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2.3 Partitioning Demos

- *Hoare 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley–McIlroy 3-way partitioning*
- *dual-pivot partitioning*

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2.3 Partitioning Demos

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Quicksort partitioning demo

Repeat until `i` and `j` pointers cross.

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K	R	A	T	E	L	E	P	U	I	M	Q	C	X	O	S
↑ lo	↑ i														↑ j

stop i scan because a[i] >= a[lo]

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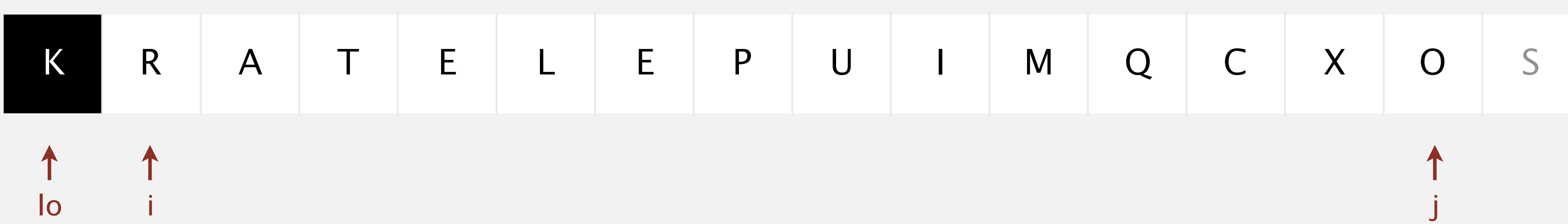

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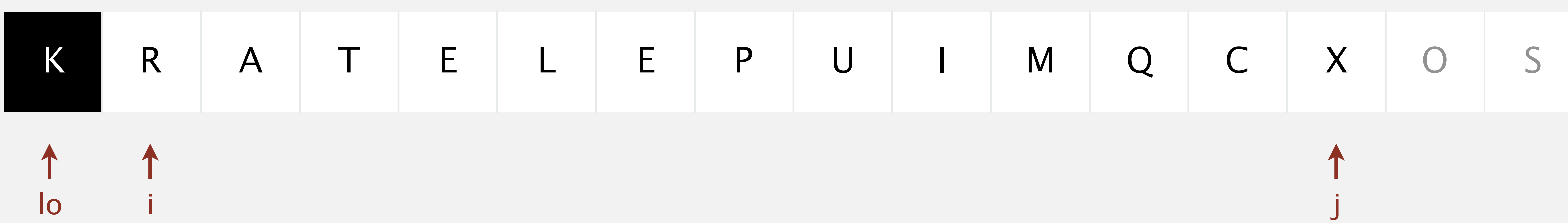

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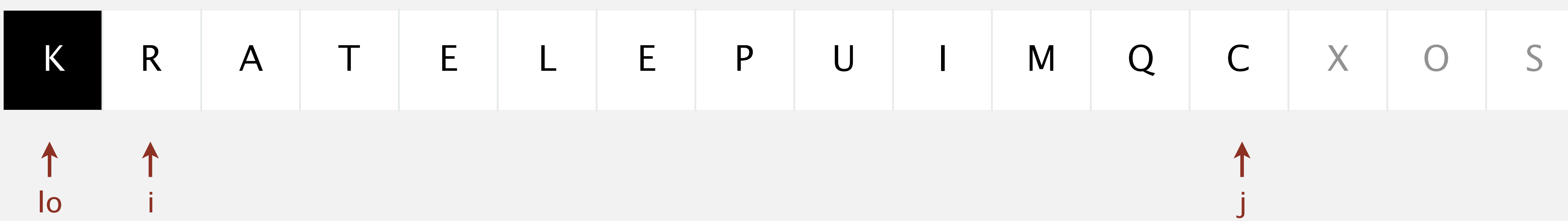

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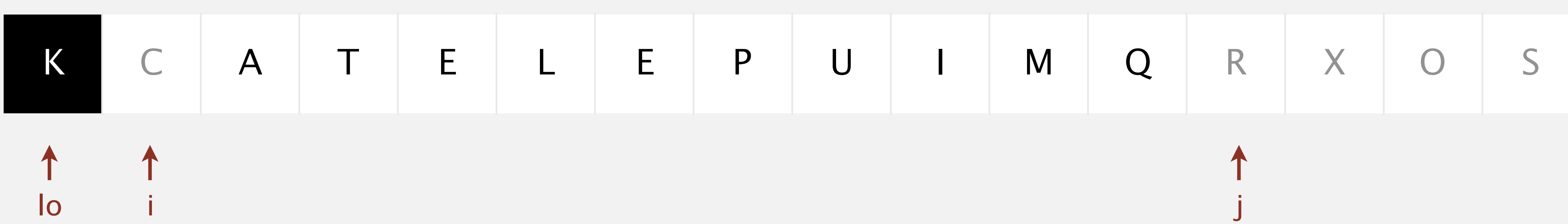

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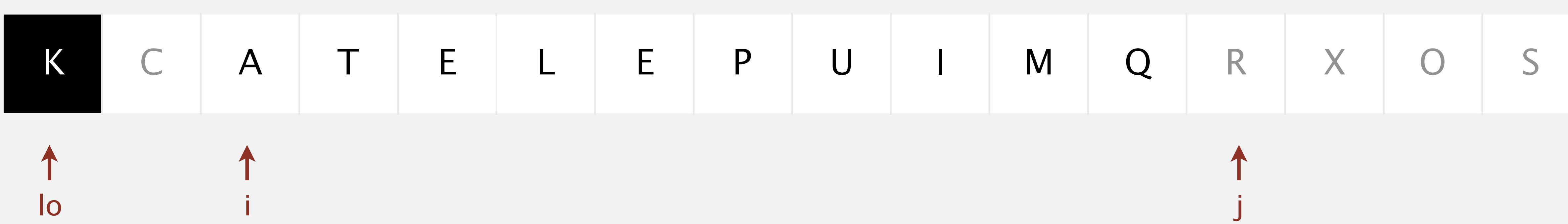

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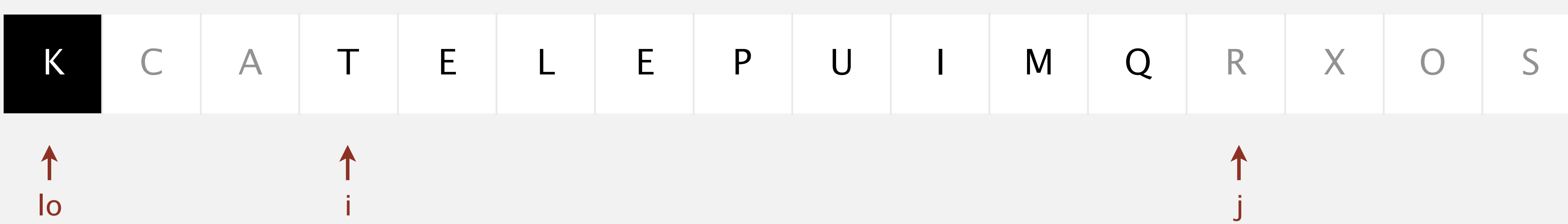

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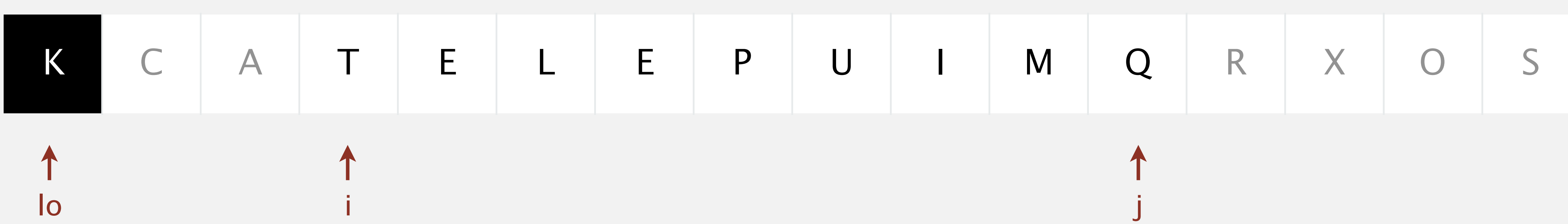

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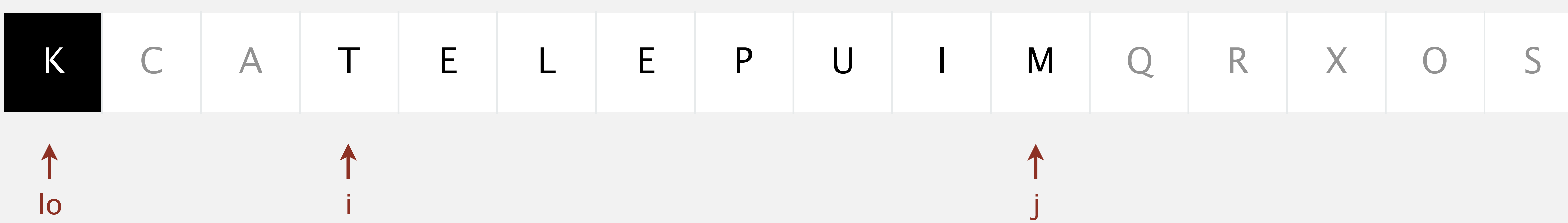

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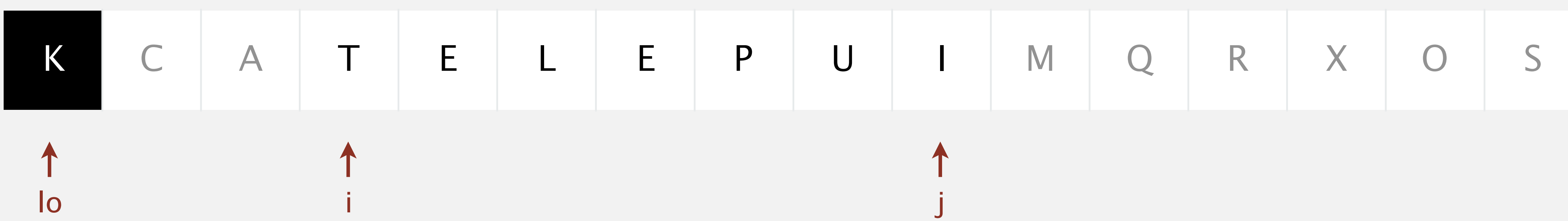

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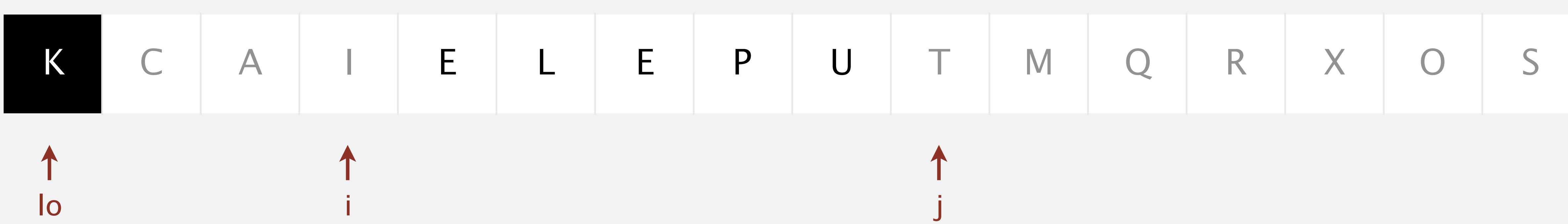

