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5.3 SUBSTRING SEARCH

- *introduction*
- *brute force*
- *Knuth–Morris–Pratt*
- *Boyer–Moore*

Last updated on 12/5/19 5:24 AM

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Substring search

Goal. Find pattern of length m in a text of length n.

typically $n \gg m$

Substring search applications

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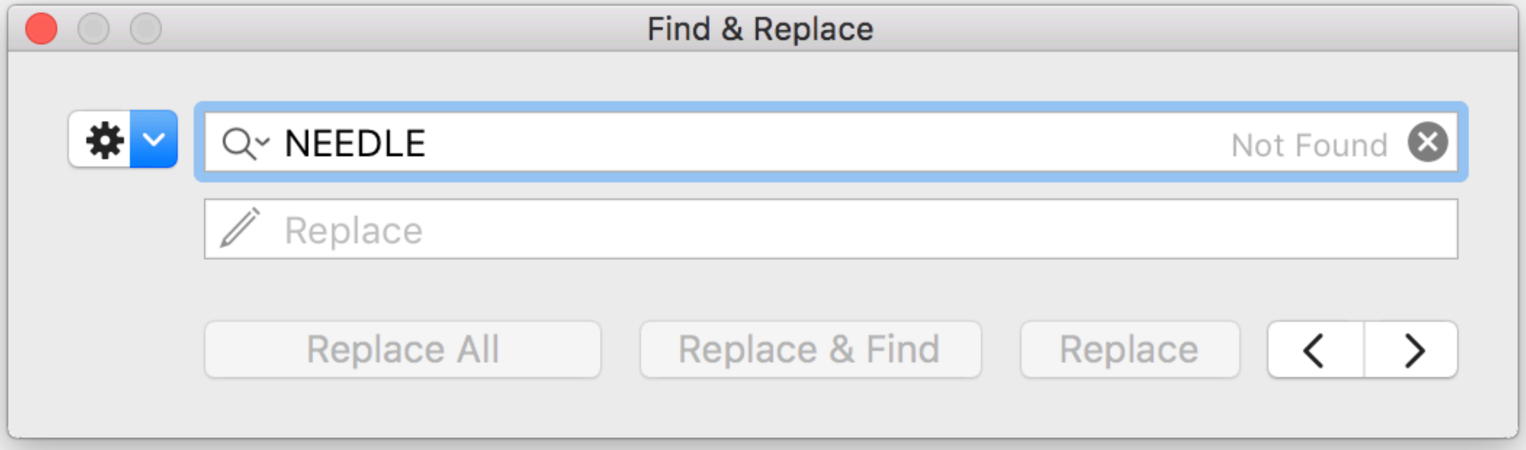
typically $n \gg m$

pattern → N E E D L E

text → I N A H A Y S T A C K N E E D L E I N A

match

Search in a word processor or IDE.



Find & Replace

NEEDLE

Not Found

Substring search applications

Goal. Find pattern of length m in a text of length n.

typically $n \gg m$

pattern → `N E E D L E`

text → `I N A H A Y S T A C K N E E D L E I N A`

match

Identify patterns indicative of spam.

- `PROFITS`
- `LOSE WE1GHT`
- `herbal Viagra`
- `There is no catch.`
- `This is a one-time mailing.`
- `This message is sent in compliance with spam regulations.`

Substring search applications

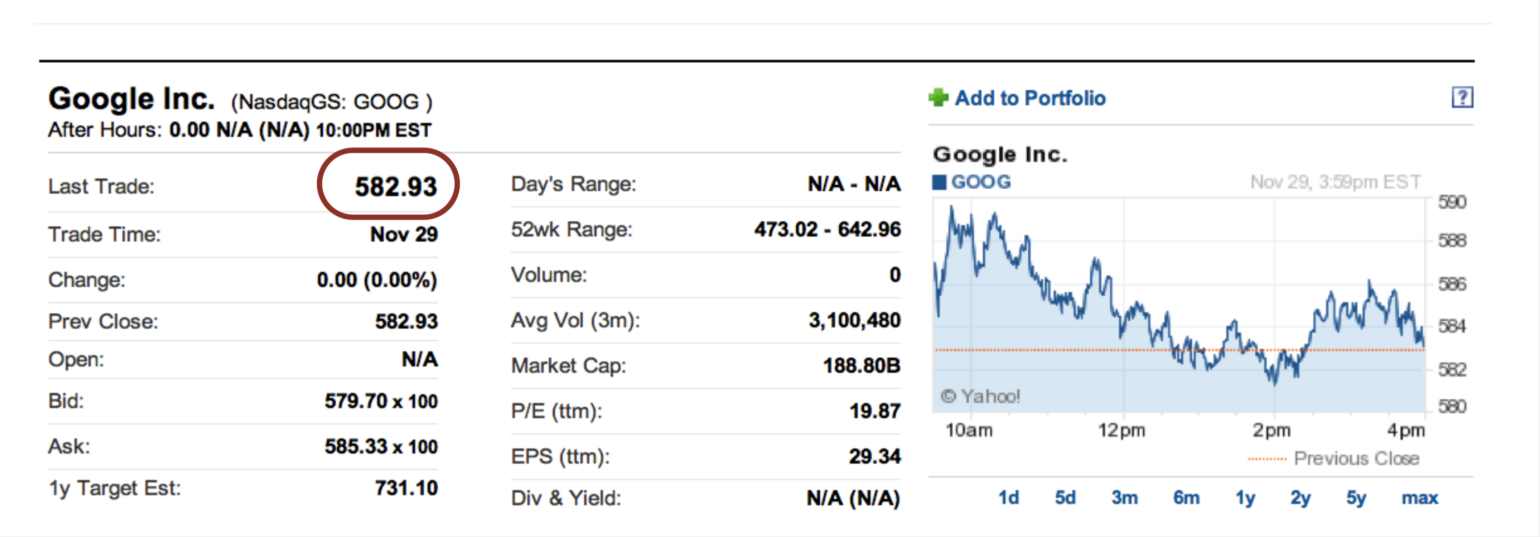
Web scraping. Extract relevant data from web page.

Ex. Find string delimited by `<b>` and `</b>` after first occurrence of pattern `Last Trade:`.

raw HTML

```
...
<tr>
<td class= "yfnc_tablehead1"
width= "48%">
Last Trade:
</td>
<td class= "yfnc_tabledata1">
<big><b>582.93</b></big>
</td></tr>
<td class= "yfnc_tablehead1"
width= "48%">
Trade Time:
</td>
<td class= "yfnc_tabledata1">
...
```

as rendered by browser



Google Inc. (NasdaqGS: GOOG)
After Hours: **0.00 N/A (N/A) 10:00PM EST**

Last Trade:	582.93	Day's Range:	N/A - N/A
Trade Time:	Nov 29	52wk Range:	473.02 - 642.96
Change:	0.00 (0.00%)	Volume:	0
Prev Close:	582.93	Avg Vol (3m):	3,100,480
Open:	N/A	Market Cap:	188.80B
Bid:	579.70 x 100	P/E (ttm):	19.87
Ask:	585.33 x 100	EPS (ttm):	29.34
1y Target Est:	731.10	Div & Yield:	N/A (N/A)

http://finance.yahoo.com/q?s=goog

Web scraping: Java implementation

Java library. The `indexOf()` method in Java's `String` data type returns the index of the first occurrence of a given string, starting at a given offset.

```
public class StockQuote
{
   public static void main(String[] args)
   {
      String name = "http://finance.yahoo.com/q?s=";
      In in = new In(name + args[0]);
      String text = in.readAll();
      int start    = text.indexOf("Last Trade:", 0);
      int from     = text.indexOf("<b>",  start);
      int to       = text.indexOf("</b>", from);
      String price = text.substring(from + 3, to);
      StdOut.println(price);
   }
}
```

```
% java StockQuote goog
582.93
```

Caveat. Must update program whenever Yahoo format changes.

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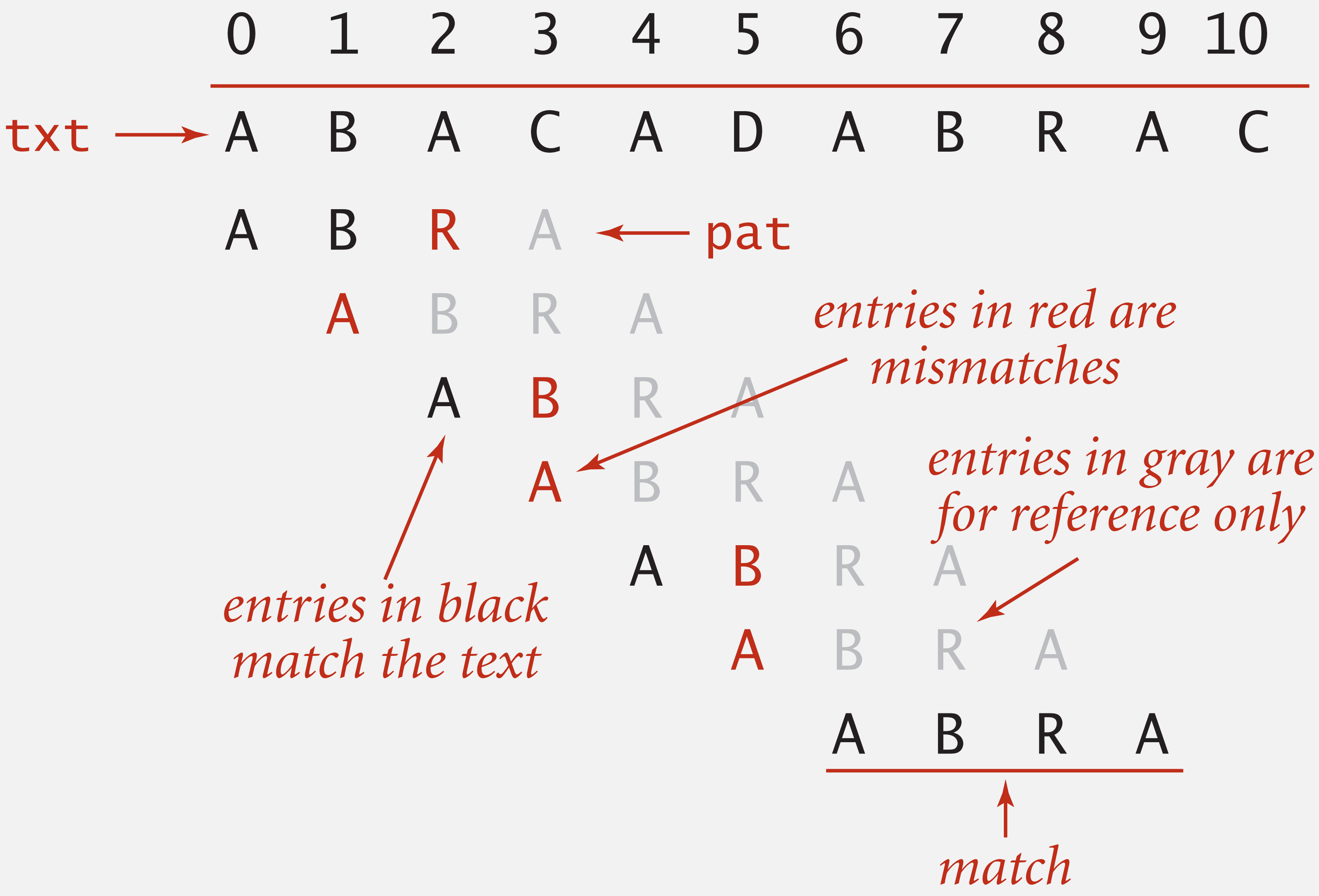
Brute-force substring search

