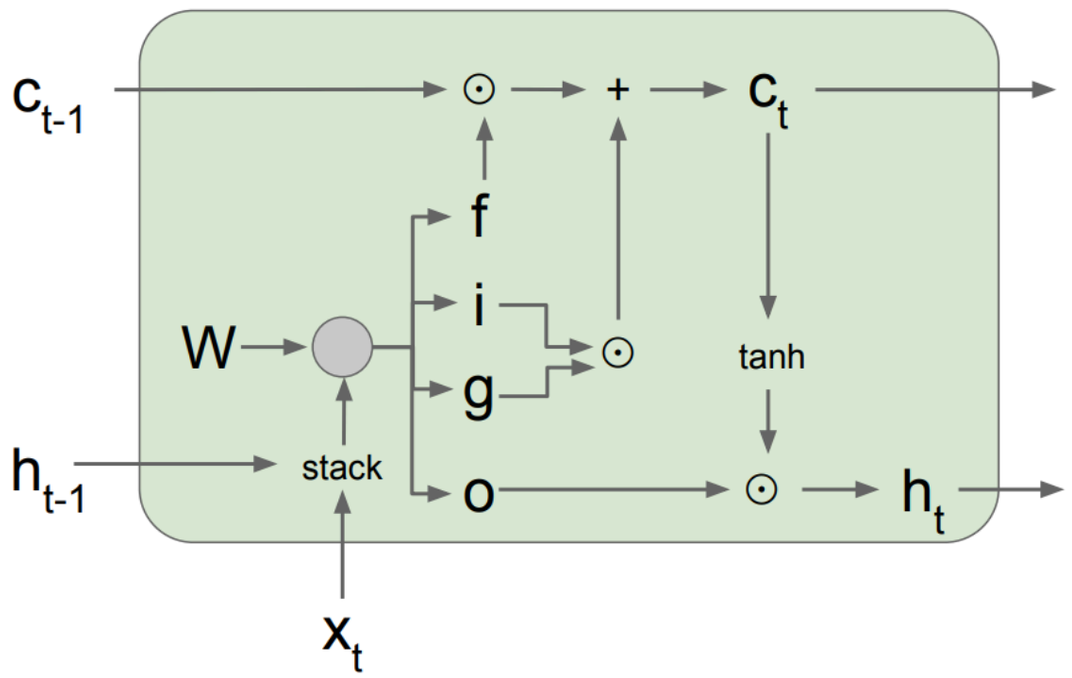
- How to fix vanishing gradient problem?
    - **LSTMs: Long short-term memory networks**
    - GRUs: Gated recurrent units

# Long Short-term Memory (LSTM)

- A type of RNN proposed by Hochreiter and Schmidhuber in 1997 as a solution to the vanishing gradients problem
- Work extremely well in practice
- **Basic idea**: turning multiplication into addition
- Use "gates" to control how much information to add/erase

$$\mathbf{h}_t = f(\mathbf{h}_{t-1}, \mathbf{x}_t) \in \mathbb{R}^d$$

- At each timestep, there is a hidden state $\mathbf{h}_t \in \mathbb{R}^d$ and also a cell state $\mathbf{c}_t \in \mathbb{R}^d$
  - $\mathbf{c}_t$ stores **long-term information**
  - We write/erase $\mathbf{c}_t$ after each step
  - We read $\mathbf{h}_t$ from $\mathbf{c}_t$



# Long Short-term Memory (LSTM)

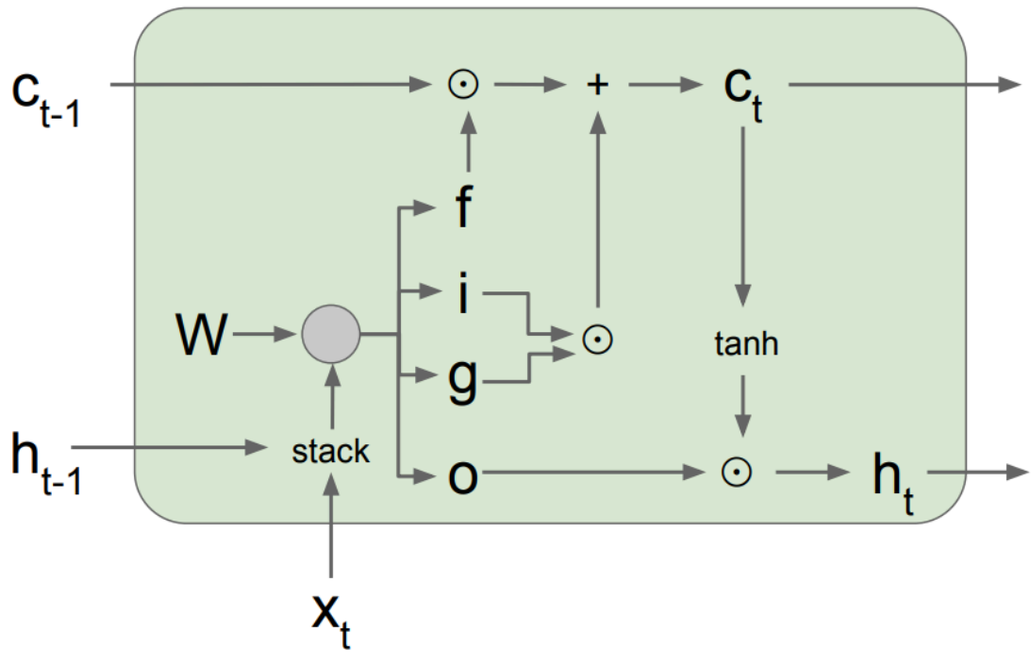
There are 4 gates:

- Input gate (how much to write):
  $\mathbf{i}_t = \sigma(\mathbf{W}^{(i)}\mathbf{h}_{t-1} + \mathbf{U}^{(i)}\mathbf{x}_t + \mathbf{b}^{(i)}) \in \mathbb{R}^d$

- Forget gate (how much to erase):
  $\mathbf{f}_t = \sigma(\mathbf{W}^{(f)}\mathbf{h}_{t-1} + \mathbf{U}^{(f)}\mathbf{x}_t + \mathbf{b}^{(f)}) \in \mathbb{R}^d$

- Output gate (how much to reveal):
  $\mathbf{o}_t = \sigma(\mathbf{W}^{(o)}\mathbf{h}_{t-1} + \mathbf{U}^{(o)}\mathbf{x}_t + \mathbf{b}^{(o)}) \in \mathbb{R}^d$

- New memory cell (what to write):
  $\tilde{\mathbf{c}}_t = \tanh(\mathbf{W}^{(c)}\mathbf{h}_{t-1} + \mathbf{U}^{(c)}\mathbf{x}_t + \mathbf{b}^{(c)}) \in \mathbb{R}^d$

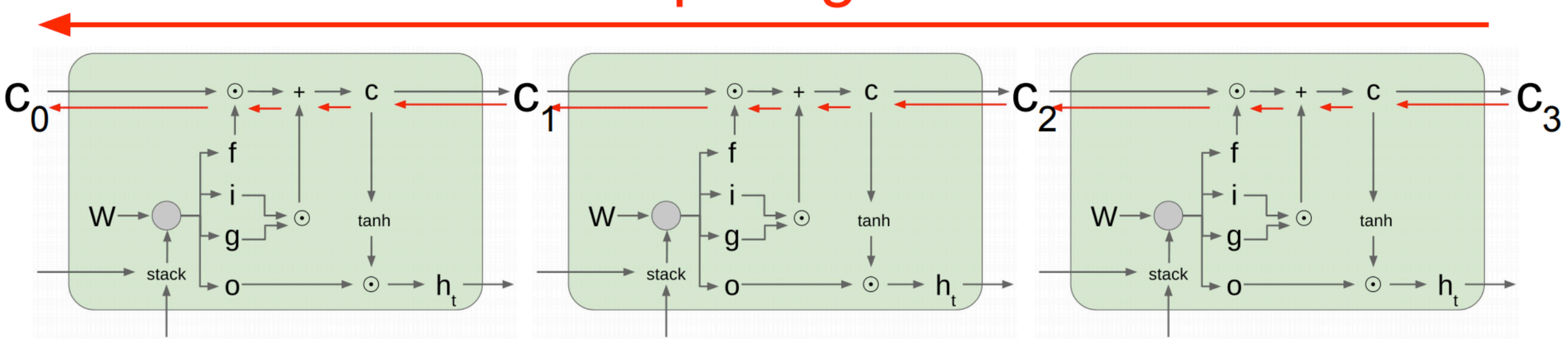



- Final memory cell: $\mathbf{c}_t = \mathbf{f}_t \odot \mathbf{c}_{t-1} + \mathbf{i}_t \odot \tilde{\mathbf{c}}_t$
- Final hidden cell: $\mathbf{h}_t = \mathbf{o}_t \odot \mathbf{c}_t$

element-wise product

How many parameters in total?

# Long Short-term Memory (LSTM)




- LSTM doesn't guarantee that there is no vanishing/exploding gradient, but it does provide an easier way for the model to learn long-distance dependencies

- LSTMs were invented in 1997 but finally got working from 2013-2015.

# Is the LSTM architecture optimal?

MUT1:

$$
\begin{aligned}
z &= \mathrm{sigm}(W_{\mathrm{xz}} x_t + b_{\mathrm{z}}) \\
r &= \mathrm{sigm}(W_{\mathrm{xr}} x_t + W_{\mathrm{hr}} h_t + b_{\mathrm{r}}) \\
h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + \tanh(x_t) + b_{\mathrm{h}}) \odot z \\
&+ h_t \odot (1 - z)
\end{aligned}
$$