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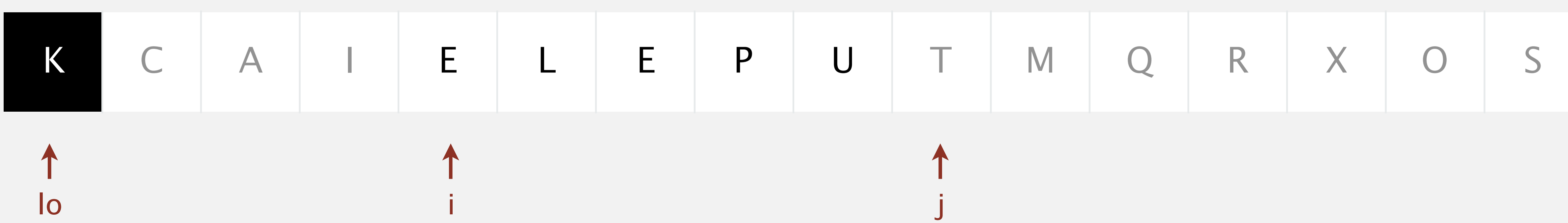

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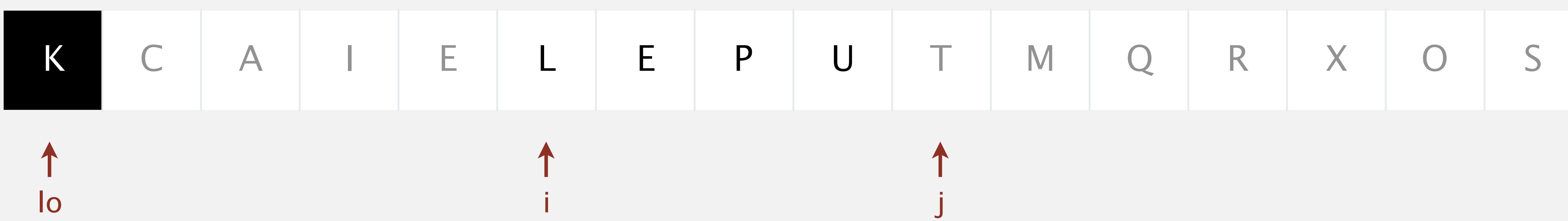

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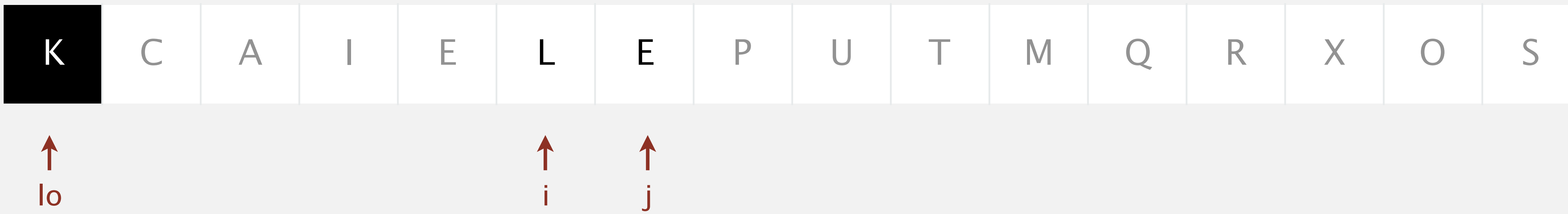

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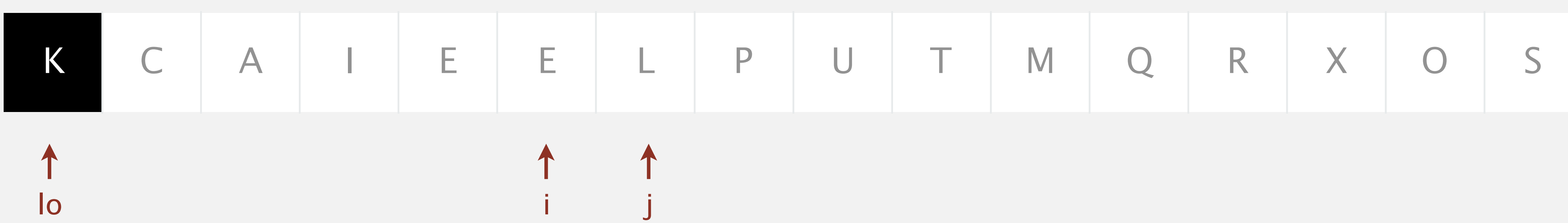

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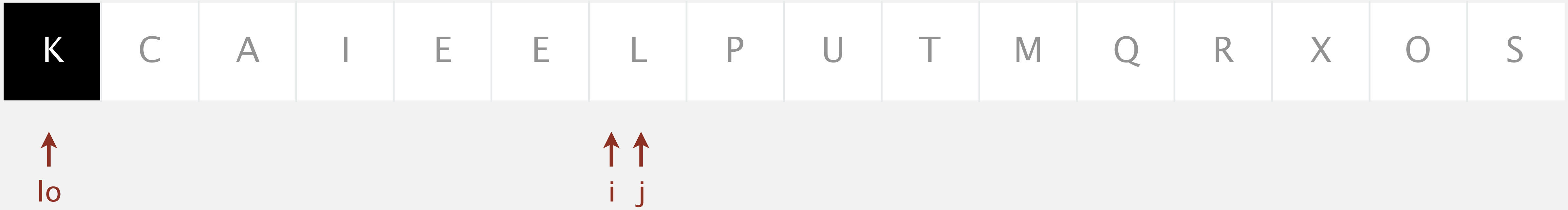

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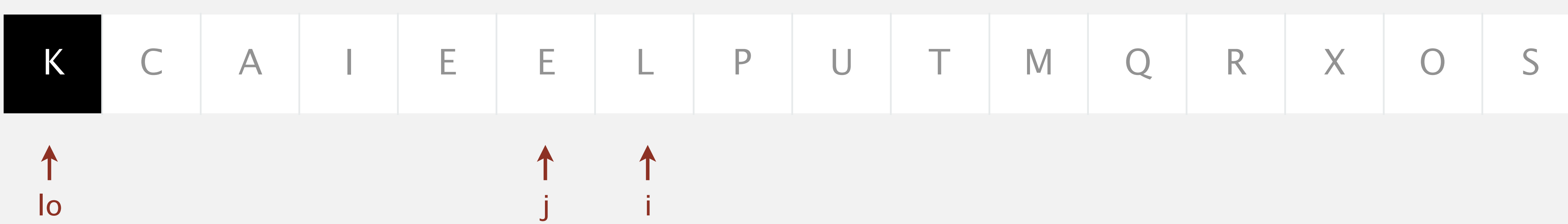


stop j scan because a[j] <= a[lo]

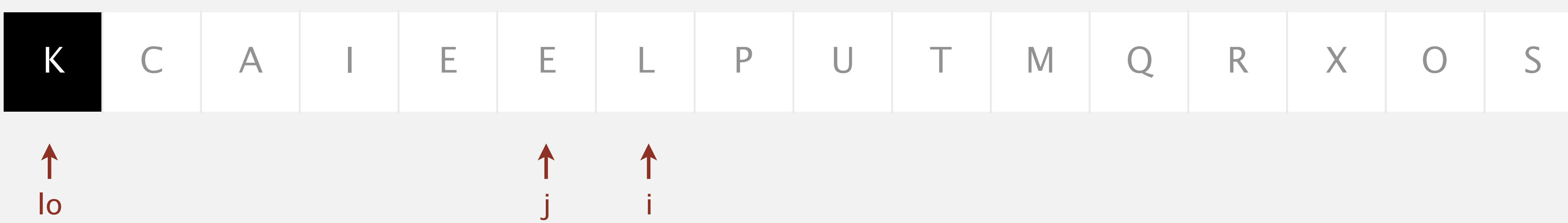
Quicksort partitioning demo

Repeat until i and j pointers cross.

- Scan i from left to right so long as (a[i] < a[lo]).
- Scan j from right to left so long as (a[j] > a[lo]).
- Exchange a[i] with a[j].

When pointers cross.

- Exchange a[lo] with a[j].



pointers cross: exchange a[lo] with a[j]

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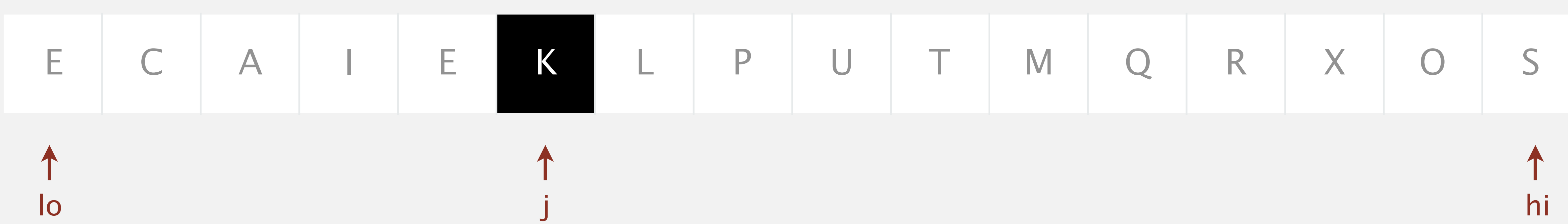

partitioned!

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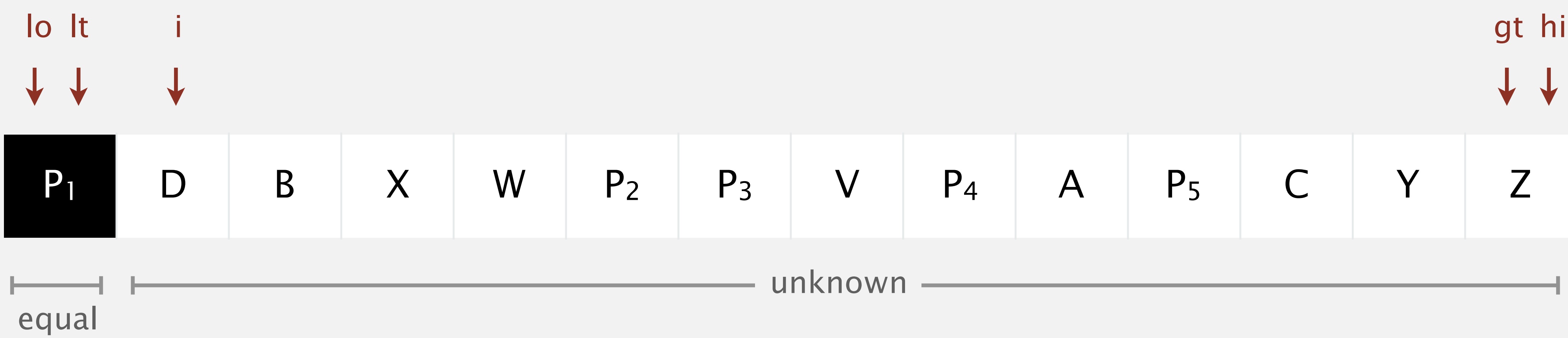