Check for pattern starting at each text position.



	0	1	2	3	4	5	6	7	8	9	10
txt	A	B	A	C	A	D	A	B	R	A	C
pat	A	B	R	A							
		A	B	R	A						
			A	B	R	A					
				A	B	R	A				
					A	B	R	A			
						A	B	R	A		
							A	B	R	A	

entries in red are mismatches

entries in gray are for reference only

entries in black match the text

match

Brute-force substring search: Java implementation

Check for pattern starting at each text position.

```
public static int search(String pat, String txt)
{
   int m = pat.length();
   int n = txt.length();

   for (int i = 0; i <= n - m; i++)  ←  for each possible offset i
   {
      for (int j = 0; j <= m; j++)  ←  check for match at offset i
      {
         if (j == m) return i;  ←  match at offset i
         if (pat.charAt(j) != txt.charAt(i+j))
            break;             ↖
                                 no match at offset i
      }                  (stop as soon as non-match is confirmed)
   }
   return n;  ←  no match
}
```

Substring search: quiz 1

What is the worst-case running time of brute-force substring search as a function of the pattern length m and text length n ?

A. $m + n$

B. m^2

C. $m\,n$

D. n^2

Algorithmic challenges in substring search

Fundamental algorithmic challenge. Linear-time guarantee.

Now is the time for all people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for many good people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for a lot of good people to come to the aid of their party. Now is the time for all of the good people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for all good Republicans to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for many or all good people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for all good Democrats to come to the aid of their party. Now is the time for all people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for many good people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for a lot of good people to come to the aid of their party. Now is the time for all of the good people to come to the aid of their party. Now is the time for all good people to come to the aid of their **attack at dawn** party. Now is the time for each person to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for all good Republicans to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for many or all good people to come to the aid of their party. Now is the time for all good people to come to the aid of their party. Now is the time for all good Democrats to come to the aid of their party.

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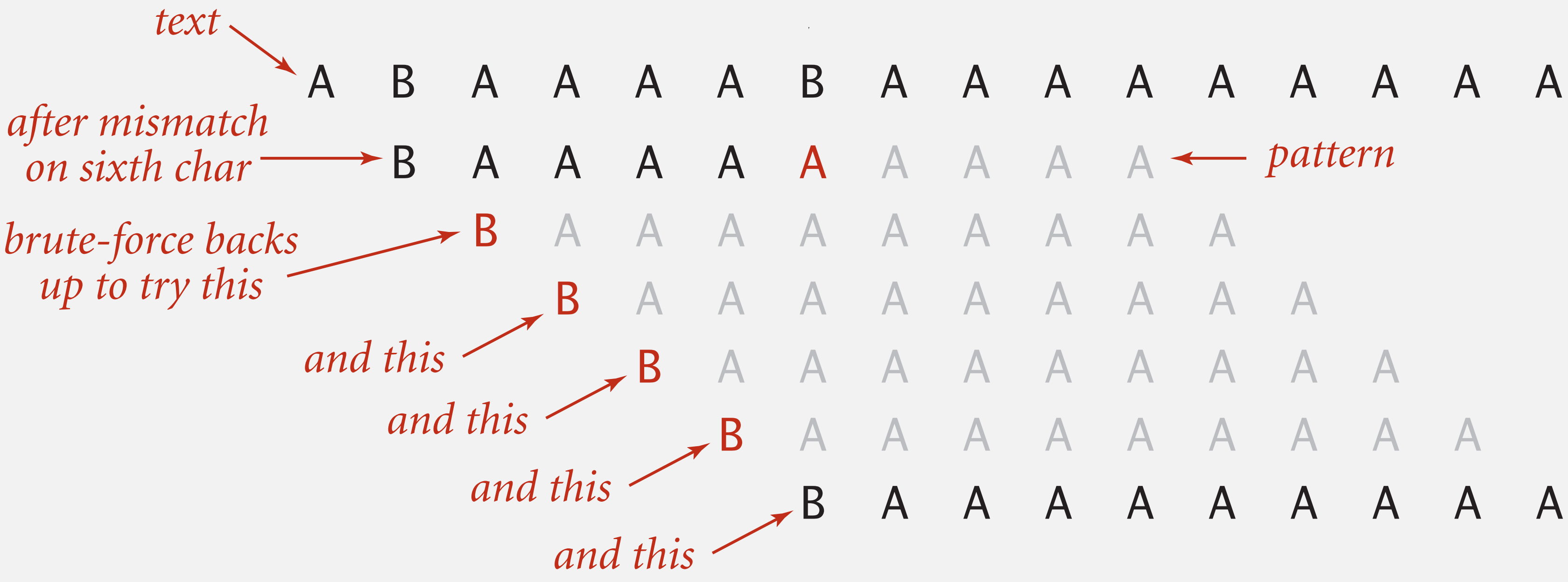
Knuth–Morris–Pratt substring search

Intuition. Suppose we are searching in text for pattern `BAAAAAAAAA`.

- Suppose we match 5 chars in pattern, with mismatch on 6th char.



text

A B A A A A B A A A A A A A A A

after mismatch on sixth char

B A A A A A A A A A

pattern

brute-force backs up to try this

B A A A A A A A A A

and this

B A A A A A A A A A

and this

B A A A A A A A A A

and this

B A A A A A A A A A

and this

B A A A A A A A A A

Knuth-Morris-Pratt substring search

Intuition. Suppose we are searching in text for pattern BAAAAAAAAA.

- Suppose we match 5 chars in pattern, with mismatch on 6th char.
- We know previous 6 chars in text must be BAAAAB.
- Don't need to compare any text character twice.

assuming { A, B } alphabet

text

A B A A A A B A A A A A A A A A

after mismatch on sixth char

B A A A A A A A A A

pattern

B A A A A A A A A A

but no backup is needed

Knuth–Morris–Pratt algorithm. Clever method to always avoid comparing a text character more than once!

Deterministic finite state automaton (DFA)

DFA is abstract string-searching machine.

- Finite number of states (including start and halt).
- Exactly one state transition for each character in alphabet.
- Accept if sequence of state transitions ever enters halt state.

internal representation

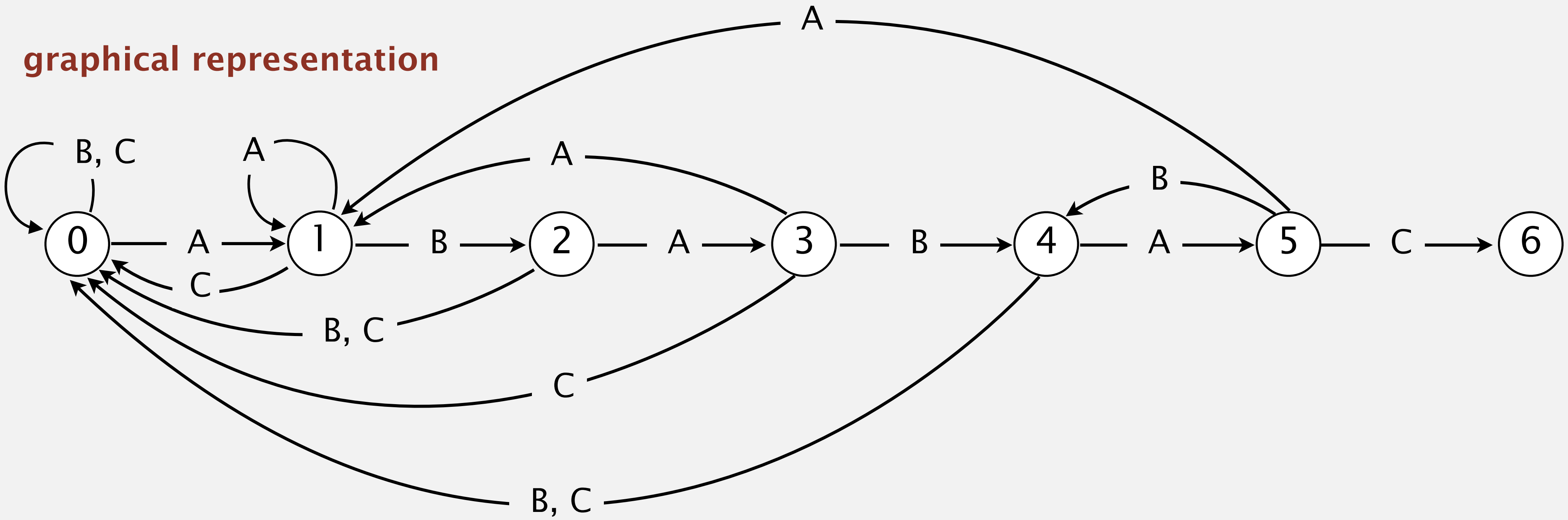
`j`		0	1	2	3	4	5
`pat.charAt(j)`		A	B	A	B	A	C
`dfa[][j]`	A	1	1	3	1	5	1
	B	0	2	0	4	0	4
	C	0	0	0	0	0	6

