COS 484: Natural Language Processing

Log-linear models

Fall 2019

Announcements

- Assignment 2 will be available soon
  - **due Monday, Oct 7, 11:59pm**
  - Start early!
- All assignments due **Mondays before lectures**

Last time

- Supervised classification:
  - Document to classify, d
  - Set of classes, $C = \{c_1, c_2, \ldots, c_k\}$
- Naive Bayes:

$$\hat{c} = \arg\max_{c} P(c)\, P(d \mid c)$$

Logistic Regression

- Powerful supervised model
- Baseline approach to most NLP tasks
- Connections with neural networks
- Binary (two classes) or multinomial (>2 classes)

Discriminative Model

- Logistic Regression is a *discriminative* model
- Naive Bayes is a *generative* model

Discriminative Model

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- Naive Bayes: $\hat{c} = \underset{c}{\mathrm{argmax}}\ P(c)\, P(d \mid c)$

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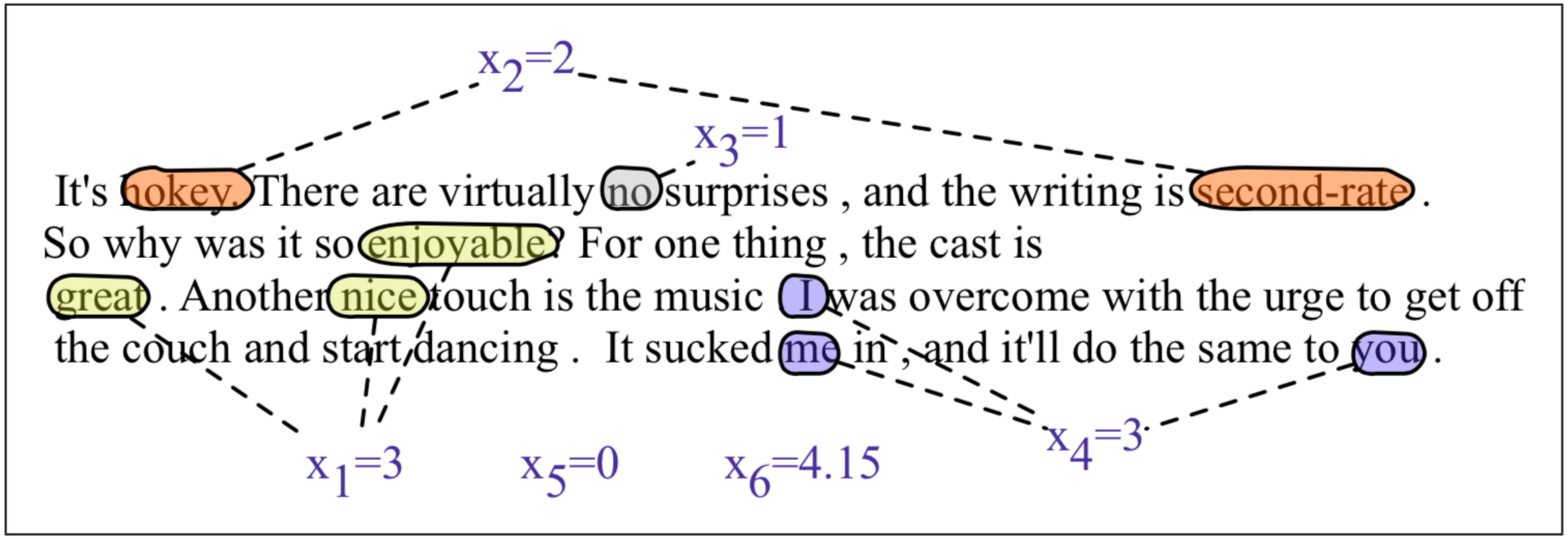
- Train phase: Learn the **parameters** of the model to minimize **loss function**

- Test phase: Apply **parameters** to predict class given a new input x

Feature representation

- Input observation: $x^{(i)}$
- Feature vector: $[x_1, x_2, \ldots, x_d]$
- Feature j of i[th] input : $x_j^{(i)}$

Sample feature vector



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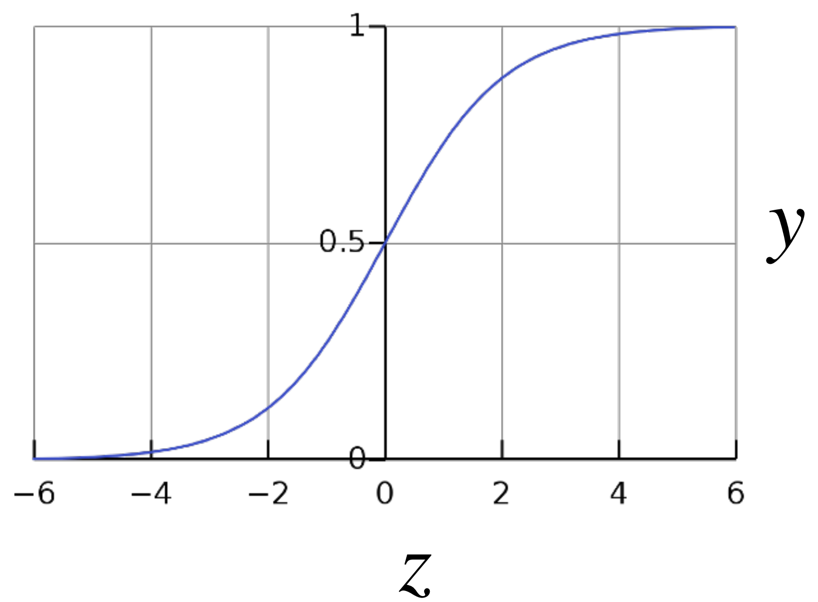
$$y = \frac{1}{1 + e^{-z}}$$