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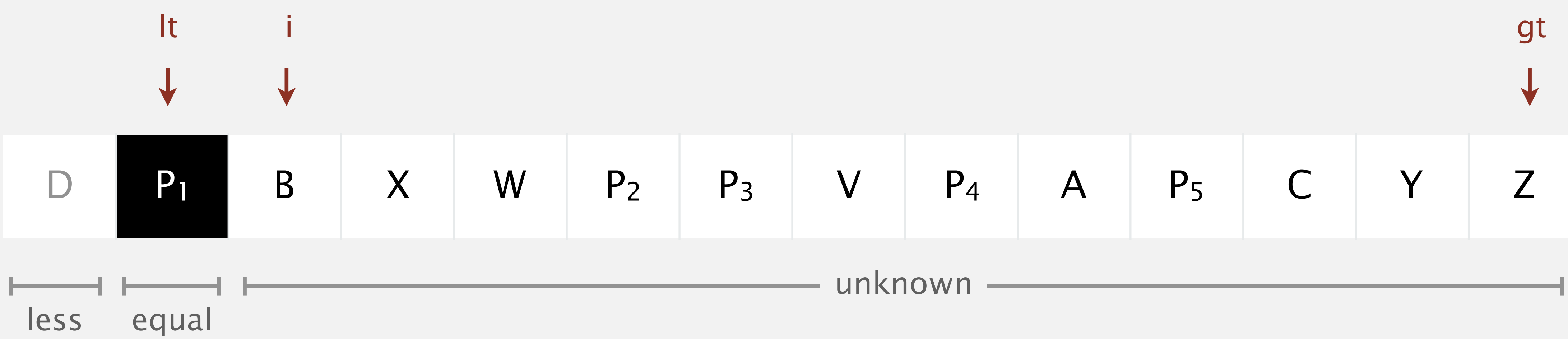

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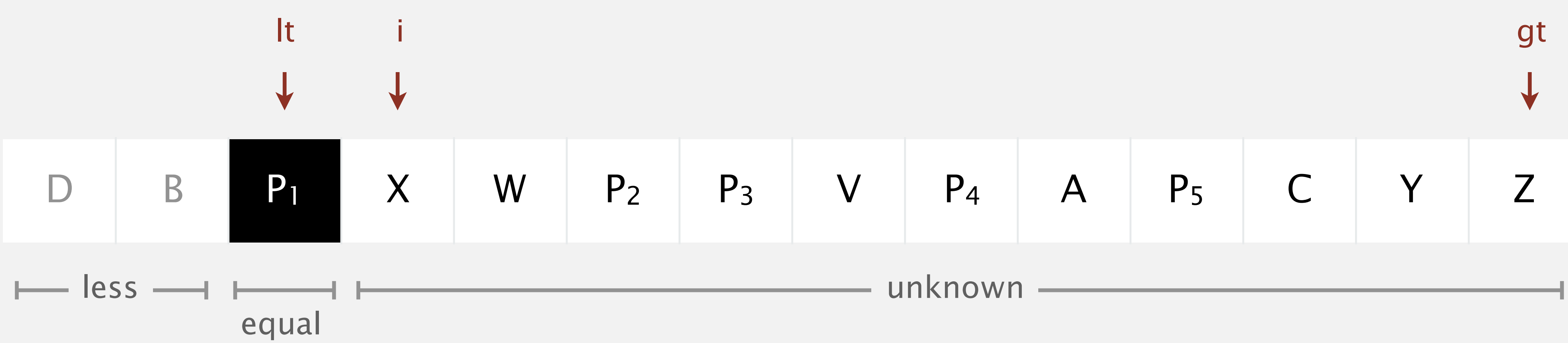

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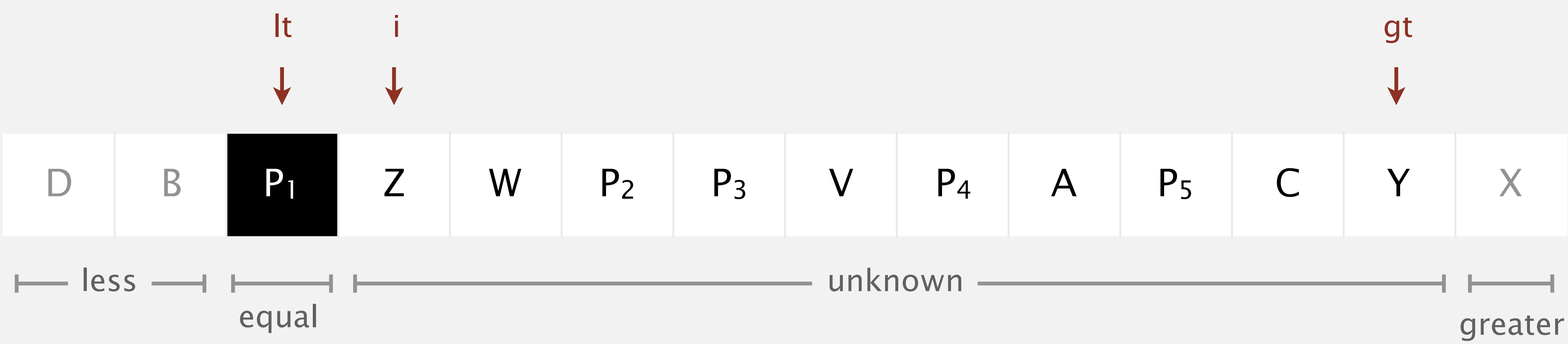

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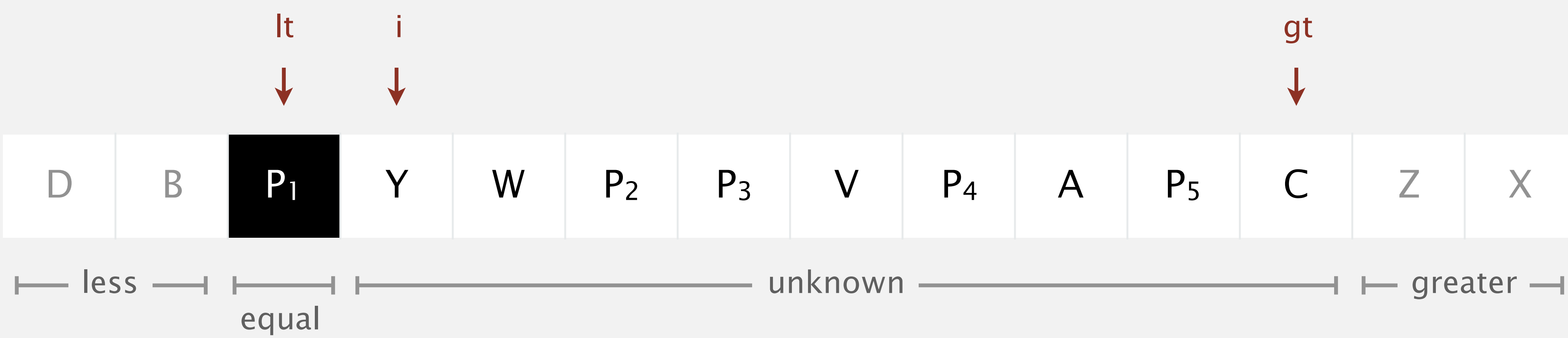

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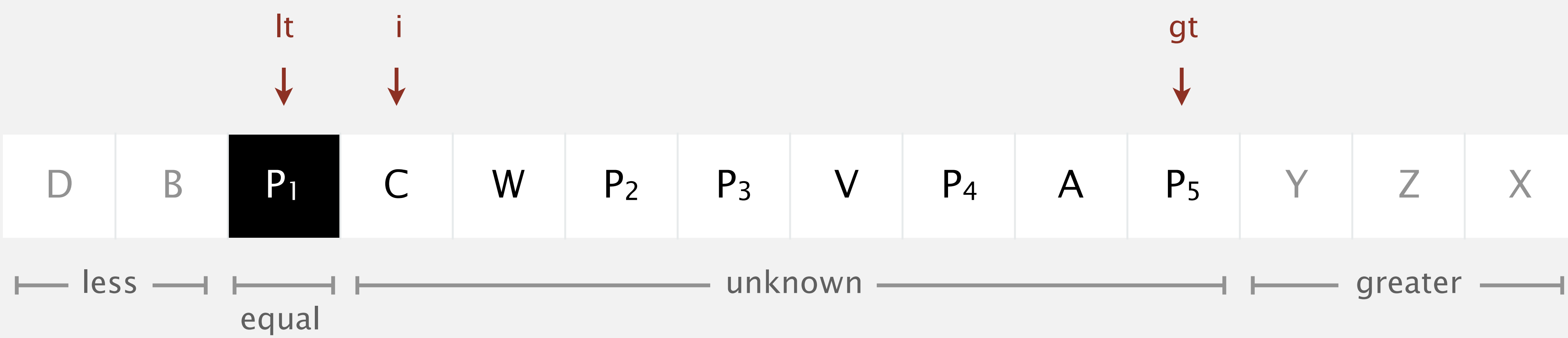

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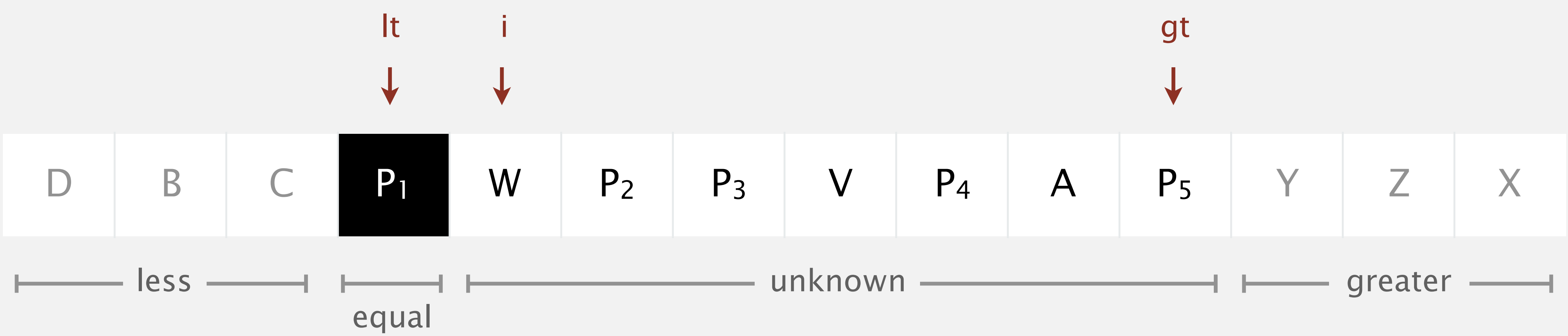

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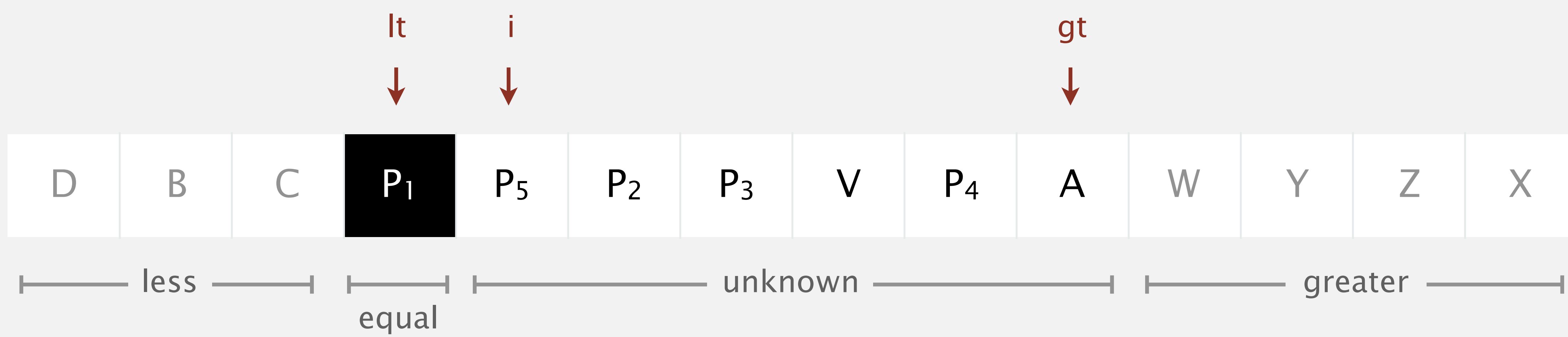