If in state `j` reading char `c`:

if `j` is 6 halt and accept

else move to state `dfa[c][j]`

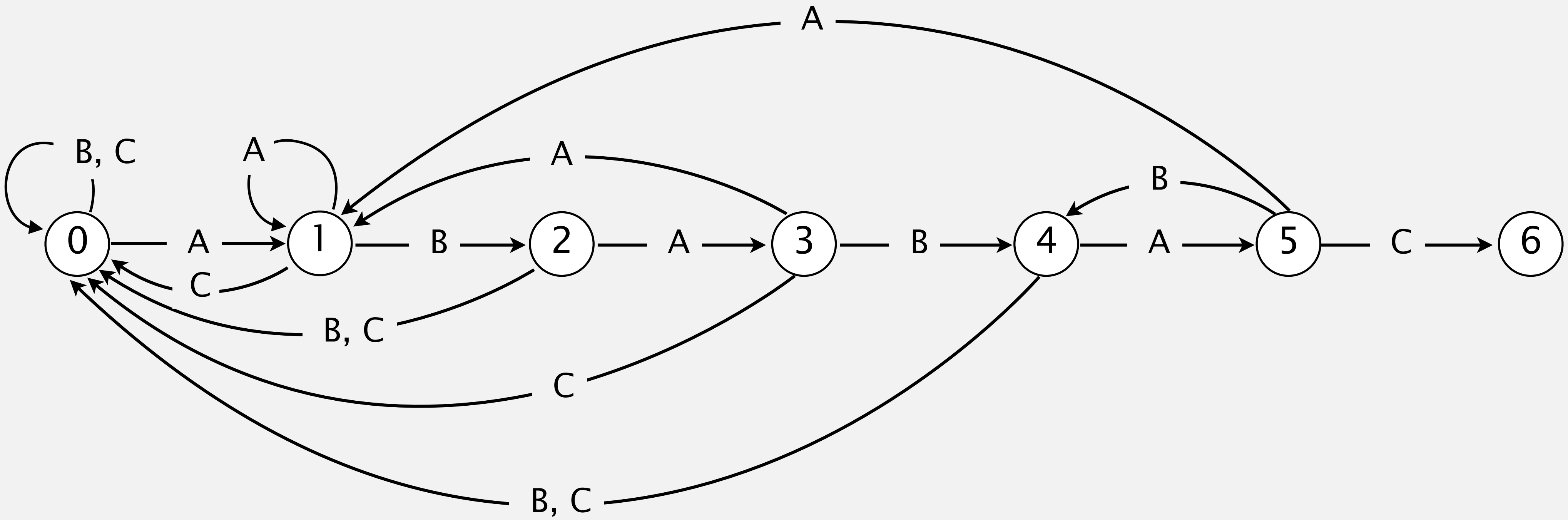
graphical representation



Knuth–Morris–Pratt demo: DFA simulation

A A B A C A A B A B A C A A

		0	1	2	3	4	5
pat.charAt(j)		A	B	A	B	A	C
	A	1	1	3	1	5	1
dfa[][j]	B	0	2	0	4	0	4
	C	0	0	0	0	0	6



Interpretation of Knuth–Morris–Pratt DFA

Q. What is interpretation of DFA state after processing `txt[i]`?

A. State = number of characters in pattern that have been matched.

length of longest prefix of `pat[]` that is a suffix of `txt[0..i]`

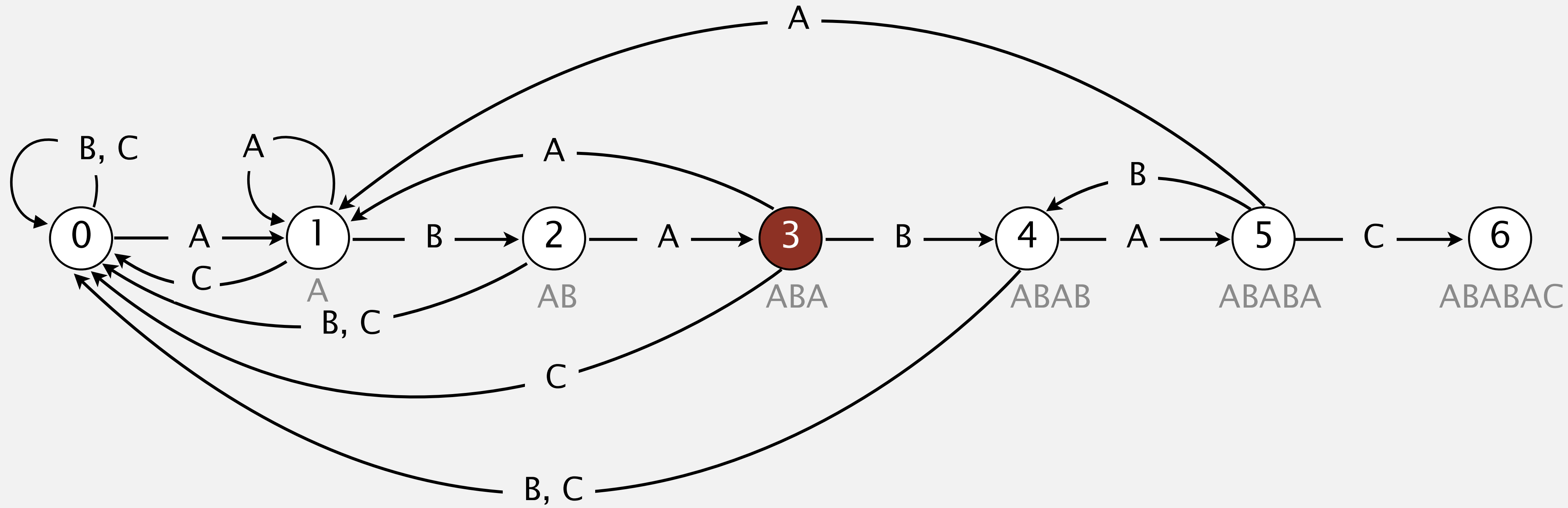
Ex. DFA is in state 3 after reading in `txt[0..6]`.

							i		
	0	1	2	3	4	5	6	7	8
`txt[]`	B	C	B	A	A	B	A	C	A

suffix of `txt[0..6]`

	0	1	2	3	4	5
`pat[]`	A	B	A	B	A	C

prefix of `pat[]`



Substring search: quiz 2

Which state is the DFA in after processing the following input?

B A A B A B A B

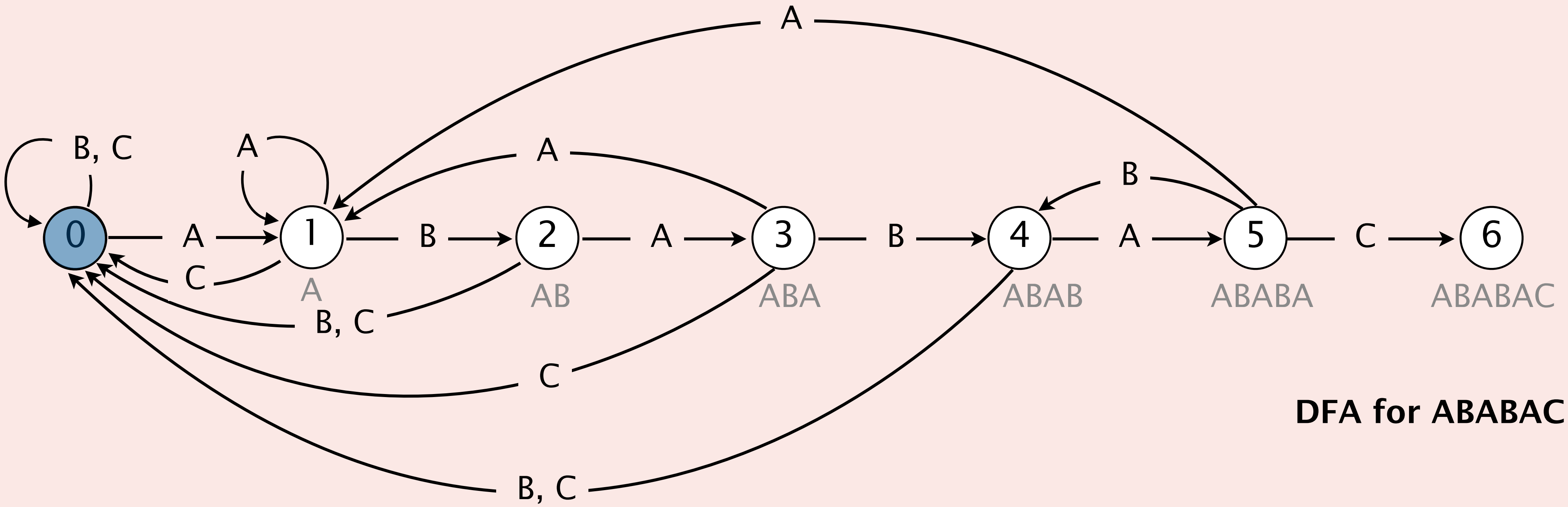
↑

A. 0

B. 1

C. 3

D. 4



DFA for ABABAC

Substring search: quiz 3

Which state is the DFA in after processing the following input?