MUT2:

$$
\begin{aligned}
z &= \mathrm{sigm}(W_{\mathrm{xz}} x_t + W_{\mathrm{hz}} h_t + b_{\mathrm{z}}) \\
r &= \mathrm{sigm}(x_t + W_{\mathrm{hr}} h_t + b_{\mathrm{r}}) \\
h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + W_{xh} x_t + b_{\mathrm{h}}) \odot z \\
&+ h_t \odot (1 - z)
\end{aligned}
$$

MUT3:

$$
\begin{aligned}
z &= \mathrm{sigm}(W_{\mathrm{xz}} x_t + W_{\mathrm{hz}} \tanh(h_t) + b_{\mathrm{z}}) \\
r &= \mathrm{sigm}(W_{\mathrm{xr}} x_t + W_{\mathrm{hr}} h_t + b_{\mathrm{r}}) \\
h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + W_{xh} x_t + b_{\mathrm{h}}) \odot z \\
&+ h_t \odot (1 - z)
\end{aligned}
$$

| Arch. | Arith. | XML | PTB |
|---|---|---|---|
| Tanh | 0.29493 | 0.32050 | 0.08782 |
| LSTM | 0.89228 | 0.42470 | 0.08912 |
| LSTM-f | 0.29292 | 0.23356 | 0.08808 |
| LSTM-i | 0.75109 | 0.41371 | 0.08662 |
| LSTM-o | 0.86747 | 0.42117 | 0.08933 |
| LSTM-b | 0.90163 | 0.44434 | 0.08952 |
| GRU | 0.89565 | 0.45963 | 0.09069 |
| MUT1 | **0.92135** | **0.47483** | 0.08968 |
| MUT2 | 0.89735 | **0.47324** | 0.09036 |
| MUT3 | 0.90728 | 0.46478 | **0.09161** |

| Arch. | 5M-tst | 10M-v | 20M-v | 20M-tst |
|---|---|---|---|---|
| Tanh | 4.811 | 4.729 | 4.635 | 4.582 (97.7) |
| LSTM | 4.699 | 4.511 | 4.437 | 4.399 (81.4) |
| LSTM-f | 4.785 | 4.752 | 4.658 | 4.606 (100.8) |
| LSTM-i | 4.755 | 4.558 | 4.480 | 4.444 (85.1) |
| LSTM-o | 4.708 | 4.496 | 4.447 | 4.411 (82.3) |
| LSTM-b | 4.698 | 4.437 | 4.423 | **4.380 (79.83)** |
| GRU | 4.684 | 4.554 | 4.559 | 4.519 (91.7) |
| MUT1 | 4.699 | 4.605 | 4.594 | 4.550 (94.6) |
| MUT2 | 4.707 | 4.539 | 4.538 | 4.503 (90.2) |
| MUT3 | 4.692 | 4.523 | 4.530 | 4.494 (89.47) |

(Jozefowicz et al, 2015): An Empirical Exploration of Recurrent Network Architectures

# Overview

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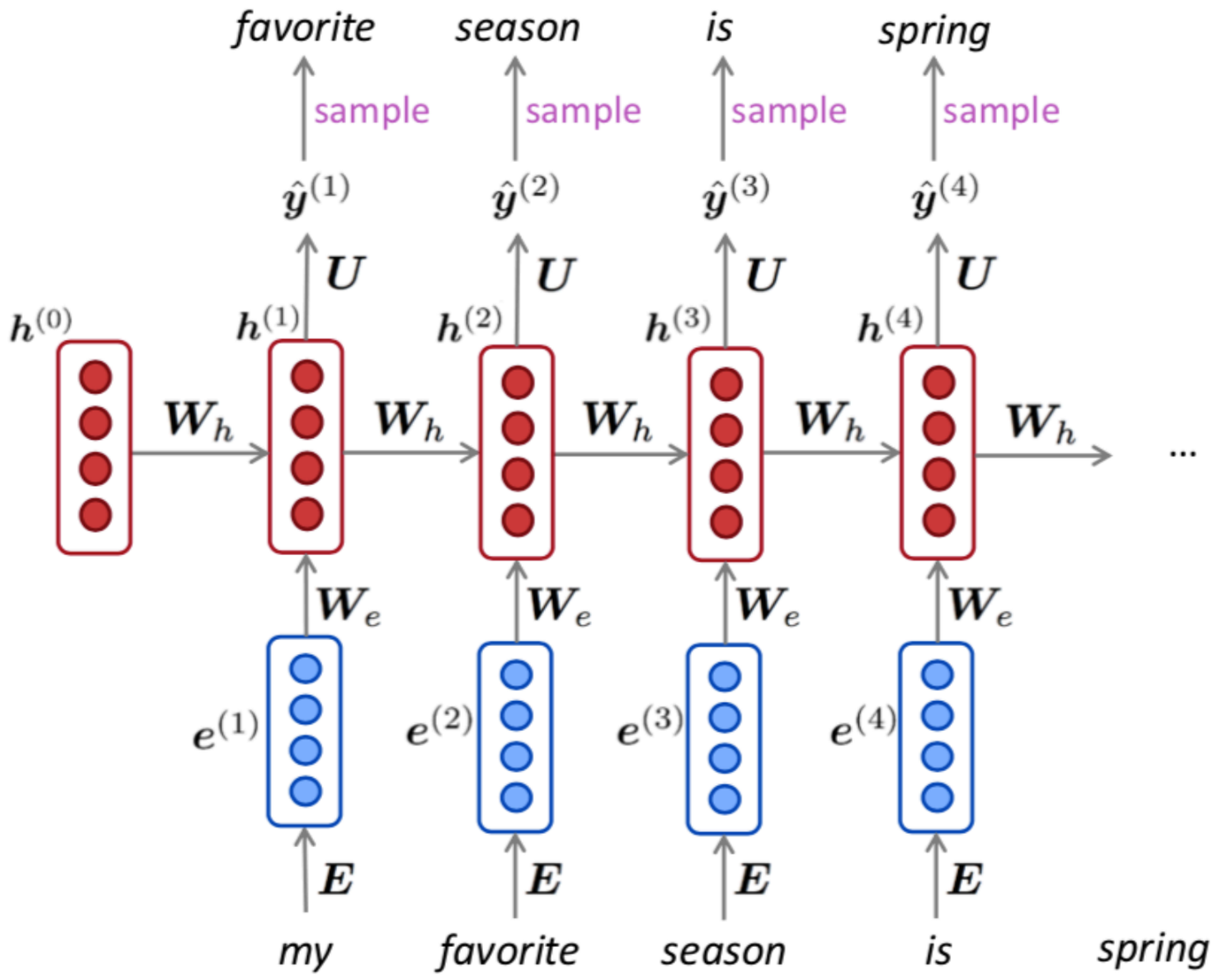
- Variants: Stacked RNNs, Bidirectional RNNs