Weights and Biases

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- **Bias:** Scalar intercept, b
- Given an instance, x: $z = \sum_{i=1}^{d} w_i x_i + b$ or $z = w \cdot x + b$

What is the bias?

- Let’s say we have a feature that is always set to 1 regardless of what the input text is.
- This is clearly not an informative feature. However, let’s say it was the only one I had…

first, how many weights do I need to learn for this feature?

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$$w \cdot x + b$$

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okay… what is the best set of weights for it?

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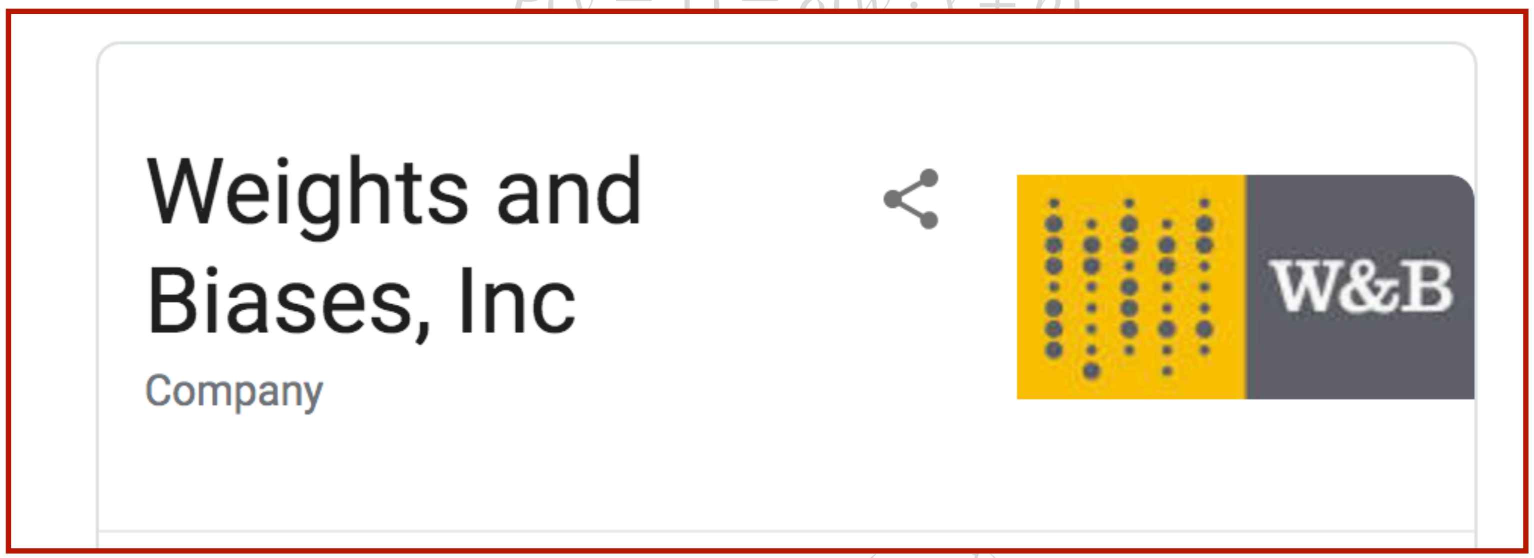

