OCaml Datatypes

COS 326
Andrew Appel
Princeton University

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OCaml So Far

• We have seen a number of basic types:
  – int
  – float
  – char
  – string
  – bool

• We have seen a few structured types:
  – pairs
  – tuples
  – options
  – lists

• In this lecture, we will see some more general ways to define our own new types and data structures
Type Abbreviations

• We have already seen some type abbreviations:

\[
\text{type point} = \text{float} \times \text{float}
\]
Type Abbreviations

• We have already seen some type abbreviations:

```haskell
type point = float * float
```

• These abbreviations can be helpful documentation:

```haskell
let distance (p1:point) (p2:point) : float =
    let square x = x *. x in
    let (x1,y1) = p1 in
    let (x2,y2) = p2 in
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

• But they add nothing of *substance* to the language
  – they are *equal* in every way to an existing type
Type Abbreviations

• We have already seen some type abbreviations:

```
type point = float * float
```

• As far as OCaml is concerned, you could have written:

```
let distance (p1: float*float) (p2: float*float) : float =
    let square x = x *. x in
    let (x1, y1) = p1 in
    let (x2, y2) = p2 in
    sqrt (square (x2 -. x1) +. square (y2 -. y1))
```

• Since the types are equal, you can substitute the definition for the name wherever you want
  – we have not added any new data structures
DATA TYPES
OCaml provides a general mechanism called a data type for defining new data structures that consist of many alternatives.

```ocaml
type my_bool = Tru | Fal
```

A value with type `my_bool` is one of two things:
- Tru, or
- Fal

Read the "|" as "or"
OCaml provides a general mechanism called a **data type** for defining new data structures that consist of many alternatives.

```ocaml
type my_bool = Tru | Fal
```

- Tru and Fal are called "constructors".
- A value with type `my_bool` is one of two things:
  - Tru, or
  - Fal
- Read the "|" as "or"
OCaml provides a general mechanism called a data type for defining new data structures that consist of many alternatives.

```ocaml
type my_bool = True | False

type color = Blue | Yellow | Green | Red
```

There's no need to stop at 2 cases; define as many alternatives as you want.
OCaml provides a general mechanism called a data type for defining new data structures that consist of many alternatives.

- **Data type**

```ocaml
type my_bool = Tru | Fal

type color = Blue | Yellow | Green | Red
```

- **Creating values:**

```ocaml
let b1 : my_bool = Tru
let b2 : my_bool = Fal
let c1 : color = Yellow
let c2 : color = Red
```

use constructors to create values
type color = Blue | Yellow | Green | Red

let c1 : color = Yellow
let c2 : color = Red

- Using data type values:

let print_color (c:color) : unit =
  match c with
  | Blue ->
  | Yellow ->
  | Green ->
  | Red ->

use pattern matching to determine which color you have; act accordingly
Data types

```ocaml
type color = Blue | Yellow | Green | Red

let c1 : color = Yellow
let c2 : color = Red

let print_color (c:color) : unit =
  match c with
  | Blue -> print_string "blue"
  | Yellow -> print_string "yellow"
  | Green -> print_string "green"
  | Red -> print_string "red"
```
Data types

```ocaml
let print_color (c:color) : unit =
  match c with
  | Blue -> print_string "blue"
  | Yellow -> print_string "yellow"
  | Green -> print_string "green"
  | Red -> print_string "red"
```

Why not just use strings to represent colors instead of defining a new type?
Data types

type color = Blue | Yellow | Green | Red

oops!:

let print_color (c:color) : unit =
match c with
| Blue -> print_string "blue"
| Yellow -> print_string "yellow"
| Red -> print_string "red"

Warning 8: this pattern-matching is not exhaustive. Here is an example of a value that is not matched: Green
Data types

type color = Blue | Yellow | Green | Red

oops!:

let print_color (c:color) : unit =
  match c with
  | Blue -> print_string "blue"
  | Yellow -> print_string "yellow"
  | Red -> print_string "red"

Warning 8: this pattern-matching is not exhaustive. Here is an example of a value that is not matched: Green

OCaml's datatype mechanism allow you to create types that contain precisely the values you want!
Data Types Can Carry Additional Values

- Data types are more than just enumerations of constants:

