COS226 Precept # 10 - Exercises Fall '19

EXERCISE 1: Compression Warm-up

A. The *compression ratio* is defined as: $\frac{\textit{compressed size}}{\textit{original size}}$. Consider a sequence of N characters, 8-bits each, what is the compression ratio achieved by *Huffman* coding in:

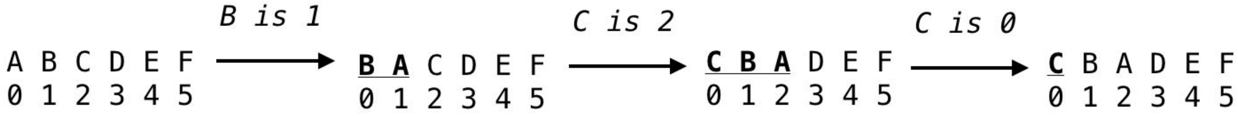
○ the best case?

○ the worst case?

B. *Move-To-Front* is a lossless encoding algorithm that works as follows:

> *Maintain an ordered sequence of the characters in the alphabet by repeatedly reading a character from the input message; printing the position in the sequence in which that character appears; and moving that character to the front of the sequence.*

```
Example. Input:    BCC
         Output:   120
```



Assuming the alphabet is `A-Z`, where A has code `0`, B code `1`, etc.:

○ Encode "`A A A B B B C C C D D D E E E`".

○ Encode "`A E A B E C A D`"

C. Move-To-Front encoding is typically used to convert a given text into one where *some characters appear much more frequently than others*. Based on the examples above, when does MoveToFront achieve this goal well?

D. The goal of the *Burrows-Wheeler* lossless transform is to convert a given text into text where *sequences of the same character occur near each other many times*.

How should Move-To-Front, Huffman and Burrows-Wheeler be used together in order to achieve a good compression ratio?

EXERCISE 2: Burrows-Wheeler Transform

A. List the *circular suffixes* of the word “W E E K E N D” and then sort them in lexicographical order.

Original

0	W E E K E N D
1	E E K E N D W
2	
3	
4	
5	
6	

Sorted

B. The Burrows-Wheeler transform is the last character of each of the sorted circular suffixes, preceded by the row number in which the original string ends up when considered in sorted order.

What is the Burrows-Wheeler Transform of “W E E K E N D”?

C. How much memory is needed to store the circular suffixes?

EXERCISE 3: Burrows-Wheeler Inverse-Transform

A. Given only the last character of each of the circular suffixes when considered in sorted order, can we infer the first character in each of these suffixes? Explain your answer.

```
? - - - - - N
? - - - - - W
? - - - - - E
? - - - - - K
? - - - - - E
? - - - - - E
? - - - - - D *
```

B. We know from Exercise 2.B that the Burrows-Wheeler transform stores where the original string is. The goal of the inverse-transform is to find the characters of the original string, i.e. the characters between W and D in row 6.

```
        s[]         t[]
       0| D - - - - - N
  +-->1| E - - - - - W
  |    2| E - - - - - E
  |    3| E - - - - - K
  |    4| K - - - - - E
  |    5| N - - - - - E
  +---6| W - - - - - D *
```

Observation. A circular suffix that ends with a W comes directly after a circular suffix that begins with a W in the original (un-sorted) circular suffix array. Look at your answer for 2.A!

Use the following (very slow) algorithm to construct the array `next[]` to keep track of where the next circular suffix is for each of the sorted circular suffixes.

(`N` is the number of circular suffixes and `used[]` is of size `R=256` and is initialized to `false`)

```
for (int i = 0; i < N; i++) {
    for (int j = 0; j < N; j++) {
       if (used[j]) continue;
       if (s[i] == t[j]) {
           used[j] = true; // disallow reuse of this character
           next[i] = j;
           break;
       }
    }
}
```

	Original
0	W E E K E N D
1	E E K E N D W
2	E K E N D W E
3	K E N D W E E
4	E N D W E E K
5	N D W E E K E
6	D W E E K E N

	S[] t[]	next[]
0	D - - - - - N	6
1	E - - - - - W	
2	E - - - - - E	
3	E - - - - - K	
4	K - - - - - E	
5	N - - - - - E	
6	W - - - - - D	

C. Trace the array `next[]` starting at row 6 to reconstruct the original string.

EXERCISE 4: Algorithm & Data Structure Design

Given a string *txt* of length N, design an algorithm or a data structure that allows search in *txt* for a given string *s* of length $m \ll N$ in *txt*. The length m is unknown in advance and is not fixed over different queries.

Performance Requirements. The running time of each search query should be in the order of m. Your solution can use up to N^2R space and can take up to N^2R of pre-processing time (not at query time), where *R* is the size of the alphabet, which is known in advance.