# Application: Text Generation




You can generate text by **repeated sampling.**
Sampled output is next step's input.

# Fun with RNNs

## Obama speeches

*Good afternoon. God bless you.*

*The United States will step up to the cost of a new challenges of the American people that will share the fact that we created the problem. They were attacked and so that they have to say that all the task of the final days of war that I will not be able to get this done. The promise of the men and women who were still going to take out the fact that the American people have fought to make sure that they have to be able to protect our part. It was a chance to stand together to completely look for the commitment to borrow from the American people. And the fact is the men and women in uniform and the millions of our country with the law system that we should be a strong stretcks of the forces that we can afford to increase our spirit of the American people and the leadership of our country who are on the Internet of American lives.*

*Thank you very much. God bless you, and God bless the United States of America.*

## Latex generation

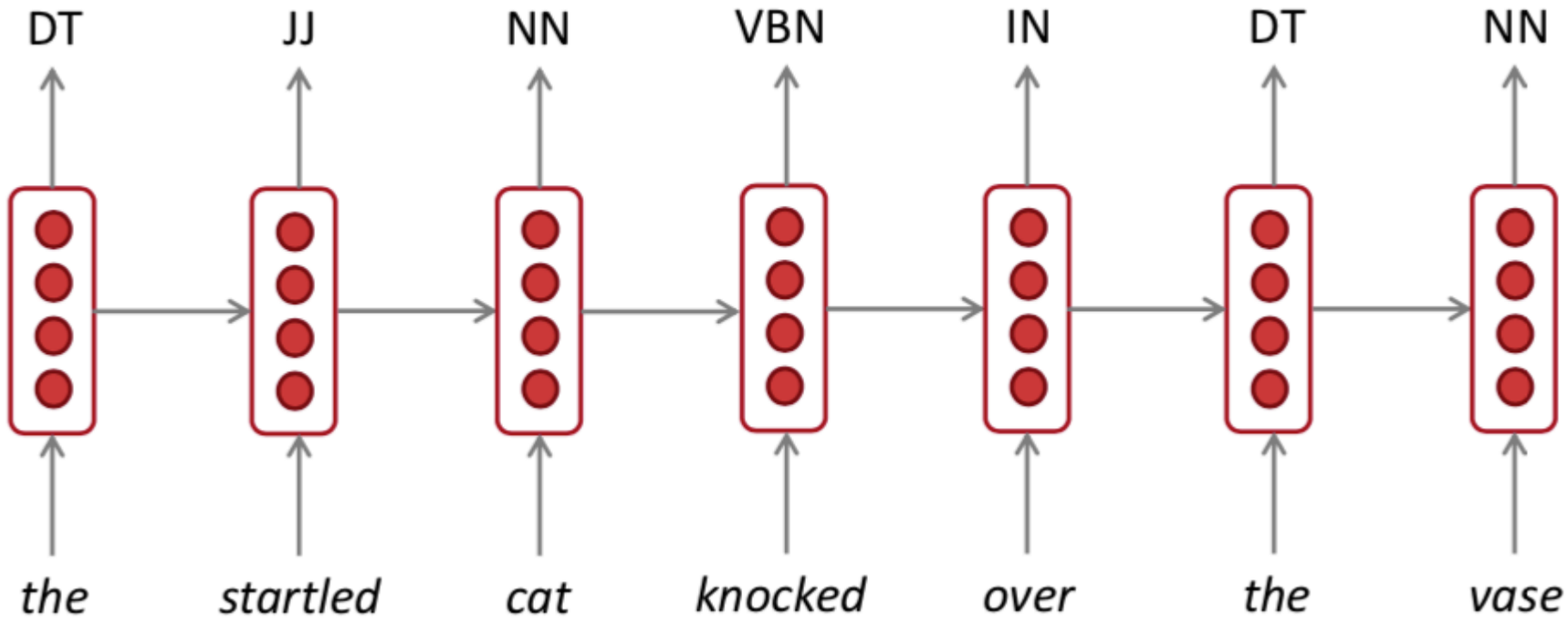
```
\begin{proof}
We may assume that $\mathcal{I}$ is an abelian sheaf on $\mathcal{C}$.
\item Given a morphism $\Delta : \mathcal{F} \to \mathcal{I}$
is an injective and let $\mathfrak q$ be an abelian sheaf on $X$.
Let $\mathcal{F}$ be a fibered complex. Let $\mathcal{F}$ be a category.
\begin{enumerate}
\item \hyperref[setain-construction-phantom]{Lemma}
\label{lemma-characterize-quasi-finite}
Let $\mathcal{F}$ be an abelian quasi-coherent sheaf on $\mathcal{C}$.
Let $\mathcal{F}$ be a coherent $\mathcal{O}_X$-module. Then
$\mathcal{F}$ is an abelian catenary over $\mathcal{C}$.
\item The following are equivalent
\begin{enumerate}
\item $\mathcal{F}$ is an $\mathcal{O}_X$-module.
\end{lemma}
```