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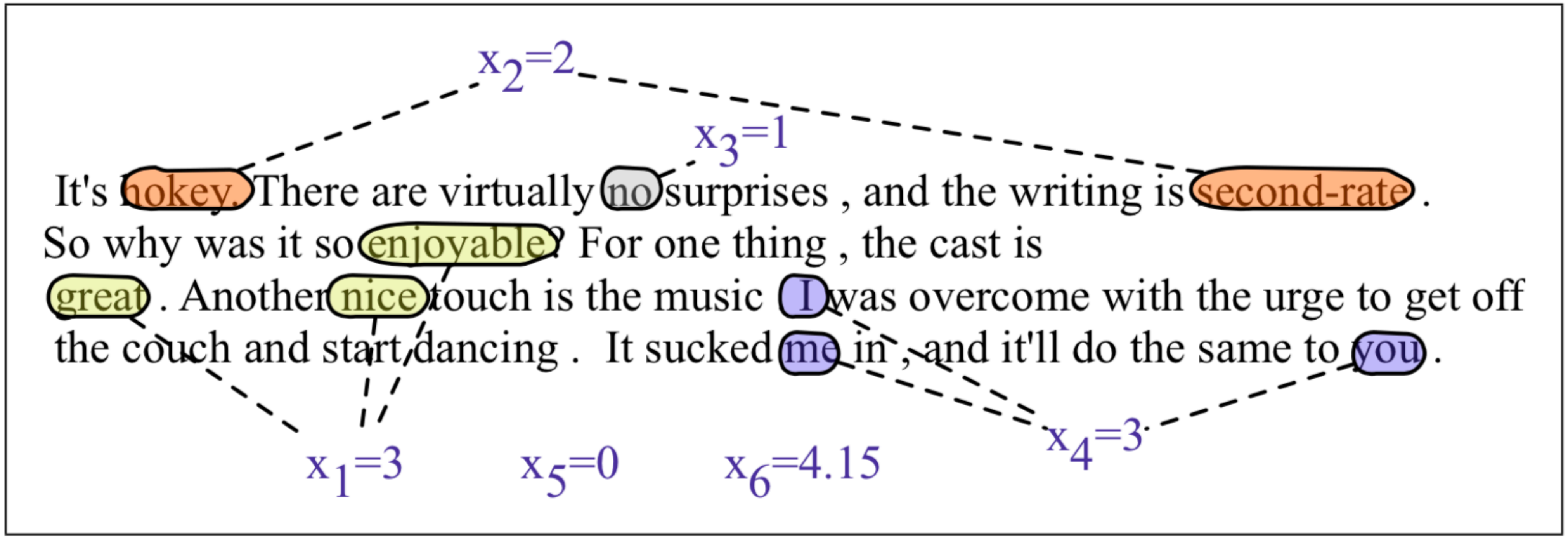

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p(+|x) = P(Y=1|x) &= \sigma(w \cdot x + b) \\
&= \sigma([2.5, -5.0, -1.2, 0.5, 2.0, 0.7] \cdot [3, 2, 1, 3, 0, 4.15] + 0.1) \\
&= \sigma(.805) \\
&= 0.69 \\
p(-|x) = P(Y=0|x) &= 1 - \sigma(w \cdot x + b) \\
&= 0.31
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- Advanced: Representation learning (we will see this later!)

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  - More robust to correlated features ("San Francisco" vs "Boston") —LR is likely to work better than NB
  - Can even have the same feature twice! (*why?*)
- **However**: NB often better on very small datasets

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  - **Optimization algorithm** for updating weights

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  - Similar to language models!
    - $\max \log P(w_t \mid w_{t-n}, \ldots, w_{t-1})$ given a corpus

Cross Entropy loss

- Assume a single data point (x, y) and two classes

- Classifier probability: $P(y \mid x) = \hat{y}^{\,y}(1 - \hat{y})^{1-y}$

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$-\log P(y|x)$

$= -\left[y \log \hat{y} + (1-y)\log(1-\hat{y})\right]$

$y = 1 \Rightarrow -\log \hat{y}$, $y = 0 \Rightarrow -\log(1-\hat{y})$

Cross Entropy loss

- Assume n data points $(x^{(i)}, y^{(i)})$
- Classifier probability: $\Pi_{i=1}^{n} P(y \mid x) = \Pi_{i=1}^{n} \hat{y}^{y}(1 - \hat{y})^{1-y}$
- CE Loss: $-\log \prod_{i=1}^{n} P(y|x)$

$$= -\sum_{i=1}^{n} \log P(y|x)$$

$$L_{CE} = -\sum_{i=1}^{n} \left[y \log \hat{y} + (1-y) \log (1-\hat{y}) \right]$$

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- If y = 0 (negative sentiment), $L_{CE} = -\log(0.31) = 1.17$

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- *Cross-entropy* between the true distribution $P(y \mid x)$ and predicted distribution $P(\hat{y} \mid x)$

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  - Find direction of steepest slope

