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ASSIGNMENT 1: PERCOLATION

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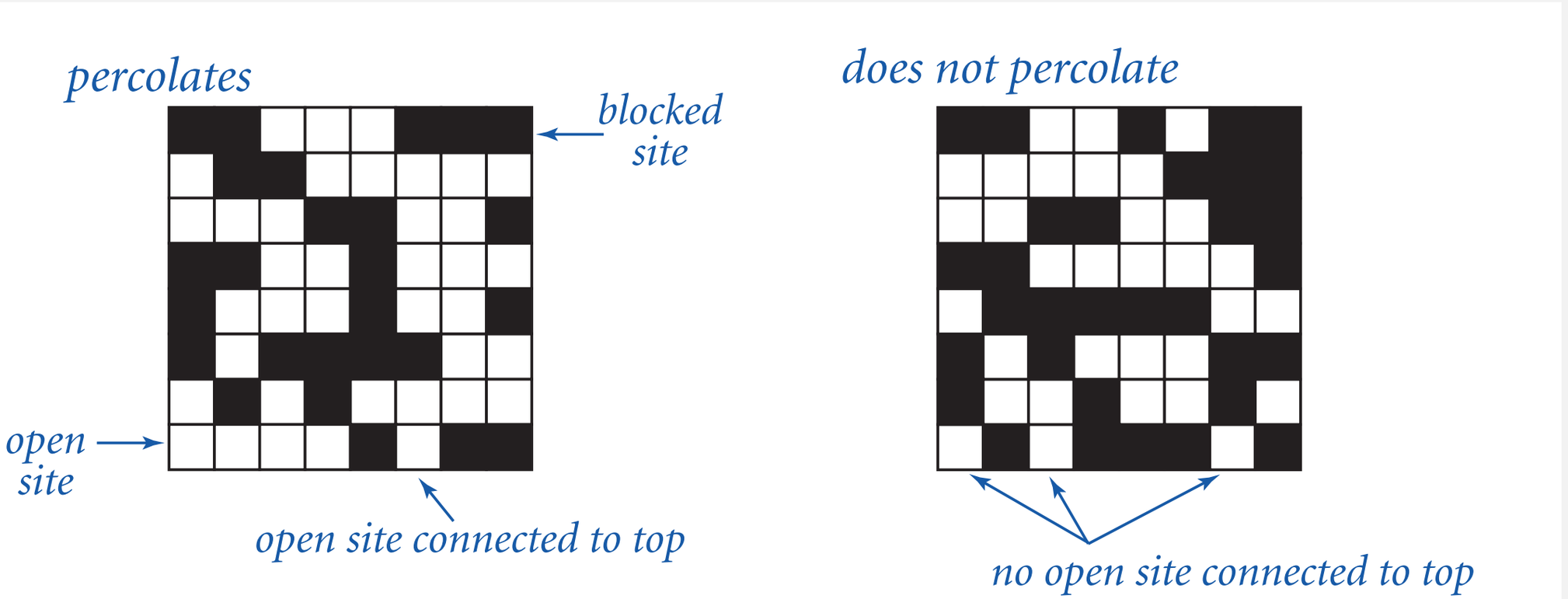
Percolation

An abstract model for many physical systems:

- n-by-n grid of sites.
- Each site is open with probability p (and blocked with probability $1 - p$).
- System percolates iff top and bottom are connected by open sites.

if and only if



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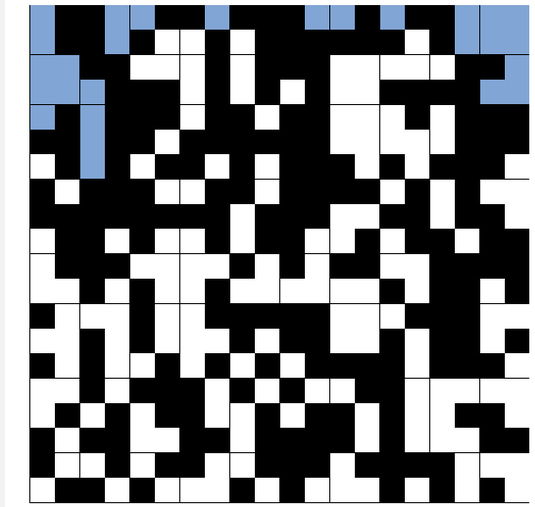
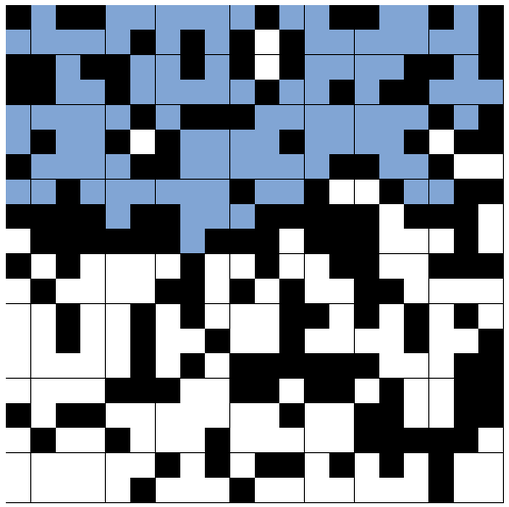
model	system	vacant site	occupied site	percolates
electricity	material	conductor	insulated	conducts
fluid flow	material	empty	blocked	porous
social interaction	population	person	empty	communicates

Likelihood of percolation

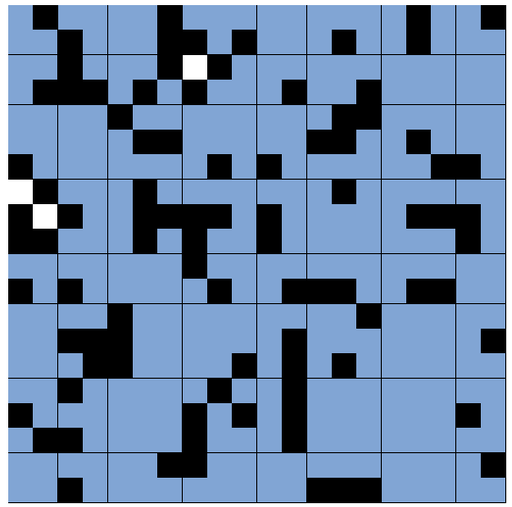
Depends on grid size n and site vacancy probability p.