Andrej Karpathy “The Unreasonable Effectiveness of Recurrent Neural Networks”

# Application: Sequence Tagging

Input: a sentence of $n$ words: $x_1, \ldots, x_n$

Output: $y_1, \ldots, y_n, y_i \in \{1,\ldots C\}$



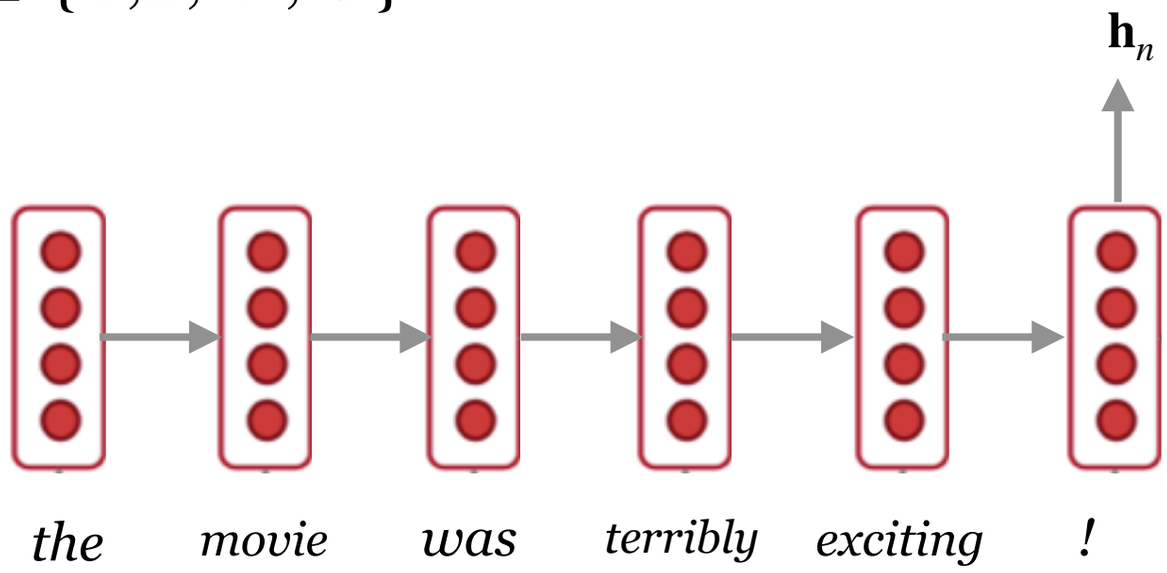

$$P(y_i = k) = softmax_k(\mathbf{W}_o \mathbf{h}_i) \quad \mathbf{W}_o \in \mathbb{R}^{C \times d}$$

