Lecture 2: Image formation and capture

COS 429: Computer Vision

Slides adapted from: Szymon Rusinkiewicz, Jia Deng
Pinhole camera: overview
Let’s design a camera

Idea 1: put a piece of film in front of an object
Do we get a reasonable image?
Let’s design a camera

Add a barrier to block off most of the rays
Pinhole camera

- Captures **pencil of rays** – all rays through a single point: aperture, center of projection, optical center, focal point, camera center
- The image is formed on the **image plane**
Camera obscura (Latin for “Dark Chamber”)

- Basic principle known to Mozi (470-390 BCE), Aristotle (384-322 BCE)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)

Source: A. Efros
Turning a room into a camera obscura

From Grand Images Through a Tiny Opening, Photo District News, February 2005

http://www.abelardomorell.net/project/camera-obscura/
Turning a room into a camera obscura

Accidental pinholes produce images that are unnoticed or misinterpreted as shadows

A. Torralba and W. Freeman, Accidental Pinhole and Pinspeck Cameras, CVPR 2012
Pinhole camera

- Each point on image plane illuminated by light from one direction

- Joseph Nicéphore Niépce: first recording onto pewter plate coated with bitumen (1826)
Dimensionality reduction: from 3D to 2D

3D world

2D image

What properties of the world are preserved?
- Straight lines, incidence

What properties are not preserved?
- Angles, lengths

Figure © Stephen E. Palmer, 2002

Slide by A. Efros
Nearer Objects Appear Bigger

size in image $\sim \frac{1}{\text{distance}}$
Perspective Projection Phenomena…
Straight Lines Remain Straight
Parallel Lines Converge at Vanishing Points
Parallel Lines Converge at Vanishing Points

Each family of parallel lines has its own vanishing point
Pinhole camera: projection of a point
To compute the projection $P'$ of a scene point $P$, form the visual ray connecting $P$ to the camera center $O$ and find where it intersects the image plane.

- All scene points that lie on this visual ray have the same projection in the image.
- Are there scene points for which this projection is undefined?
The coordinate system

- The optical center (O) is at the origin
- The image plane is parallel to xy-plane or perpendicular to the z-axis, which is the optical axis
Projection equations
Projection equations

- Using similar triangles:

\[
\frac{x}{z} = \frac{x'}{f}
\]
Projection equations

Using similar triangles:

\[
\frac{x}{z} = \frac{x'}{f} \quad \frac{y}{z} = \frac{y'}{f}
\]
Projection equations

Using similar triangles:

\[
\frac{x}{z} = \frac{x'}{f} \quad \frac{y}{z} = \frac{y'}{f}
\]

Thus:

\[(x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z})\]
Fronto-parallel planes

- What happens to the projection of a pattern on a plane parallel to the image plane?
  - All points on that plane are at a fixed depth $z$
  - The pattern gets scaled by a factor of $f/z$, but angles and ratios of lengths/areas are preserved

\[(x, y, z) \rightarrow \left( f \frac{x}{z}, f \frac{y}{z} \right)\]
Pinhole camera: projection of a line
Vanishing points

• All parallel lines converge to a *vanishing point*
  • Each direction in space is associated with its own vanishing point
  • Exception: *directions parallel to the image plane*
Constructing the vanishing point of a line

- **Claim**: Each direction in space is associated with one vanishing point
  - Any point on the line: \((x, y, z) + \alpha(dx, dy, 1)\)
  - This point is projected to: \(\left( f \frac{x + \alpha dx}{z + \alpha}, f \frac{y + \alpha dy}{z + \alpha} \right)\)
  - The limit as \(\alpha \to \infty\): \((f dx, f dy)\)
  - Thus the vanishing point is independent of the location \((x, y, z)\) and uniquely determined by the direction \((dx, dy, 1)\)
• To find the vanishing point, shoot a ray from camera center along the same direction. Find the intersection with the image plane.

• How does the vanishing point move if the camera is moved without rotation?
Perspective cues
Perspective cues
Perspective cues
Pinhole camera: projection beyond a single line
Vanishing lines of planes

- Vanishing points of co-planar directions form a vanishing line (exercise for fun: show this algebraically)
- **Horizon**: vanishing line of the ground plane
  - All points at the same height as the camera project to the horizon
  - Points higher (resp. lower) than the camera project above (resp. below) the horizon
  - Provides way of comparing height of objects

Adapted from Steve Seitz
Comparing heights

Vanishing Point

Slide by Steve Seitz
Measuring height

What is the height of the camera?
Quiz 1

Which is higher? The parachutist or the person taking the picture?