Optimization

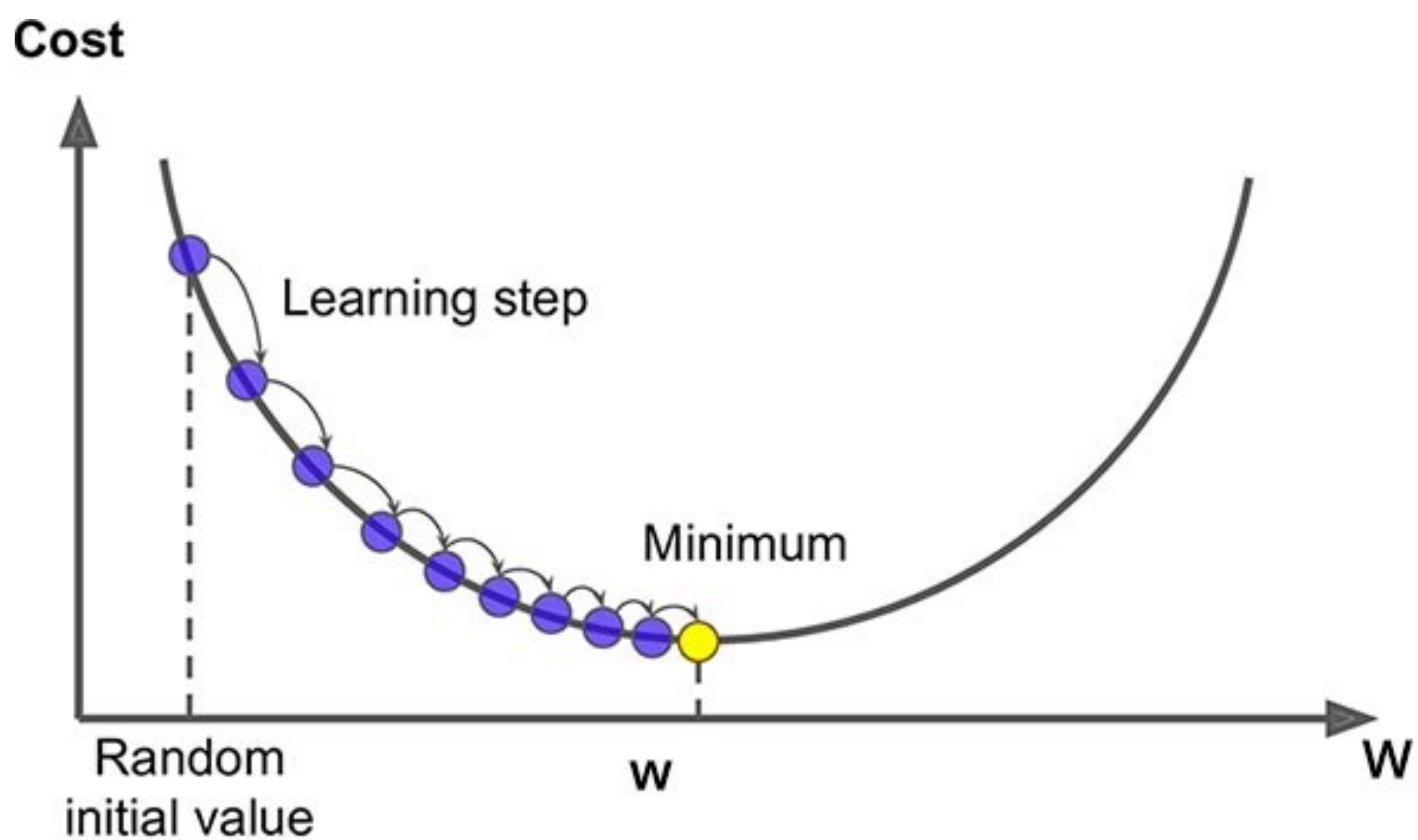
- We have our **classification function** and **loss function** - how do we find the best w and b?

$$\theta = [w; b]$$

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{n} \sum_{i=1}^{n} L_{CE}(y^{(i)}, x^{(i)}; \theta)$$

- Gradient descent:
  - Find direction of steepest slope
  - Move in the opposite direction

Gradient descent (1-D)



$$\theta^{t+1} = \theta^{t} - \eta \frac{d}{d\theta} f(x;\theta)$$

Gradient descent for LR

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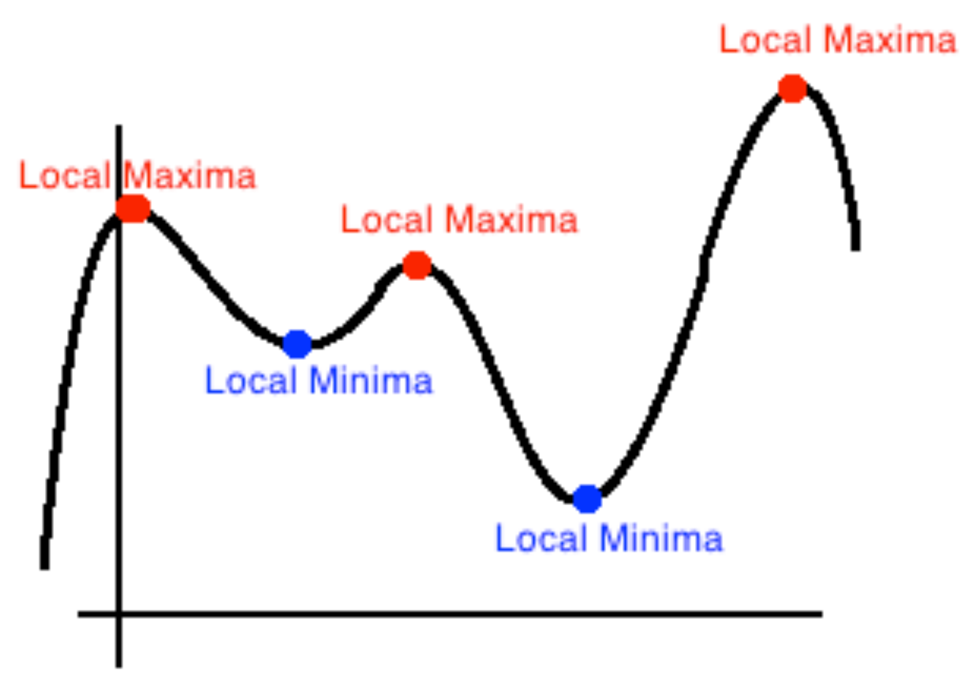
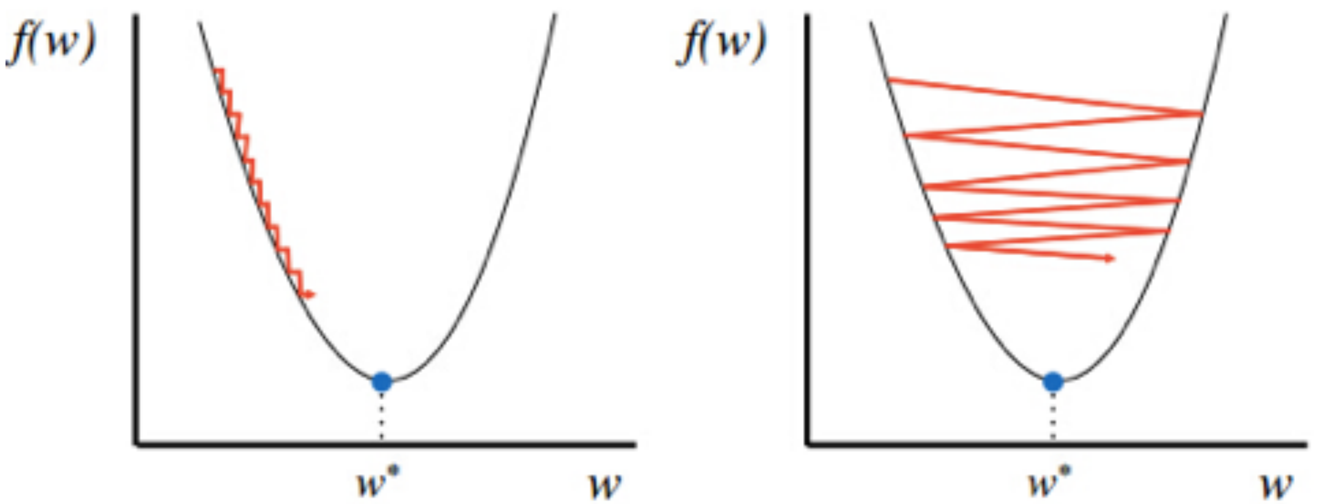