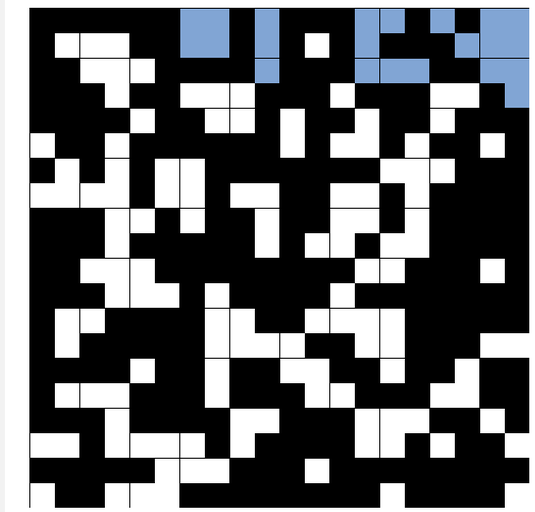
p low (0.4)
does not percolate

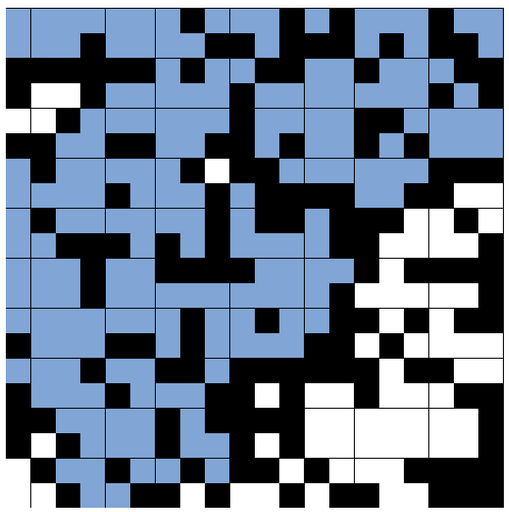


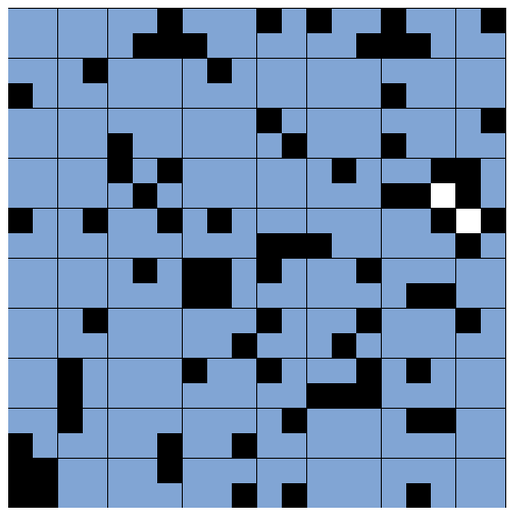
p medium (0.6)
percolates?



p high (0.8)
percolates







empty open site
(not connected to top)

full open site
(connected to top)

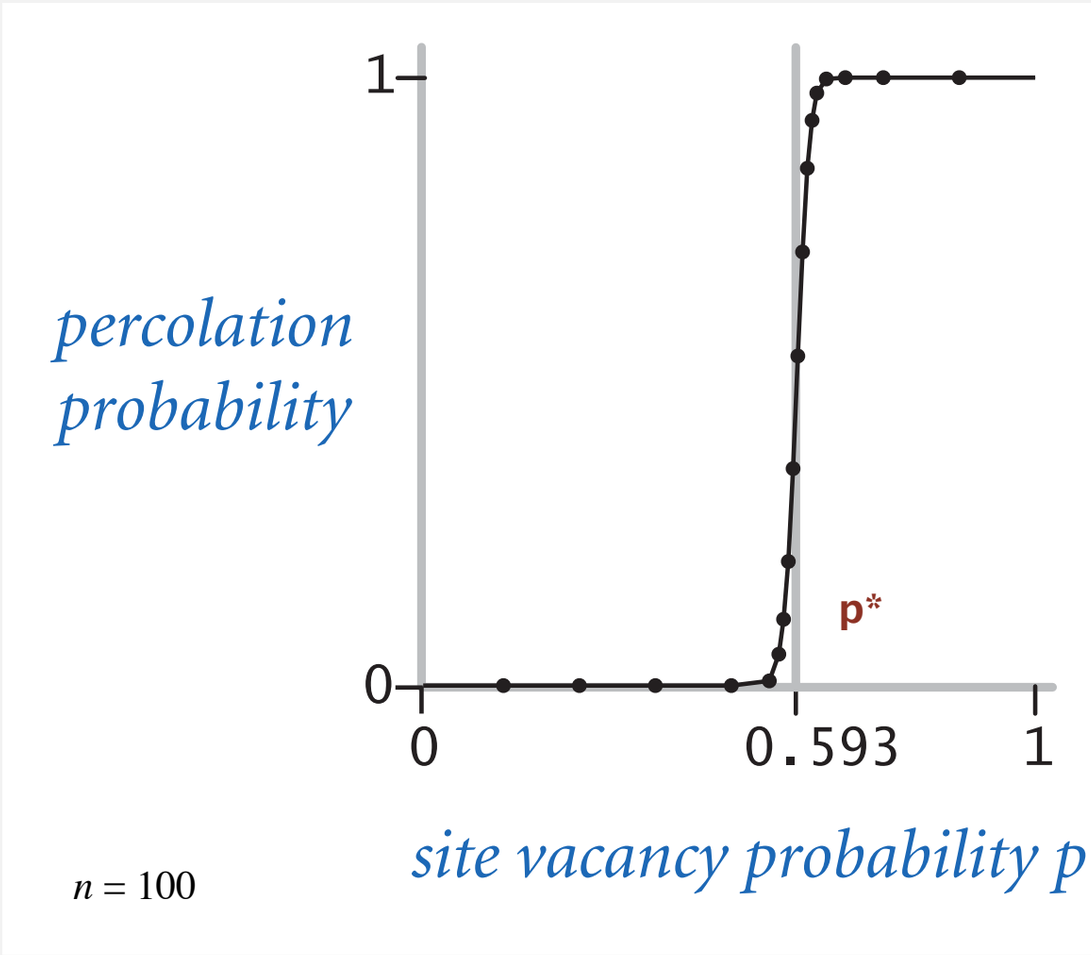
blocked site

Percolation phase transition

When n is large, theory guarantees a sharp threshold p^*.

- $p > p^*$: almost certainly percolates.
- $p < p^*$: almost certainly does not percolate.

Q. What is the value of p^* ?



Monte Carlo simulation

Barrier. Determining the exact threshold p^* is beyond mathematical reach.