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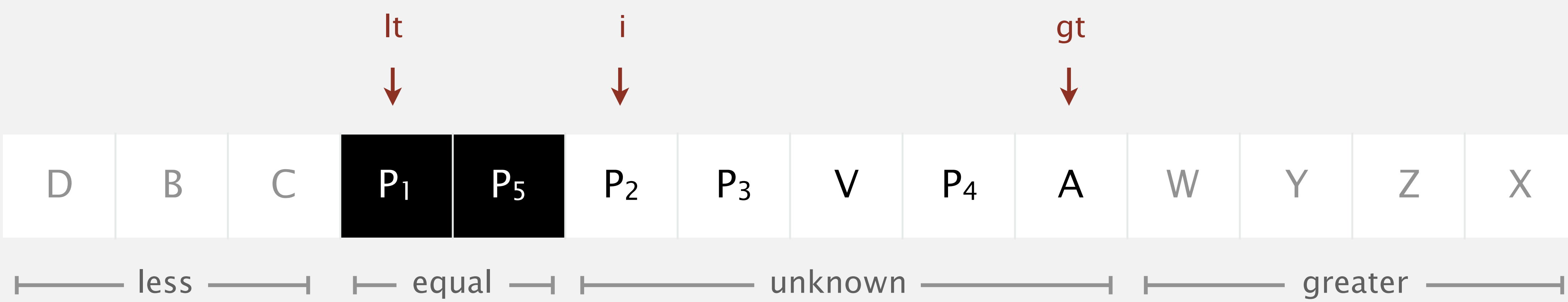

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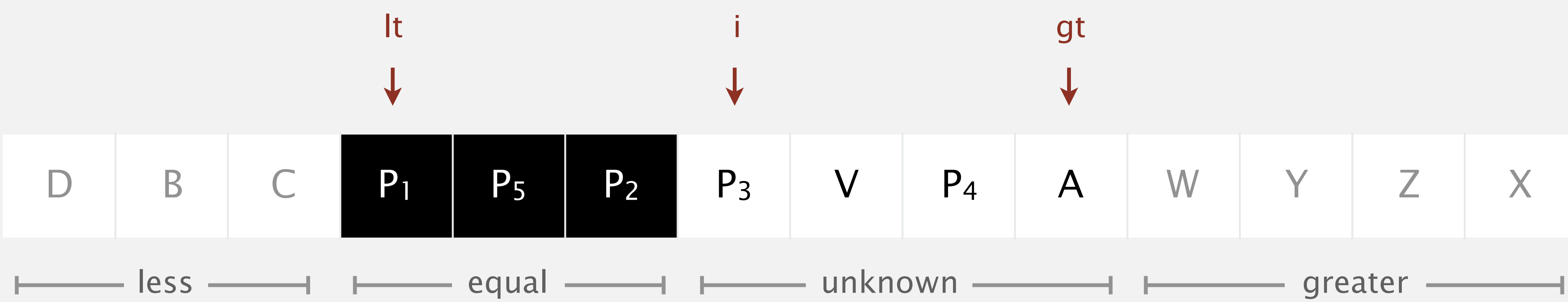

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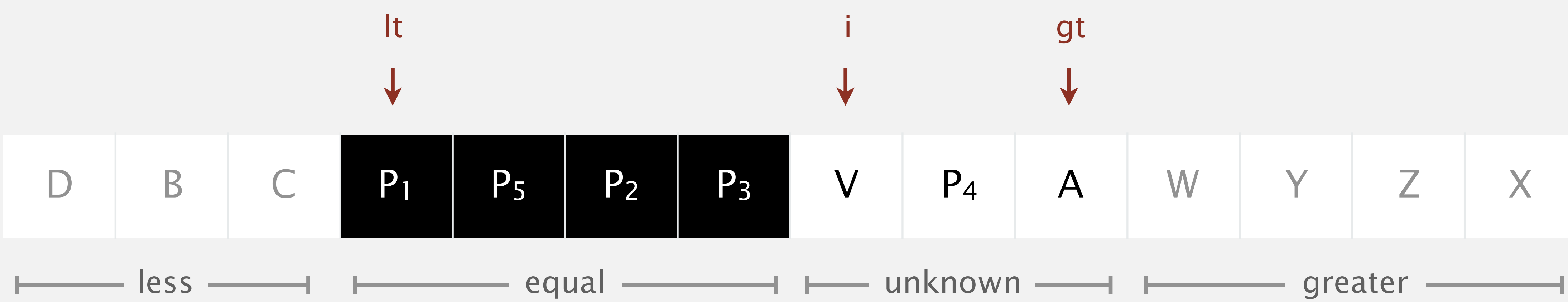

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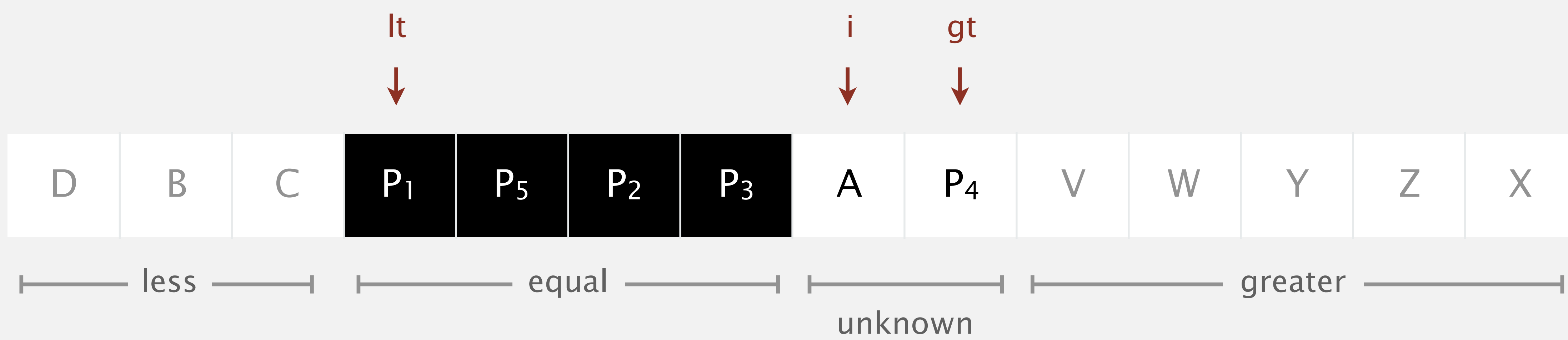

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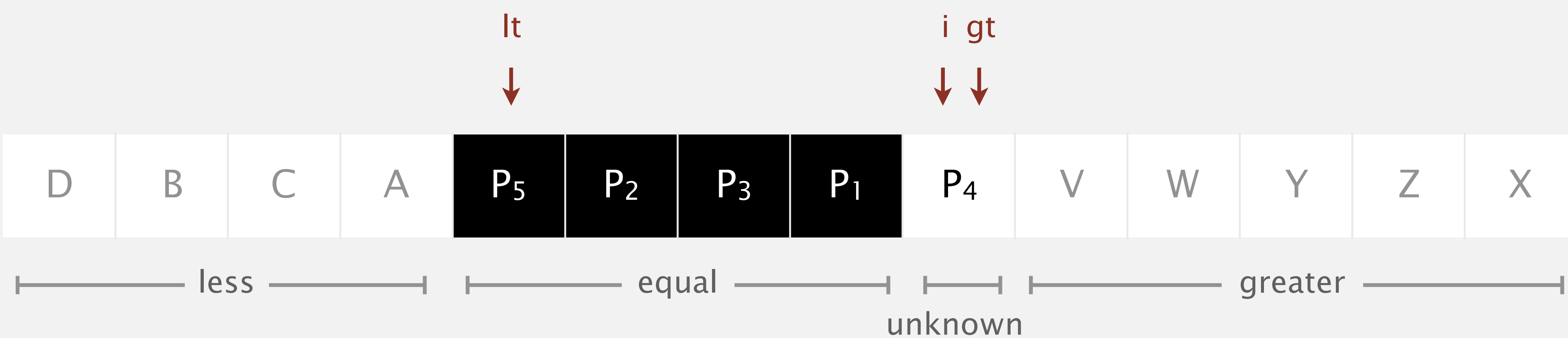

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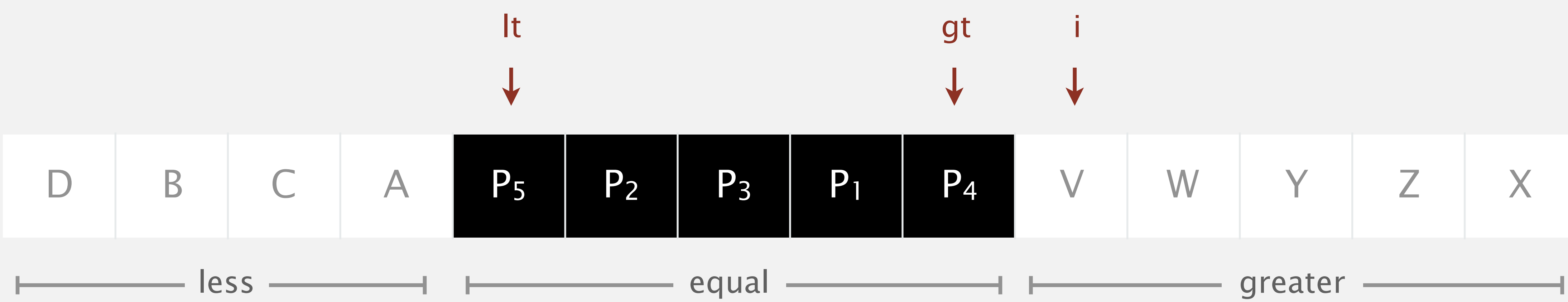

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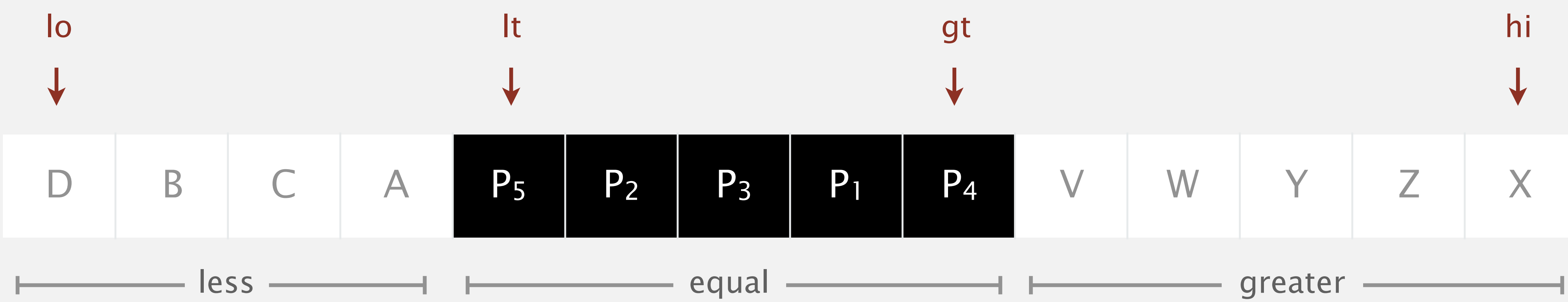