```python
type point = float * float

type simple_shape =
    Circle of point * float
| Square of point * float
```

- Read as: a `simple_shape` is either:
  - a `Circle`, which contains a pair of a `point` and `float`, or
  - a `Square`, which contains a pair of a `point` and `float`
Data Types Can Carry Additional Values

- Data types are more than just enumerations of constants:

```ocaml
type point = float * float

type simple_shape =
    Circle of point * float |
    Square of point * float

let origin : point = (0.0, 0.0)

let circ1  : simple_shape = Circle (origin, 1.0)
let circ2  : simple_shape = Circle ((1.0, 1.0), 5.0)
let square : simple_shape = Square (origin, 2.3)
```
Data Types Can Carry Additional Values

- Data types are more than just enumerations of constants:

```ocaml
type point = float * float

type simple_shape =
  | Circle of point * float
  | Square of point * float

let simple_area (s:simple_shape) : float =
  match s with
  | Circle (_, radius) -> 3.14 *. radius *. radius
  | Square (_, side) -> side *. side
```
Compare

- Data types are more than just enumerations of constants:

```ocaml
type point = float * float

type simple_shape =
  Circle of point * float
| Square of point * float

let simple_area (s:simple_shape) : float =
  match s with
  | Circle (_, radius) -> 3.14 *. radius *. radius
  | Square (_, side) -> side *. side