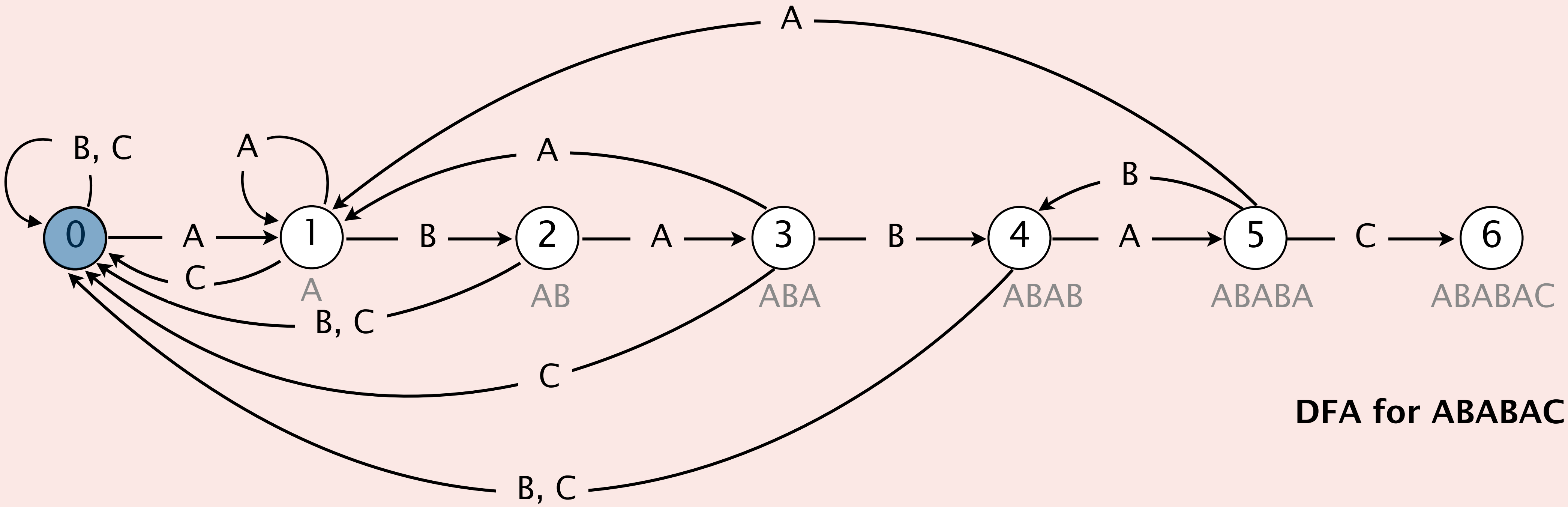
A B A A B B A B A B B A B A A B A A B A A A B A B A B A A B A A B A A B A B A B

A. 0

B. 1

C. 3

D. 4



DFA for ABABAC

Knuth–Morris–Pratt substring search: Java implementation

Key differences from brute-force implementation.

- Need to precompute `dfa[][]` from pattern.
- Each text character compared (at most) once.

```
public int search(String txt)
{
   int m = pat.length(), n = txt.length();
   int j = 0;
   for (int i = 0; i < n; i++)
   {
      if (j == m) return i - m;
      j = dfa[txt.charAt(i)][j];
   }
   return n;
}
```

current state

halt state (match found)

no match

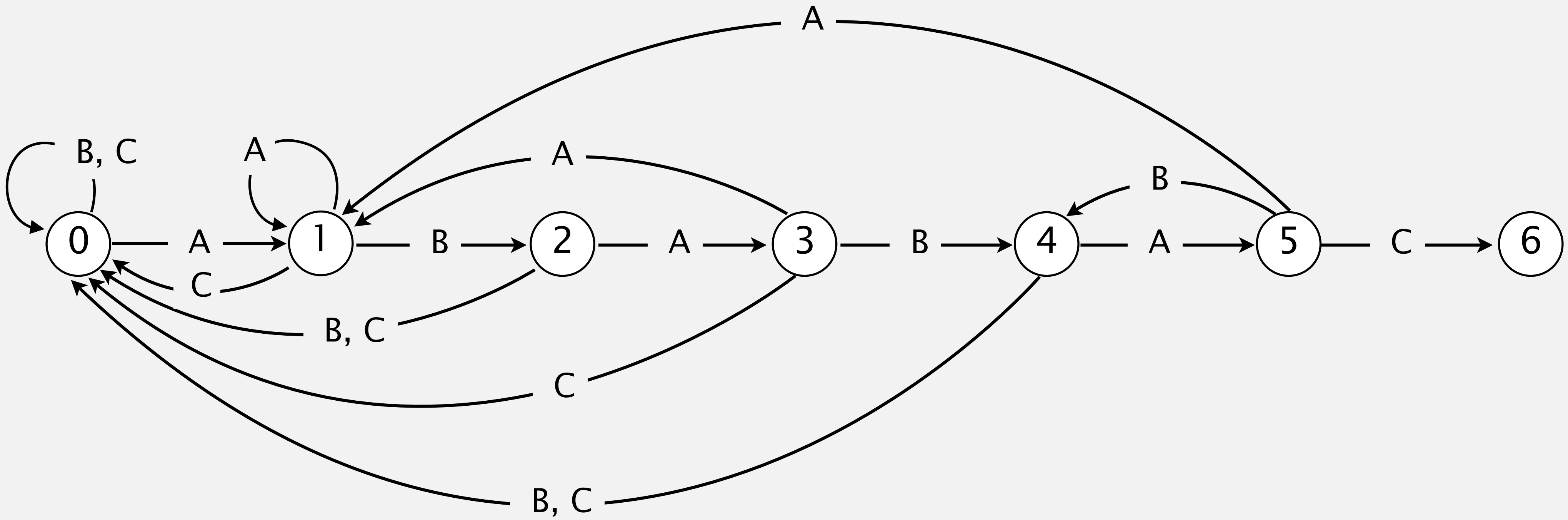
Running time.

- Simulate DFA on text: at most n character accesses.
- Build DFA: how to do efficiently? [warning: tricky algorithm ahead]

Knuth-Morris-Pratt demo: DFA construction

		0	1	2	3	4	5
pat.charAt(j)		A	B	A	B	A	C
dfa[][j]	A	1	1	3	1	5	1
	B	0	2	0	4	0	4
	C	0	0	0	0	0	6

Constructing the DFA for KMP substring search for A B A B A C



How to build DFA from pattern?

Include one state for each character in pattern (plus accept state).

		0	1	2	3	4	5
`pat.charAt(j)`		A	B	A	B	A	C
`dfa[][j]`	A						
	B						
	C						

How to build DFA from pattern?

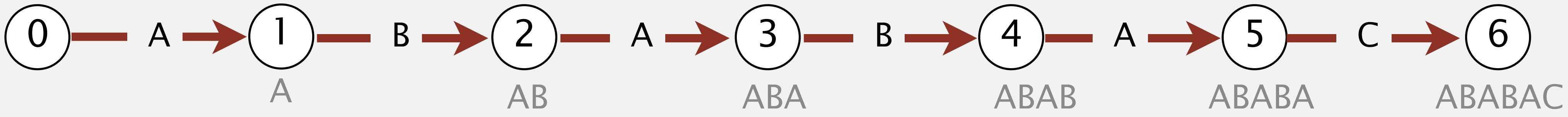
Match transition. If in state `j` and next char `c == pat.charAt(j)`, go to `j+1`.

first j characters of pattern have already been matched

next char matches

now first j +1 characters of pattern have been matched

		0	1	2	3	4	5
`pat.charAt(j)`		A	B	A	B	A	C
	A	1		3		5	
`dfa[][j]`	B		2		4		
	C						6



How to build DFA from pattern?

Mismatch transition. If in state `j` and next char `c != pat.charAt(j)`, then the last `j-1` characters of input are `pat[1..j-1]`, followed by `c`.

To compute `dfa[c][j]`: Simulate `pat[1..j-1]` on DFA and take transition `c`.

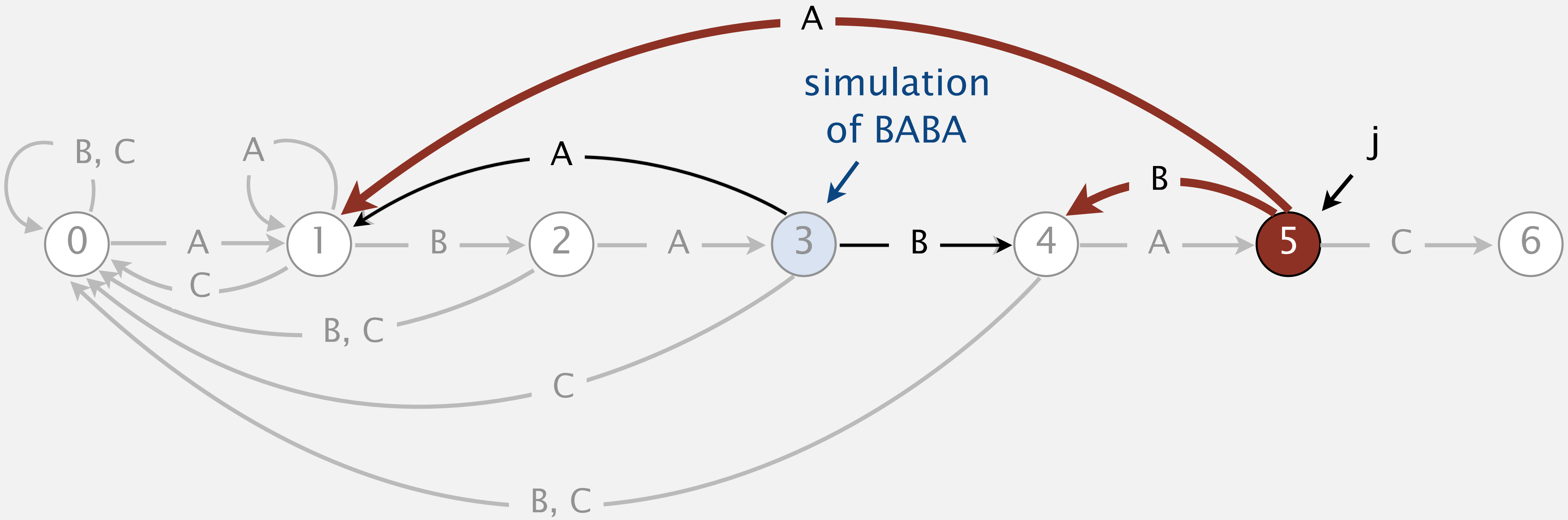
Running time. Seems to require `j` steps.

still under construction (!)

