EXERCISE 1: Seam Carving

Consider the given 3x4 image and the corresponding energies matrix.

- A *Vertical Seam* is a path of pixels connected from the top row to the bottom row , where a pixel at column x and row y can only be connected to the pixels $(x-1, y+1)$, $(x, y+1)$ and $(x+1, y+1)$.
- The *Seam Energy* is the sum of the energies of the pixels in the seam.
- A *Minimum Energy Vertical Seam* is the vertical seam with the minimum energy.

(15,10,16)	(31,15,19)	(15,10,3)
(5,18,0)	(80,18,0)	(120,100,80)
(35,20,12)	(36,17,13)	(15,10,3)
(5,1,13)	(13,1,16)	(120,110,40)

RGB Values of the 3x4 Image

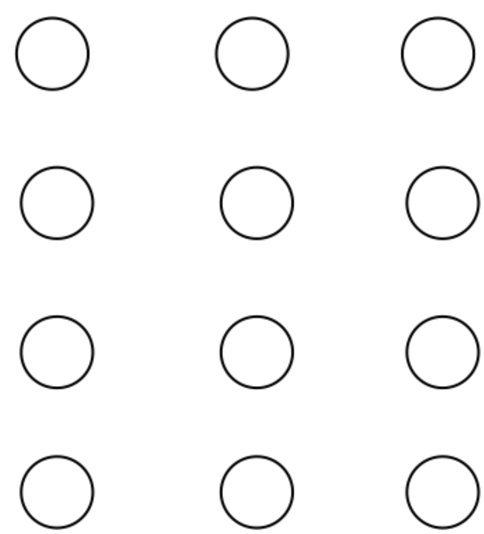
32	72	45
123	163	75
32	75	41
156	161	9

Energy Values (Rounded)

A. Mark the *minimum energy vertical seam* in the given energies matrix. What is the energy of this seam?

B. In order to find the minimum energy vertical seam, you will have to find the shortest path from any pixel in the top row to any pixel in the bottom row.

Draw the implicit graph which the energies matrix represents. Show all the edges and edge weights.



C. Assume that the image is of size $W \times H$, what is the order of growth of the running time of finding the minimum energy vertical Seam using ***Dijkstra's*** algorithm (use W and H)?

EXERCISE 2: Algorithm Properties

For each of the following statements, argue for why it is true or provide a counterexample if it is false.

A. *Incrementing* all the weights of edges in a graph by the same positive constant does not affect the *shortest path*.

B. *Incrementing* all the weights of edges in a graph by the same constant does not affect the *minimum spanning tree*.

C. *Multiplying* all the weights of edges in a graph by the same positive constant does not affect the *shortest path*.

D. The path between two vertices in a minimum spanning tree is always a shortest path between the two vertices on the full graph.

E. Prim and Kruskal's algorithm always correctly compute the *minimum spanning tree* when there are *negative edge weights* in the graph.

F. The following algorithm always correctly computes a minimum spanning tree:
For every vertex in the graph, pick the incident edge with the minimum weight and add it to the MST.