let simple_area (s:my_shape) : float =
  (3.14 *. radius *. radius) ?? or ?? (side *. side)
```
type point = float * float

type shape =
    Square of float
    | Ellipse of float * float
    | RtTriangle of float * float
    | Polygon of point list

Square $s = \square$

RtTriangle $(s_1, s_2) = \triangle$

Ellipse $(r_1, r_2) = \bigcirc$

Polygon $[v_1; ...; v_5] =$
type point = float * float

type radius = float

type side = float

type shape =
  Square of side
  | Ellipse of radius * radius
  | RtTriangle of side * side
  | Polygon of point list

Type abbreviations can aid readability

Square $s = \square$

RtTriangle $(s_1, s_2) =$

Ellipse $(r_1, r_2) =$

RtTriangle $[v_1; ...; v_5] =$
type point = float * float
type radius = float
type side = float

type shape =
    Square of side
| Ellipse of radius * radius
| RtTriangle of side * side
| Polygon of point list

let sq : shape = Square 17.0
let ell : shape = Ellipse (1.0, 2.0)
let rt : shape = RtTriangle (1.0, 1.0)
let poly : shape = Polygon [(0., 0.); (1., 0.); (0.; 1.)]

they are all shapes;
they are constructed in
different ways

Square builds a shape
from a single side

RtTriangle builds a shape
from a pair of sides

Polygon builds a shape
from a list of points
(where each point is itself a pair)
More General Shapes

type point = float * float
type radius = float
type side = float

type shape =
  Square of side
  | Ellipse of radius * radius
  | RtTriangle of side * side
  | Polygon of point list

let area (s : shape) : float =
  match s with
  | Square s ->
  | Ellipse (r1, r2) ->
  | RtTriangle (s1, s2) ->
  | Polygon ps ->

a data type also defines a pattern for matching
type point = float * float
type radius = float
type side = float

type shape =
    Square of side
| Ellipse of radius * radius
| RtTriangle of side * side
| Polygon of point list

let area (s : shape) : float =
    match s with
    | Square s ->
    | Ellipse (r1, r2) ->
    | RtTriangle (s1, s2) ->
    | Polygon ps ->
More General Shapes

type point = float * float

type radius = float

type side = float

type shape =
  Square of side
  | Ellipse of radius * radius
  | RtTriangle of side * side
  | Polygon of point list

let area (s : shape) : float =
  match s with
  | Square s -> s *. s
  | Ellipse (r1, r2) -> pi *. r1 *. r2
  | RtTriangle (s1, s2) -> s1 *. s2 /. 2.
  | Polygon ps -> ???

a data type also defines a pattern for matching
Computing Area

• How do we compute polygon area?
• For convex polygons:
  – Case: the polygon has fewer than 3 points:
    • it has 0 area! (it is a line or a point or nothing at all)
  – Case: the polygon has 3 or more points:
    • Compute the area of the triangle formed by the first 3 vertices
    • Delete the second vertex to form a new polygon
    • Sum the area of the triangle and the new polygon
How do we compute polygon area?

For convex polygons:

- Case: the polygon has fewer than 3 points:
  - it has 0 area! (it is a line or a point or nothing at all)

- Case: the polygon has 3 or more points:
  - Compute the area of the triangle formed by the first 3 vertices
  - Delete the second vertex to form a new polygon
  - Sum the area of the triangle and the new polygon

Note: This is a beautiful inductive algorithm:

- the area of a polygon with \( n \) points is computed in terms of a smaller polygon with only \( n-1 \) points!
let area (s : shape) : float = 
    match s with 
    | Square s -> s *. s 
    | Ellipse (r1, r2) -> r1 *. r2 
    | RtTriangle (s1, s2) -> s1 *. s2 / 2. 
    | Polygon ps -> poly_area ps

let poly_area (ps : point list) : float = 
    match ps with 
    | p1 :: p2 :: p3 :: tail -> 
        tri_area p1 p2 p3 +. poly_area (p1::p3::tail) 
    | _ -> 0.
let tri_area (p1:point) (p2:point) (p3:point) : float =
  let a = distance p1 p2 in
  let b = distance p2 p3 in
  let c = distance p3 p1 in
  let s = 0.5 *. (a +. b +. c) in
  sqrt (s *. (s -. a) *. (s -. b) *. (s -. c))

let rec poly_area (ps : point list) : float =
  match ps with
  | p1 :: p2 :: p3 :: tail ->
    tri_area p1 p2 p3 +. poly_area (p1::p3::tail)
  | _ -> 0.

let area (s : shape) : float =
  match s with
  | Square s -> s *. s
  | Ellipse (r1, r2) -> pi *. r1 *. r2
  | RtTriangle (s1, s2) -> s1 *. s2 /. 2.
  | Polygon ps -> poly_area ps
INDUCTIVE DATA TYPES
Inductive data types

- We can use data types to define inductive data
- A binary tree is:
  - a Leaf containing no data
  - a Node containing a key, a value, a left subtree and a right subtree
Inductive data types

• We can use data types to define inductive data
• A binary tree is:
  – a **Leaf** containing no data
  – a **Node** containing a **key**, a **value**, a left **subtree** and a right **subtree**

```ocaml
type key = string
type value = int

type tree =  
  Leaf  
  | Node of key * value * tree * tree
```
Inductive data types

type key = int
type value = string

type tree =
  Leaf
| Node of key * value * tree * tree

let rec insert (t:tree) (k:key) (v:value) : tree =
Inductive data types