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2.3 Partitioning Demos

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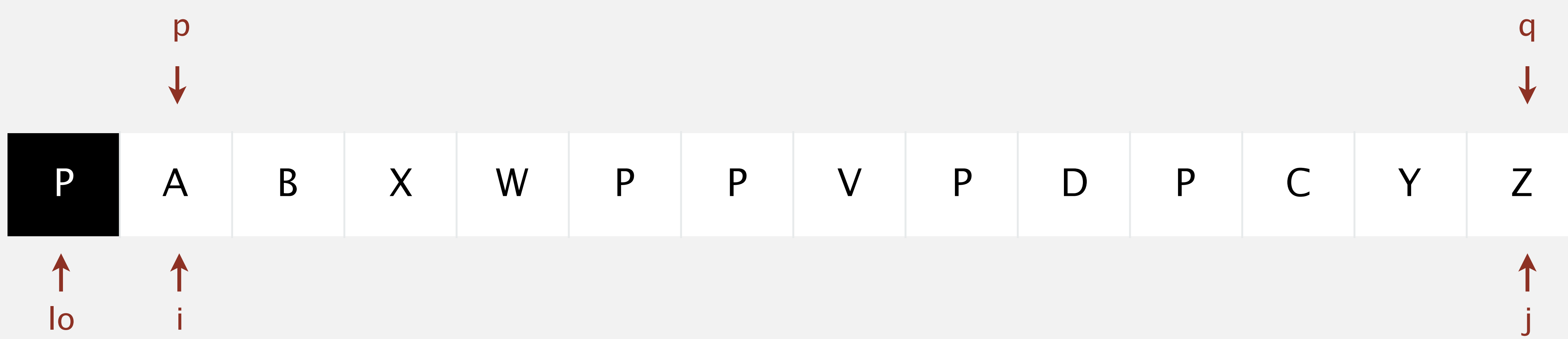
Bentley–McIlroy 3-way partitioning demo

Phase I. Repeat until `i` and `j` pointers cross.

- Scan `i` from left to right so long as `(a[i] < a[lo])`.
- Scan `j` from right to left so long as `(a[j] > a[lo])`.
- Exchange `a[i]` with `a[j]`.
- If `(a[i] == a[lo])`, exchange `a[i]` with `a[p]` and increment `p`.
- If `(a[j] == a[lo])`, exchange `a[j]` with `a[q]` and decrement `q`.



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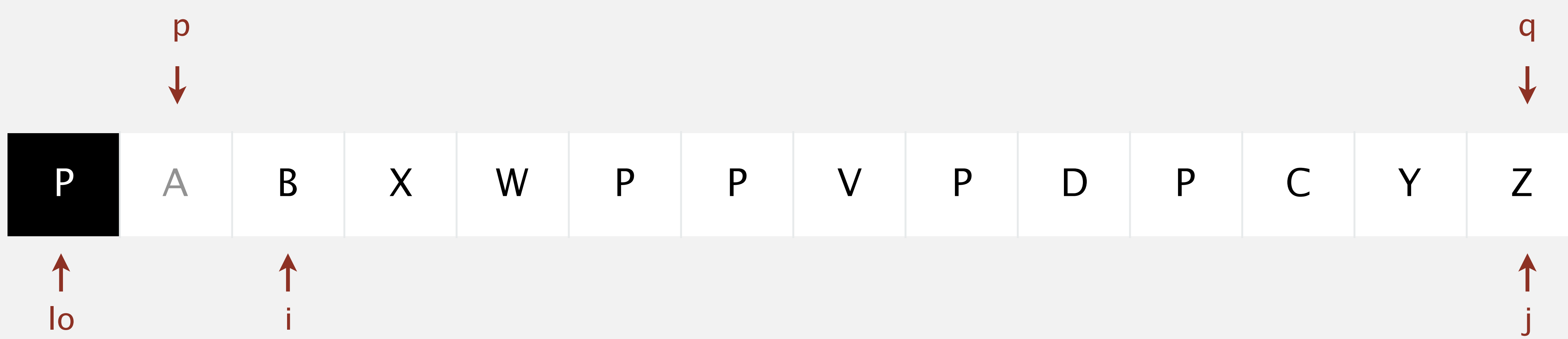
- Scan `j` from right to left so long as `(a[j] > a[lo])`.
- Exchange `a[i]` with `a[j]`.
- If `(a[i] == a[lo])`, exchange `a[i]` with `a[p]` and increment `p`.
- If `(a[j] == a[lo])`, exchange `a[j]` with `a[q]` and decrement `q`.



Bentley–McIlroy 3-way partitioning demo

Phase I. Repeat until `i` and `j` pointers cross.

- Scan `i` from left to right so long as `(a[i] < a[lo])`.
- Scan `j` from right to left so long as `(a[j] > a[lo])`.
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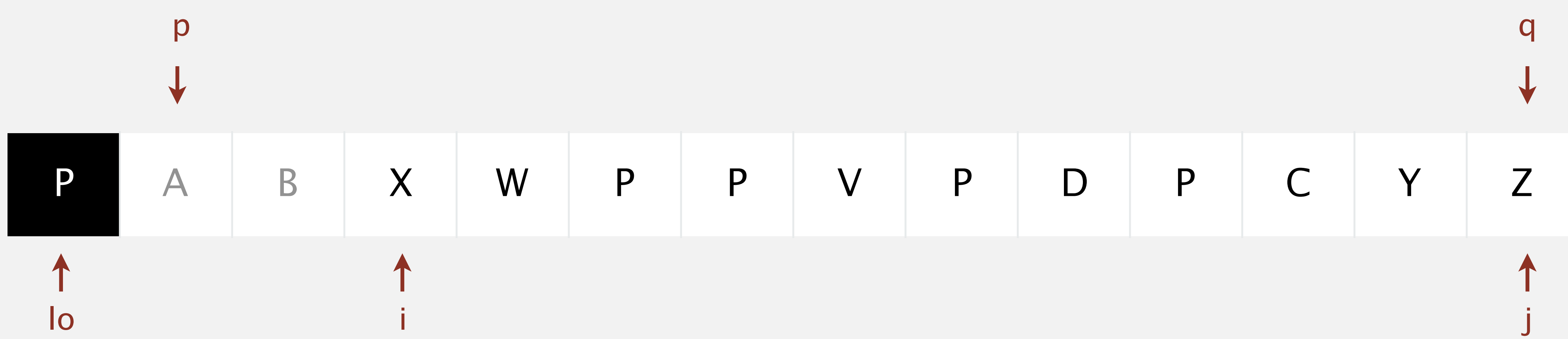

Bentley–McIlroy 3-way partitioning demo

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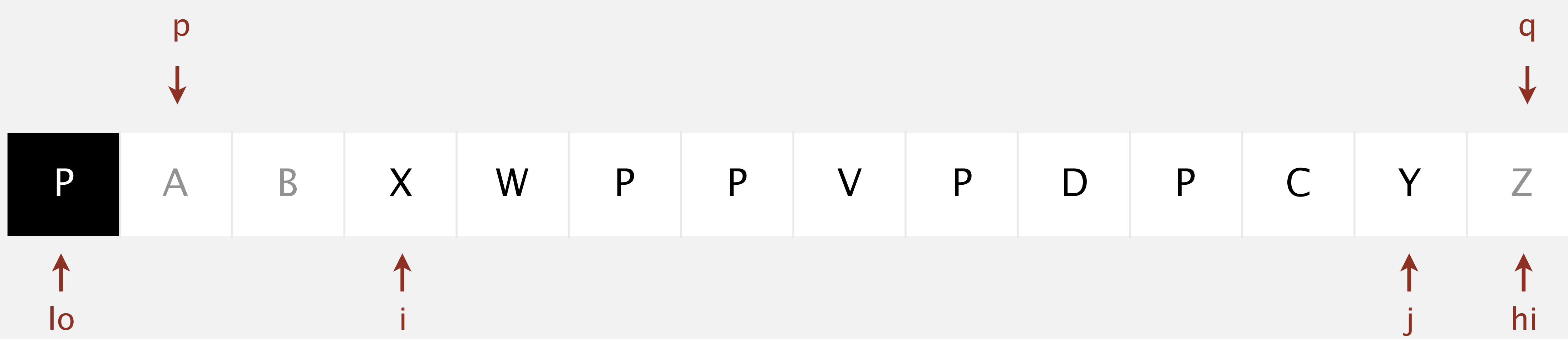

Bentley–McIlroy 3-way partitioning demo

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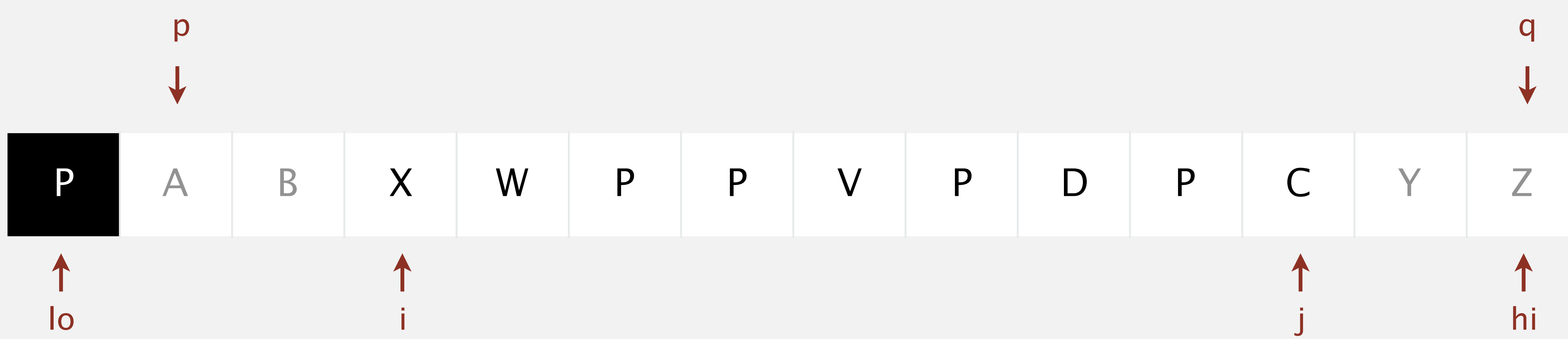

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Bentley–McIlroy 3-way partitioning demo

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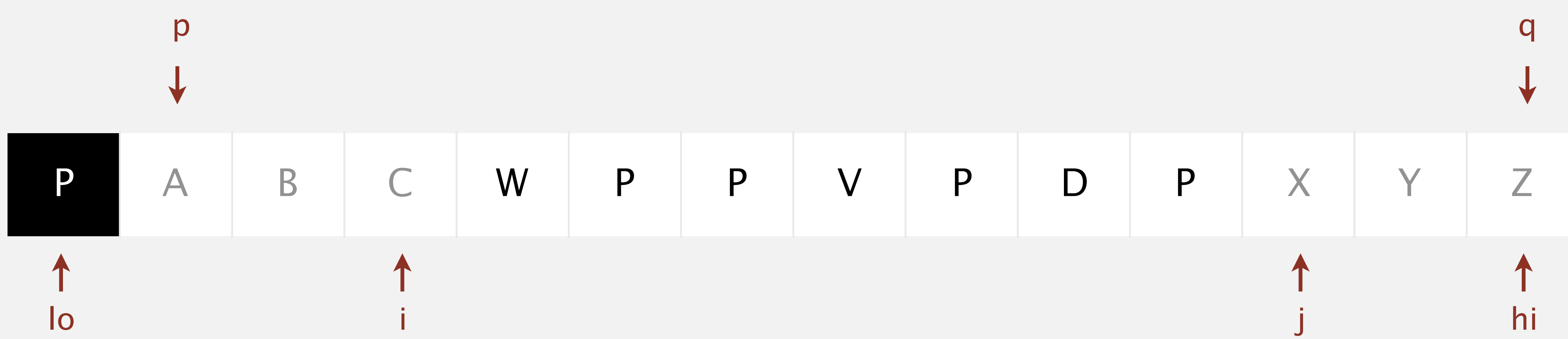