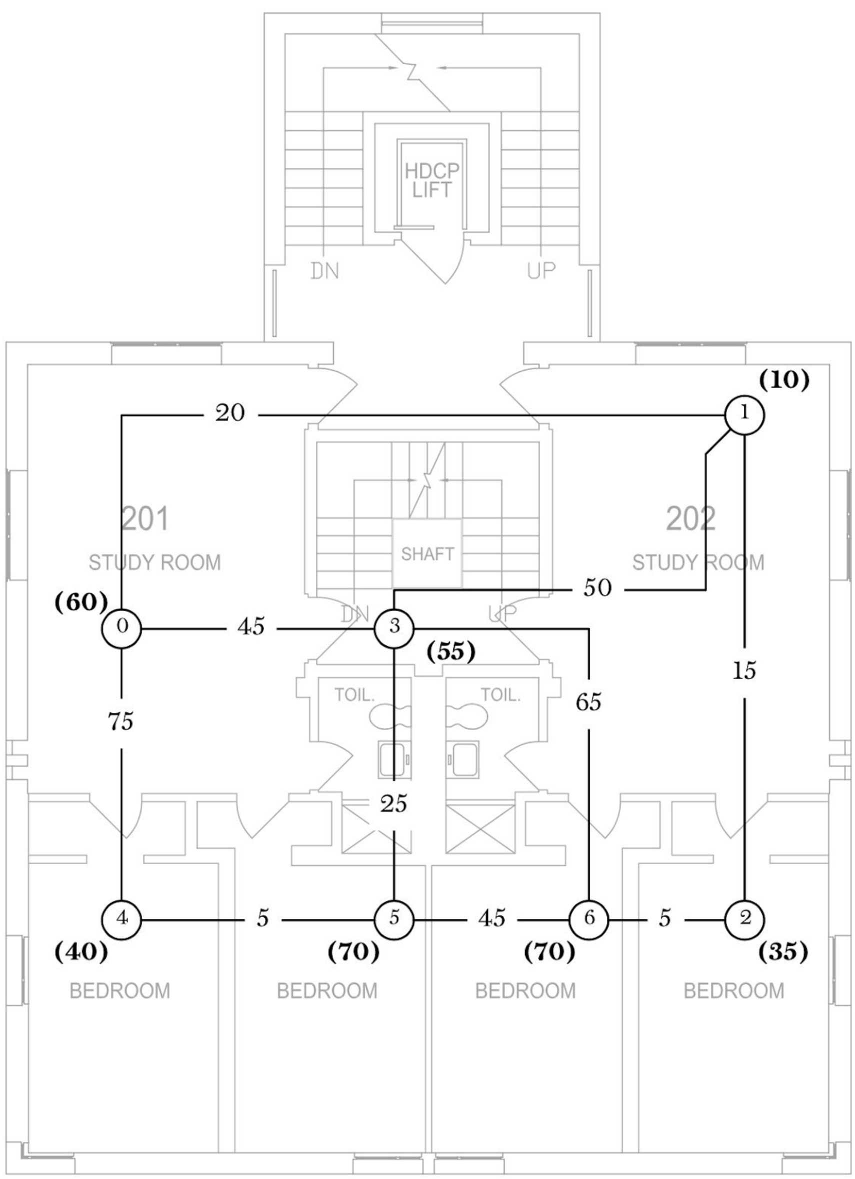
EXERCISE 3: Dorm Room and Routers

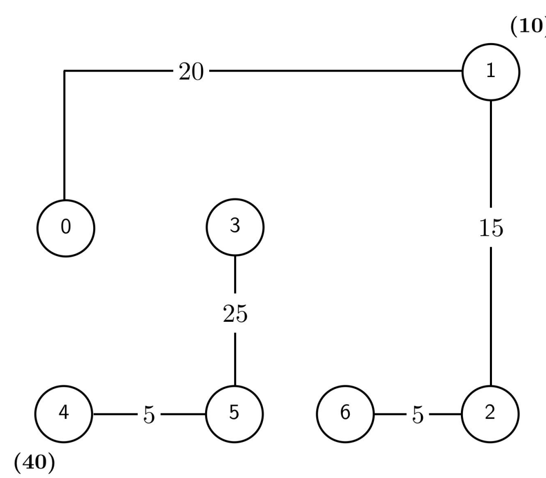
There are N rooms, each of which needs an internet connection. A room i has internet access if either of the following is true:

- There is a router installed in room i (this costs $r_i > 0$).
- The room i is connected by some fiber path to another room j which itself has internet access (putting down fiber between room i and j costs $f_{ij} > 0$).

The goal of this problem is to determine in which rooms to install a router, and in which pair of rooms to connect together with fiber, so as to minimize the total cost.

Formulate the problem as a *minimum spanning tree* problem, given a graph $G = (V, E)$ with vertices $V = \{v_1, ..., v_n\}$ and the previously mentioned costs, r_i and f_{ij}. You may use the below example to test your formulation.





Solution

For example, this instance contains 7 dorm rooms and 10 possible connections. The router installation costs are indicated in bold and parentheses; the fiber costs are given on the edges. The optimal solution, which costs 120, *installs a router in rooms 1 and 4 (for a cost of 10 + 40) and builds the shown fiber connections.*