$$L = -\frac{1}{n} \sum_{i=1}^{n} \log P(y_i = k)$$

# Application: Text Classification

Input: a sentence of $n$ words

Output: $y \in \{1,2,\ldots,C\}$

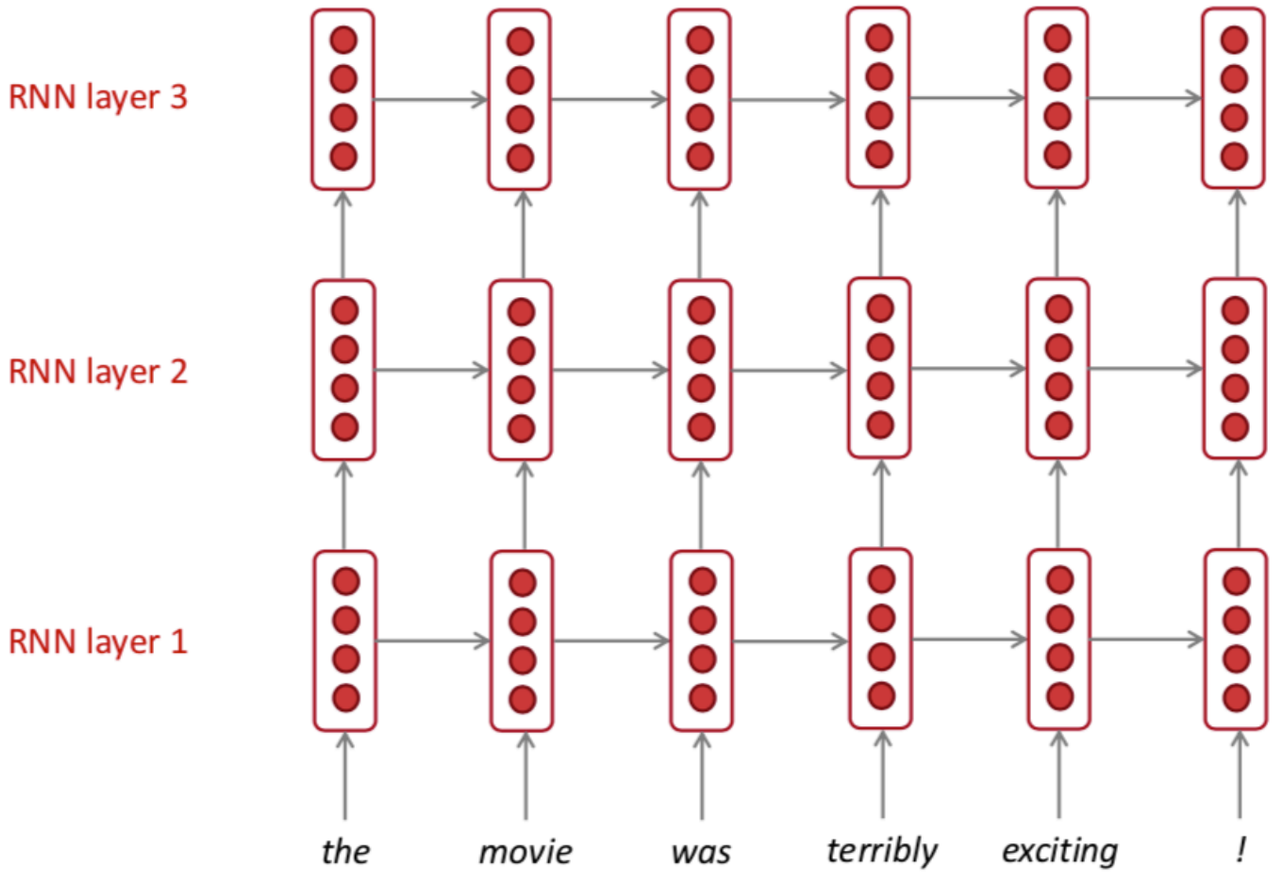



$$P(y = k) = softmax_k(\mathbf{W}_o\mathbf{h}_n) \qquad \mathbf{W}_o \in \mathbb{R}^{C \times d}$$

# Multi-layer RNNs

- RNNs are already “deep” on one dimension (unroll over time steps)
- We can also make them “deep” in another dimension by applying multiple RNNs
- Multi-layer RNNs are also called stacked RNNs.

# Multi-layer RNNs

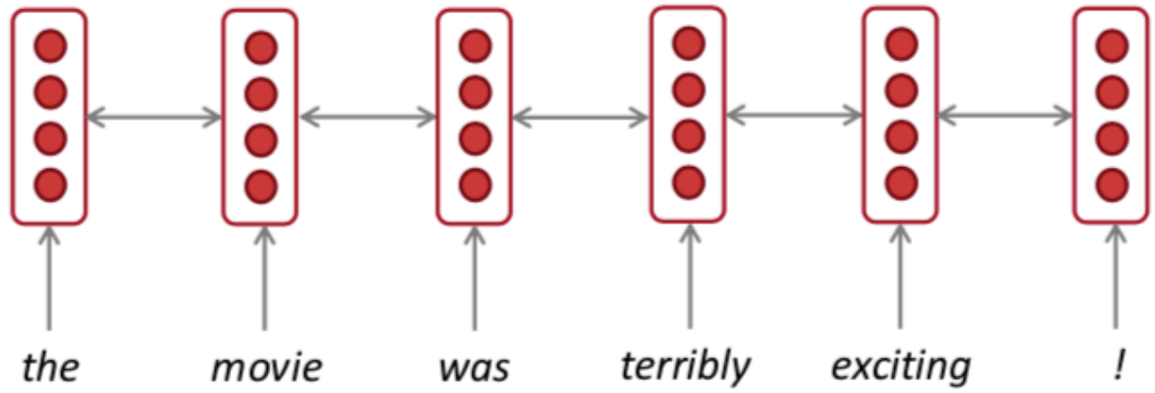



The hidden states from RNN layer $i$ are the inputs to RNN layer $i + 1$

- In practice, using 2 to 4 layers is common (usually better than 1 layer)
- Transformer-based networks can be up to 24 layers with lots of skip-connections.