Learning Rate

- Updates: $\theta^{t+1} = \theta^{t} - \eta \frac{d}{d\theta} f(x;\theta)$

- Magnitude of movement along gradient

- Higher/faster learning rate = larger updates to parameters



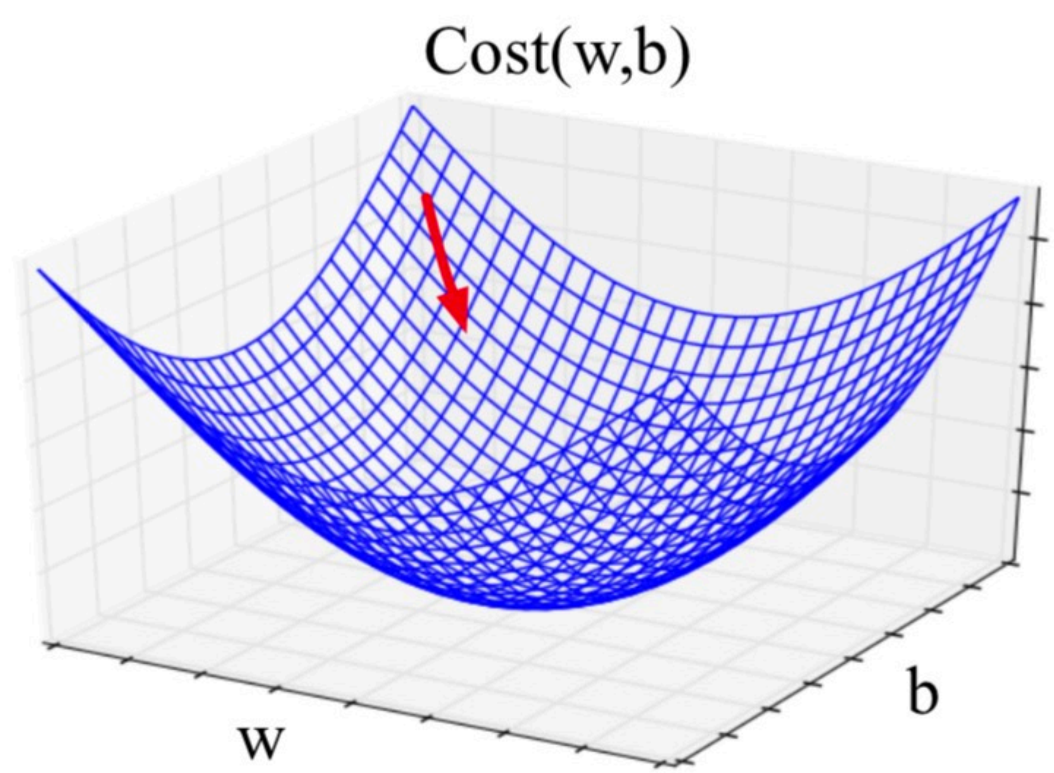
Too small: converge very slowly

Too big: overshoot and even diverge

Gradient descent with vector weights

- In LR: weight w is a vector
- Express slope as a partial derivative of loss w.r.t each weight:

$$\nabla_\theta L(f(x;\theta),y)) \;=\; \begin{bmatrix} \frac{\partial}{\partial w_1} L(f(x;\theta),y) \\ \frac{\partial}{\partial w_2} L(f(x;\theta),y) \\ \vdots \\ \frac{\partial}{\partial w_n} L(f(x;\theta),y) \end{bmatrix}$$



