



<https://algs4.cs.princeton.edu>

## ALGORITHM DESIGN

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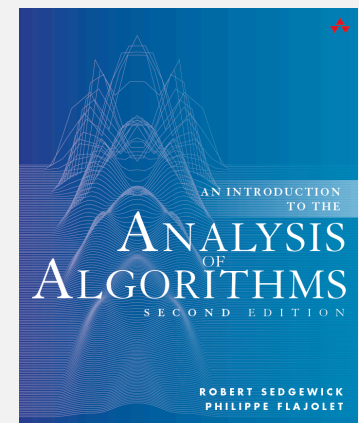
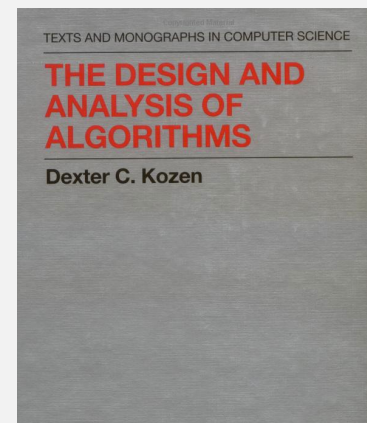
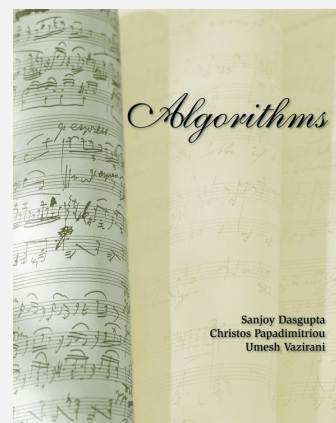
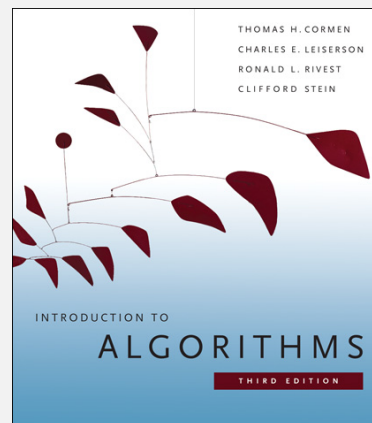
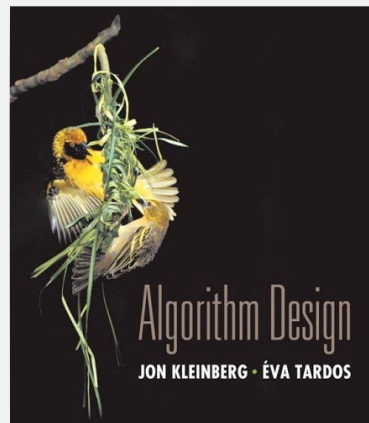
- ▶ *analysis of algorithms*
- ▶ *greedy*
- ▶ *network flow*
- ▶ *dynamic programming*
- ▶ *divide-and-conquer*
- ▶ *randomized algorithms*

# Algorithm design

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## Algorithm design patterns.

- Analysis of algorithms.
- Greedy.
- Network flow.
- Dynamic programming.
- Divide-and-conquer.
- Randomized algorithms.



Want more? See COS 340, COS 343, **COS 423**, COS 445, COS 451, COS 488, ....

# Interview questions

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# Egg drop

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**Goal.** Find  $T$  using fewest number of tosses.

**Variant 0.** 1 egg.

**Variant 1.**  $\infty$  eggs.

**Variant 2.**  $\infty$  eggs and  $\sim 2 \lg T$  tosses.

**Variant 3.** 2 eggs.





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# Greedy algorithms

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Make locally optimal choices at each step.