# Bidirectional RNNs

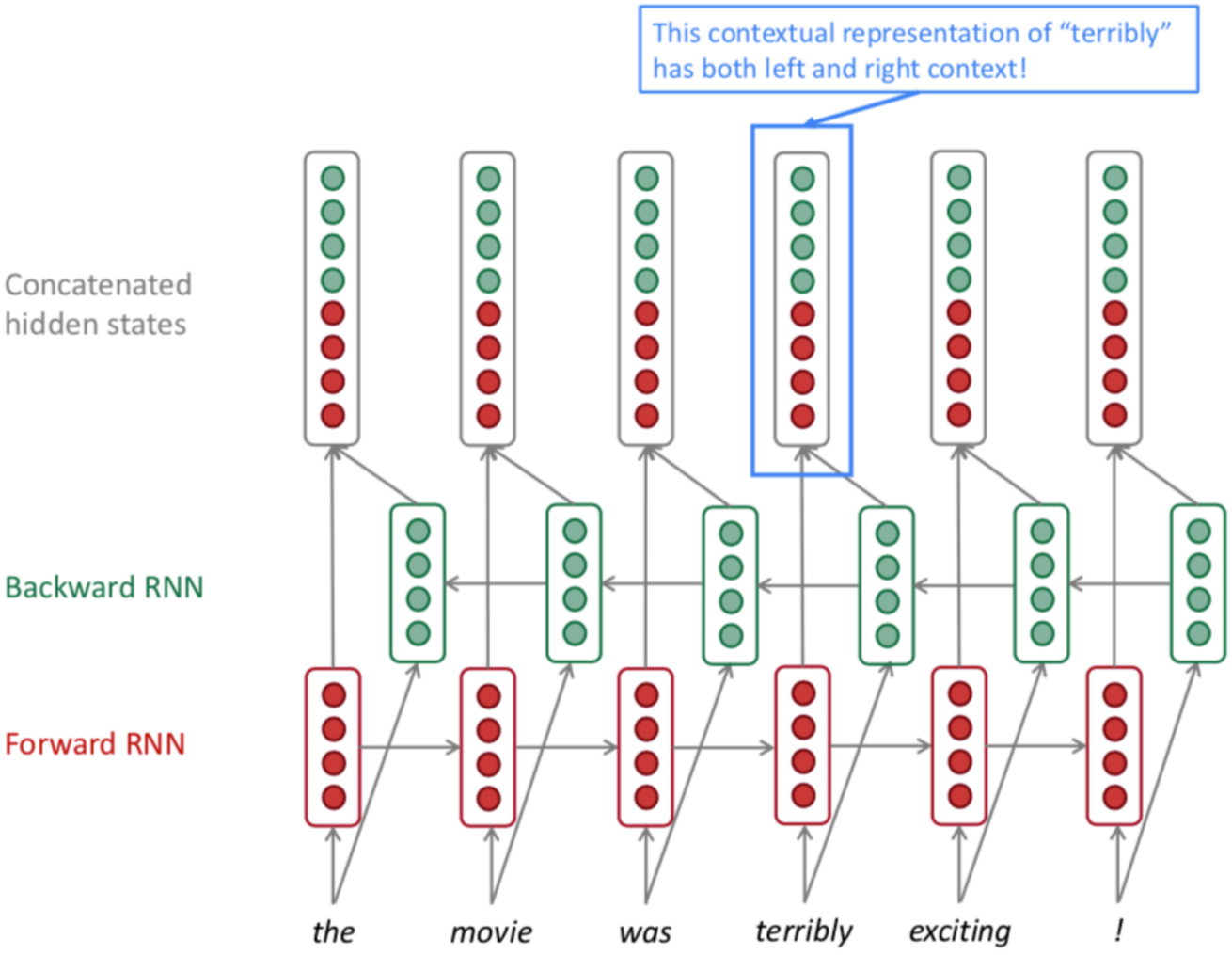
- Bidirectionality is important in language representations:




*terribly*:

- left context “the movie was”
- right context “exciting !”