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Gradient for logistic regression

Gradient for logistic regression

- $L_{CE} = -\sum_{i=1}^{n} [y^{(i)} \log \sigma(w \cdot x^{(i)} + b) + (1 - y^{(i)}) \log(1 - \sigma(w \cdot x^{(i)} + b))]$

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Diff between true y and prediction

input feature value

- $\dfrac{dL_{CE}(w, b)}{db} = \sum_{i=1}^{n} [\sigma(w \cdot x^{(i)} + b) - y^{(i)}]$

Stochastic Gradient Descent

- Online optimization
- Compute loss and minimize after each training example
- **function** STOCHASTIC GRADIENT DESCENT($L()$, $f()$, x, y) **returns** θ

    \# where: L is the loss function
    \# f is a function parameterized by θ
    \# x is the set of training inputs $x^{(1)}, x^{(2)}, \ldots, x^{(n)}$
    \# y is the set of training outputs (labels) $y^{(1)}, y^{(2)}, \ldots, y^{(n)}$

    $\theta \leftarrow 0$
    repeat til done # see caption
      For each training tuple $(x^{(i)}, y^{(i)})$ (in random order)
        1. Optional (for reporting): # How are we doing on this tuple?
          Compute $\hat{y}^{(i)} = f(x^{(i)}; \theta)$ # What is our estimated output $\hat{y}$?
          Compute the loss $L(\hat{y}^{(i)}, y^{(i)})$ # How far off is $\hat{y}^{(i)}$) from the true output $y^{(i)}$?
        2. $g \leftarrow \nabla_\theta L(f(x^{(i)}; \theta), y^{(i)})$ # How should we move θ to maximize loss?
        3. $\theta \leftarrow \theta - \eta\, g$ # Go the other way instead
    return θ

Per Instance Loss →

Stochastic Gradient Descent

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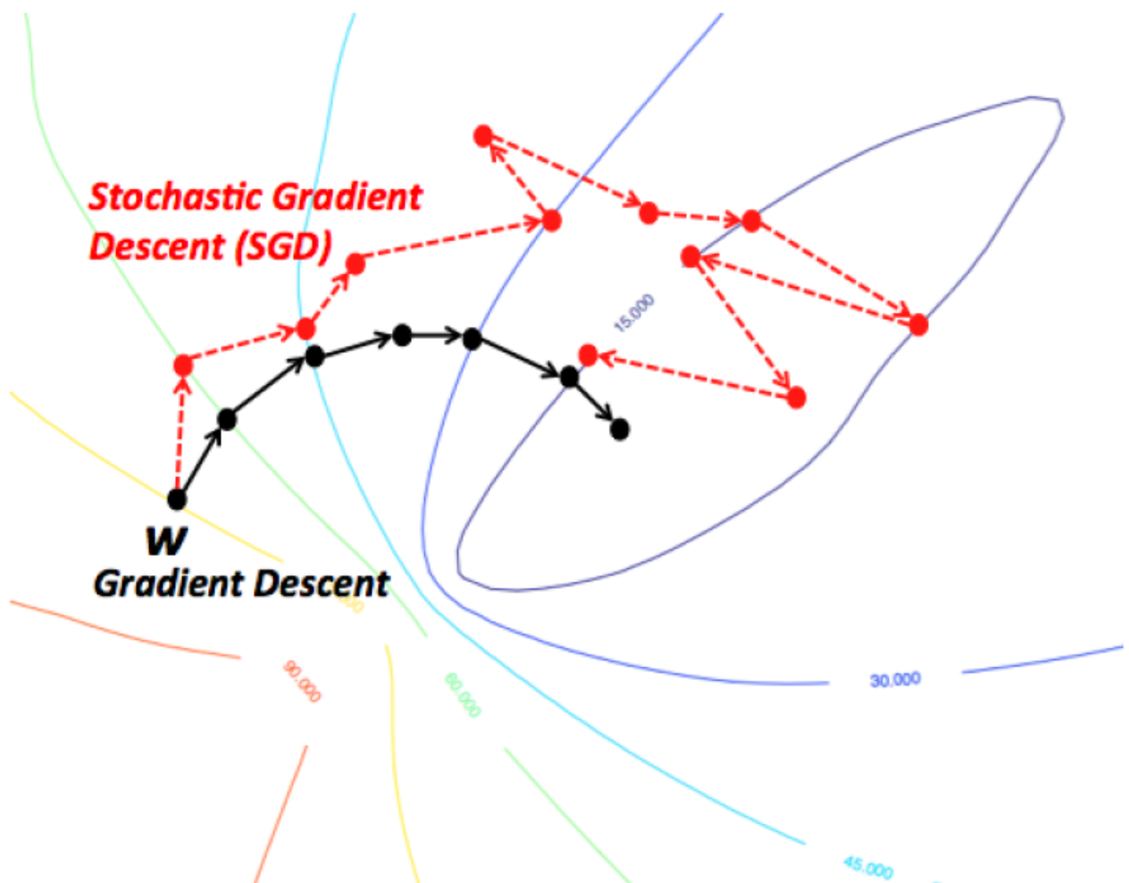
- 