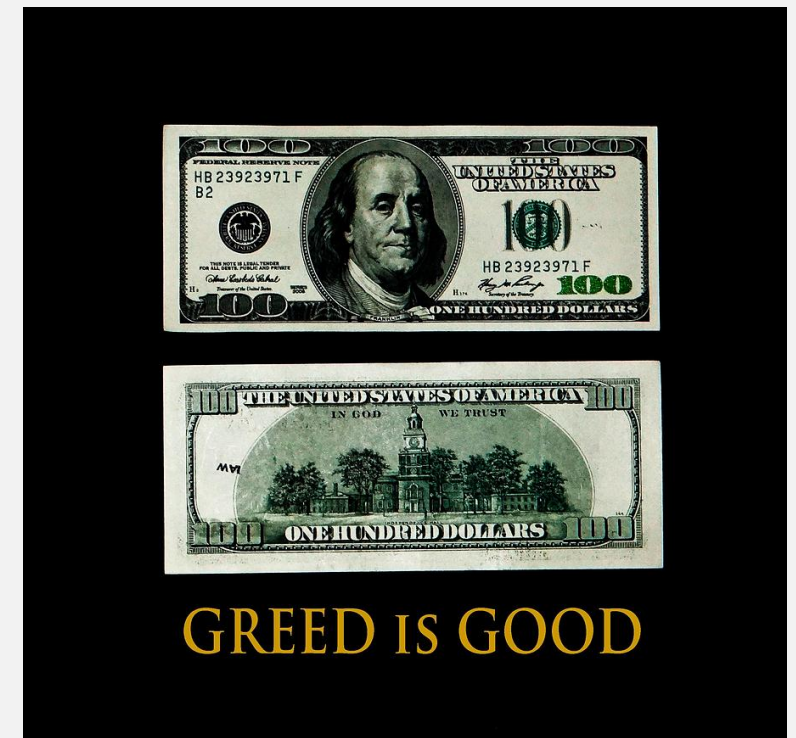
## Familiar examples.

- Huffman coding.
- Prim's algorithm.
- Kruskal's algorithm.
- Dijkstra's algorithm.

## More classic examples.

- U.S. coin changing.
- Activity scheduling.
- Gale–Shapley stable marriage.
- ...

**Caveat.** Greedy algorithm rarely leads to globally optimal solution.  
(but is often used anyway, especially for intractable problems)



## Document search

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Given a document that is a sequence of  $n$  words, and a query that is a sequence of  $m$  words, find the smallest range in the document that includes the  $m$  query words (in the same order).

**Ex.** Query = “textbook programming computer”

This book is intended to survey the most important computer algorithms in use today, and to teach fundamental techniques to the growing number of people in need of knowing them. It is intended for use as a **textbook for a second course in computer science, after students have acquired basic programming skills and familiarity with computer systems.** The book also may be useful for self-study or as a reference for people engaged in the development of computer systems or applications programs, since it contains implementations of useful algorithms and detailed information on performance characteristics and clients.





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# Network flow

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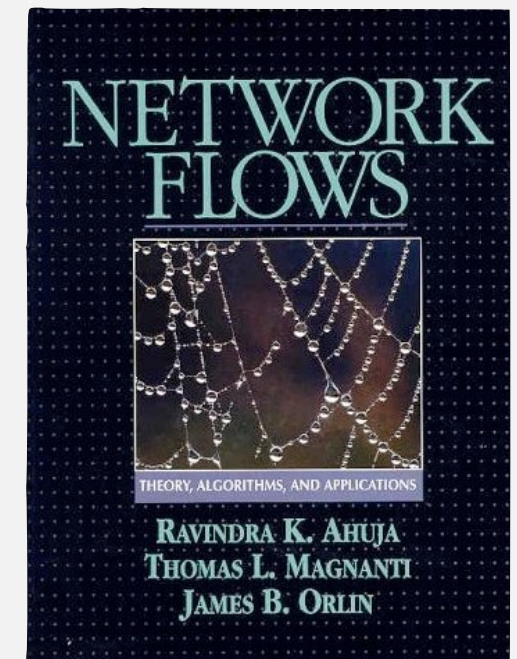
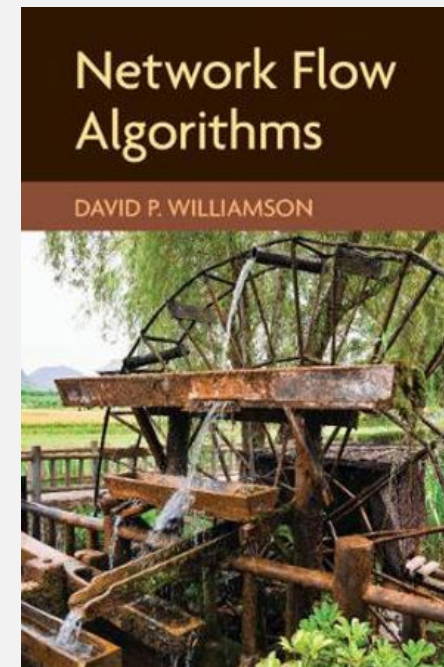
Classic problems on edge-weighted graphs.