Bentley–McIlroy 3-way partitioning demo

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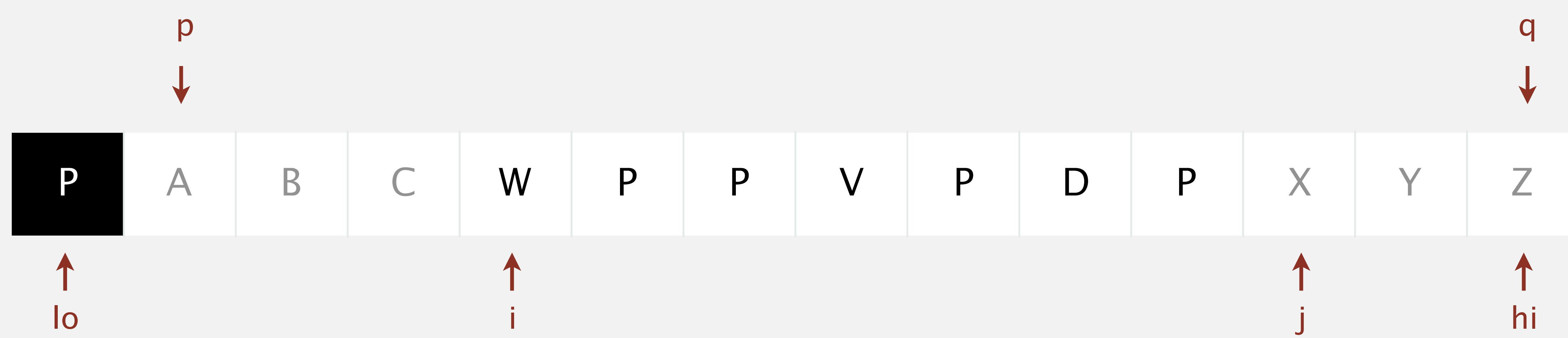

Bentley–McIlroy 3-way partitioning demo

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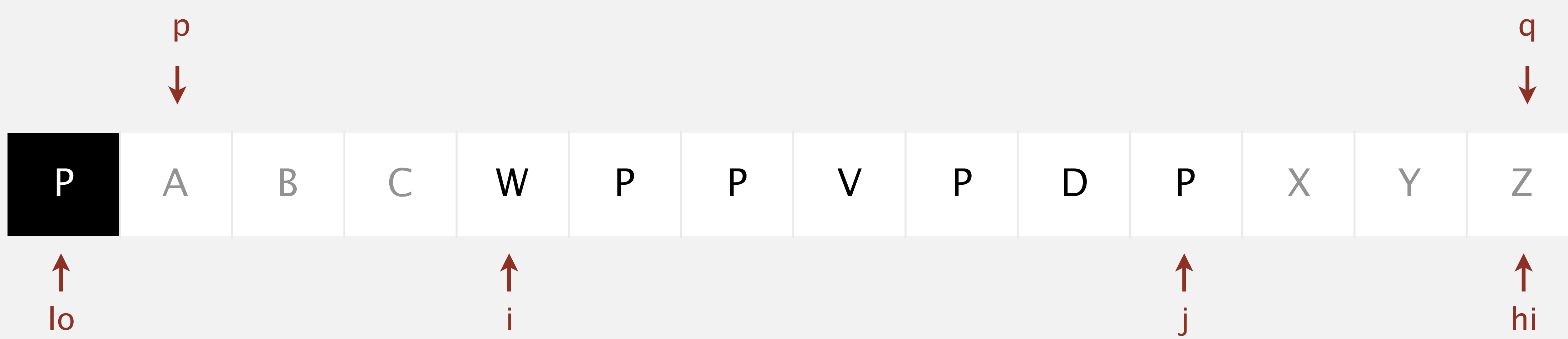

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Bentley–McIlroy 3-way partitioning demo

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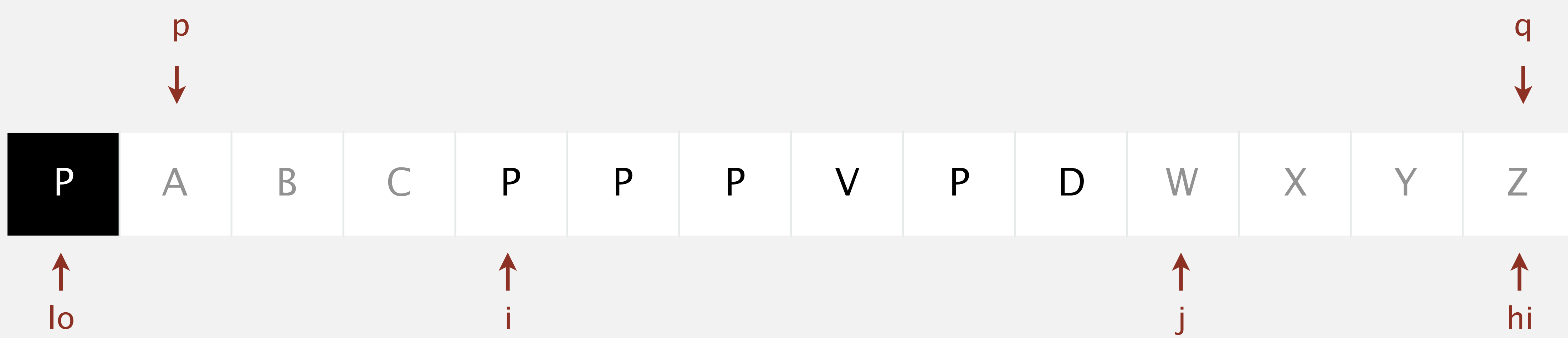

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Bentley–McIlroy 3-way partitioning demo

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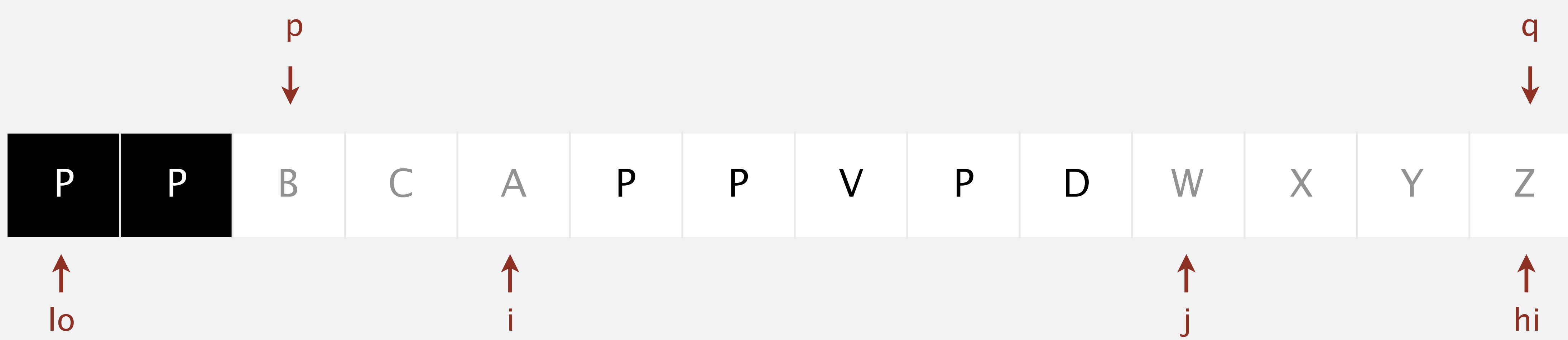

Bentley–McIlroy 3-way partitioning demo

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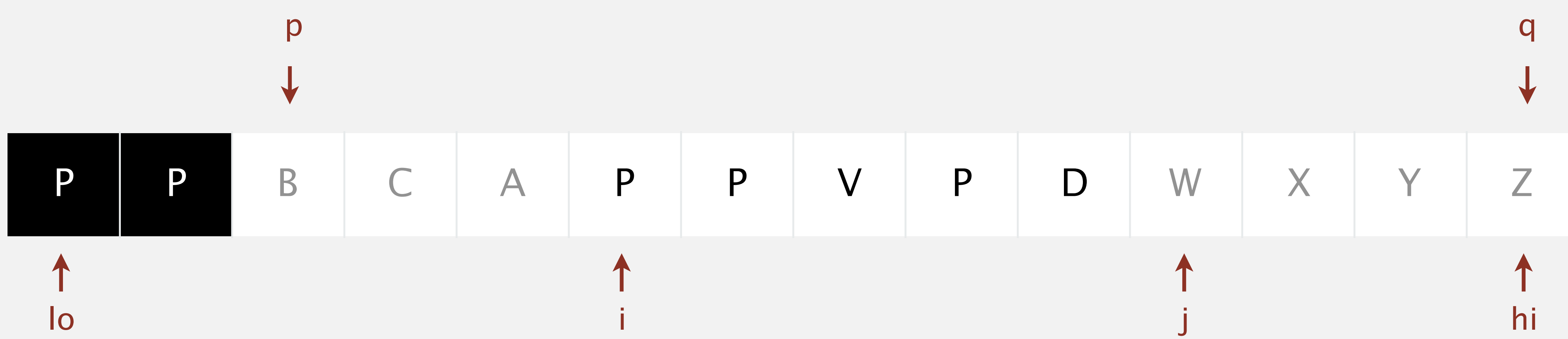

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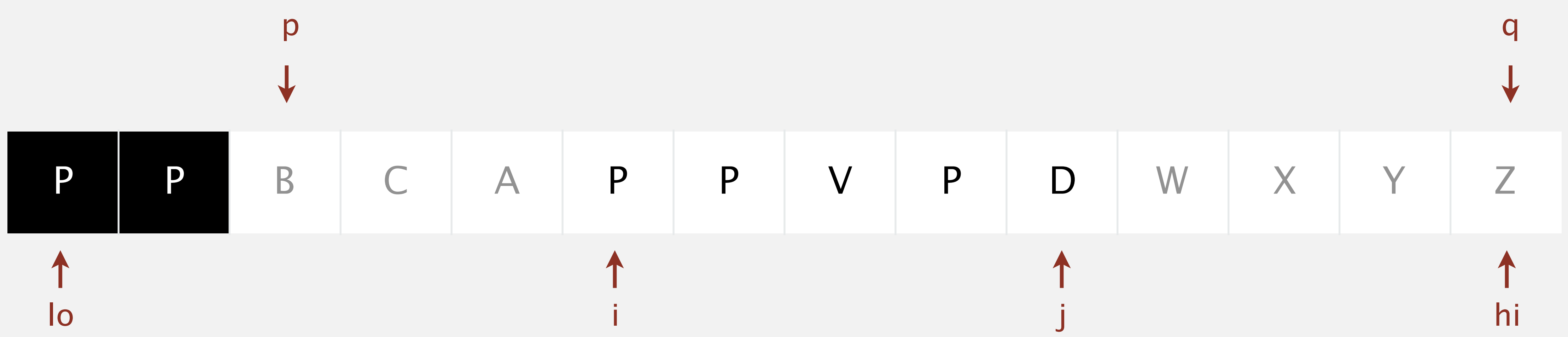