Image source: S. Seitz
How does the location of the horizon change if the person taking the picture raise the camera but keep its orientation?
Are the widths of the projected columns equal?

- The exterior columns are wider
- This is not an optical illusion, and is not due to lens flaws
- Phenomenon pointed out by Da Vinci

Source: F. Durand
Perspective distortion

• What is the shape of the projection of a sphere?
What is the shape of the projection of a sphere?
Perspective distortion: People
Building a real camera
Pinhole camera
Home-made pinhole camera

http://www.debevec.org/Pinhole/

Slide by A. Efros
Aperture

- Controls amount of light
Shrinking the aperture

Why not make the aperture as small as possible?
  - Less light gets through
  - Diffraction effects…
Shrinking the aperture

- 2 mm
- 1 mm
- 0.6 mm
- 0.35 mm
- 0.15 mm
- 0.07 mm
Adding a lens

A lens focuses light onto the film

- Thin lens model:
  - Rays passing through the center are not deviated (pinhole projection model still holds)
A lens focuses light onto the film

- Thin lens model:
  - Rays passing through the center are not deviated (pinhole projection model still holds)
  - All parallel rays (along the principal axis) converge to one point on a plane located at the *focal length* $f$
A lens focuses light onto the film

- There is a specific distance at which objects are “in focus”
  - Other points project to a “circle of confusion” in the image
Thin lens formula

- What is the relation between the focal length ($f$), the distance of the object from the optical center ($D$), and the distance at which the object will be in focus ($D'$)?
Thin lens formula

Similar triangles everywhere!
Similar triangles everywhere! \[ \frac{y}{y'} = \frac{D}{D'} \]
Thin lens formula

Similar triangles everywhere!

\[ \frac{y}{y'} = \frac{D}{D'} \]

\[ \frac{y}{y'} = \frac{f}{(D' - f)} \]
Thin lens formula

\[ \frac{1}{D'} + \frac{1}{D} = \frac{1}{f} \]

Any point satisfying the thin lens equation is in focus.
Focus and Depth of Field

- For a given $D$, “perfect” focus at only one $D'$
- In practice, OK for some range of depths
  - Circle of confusion smaller than a pixel
Depth of Field

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm
Controlling depth of field

Changing the aperture size affects depth of field

- A smaller aperture increases the range in which the object is approximately in focus
- But small aperture reduces amount of light – need to increase exposure

Varying the aperture

Large aperture = small DOF

Small aperture = large DOF
Aperture

- Aperture typically given as “f-number”
- What is $f/4$?
  - Aperture diameter is $\frac{1}{4}$ the focal length
- One “f-stop” equals change of $f$-number by $\sqrt{2}$
  - Equals change in aperture area by factor of 2
  - Equals change in amount of light by factor of 2
  - Example: $f/2 \rightarrow f/2.8 \rightarrow f/4$ (each one doubles light)
Building a real camera: field of view
Field of View

Graph showing the field of view for different focal lengths:
- 1000 mm: 2.5°
- 500 mm: 6°
- 300 mm: 8°
- 135 mm: 18°
- 85 mm: 28°
- 50 mm: 47°
- 28 mm: 75°
- 17 mm: 104°

Images illustrating the field of view:
- 17mm
- 28mm
- 50mm
- 85mm

Slide by A. Efros
Field of View

FOV depends on focal length and size of the camera retina

$$\theta = 2 \tan^{-1} \left( \frac{d}{2f} \right) \approx \frac{d}{f}$$

Larger focal length = smaller FOV

Slide by A. Efros
Field of View / Focal Length

Large FOV, small $f$
Camera close to car

Small FOV, large $f$
Camera far from the car

Sources: A. Efros, F. Durand
A bit about digital cameras
A digital camera replaces film with a sensor array

- Each cell in the array is light-sensitive diode that converts photons to electrons
- Two common types
  - Charge Coupled Device (CCD)
  - Complementary metal oxide semiconductor (CMOS)

Color sensing: Color filter array

Bayer grid

Estimate missing components from neighboring values (demosaicing)

Why more green?

Human Luminance Sensitivity Function

Source: Steve Seitz
Next class: convolution and filtering