## Familiar examples.

- Shortest paths.
- Bipartite matching.
- Maxflow and mincut.
- Minimum spanning tree.

## Other classic examples.

- Minimum-cost arborescence.
- Non-bipartite matching.
- Assignment problem.
- Minimum-cost flow.
- ...



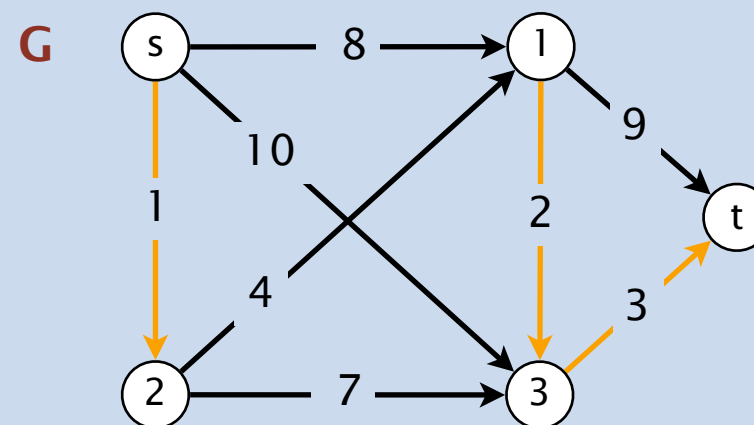
“reduction”  
↙

**Applications.** Many many problems can be modeled using network flow.

# Shortest path with orange and black edges

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**Goal.** Given a digraph, where each edge has a positive weight and is orange or black, find shortest path from  $s$  to  $t$  that uses at most  $k$  orange edges.



$$k = 0: s \rightarrow 1 \rightarrow t \quad (17)$$

$$k = 1: s \rightarrow 3 \rightarrow t \quad (13)$$

$$k = 2: s \rightarrow 2 \rightarrow 3 \rightarrow t \quad (11)$$

$$k = 3: s \rightarrow 2 \rightarrow 1 \rightarrow 3 \rightarrow t \quad (10)$$



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# Dynamic programming

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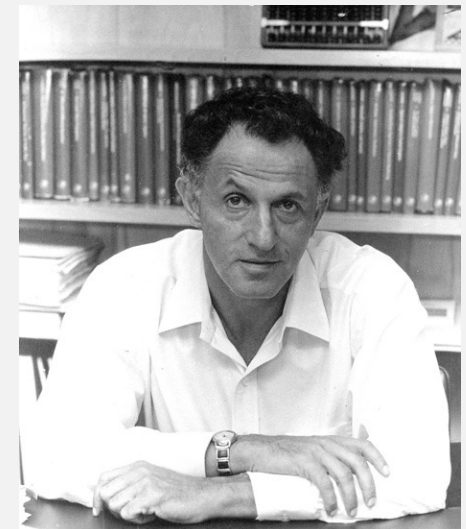
- Break up problem into a series of overlapping subproblems.
- Build up solutions to larger and larger subproblems.  
(caching solutions to subproblems in a table for later reuse)

## Familiar examples.

- Shortest paths in DAGs.
- Seam carving.
- Bellman–Ford.