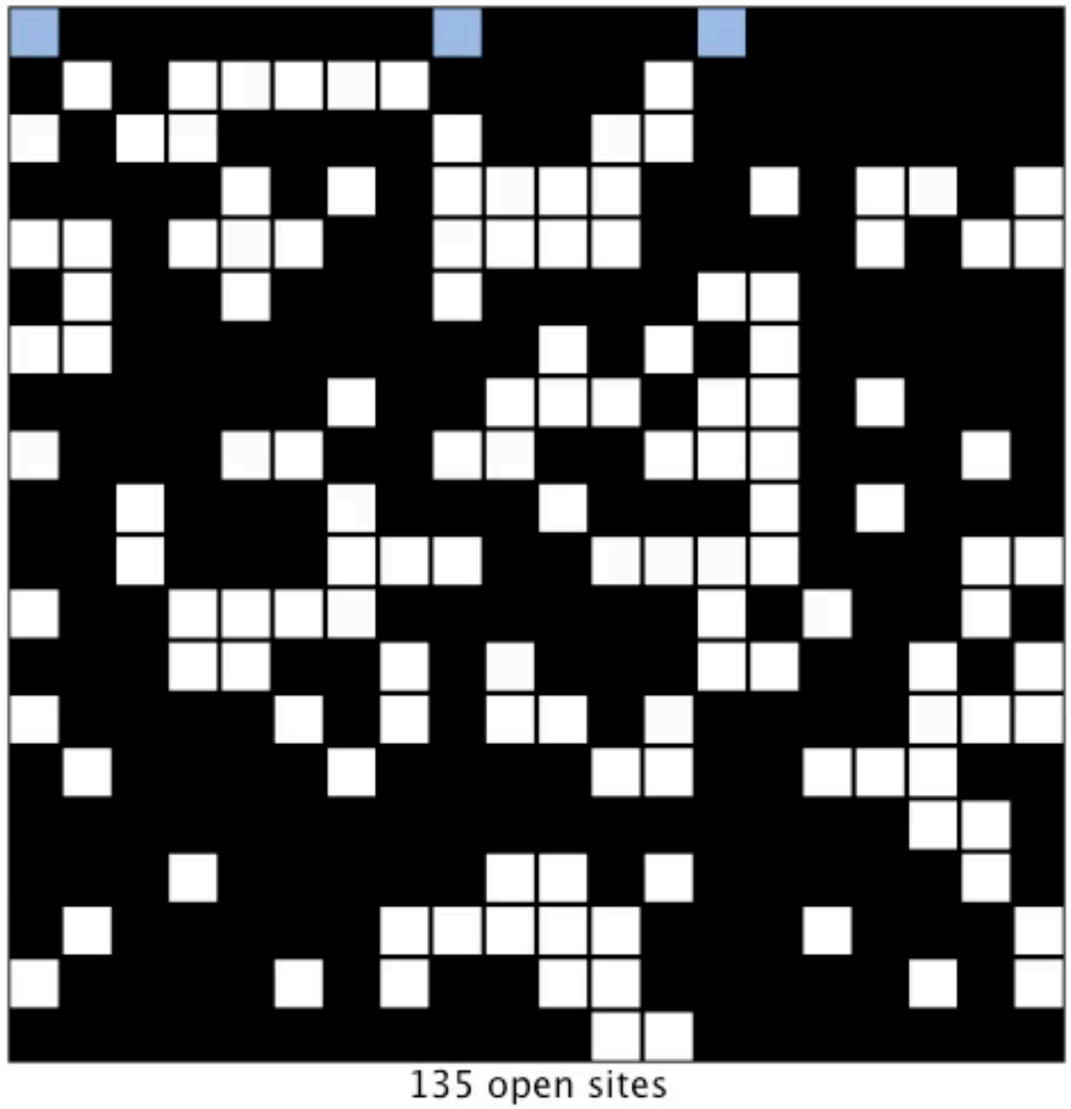
Computational approach.

- Conduct many random experiments.
- Compute statistics.
- Obtain estimate of p^*.

Casino de Monte-Carlo

Monte Carlo simulation

- Initialize all sites in an n-by-n grid to be blocked.
- Declare random sites open until top connected to bottom.
- Vacancy percentage estimates p*.
- Repeat many times to get more accurate estimate.



$$\hat{p} = \frac{204}{400} = 0.51$$

$n = 20$

full open site
(connected to top)

empty open site
(not connected to top)