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Bentley–McIlroy 3-way partitioning demo

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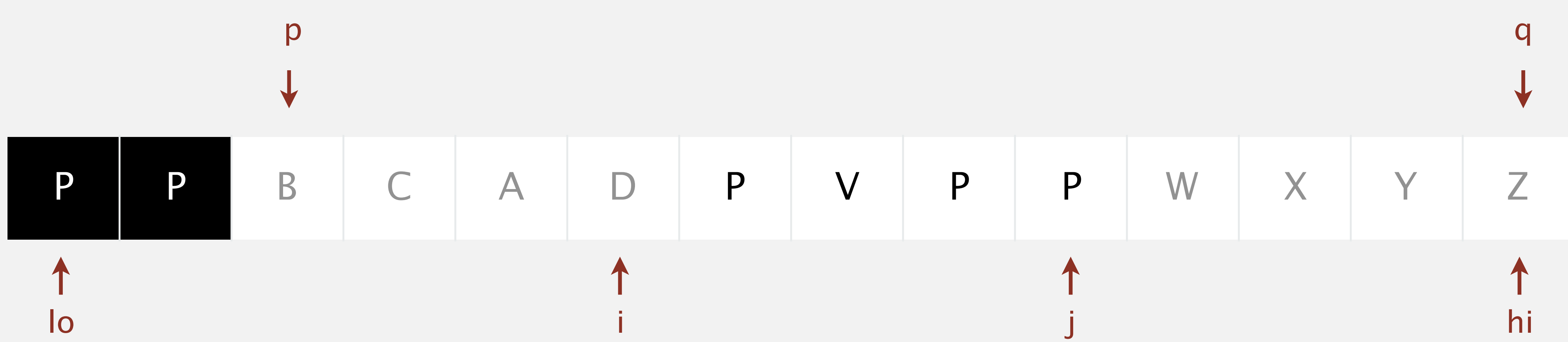

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Bentley–McIlroy 3-way partitioning demo

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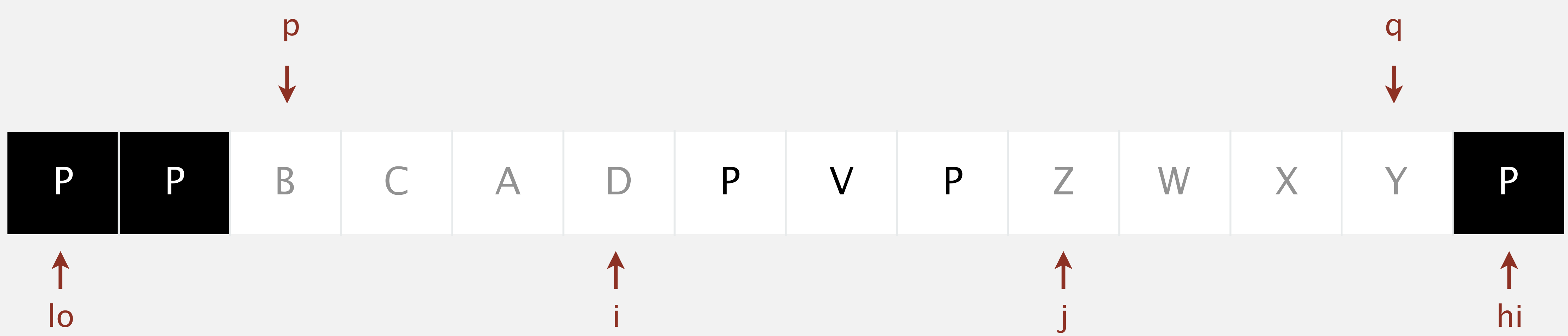

Bentley–McIlroy 3-way partitioning demo

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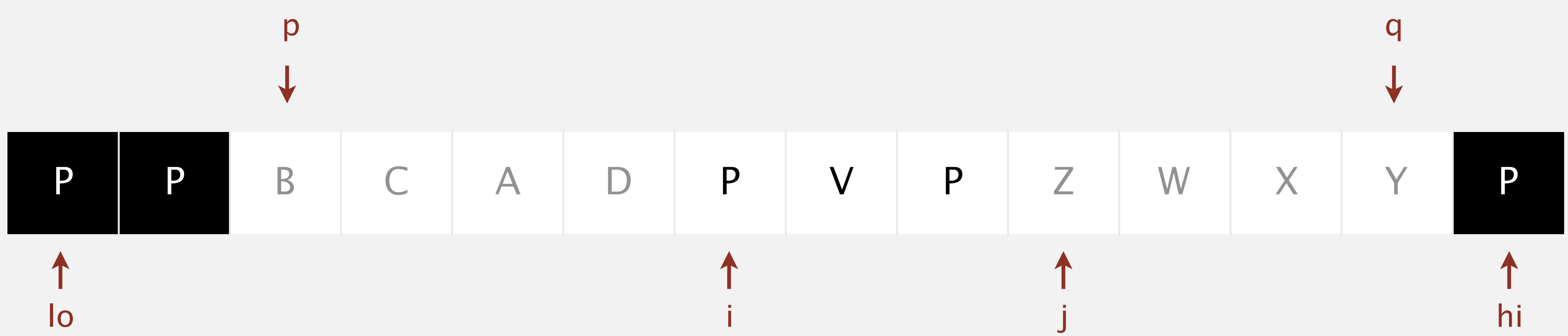

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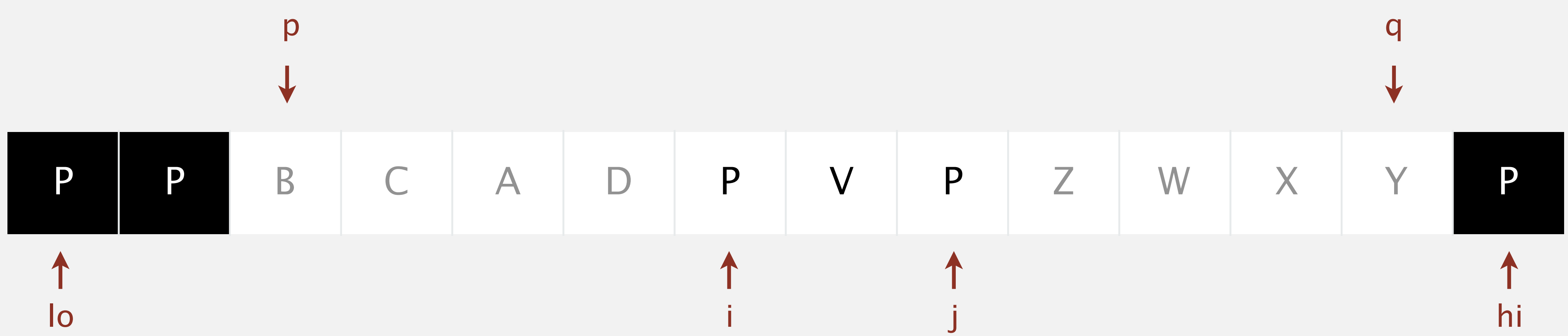

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Bentley–McIlroy 3-way partitioning demo

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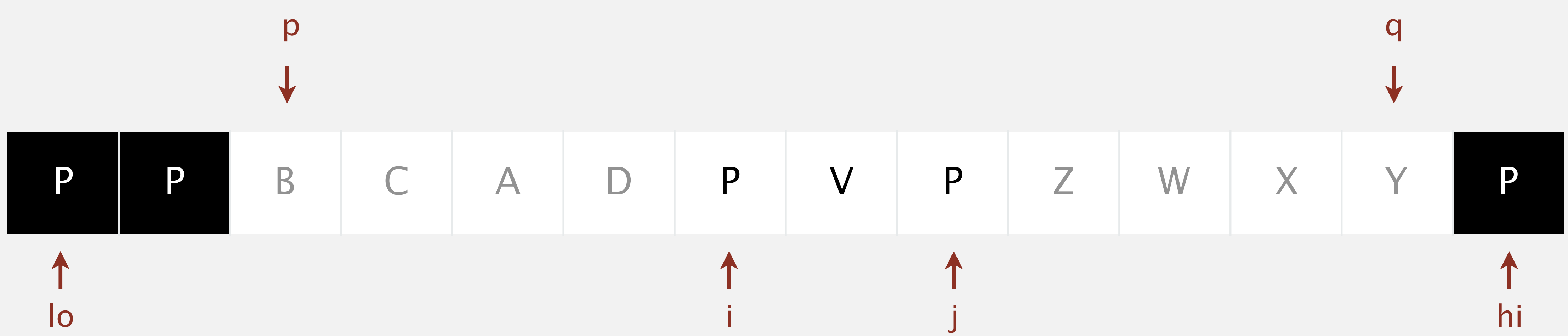

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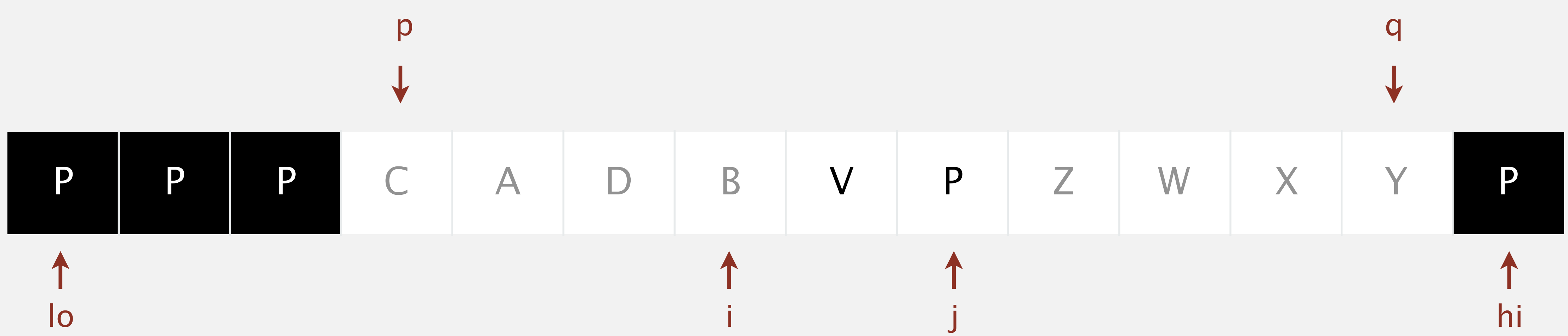

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Bentley–McIlroy 3-way partitioning demo

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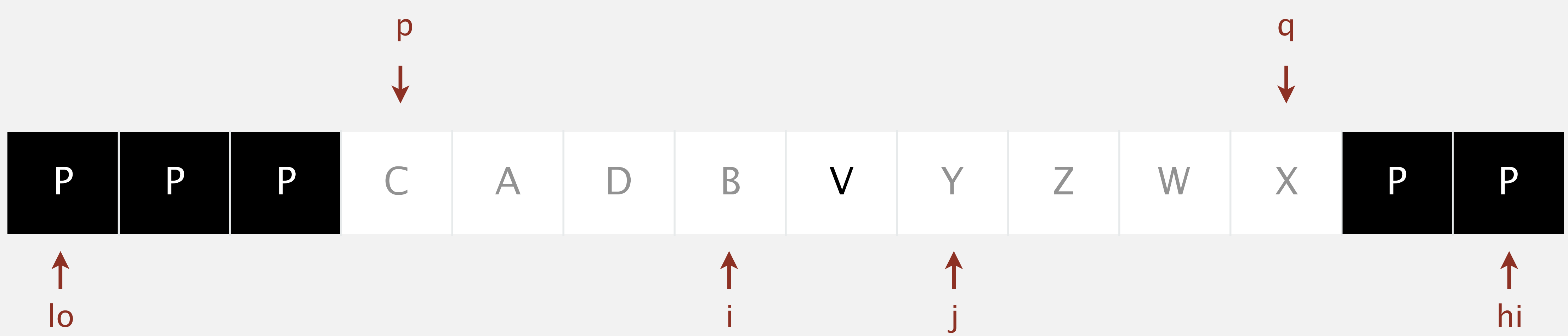