Ex. `dfa['A'][5] = 1`　`dfa['B'][5] = 4`

simulate BABAA　simulate BABAB

j	0	1	2	3	4	5
`pat.charAt(j)`	A	B	A	B	A	C



How to build DFA from pattern?

Mismatch transition. If in state `j` and next char `c != pat.charAt(j)`, then the last `j-1` characters of input are `pat[1..j-1]`, followed by `c`.

state x

To compute `dfa[c][j]`: Simulate `pat[1..j-1]` on DFA and take transition `c`.

Running time. Takes only constant time if we maintain state x.

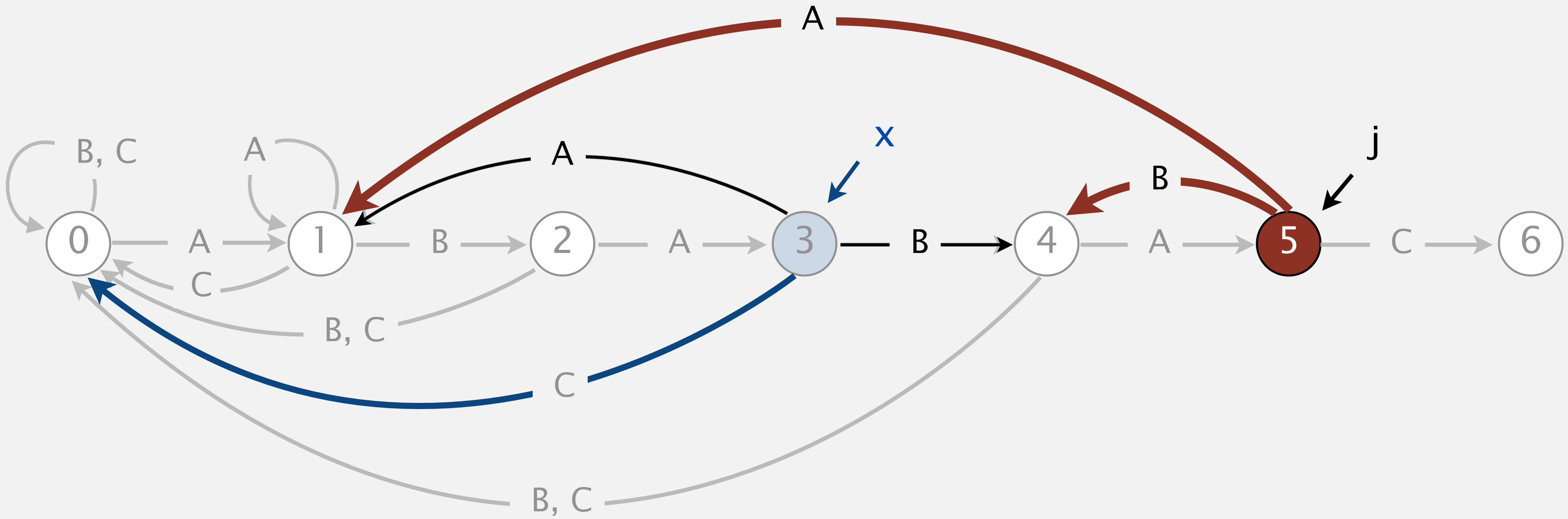
Ex. `dfa['A'][5] = 1`   `dfa['B'][5] = 4`   `x' = 0`

from state x,
take transition 'A'
= dfa['A'][x]

from state x,
take transition 'B'
= dfa['B'][x]

from state x,
take transition 'C'
= dfa['C'][x]

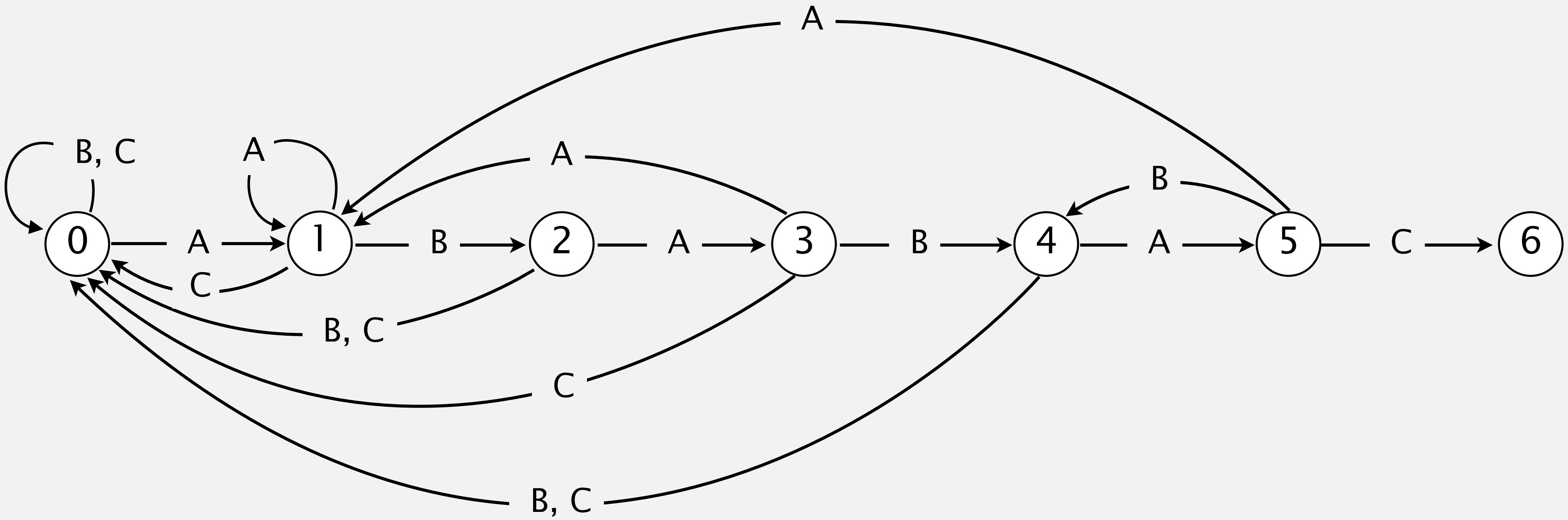
0	1	2	3	4	5
A	B	A	B	A	C



Knuth-Morris-Pratt demo: DFA construction in linear time

		0	1	2	3	4	5
`pat.charAt(j)`		A	B	A	B	A	C
`dfa[][j]`	A	1	1	3	1	5	1
	B	0	2	0	4	0	4
	C	0	0	0	0	0	6

Constructing the DFA for KMP substring search for A B A B A C



Constructing the DFA for KMP substring search: Java implementation

For each state `j`:

- Copy `dfa[][x]` to `dfa[][j]` for mismatch case.
- Set `dfa[pat.charAt(j)][j]` to `j+1` for match case.
- Update `x`.

```
public KMP(String pat)
{
   this.pat = pat;
   m = pat.length();
   dfa = new int[R][m];
   dfa[pat.charAt(0)][0] = 1;
   for (int x = 0, j = 1; j < m; j++)
   {
      for (char c = 0; c < R; c++)
         dfa[c][j] = dfa[c][x];
      dfa[pat.charAt(j)][j] = j+1;
      x = dfa[pat.charAt(j)][x];
   }
}
```

copy mismatch cases

set match case

update restart state

Running time. m character accesses (but space/time proportional to $R\, m$).

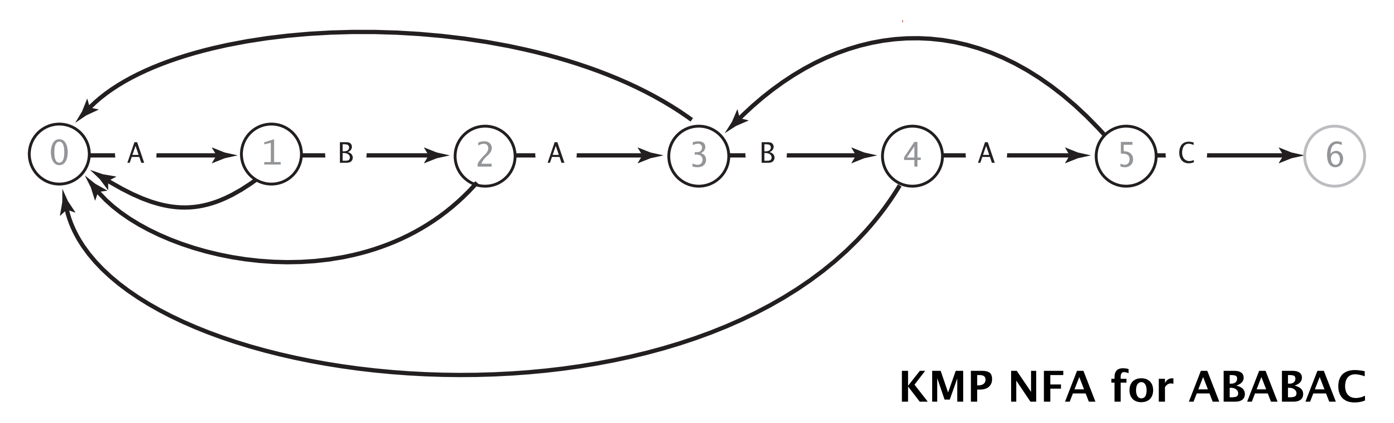
KMP substring search analysis

Proposition. KMP substring search accesses no more than $m + n$ chars to search for a pattern of length m in a text of length n.

Pf. Each pattern character accessed once when constructing the DFA; each text character accessed (at most) once when simulating the DFA.

Proposition. KMP constructs `dfa[][]` in time and space proportional to $R\,m$.