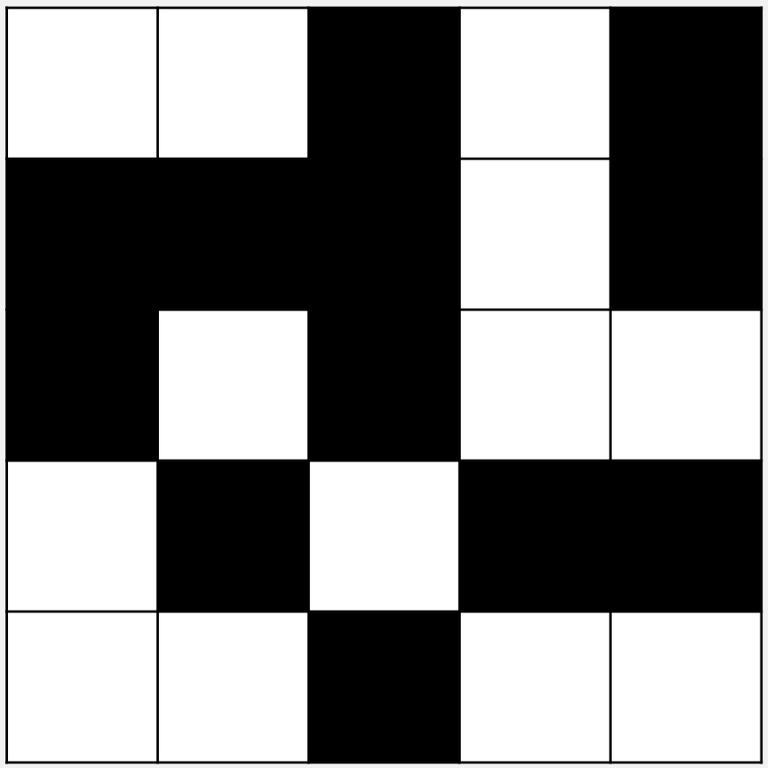
blocked site

Dynamic-connectivity solution to estimate percolation threshold

Q. How to check whether an n-by-n system percolates?

A. Model as a dynamic-connectivity problem problem and use union–find.

$n = 5$



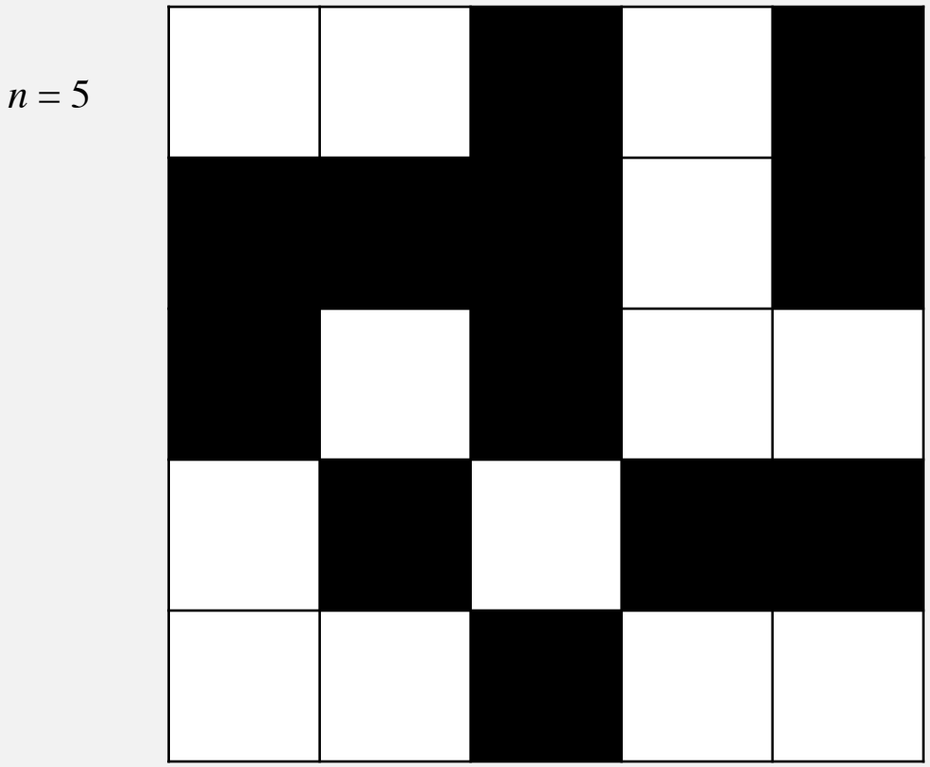
open site

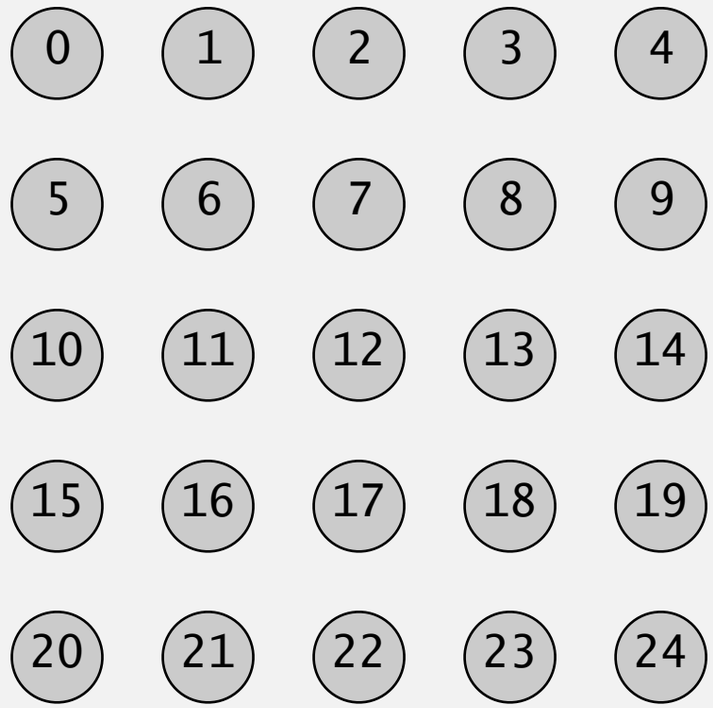
blocked site

Dynamic-connectivity solution to estimate percolation threshold

Q. How to check whether an n-by-n system percolates?

- Create an element for each site, named 0 to $n^2 - 1$.