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L2 regularization

- $R(\theta) = ||\theta||^2 = \sum_{j=1}^{d} \theta_j^2$

- Euclidean distance of weight vector θ from origin

- L2 regularized objective:

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L1 Regularization

- $R(\theta) = ||\theta||_1 = \sum_{j=1}^{d} |\theta_j|$

- Manhattan distance of weight vector θ from origin

- L1 regularized objective:

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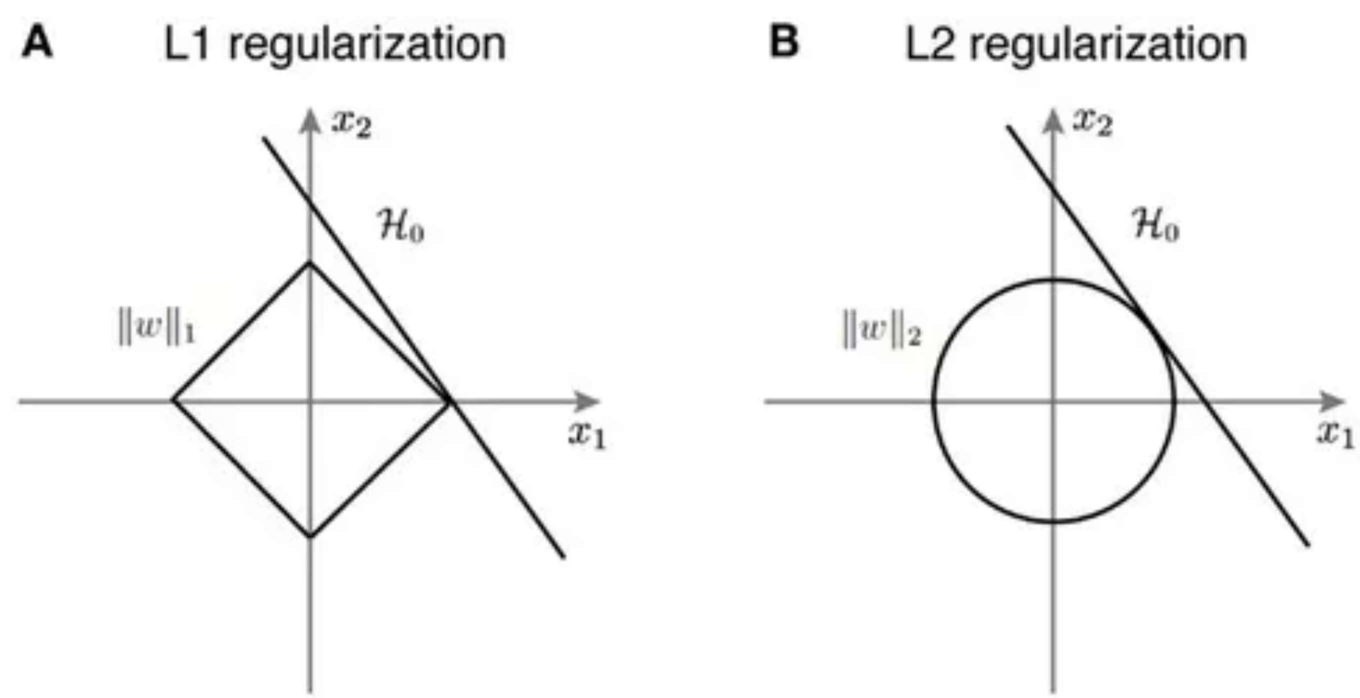

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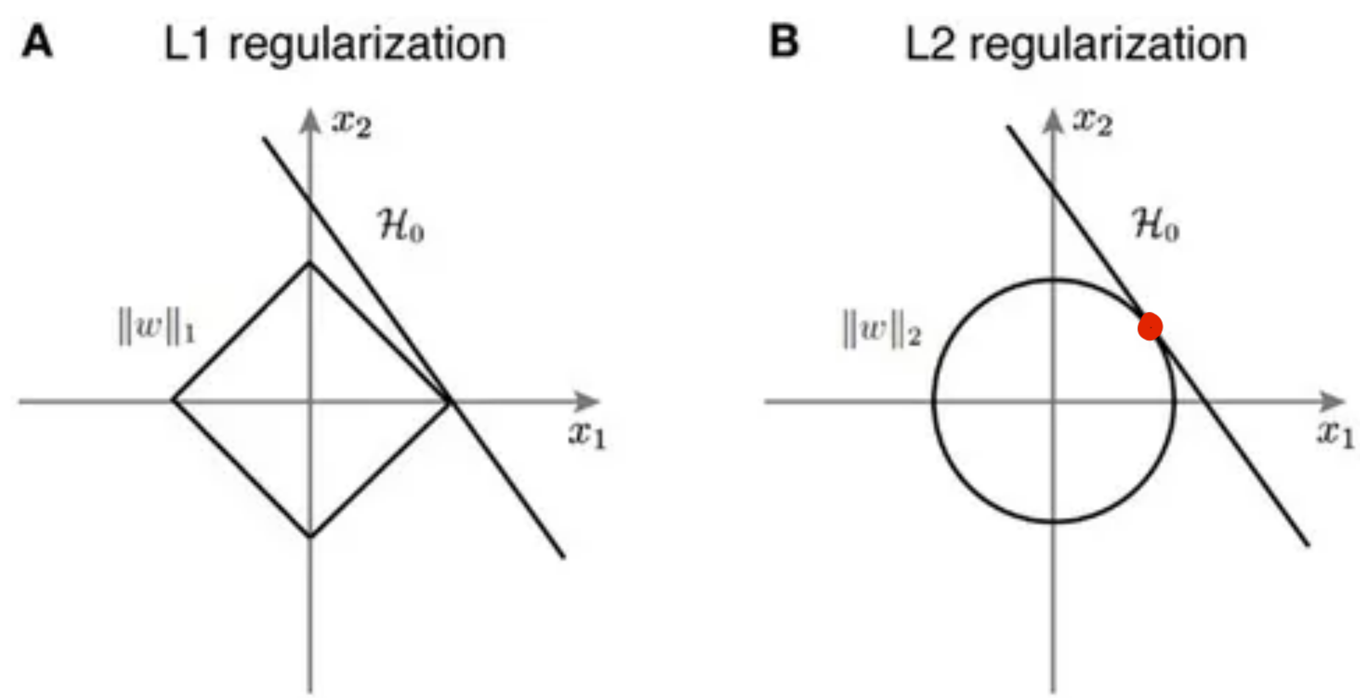