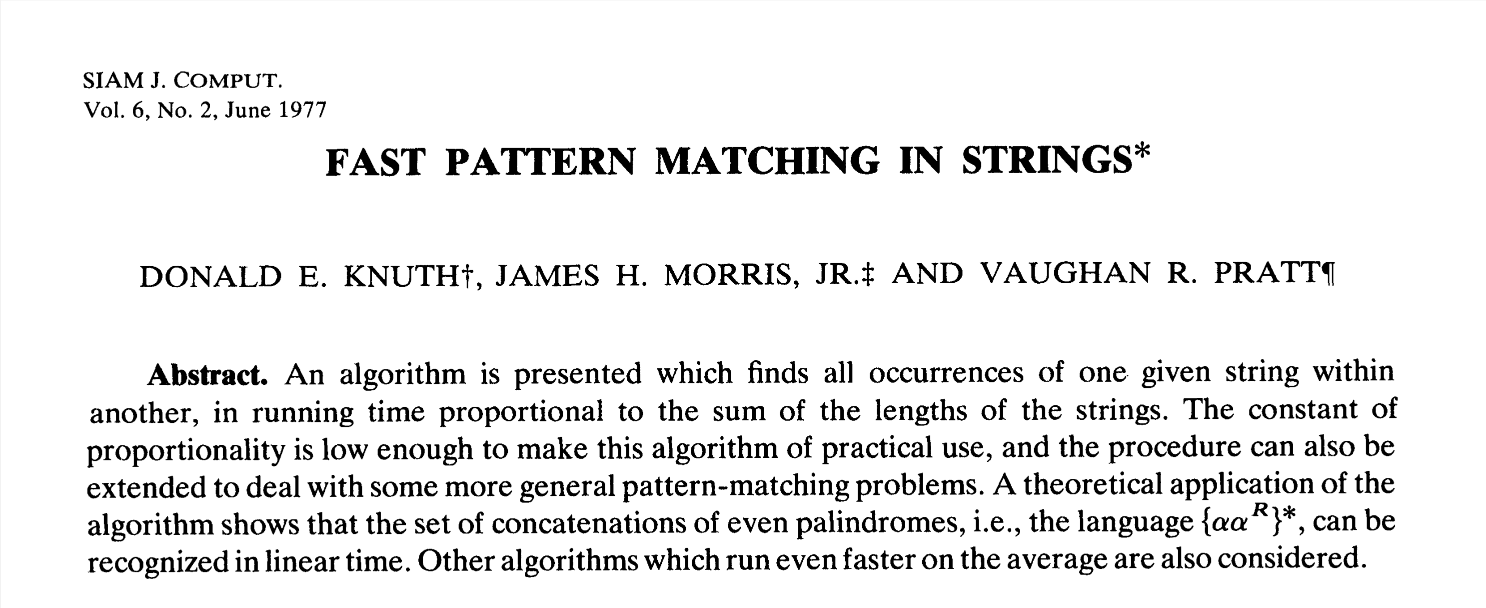
Larger alphabets. Improved version of KMP constructs `nfa[]` in time and space proportional to m.



KMP NFA for ABABAC

Knuth–Morris–Pratt: brief history

- Independently discovered by two theoreticians and a hacker.
  - Knuth: inspired by esoteric theorem, discovered linear algorithm
  - Pratt: made running time independent of alphabet size
  - Morris: built a text editor for the CDC 6400 computer
- Theory meets practice.



SIAM J. COMPUT.
Vol. 6, No. 2, June 1977

FAST PATTERN MATCHING IN STRINGS*

DONALD E. KNUTH†, JAMES H. MORRIS, JR.‡ AND VAUGHAN R. PRATT¶

Abstract. An algorithm is presented which finds all occurrences of one given string within another, in running time proportional to the sum of the lengths of the strings. The constant of proportionality is low enough to make this algorithm of practical use, and the procedure can also be extended to deal with some more general pattern-matching problems. A theoretical application of the algorithm shows that the set of concatenations of even palindromes, i.e., the language $\{\alpha\alpha^R\}^*$, can be recognized in linear time. Other algorithms which run even faster on the average are also considered.

Don Knuth

Jim Morris

Vaughan Pratt

CYCLIC ROTATION

A string s is a cyclic rotation of t if s and t have the same length and s is a suffix of t followed by a prefix of t.

yes	yes	no
ROTATEDSTRING	ABABABBABBABA	ROTATEDSTRING
STRINGROTATED	BABBABBABAABA	GNIRTSDETATOR

Problem. Given two binary strings s and t, design a linear-time algorithm to determine if s is a cyclic rotation of t.

Algorithms

Robert Sedgewick | Kevin Wayne

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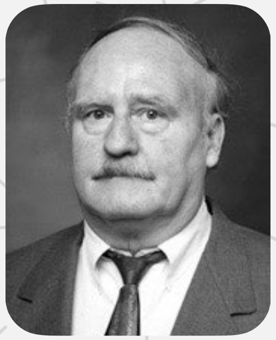


Robert Boyer

J. Strother Moore

Boyer-Moore: mismatched character heuristic

Intuition.

- Scan characters in pattern from right to left.
- Can skip as many as m text chars when finding one not in the pattern.

i	j	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
text →		F	I	N	D	I	N	A	H	A	Y	S	T	A	C	K	N	E	E	D	L	E	I	N	A
0	5	N	E	E	D	L	E																		
5	5						N	E	E	D	L	E													
11	4												N	E	E	D	L	E							
15	0																N	E	E	D	L	E			

← *pattern*

no S in pattern

align N in text with N in pattern

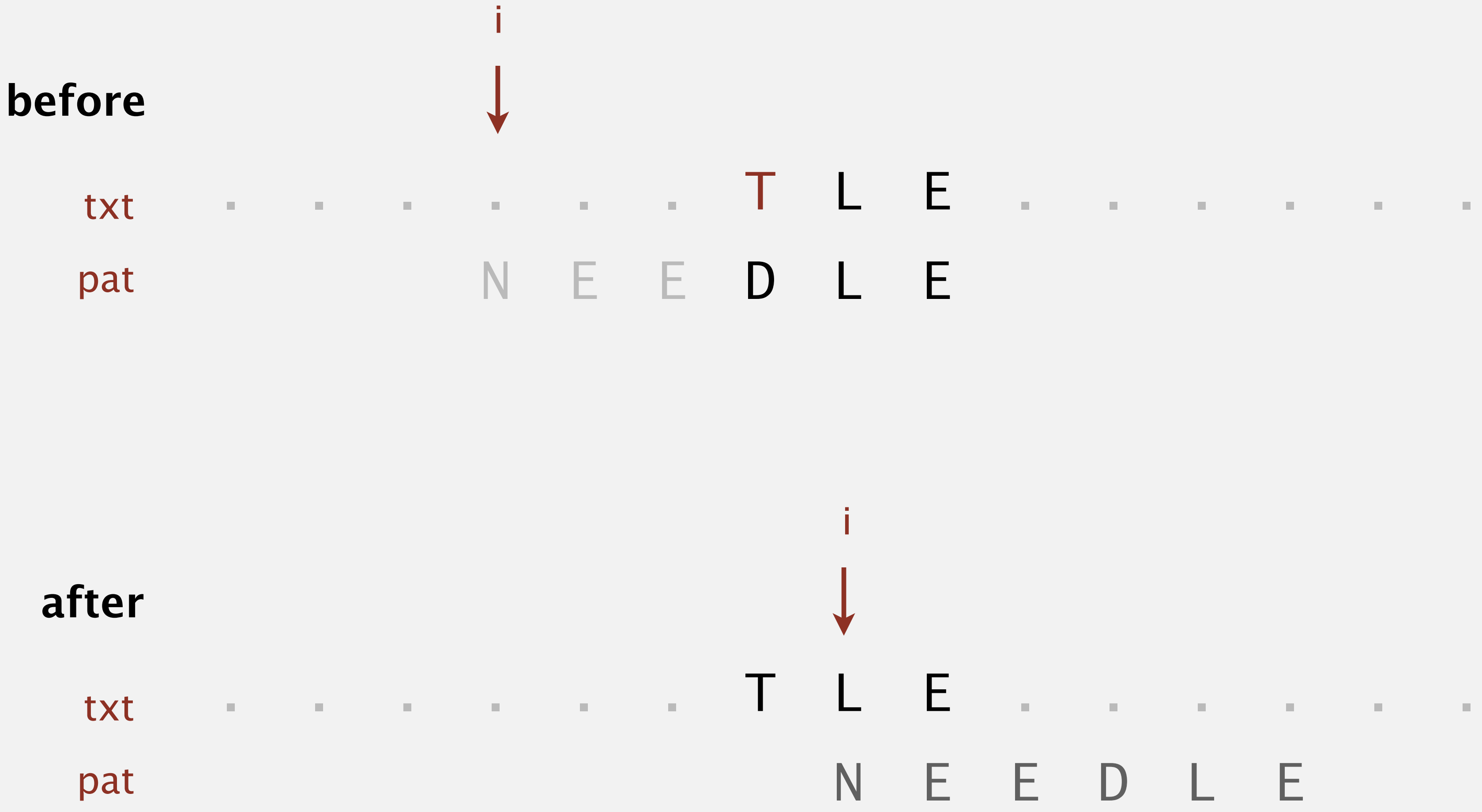
align N in text with N in pattern

return `i = 15`

Boyer-Moore: mismatched character heuristic

Q. How much to skip?

Case 1. Mismatch character not in pattern.