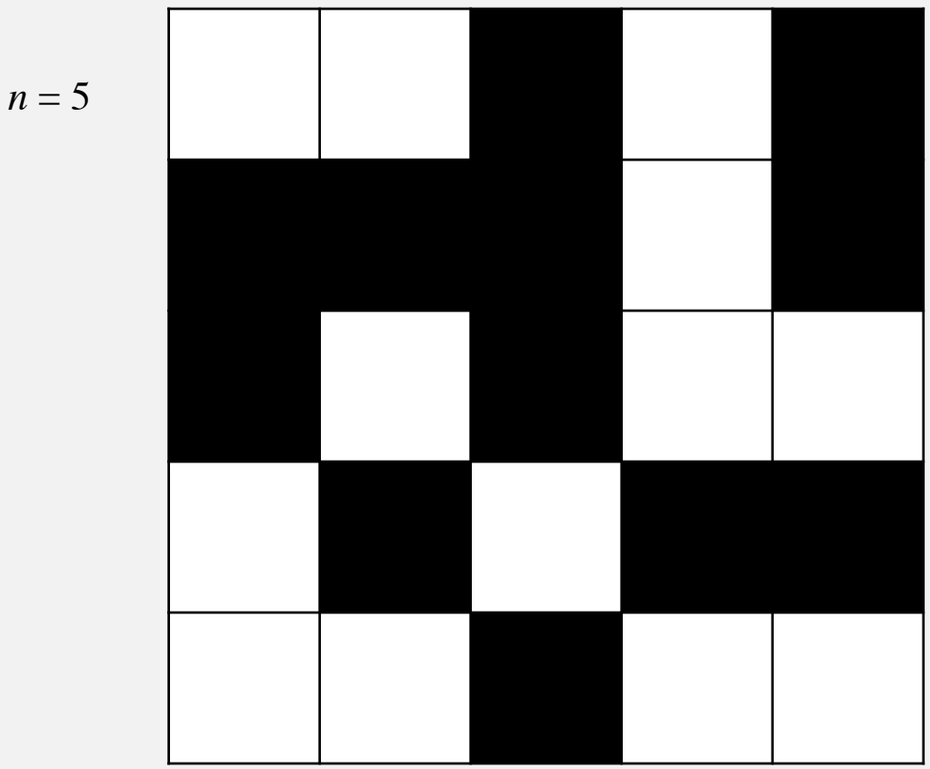


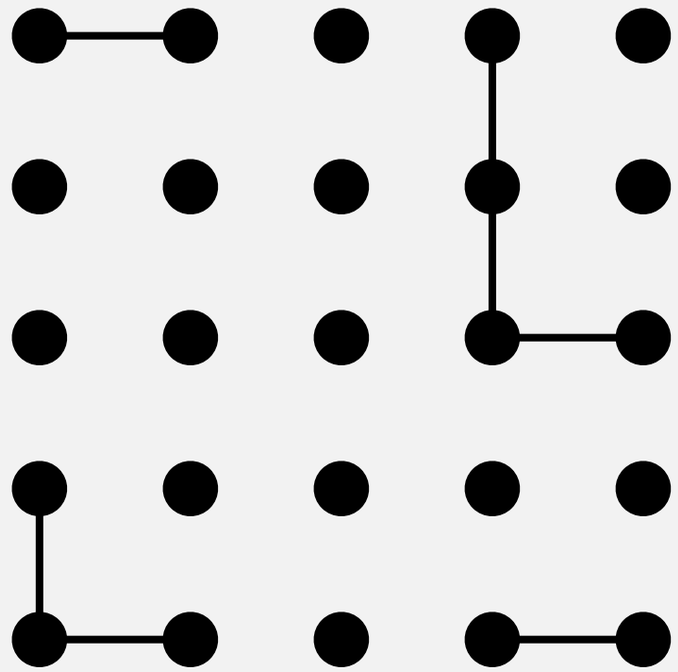
Dynamic-connectivity solution to estimate percolation threshold

Q. How to check whether an n-by-n system percolates?

- Create an element for each site, named 0 to $n^2 - 1$.
- Add edge between two adjacent sites if they both open.

4 possible neighbors: left, right, top, bottom





open site

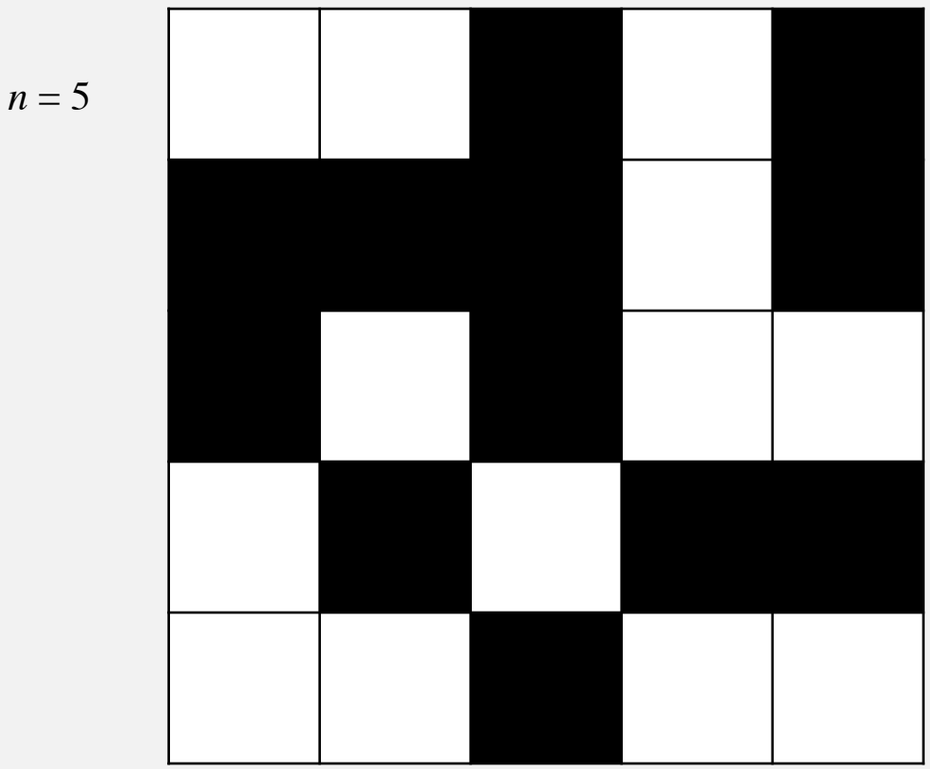
blocked site

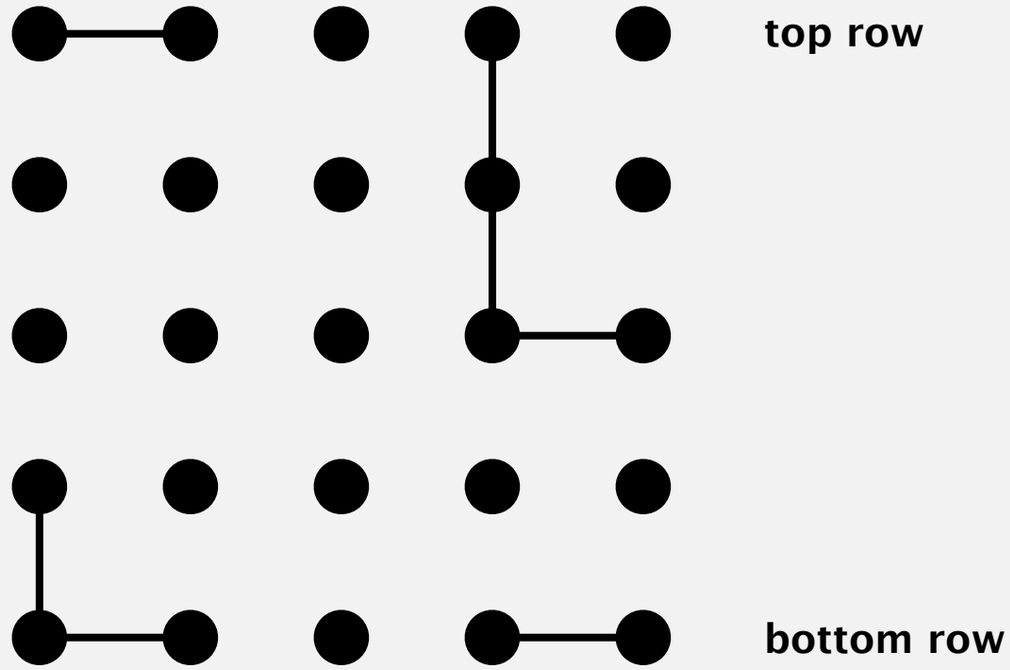
Dynamic-connectivity solution to estimate percolation threshold

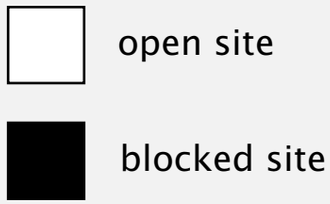
Q. How to check whether an n-by-n system percolates?