```ocaml
type key = int
type value = string

type tree =
  Leaf
| Node of key * value * tree * tree

let rec insert (t:tree) (k:key) (v:value) : tree =
  match t with
  | Leaf ->
  | Node (k', v', left, right) ->
```

Again, the type definition specifies the cases you must consider.
type key = int
type value = string

type tree =
  Leaf
| Node of key * value * tree * tree

let rec insert (t:tree) (k:key) (v:value) : tree =
  match t with
  | Leaf -> Node (k, v, Leaf, Leaf)
  | Node (k', v', left, right) ->
type key = int
type value = string

type tree =
  Leaf
| Node of key * value * tree * tree

let rec insert (t:tree) (k:key) (v:value) : tree =
  match t with
  | Leaf -> Node (k, v, Leaf, Leaf)
  | Node (k', v', left, right) ->
    if k < k' then
      Node (k', v', insert left k v, right)
    else if k > k' then
      Node (k', v', left, insert right k v)
    else
      Node (k, v, left, right)
Inductive data types

type key = int
type value = string
type tree =
  Leaf
| Node of key * value * tree * tree

let rec insert (t:tree) (k:key) (v:value) : tree =
match t with
  | Leaf -> Node (k, v, Leaf, Leaf)
  | Node (k', v', left, right) ->
    if k < k' then
      Node (k', v', insert left k v, right)
    else if k > k' then
      Node (k', v', left, insert right k v)
    else
      Node (k, v, left, right)
Inductive data types

```ml
let rec insert (t:tree) (k:key) (v:value) : tree =
  match t with
  | Leaf -> Node (k, v, Leaf, Leaf)
  | Node (k', v', left, right) ->
    if k < k' then
      Node (k', v', insert left k v, right)
    else if k > k' then
      Node (k', v', left, insert right k v)
    else
      Node (k, v, left, right)
```

Note on memory use
Inductive data types: Another Example

- Recall, we used the type "int" to represent natural numbers
  - but that was kind of broken: it also contained negative numbers
  - we had to use a dynamic test to guard entry to a function:

```ocaml
let double (n : int) : int =
  if n < 0 then
    raise (Failure "negative input!"
  else
    double_nat n
```

- it would be nice if there was a way to define the natural numbers exactly, and use OCaml's type system to guarantee no client ever attempts to double a negative number
Inductive data types

• Recall, a natural number n is either:
  – zero, or
  – m + 1

• We use a data type to represent this definition exactly:
Inductive data types

• Recall, a natural number \( n \) is either:
  – zero, or
  – \( m + 1 \)

• We use a data type to represent this definition exactly:

\[
\text{type nat} = \text{Zero} \mid \text{Succ of nat}
\]
Inductive data types

• Recall, a natural number \( n \) is either:
  – zero, or
  – \( m + 1 \)

• We use a data type to represent this definition exactly:

```ocaml
type nat = Zero | Succ of nat

let rec nat_to_int (n : nat) : int =
  match n with
  Zero -> 0
| Succ n -> 1 + nat_to_int n
```
Inductive data types

• Recall, a natural number \( n \) is either:
  – zero, or
  – \( m + 1 \)

• We use a data type to represent this definition exactly:

\[
\text{type nat } = \text{ Zero } | \text{ Succ of nat }
\]

let rec nat_to_int (n : nat) : int =
  match n with
  | Zero -> 0
  | Succ n -> 1 + nat_to_int n

let rec double_nat (n : nat) : nat =
  match n with
  | Zero -> Zero
  | Succ m -> Succ (Succ(double_nat m))
Summary of Part I

• OCaml data types: a powerful mechanism for defining complex data structures:
  – They are precise
    • contain exactly the elements you want, not more elements
  – They are general
    • recursive, non-recursive (mutually recursive and polymorphic)
  – The type checker helps you detect errors
    • missing cases in your functions
OCaml Datatypess Part II: An Exercise in Type Design
IBM developed GML (Generalize Markup Language) in 1969

- Precursor to SGML, HTML and XML

---

:hl. Chapter 1: Introduction
:p. GML supported hierarchical containers, such as
:ol
:li. Ordered lists (like this one),
:li. Unordered lists, and
:li. Definition lists
:eol.

as well as simple structures.
:p. Markup Minimization (later generalized and formalized in SGML), allowed the end-tags to be omitted for the “hl” and “p” elements.
To process a GML document, an OCaml program would:

- **Read** a series of characters from a text file & **Parse** GML structure
- **Represent** the information content as an OCaml data structure
- **Analyze or transform** the data structure
- **Print/Store/Communicate** results

We will focus on how to **represent** and **transform** the information content of a GML document.
Example Type Design

• A **GML document** consists of:
  – a list of **elements**

• An **element** is either:
  – a **word** or **markup** applied to an element

• **Markup** is either:
  – italicize, **bold**, or a **font name**
• A **GML document** consists of:
  – a list of **elements**

• An **element** is either:
  – a **word** or **markup** applied to an element

• **Markup** is either:
  – italicize, **bold**, or **a font name**