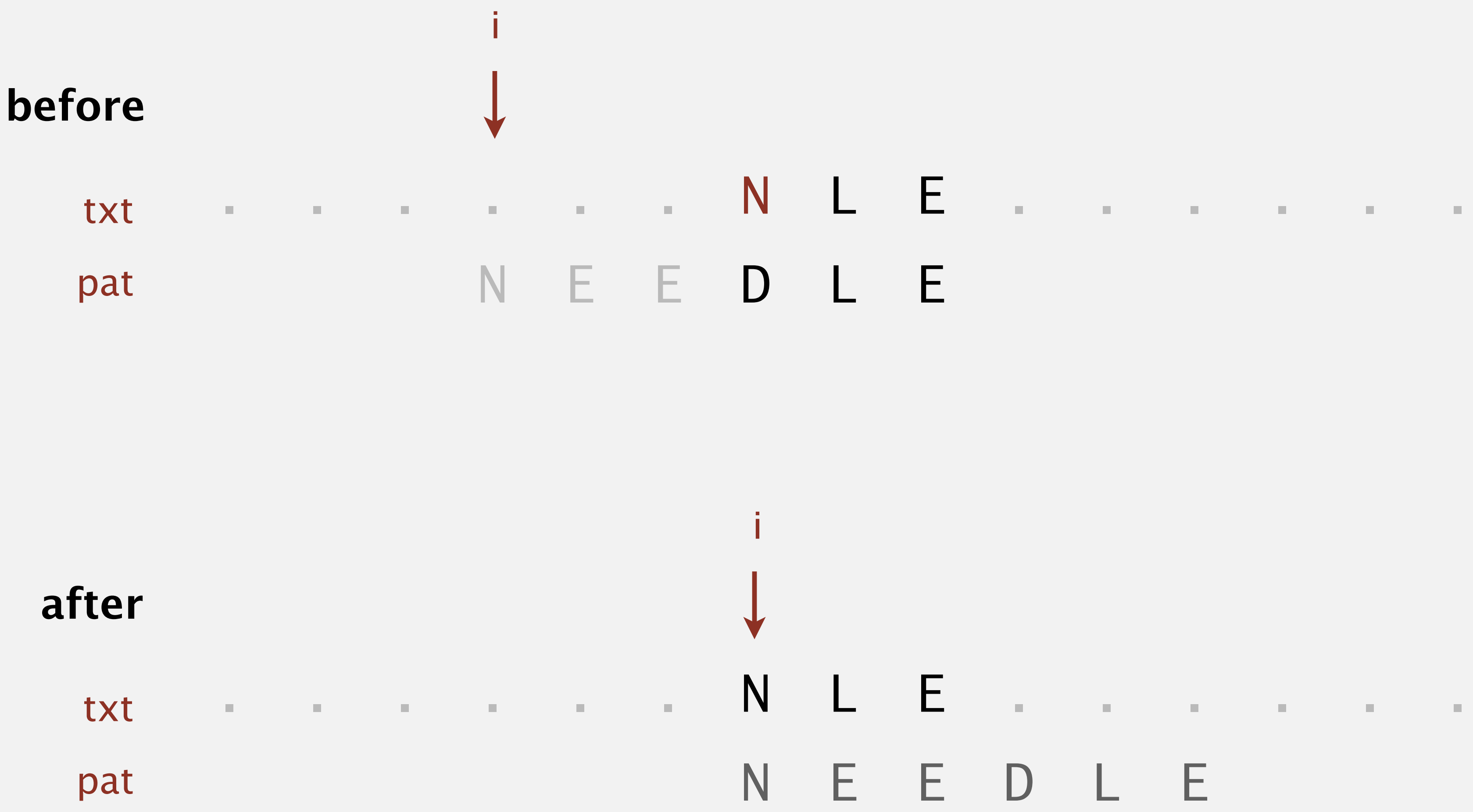


mismatch character T not in pattern: increment i one character beyond T

Boyer-Moore: mismatched character heuristic

Q. How much to skip?

Case 2a. Mismatch character in pattern.



before

i

txt . . . . . . N L E . . . . . .

pat N E E D L E

after

i

txt . . . . . . N L E . . . . . .

pat N E E D L E

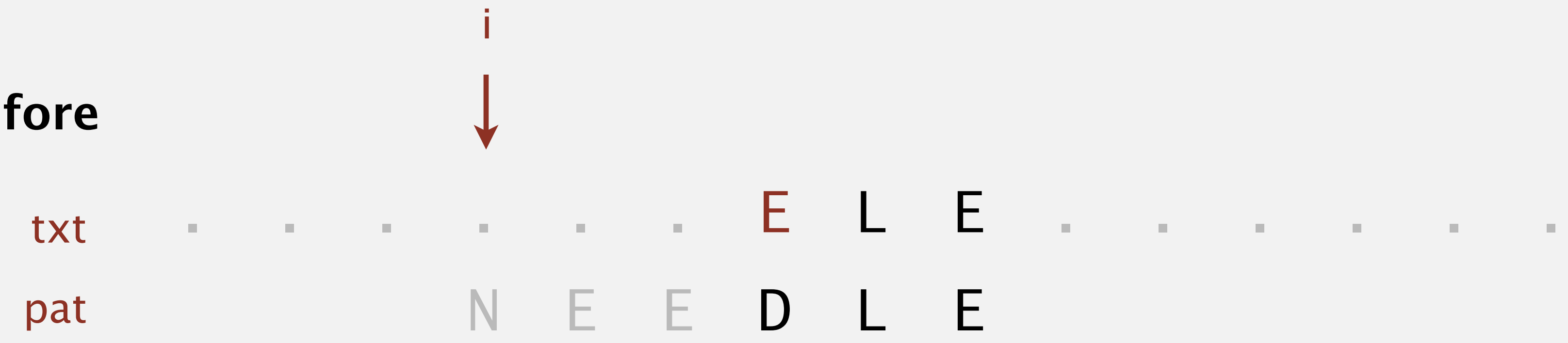
mismatch character N in pattern: align text N with rightmost (why?) pattern N

Boyer-Moore: mismatched character heuristic

Q. How much to skip?

Case 2b. Mismatch character in pattern (but heuristic no help).

before



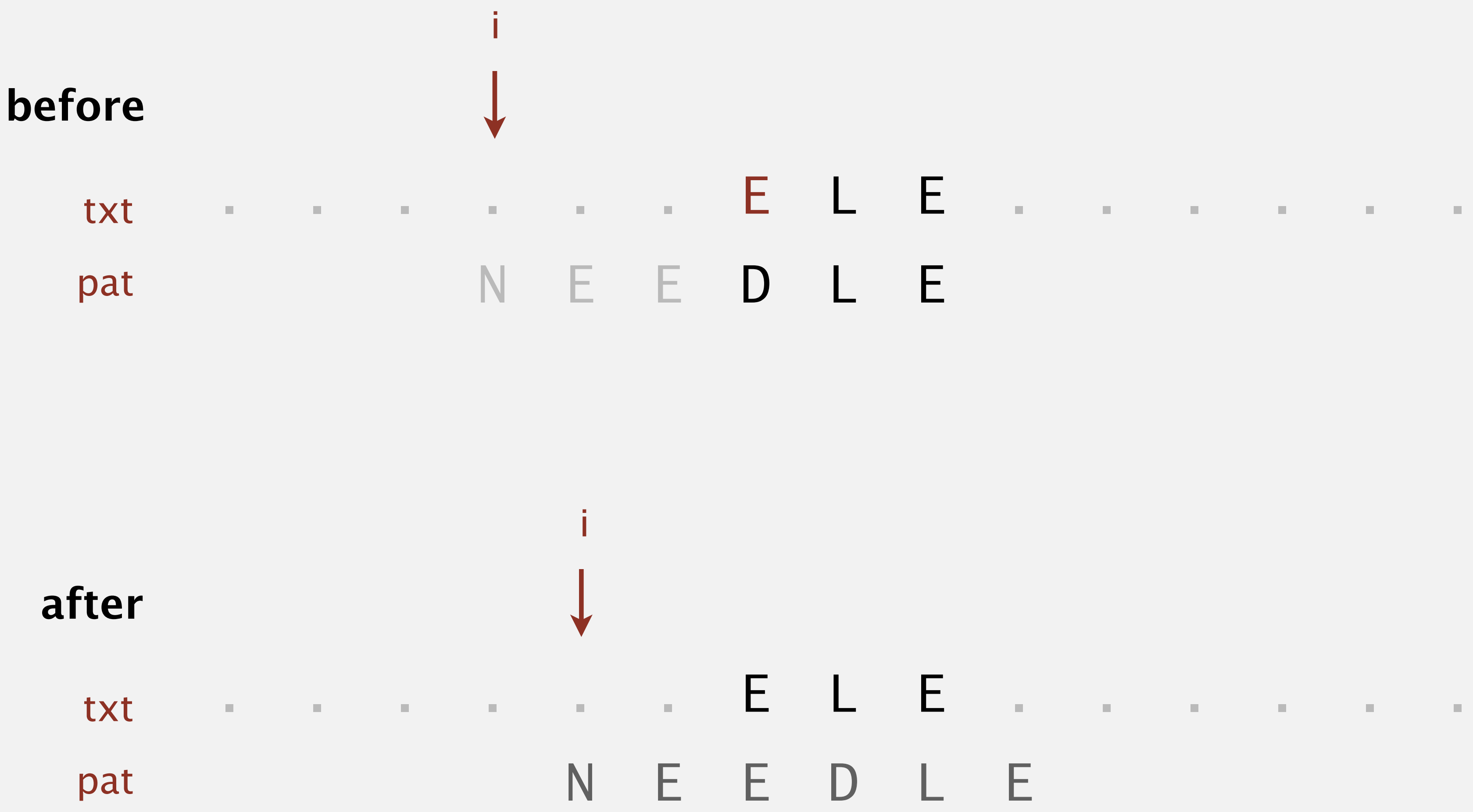
aligned with rightmost E?

mismatch character E in pattern: align text E with rightmost pattern E ?

Boyer-Moore: mismatched character heuristic

Q. How much to skip?

Case 2b. Mismatch character in pattern (but heuristic no help).

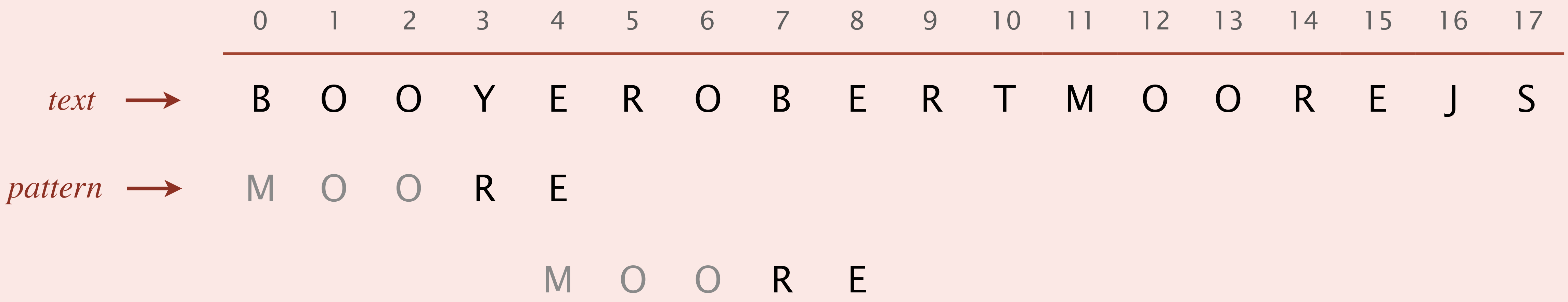


mismatch character E in pattern: increment i by 1

Substring search: quiz 5

Which text character is compared next with pattern character E?