- Create an element for each site, named 0 to $n^2 - 1$.
- Add edge between two adjacent sites if they both open.
- Percolates iff any site on bottom row is connected to any site on top row.

brute-force algorithm: n^2 connected queries





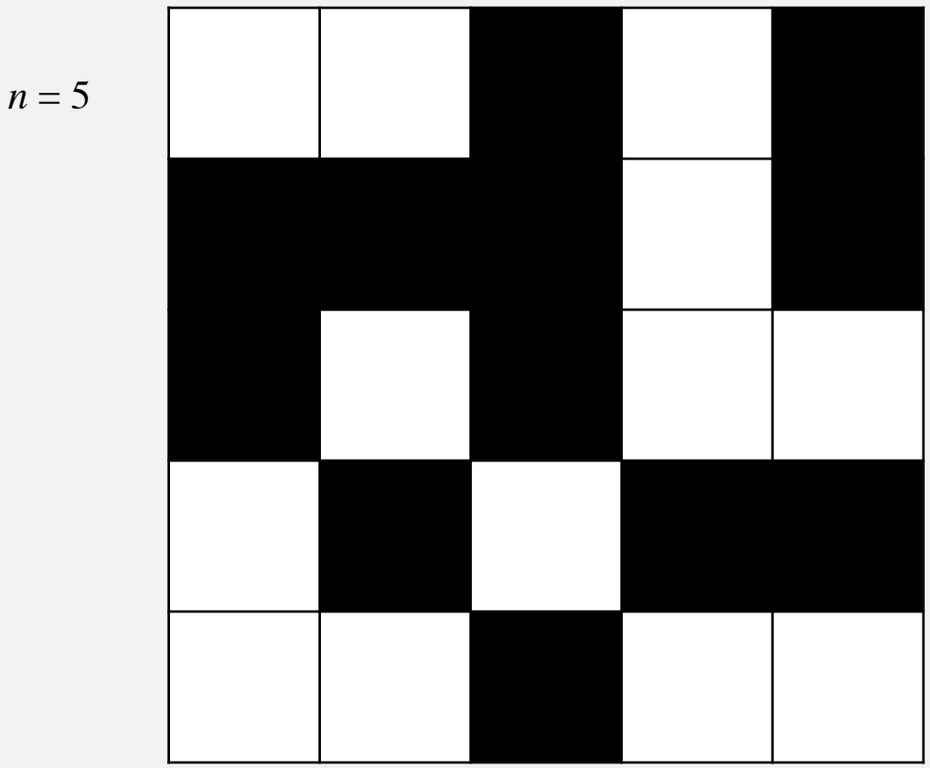


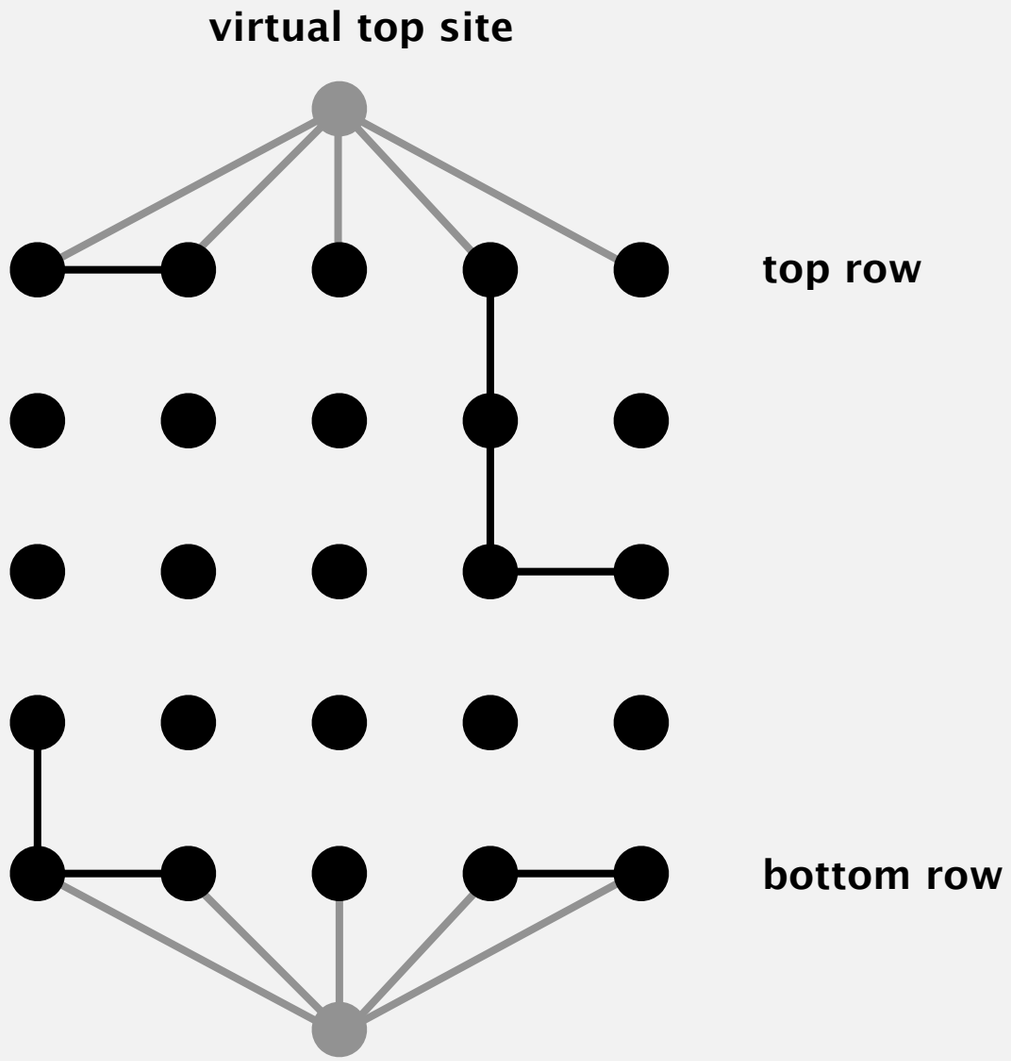
Dynamic-connectivity solution to estimate percolation threshold

Clever trick. Introduce 2 virtual sites (and edges to top and bottom).