## More classic examples.

- Unix diff.
- Viterbi algorithm for hidden Markov models.
- Smith–Waterman for DNA sequence alignment.
- CKY algorithm for parsing context-free grammars.
- ...

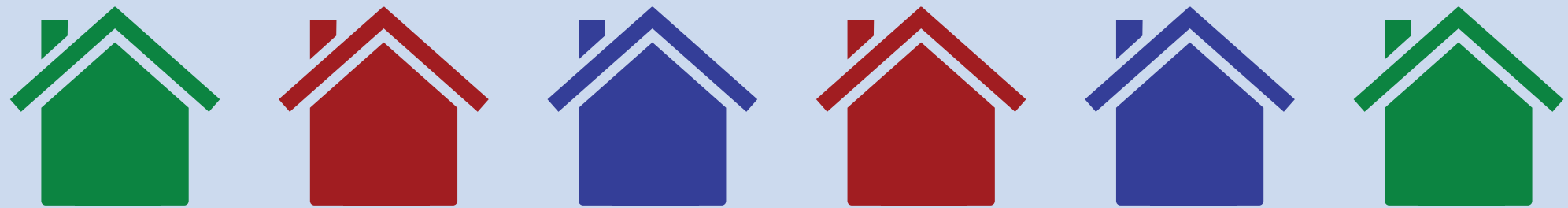


THE THEORY OF DYNAMIC PROGRAMMING  
RICHARD BELLMAN

# House coloring problem

**Goal.** Paint a row of  $n$  houses red, green, or blue so that

- No two adjacent houses have the same color.
- Minimize total cost, where  $cost(i, color)$  is cost to paint  $i$  given color.



	A	B	C	D	E	F
Red	7	6	7	8	9	20
Green	3	8	9	22	12	8
Blue	16	10	4	2	5	7

cost to paint house  $i$  the given color

$$(3 + 6 + 4 + 8 + 5 + 8 = 34)$$





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# Divide and conquer

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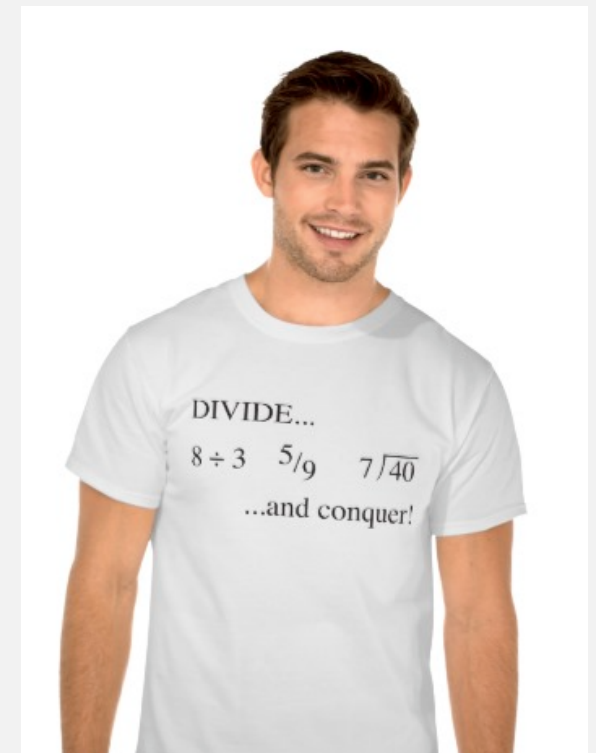
- Break up problem into two or more independent subproblems.
- Solve each subproblem recursively.
- Combine solutions to subproblems to form solution to original problem.

## Familiar examples.

- Mergesort.
- Quicksort.