- **A.** R (index 5)
- **B.** O (index 6)
- **C.** O (index 12)
- **D.** O (index 13)

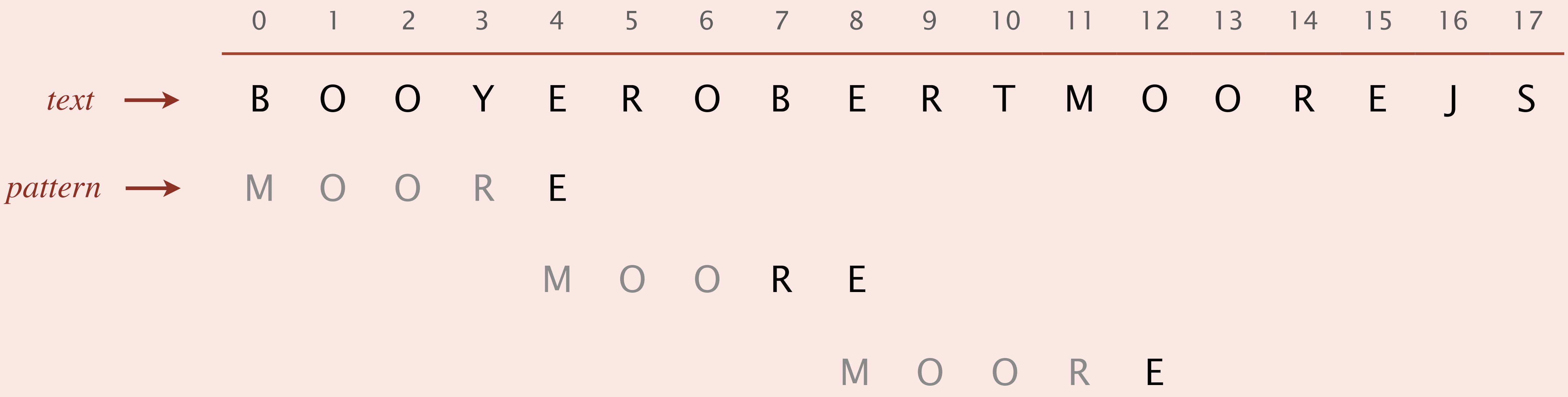


	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
text →	B	O	O	Y	E	R	O	B	E	R	T	M	O	O	R	E	J	S
pattern →	M	O	O	R	E													
					M	O	O	R	E									

Substring search: quiz 6

Which text character is compared next with pattern character E?

- **A.** O
- **B.** R
- **C.** E
- **D.** J



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
text →	B	O	O	Y	E	R	O	B	E	R	T	M	O	O	R	E	J	S
pattern →	M	O	O	R	E													
					M	O	O	R	E									
									M	O	O	R	E					

Boyer-Moore: mismatched character heuristic

Q. How much to skip?

A. Precompute index of rightmost occurrence of character `c` in pattern.
(−1 if character not in pattern)

```
right = new int[R];
for (int c = 0; c < R; c++)
   right[c] = -1;
for (int j = 0; j < m; j++)
   right[pat.charAt(j)] = j;
```

		N	E	E	D	L	E	
c		0	1	2	3	4	5	right[c]
A	-1	-1	-1	-1	-1	-1	-1	-1
B	-1	-1	-1	-1	-1	-1	-1	-1
C	-1	-1	-1	-1	-1	-1	-1	-1
D	-1	-1	-1	-1	③	3	3	3
E	-1	-1	①	②	2	2	⑤	5
...								-1
L	-1	-1	-1	-1	-1	④	4	4
M	-1	-1	-1	-1	-1	-1	-1	-1
N	-1	0	0	0	0	0	0	0
...								-1

Boyer-Moore skip table computation

Boyer-Moore: Java implementation

```
public int search(String txt)
{
   int n = txt.length();
   int m = pat.length();
   int skip;
   for (int i = 0; i <= n-m; i += skip)
   {
      skip = 0;
      for (int j = m-1; j >= 0; j--)
      {
         if (pat.charAt(j) != txt.charAt(i+j))
         {
            skip = Math.max(1, j - right[txt.charAt(i+j)]);
            break;
         }
      }
      if (skip == 0) return i;

   }
   return n;
}
```

compute skip value

in case other term is zero or negative

match

Boyer–Moore: analysis

Property. Substring search with the Boyer–Moore mismatched character heuristic takes about $\sim n / m$ character compares to search for a pattern of length m in a text of length n.

sublinear!

Worst-case. Can be as bad as $\sim m\, n$.

i	skip	0	1	2	3	4	5	6	7	8	9
	txt →	B	B	B	B	B	B	B	B	B	B
0	0	A	B	B	B	B	← pat				
1	1		A	B	B	B	B				
2	1			A	B	B	B	B			
3	1				A	B	B	B	B		
4	1					A	B	B	B	B	
5	1						A	B	B	B	B

Boyer–Moore variant. Can improve worst case to $\sim 3\, n$ character compares by adding a KMP-like rule to guard against repetitive patterns.

Substring search: quiz 7

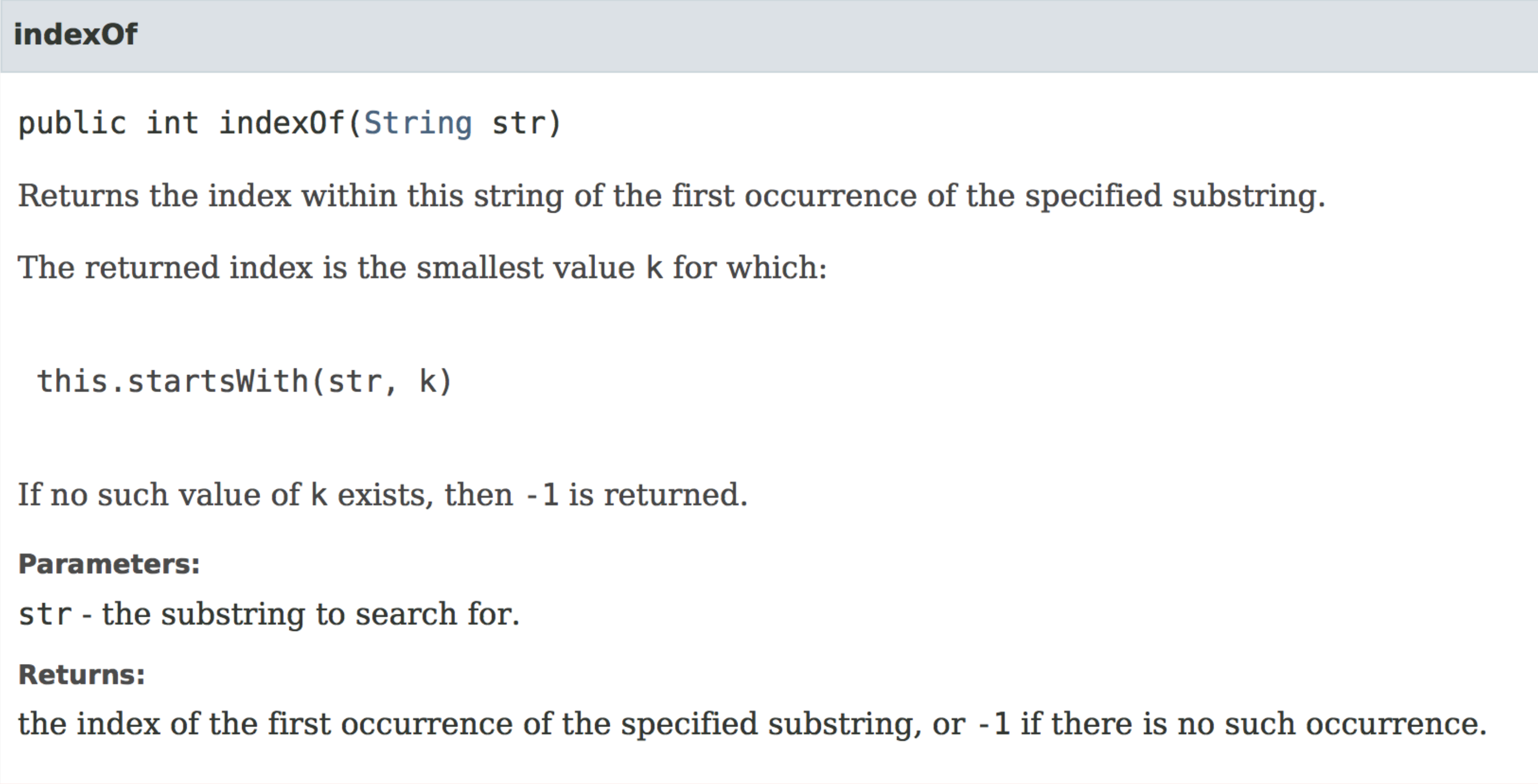
Which substring search algorithm does Java's indexOf() method use?

A. Brute-force search

B. Knuth–Morris–Pratt

C. Boyer–Moore

D. Rabin–Karp



indexOf

```
public int indexOf(String str)
```

Returns the index within this string of the first occurrence of the specified substring.

The returned index is the smallest value `k` for which:

```
 this.startsWith(str, k)
```

If no such value of `k` exists, then `-1` is returned.

Parameters:

`str` - the substring to search for.

Returns:

the index of the first occurrence of the specified substring, or `-1` if there is no such occurrence.