- Percolates iff virtual top site is connected to virtual bottom site.

more efficient algorithm: only 1 connected query



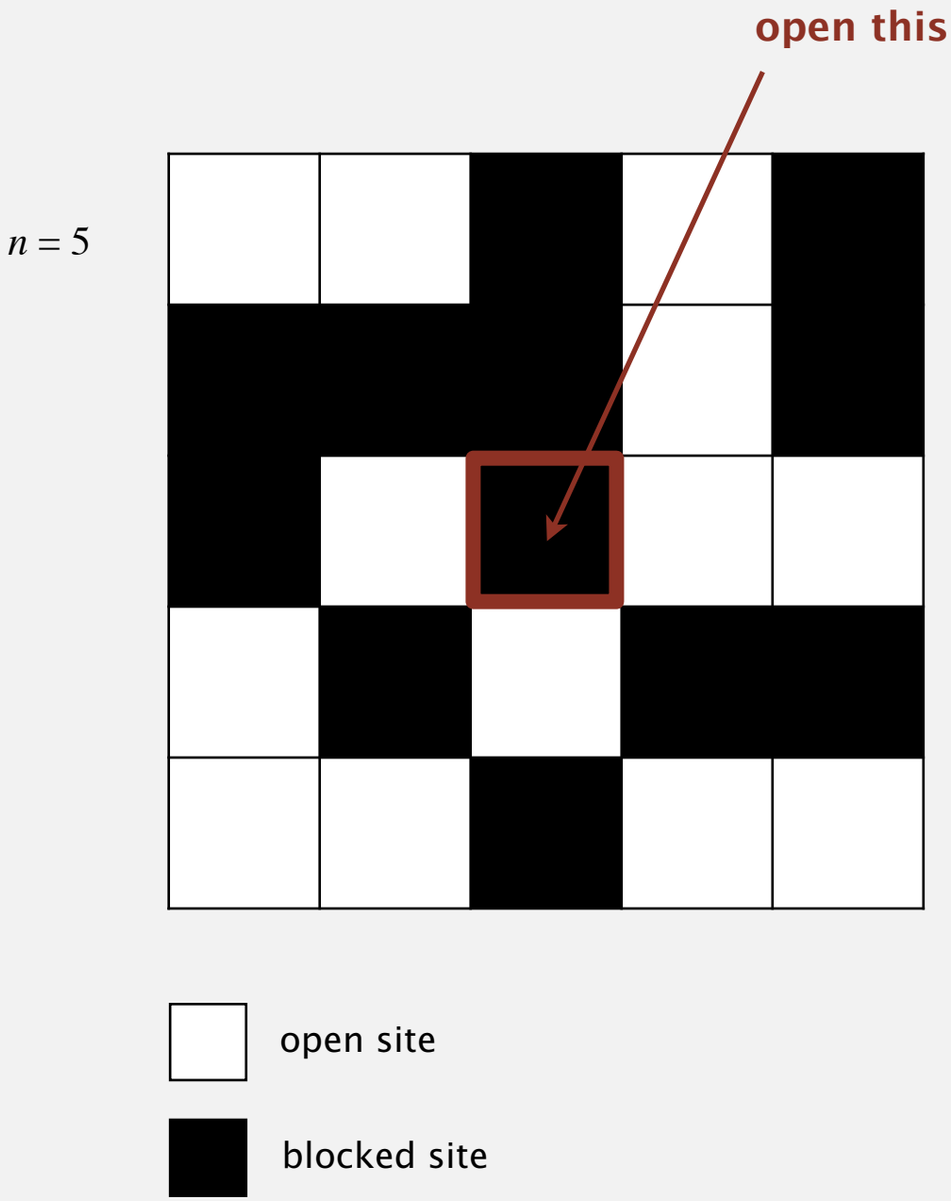


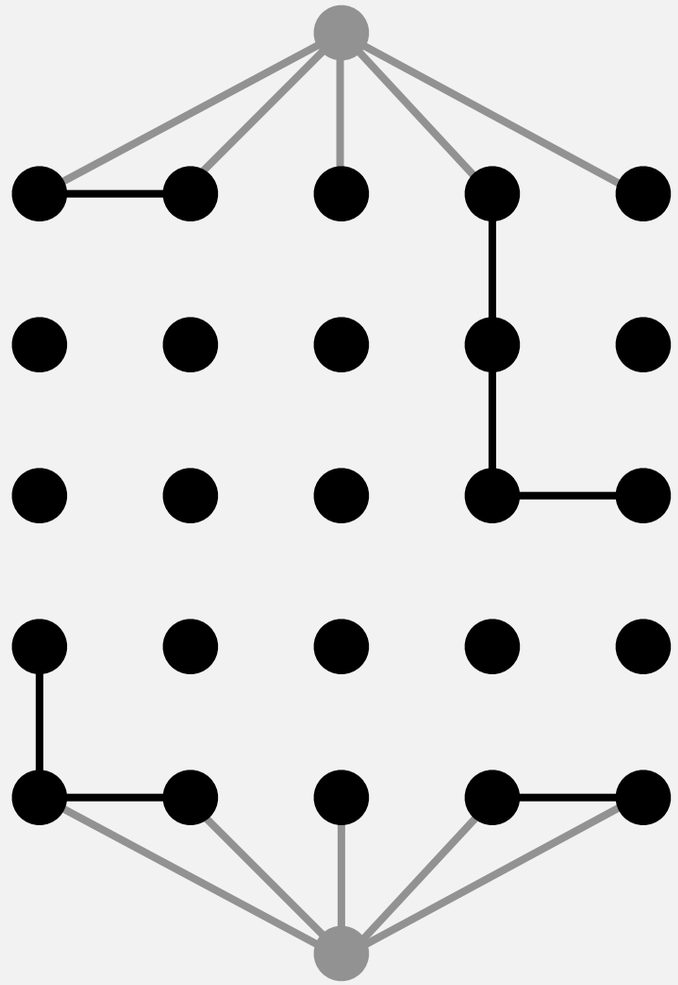
open site

blocked site

Dynamic-connectivity solution to estimate percolation threshold

Q. How to model opening a new site?



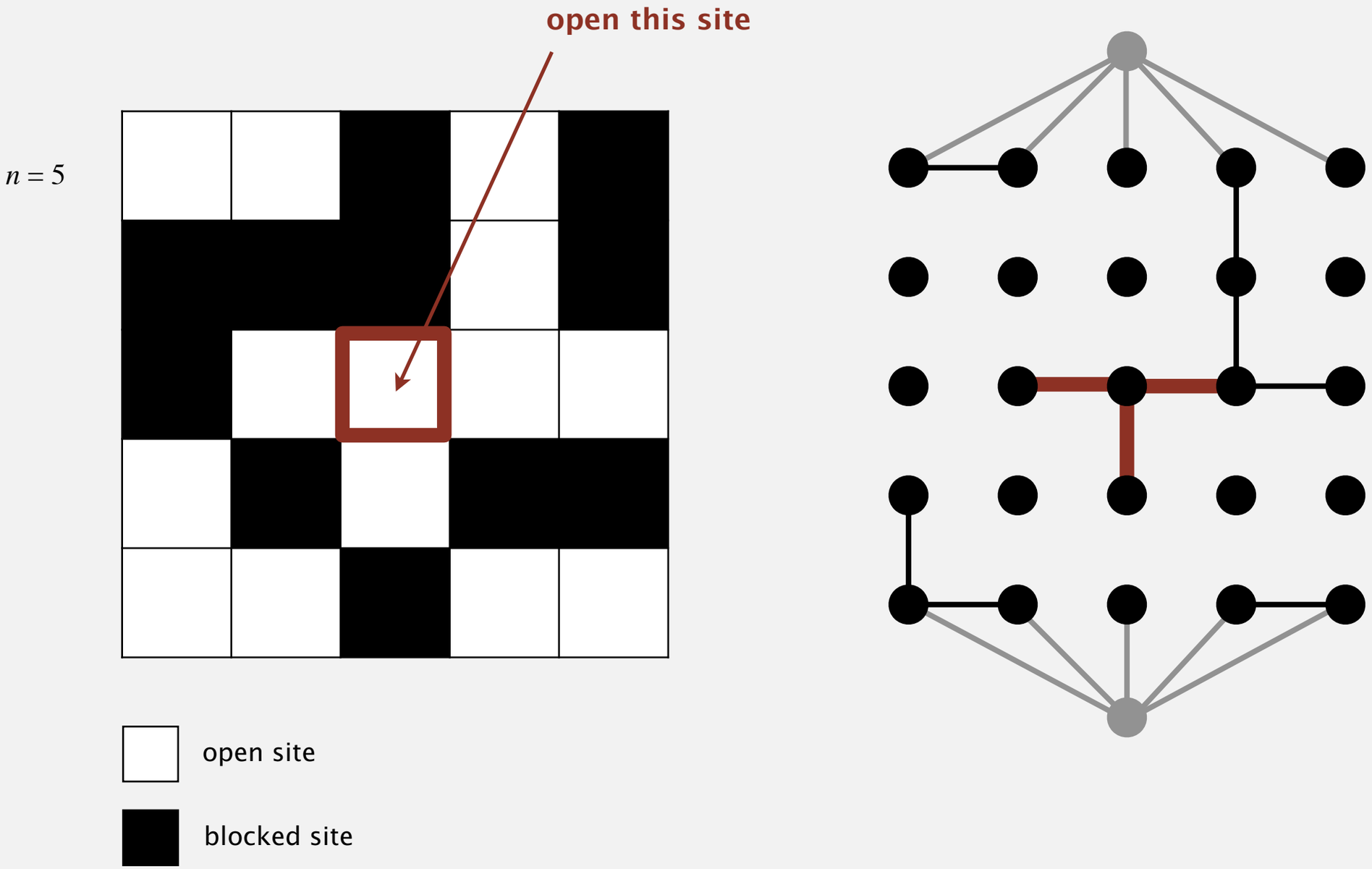