```plaintext
type markup = Ital | Bold | Font of string

type elt =
  Words of string list
| Formatted of markup * elt

type doc = elt list
```
Example Data

type markup = Ital | Bold | Font of string

type elt =
    Words of string list
| Formatted of markup * elt

type doc = elt list

let d = [ Formatted (Bold,
       Formatted (Font "Arial",
         Words ["Chapter";"One"]));

       Words ["It"; "was"; "a"; "dark";
         ";"; "stormy; "night."; "A"];

       Formatted (Ital, Words["shot"]);

       Words ["rang"; "out."] ];;;
Challenge

• Change all of the “Arial” fonts in a document to “Courier”.
• Of course, when we program functionally, we implement change via a function that
  – receives one data structure as input
  – builds a new (different) data structure as an output
Challenge

- Change all of the “Arial” fonts in a document to “Courier”.

```ocaml
type markup = Ital | Bold | Font of string

type elt =
  Words of string list
| Formatted of markup * elt

type doc = elt list
```
Challenge

• Change all of the “Arial” fonts in a document to “Courier”.

```ocaml
type markup = Ital | Bold | Font of string

type elt =
    Words of string list
  | Formatted of markup * elt

type doc = elt list

let rec chfonts (elts:doc) : doc =
```

• Technique: approach the problem top down, work on doc first:
Challenge

• Change all of the “Arial” fonts in a document to “Courier”.

```ocaml
type markup = Ital | Bold | Font of string

type elt =
| Words of string list
| Formatted of markup * elt

type doc = elt list
```

• Technique: approach the problem top down, work on `doc` first:

```ocaml
let rec chfonts (elts:doc) : doc =
    match elts with
    | [] ->
    | hd::tl ->
```
Challenge

- Change all of the “Arial” fonts in a document to “Courier”.

```ocaml
type markup = Ital | Bold | Font of string

type elt =
    Words of string list
  | Formatted of markup * elt

type doc = elt list

let rec chfonts (elts:doc) : doc =
    match elts with
    | [] -> []
    | hd::tl -> (chfont hd)::(chfonts tl)
```
Changing fonts in an element

• Change all of the “Arial” fonts in a document to “Courier”.

```ocaml
type markup = Ital | Bold | Font of string

type elt =
    Words of string list
  | Formatted of markup * elt

type doc = elt list
```

• Next work on changing the font of an element:

```ocaml
let rec chfont (e:elt) : elt =
```
Changing fonts in an element

- Change all of the “Arial” fonts in a document to “Courier”.

```ocaml
let rec chfont (e:elt) : elt =
  match e with
  | Words ws ->
  | Formatted (m,e) ->
```

- Next work on changing the font of an element:
Changing fonts in an element

- Change all of the “Arial” fonts in a document to “Courier”.

```ocaml
let rec chfont (e:elt) : elt =
  match e with
  | Words ws -> Words ws
  | Formatted(m,e) ->

type markup = Ital | Bold | Font of string

type elt =
  Words of string list
| Formatted of markup * elt

type doc = elt list
```

- Next work on changing the font of an element:
Changing fonts in an element

• Change all of the “Arial” fonts in a document to “Courier”.

```ocaml
type markup = Ital | Bold | Font of string

type elt =
  Words of string list
| Formatted of markup * elt

type doc = elt list
```

• Next work on changing the font of an element:

```ocaml
let rec chfont (e:elt) : elt =
  match e with
  | Words ws -> Words ws
  | Formatted(m,e) -> Formatted(chmarkup m, chfont e)
```
Changing fonts in an element

- Change all of the “Arial” fonts in a document to “Courier”.