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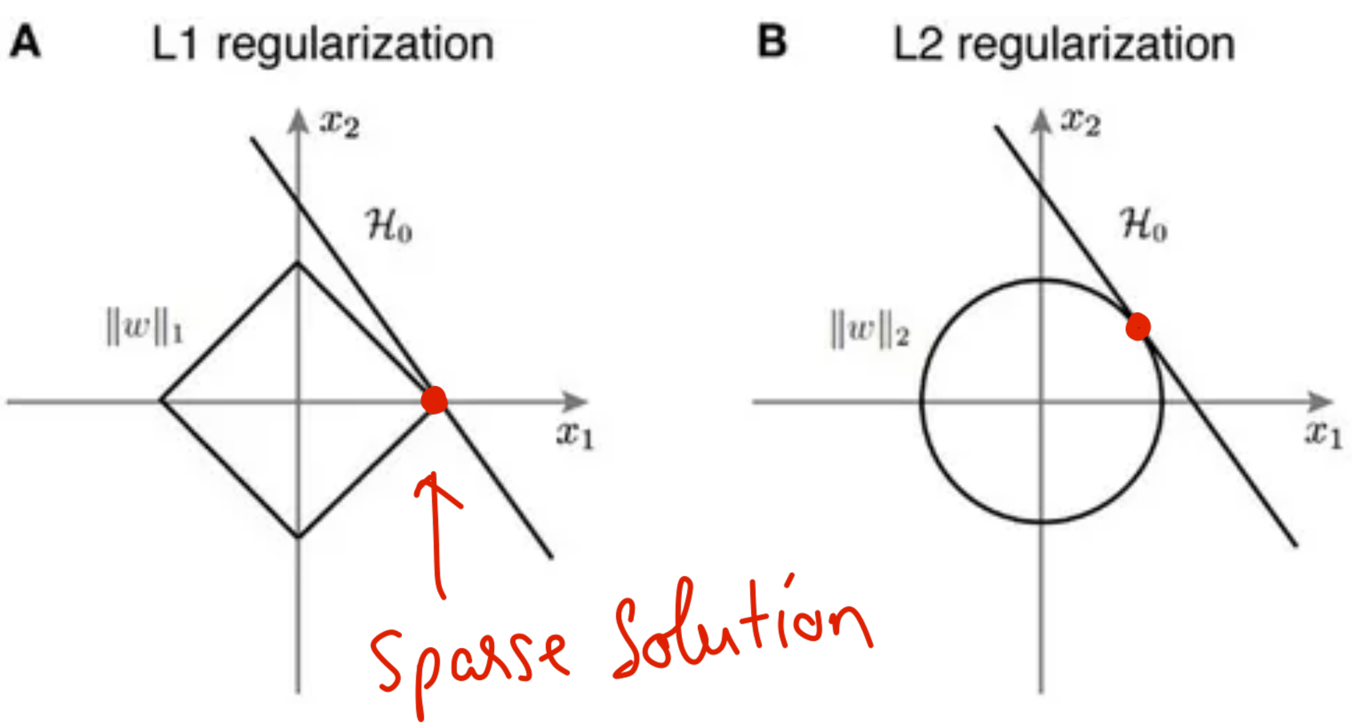

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← Normalization

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$$\log P(y = c \mid x) \propto w_c \cdot x + b_c$$

(log-linear)

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Var	Definition	Wt
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$f_1(+,x)$	$\begin{cases} 1 & \text{if "!"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	2.6
$f_1(-,x)$	$\begin{cases} 1 & \text{if "!"} \in \text{doc} \\ 0 & \text{otherwise} \end{cases}$	1.3

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$$\frac{dL_{CE}}{dw_c} = -(1\{y = c\} - P(y = c \,|\, x))x_c$$

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