Dynamic-connectivity solution to estimate percolation threshold

Q. How to model opening a new site?

A. Mark new site as open; add edge to any adjacent site that is open.

adds up to 4 edges

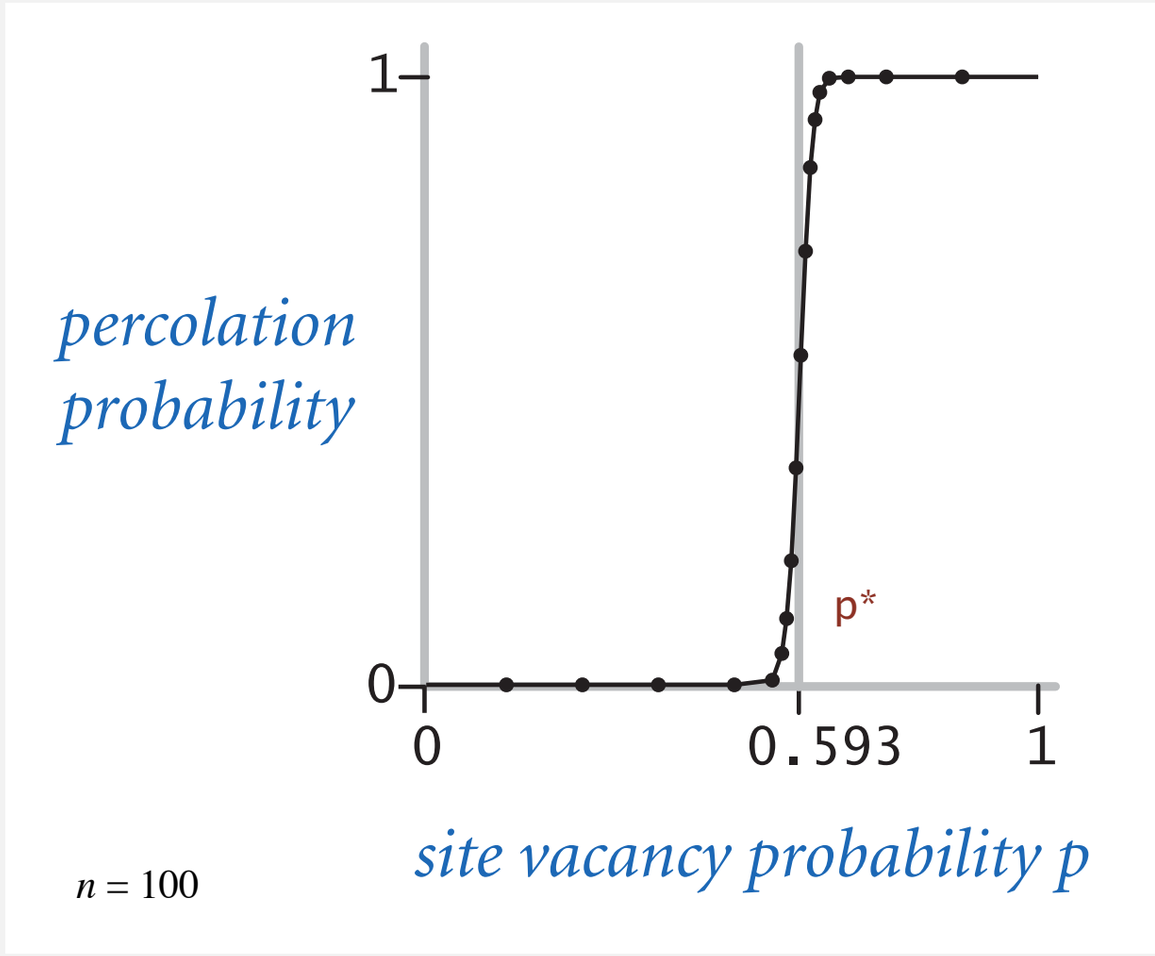


Percolation threshold

Q. What is percolation threshold p^* ?

A. About 0.592746 for large square lattices.

constant known only via simulation



Fast algorithm enables accurate answer to scientific question.