## More classic examples.

- Closest pair.
- Convolution and FFT.
- Matrix multiplication.
- Integer multiplication.
- ...



needs to take COS 226?

**Prototypical usage.** Turn brute-force  $n^2$  algorithm into  $n \log n$  algorithm.



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# Randomized algorithms

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Algorithm that uses random coin flips to guide its behavior.

## Familiar examples.

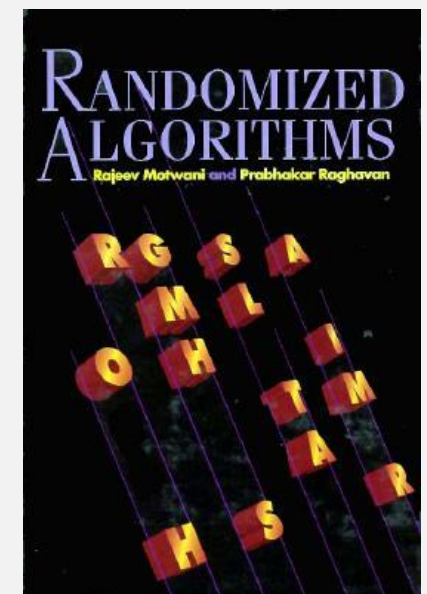
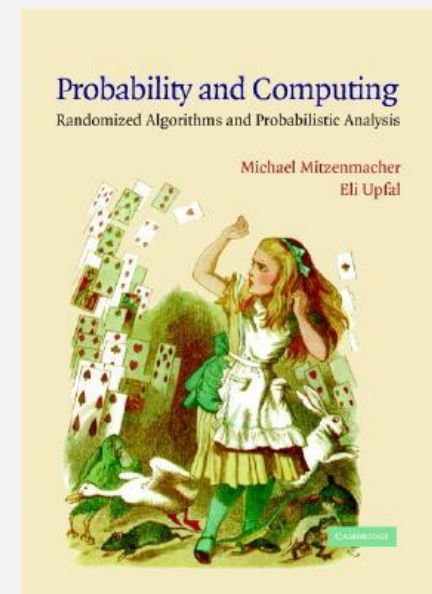
- Quicksort.
- Quickselect.



## More classic examples.

- Rabin–Karp substring search.
- Miller–Rabin primality testing.
- Polynomial identity testing.
- Volume of convex body.
- Universal hashing.
- Global min cut.

...



# Nuts and bolts

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**Problem.** A disorganized carpenter has a mixed pile of  $n$  nuts and  $n$  bolts.

- The goal is to find the corresponding pairs of nuts and bolts.
- Each nut fits exactly one bolt and each bolt fits exactly one nut.
- By fitting a nut and a bolt together, the carpenter can see which one is bigger (but cannot directly compare either two nuts or two bolts).



**Brute-force  $n^2$  solution.** Compare each bolt to each nut.

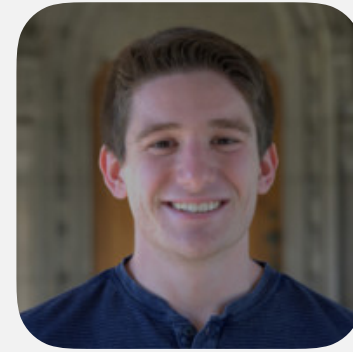
**Challenge.** Design an  $n \log n$  algorithm.



# Credits

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Faculty lead preceptors, Turing preceptor, and graduate student AIs.



Undergraduate graders and lab TAs. Apply to be one next semester!

Ed tech. Several developed here at Princeton!





A farewell video (from P04, Fall 2018)

A group of people, some wearing white shirts, are gathered in a grassy field. In the background, there is a brick building with several windows and a tree. The scene is slightly blurred, suggesting movement or a video recording.

**COS 22.6 P04  
Presents..**

*“ Algorithms and data structures are love.  
Algorithms and data structures are life. ”*

*— anonymous COS 226 student*