```haskell
type markup = Ital | Bold | Font of string

let chmarkup (m:markup) : markup =
```

- Next work on changing a markup:
Changing fonts in an element

- Change all of the "Arial" fonts in a document to "Courier".

```
type markup = Ital | Bold | Font of string

type elt =
  Words of string list
| Formatted of markup * elt

type doc = elt list
```

- Next work on changing a markup:

```
let chmarkup (m:markup) : markup =
    match m with
    | Font "Arial" -> Font "Courier"
    | _ -> m
```
Summary: Changing fonts in an element

- Change all of the “Arial” fonts in a document to “Courier”
- Lesson: function structure follows type structure

```ml
let chmarkup (m:markup) : markup =
  match m with
  | Font “Arial” -> Font “Courier”
  | _ -> m

let rec chfont (e:elt) : elt =
  match e with
  | Words ws -> Words ws
  | Formatted(m,e) -> Formatted(chmarkup m, chfont e)

let rec chfonts (elts:doc) : doc =
  match elts with
  | [] -> []
  | hd::tl -> (chfont hd)::(chfonts tl)
```
• Consider again our definition of markup and markup change:

```plaintext
type markup =
    Ital | Bold | Font of string

let chmarkup (m:markup) : markup =
    match m with
    | Font "Arial" -> Font "Courier"
    | _ -> m
```
What if we make a change:

```ocaml
type markup =
  Ital | Bold | Font of string | TTFont of string

let chmarkup (m:markup) : markup =
  match m with
  | Font "Arial" -> Font "Courier"
  | _ -> m
```

the underscore silently catches all possible alternatives.

this may not be what we want -- perhaps there is an Arial TT font.

it is better if we are alerted of all functions whose implementation may need to change.
• Original code:

```ocaml
type markup = 
    Ital | Bold | Font of string

let chmarkup (m:markup) : markup = 
    match m with
    | Font "Arial" -> Font "Courier"
    | Ital | Bold -> m
```
Better Style

• Updated code:

```ml
type markup =
  Ital | Bold | Font of string | TTFont of string

let chmarkup (m:markup) : markup =
  match m with
  | Font "Arial" -> Font "Courier"
  | Ital | Bold -> m
```

..match m with
  | Font "Arial" -> Font "Courier"
  | Ital | Bold -> m..

Warning 8: this pattern-matching is not exhaustive. Here is an example of a value that is not matched: TTFont _
• Updated code, fixed:

```haskell
type markup =
    Ital | Bold | Font of string | TTFont of string

let chmarkup (m:markup) : markup =
    match m with
    | Font "Arial" -> Font "Courier"
    | TTFont "Arial" -> TTFont "Courier"
    | Font s -> Font s
    | TTFont s -> TTFont s
    | Ital | Bold -> m
```

• **Lesson:** use the type checker where possible to help you maintain your code
A couple of practice problems

• Write a function that gets rid of immediately redundant markup in a document.
  – \texttt{Formatted(\textit{Ital}, Formatted(\textit{Ital},e))} can be simplified to \texttt{Formatted(\textit{Ital},e)}
  – write maps and folds over markups

• Design a datatype to describe bibliography entries for publications. Some publications are journal articles, others are books, and others are conference papers. Journals have a name, number and issue; books have an ISBN number; All of these entries should have a title and author.
  – design a sorting function
  – design maps and folds over your bibliography entries
To Summarize

- Design recipe for writing OCaml code:
  - write down English specifications
    - try to break problem into obvious sub-problems
  - write down some sample test cases
  - write down the signature (types) for the code
  - use the signature to guide construction of the code:
    - tear apart inputs using pattern matching
      - make sure to cover all of the cases! (OCaml will tell you)
    - handle each case, building results using data constructor
      - this is where human intelligence comes into play
      - the “skeleton” given by types can almost be done automatically!
    - clean up your code
  - use your sample tests (and ideally others) to ensure correctness