# Bidirectional RNNs




$$\mathbf{h}_t = f(\mathbf{h}_{t-1}, \mathbf{x}_t) \in \mathbb{R}^d$$

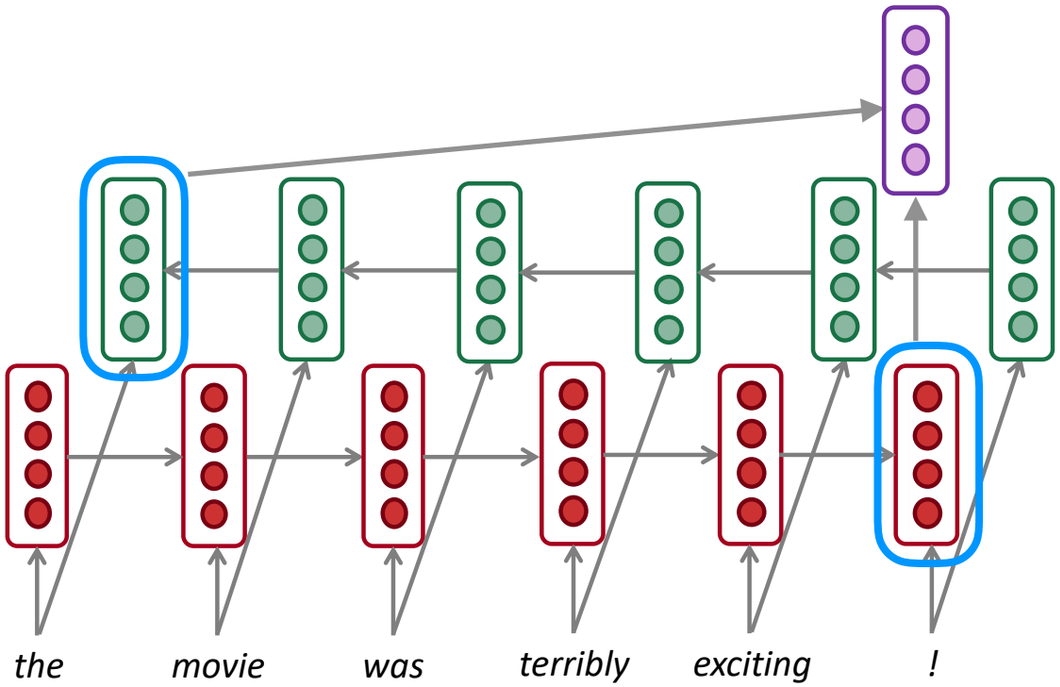
$$\overrightarrow{\mathbf{h}}_t = f_1(\overrightarrow{\mathbf{h}}_{t-1}, \mathbf{x}_t), t = 1,2,\ldots n$$

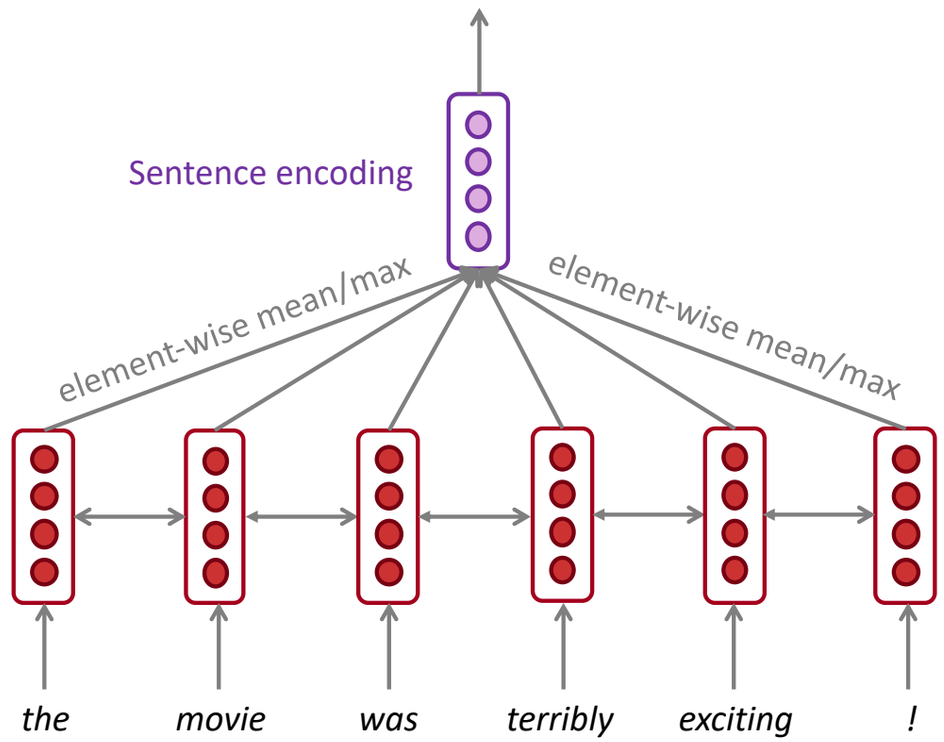
$$\overleftarrow{\mathbf{h}}_t = f_2(\overleftarrow{\mathbf{h}}_{t+1}, \mathbf{x}_t), t = n, n-1,\ldots 1$$

$$\mathbf{h}_t = [\overleftarrow{\mathbf{h}}_t, \overrightarrow{\mathbf{h}}_t] \in \mathbb{R}^{2d}$$

# Bidirectional RNNs

- Sequence tagging: Yes!
- Text classification: Yes! With slight modifications.




- Text generation: No. Why?