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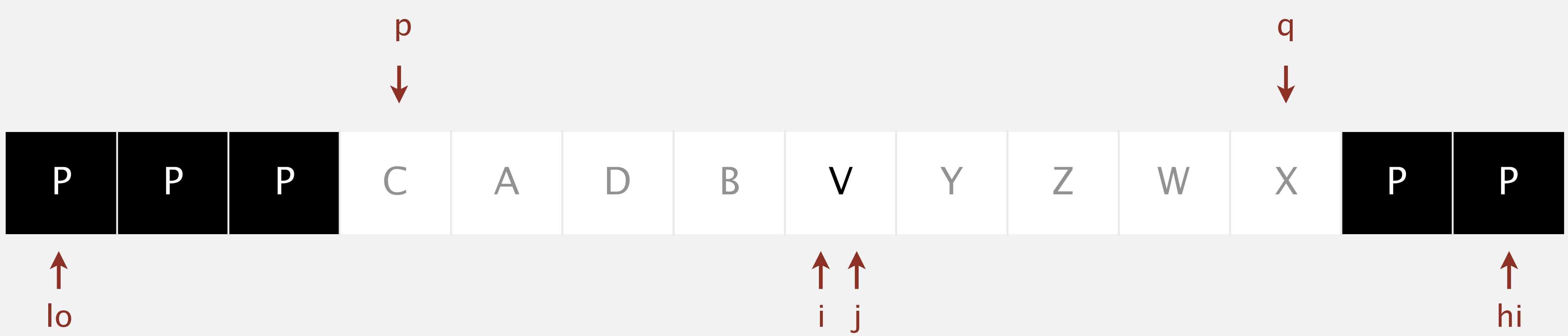

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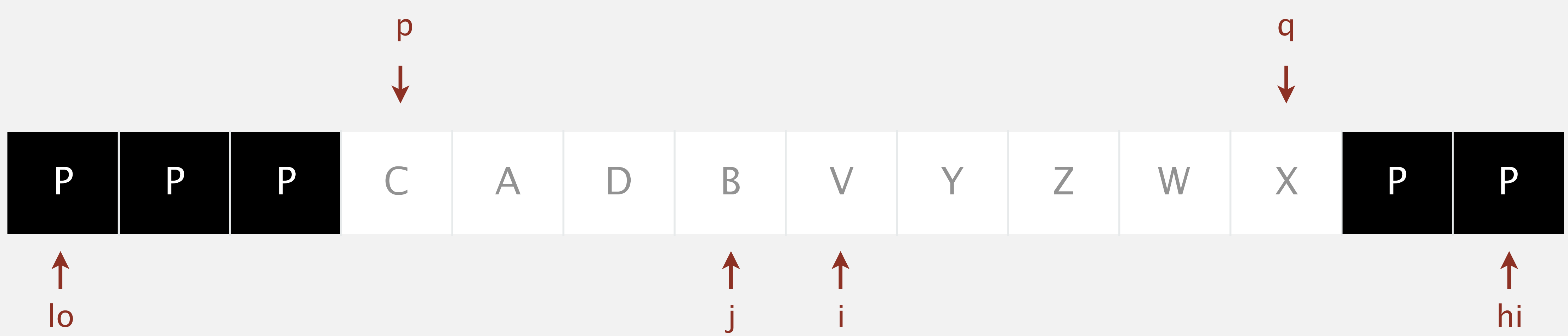
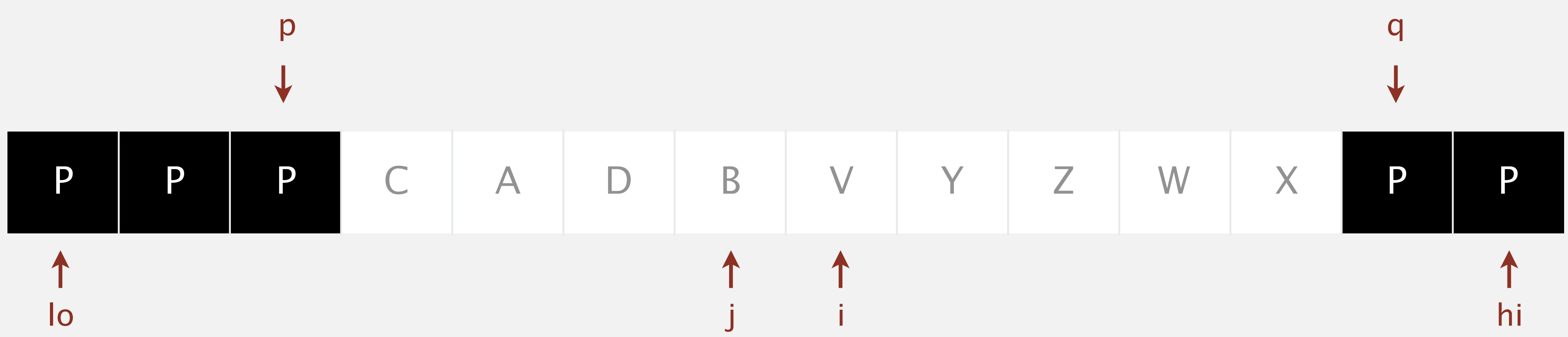


pointers cross

Bentley–McIlroy 3-way partitioning demo

Phase II. Swap equal keys to the center.

- Scan `j` and `p` from right to left and exchange `a[j]` with `a[p]`.
- Scan `i` and `q` from left to right and exchange `a[i]` with `a[q]`.



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Bentley–McIlroy 3-way partitioning demo

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- Scan `j` and `p` from right to left and exchange `a[j]` with `a[p]`.
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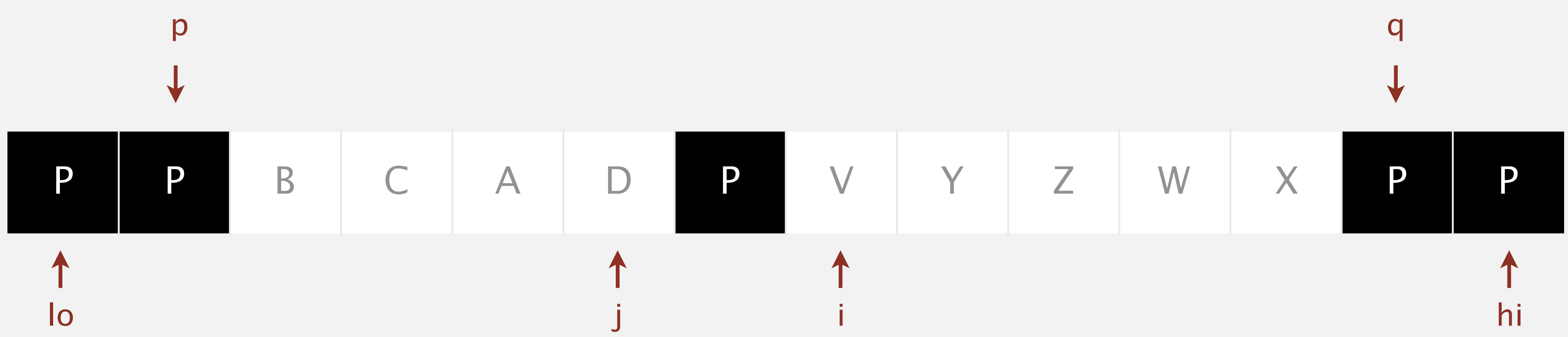

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Bentley–McIlroy 3-way partitioning demo

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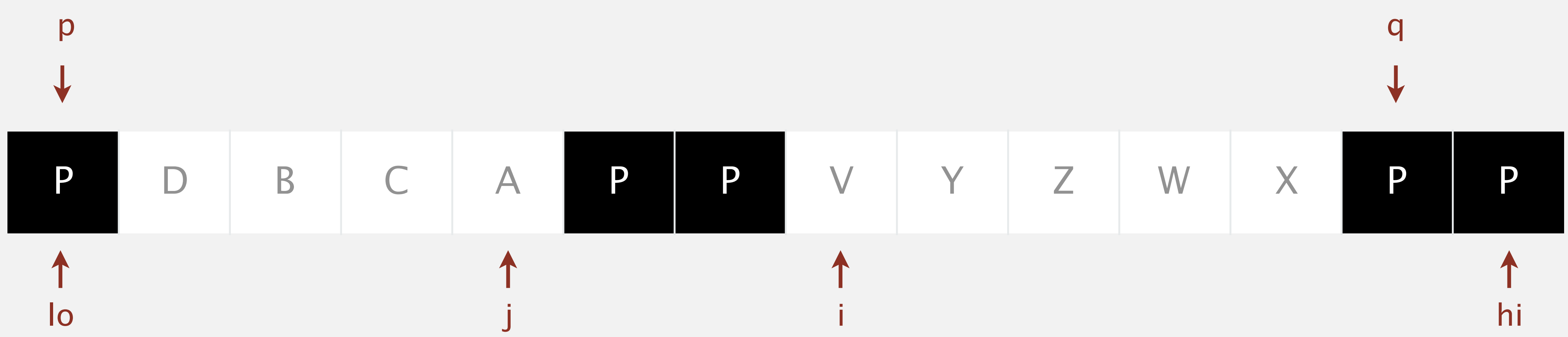

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Bentley–McIlroy 3-way partitioning demo

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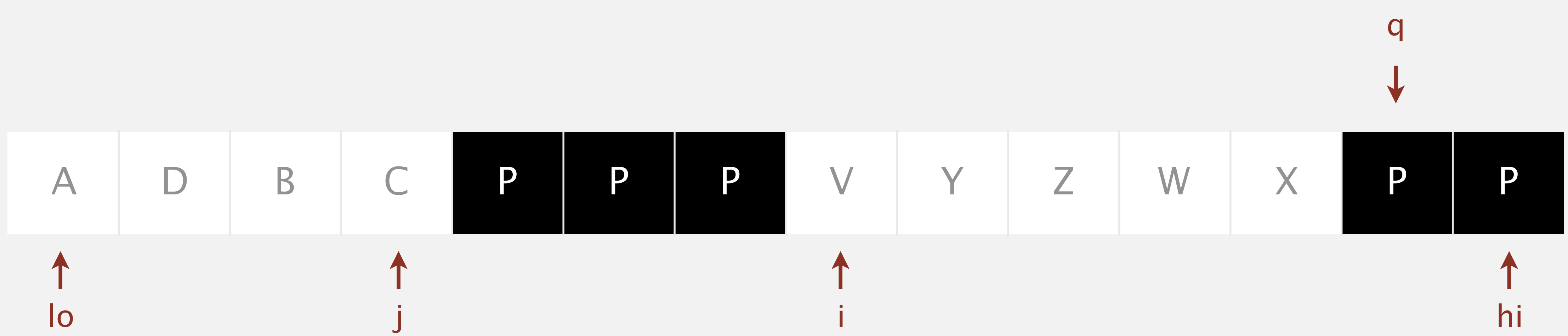

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Bentley–McIlroy 3-way partitioning demo

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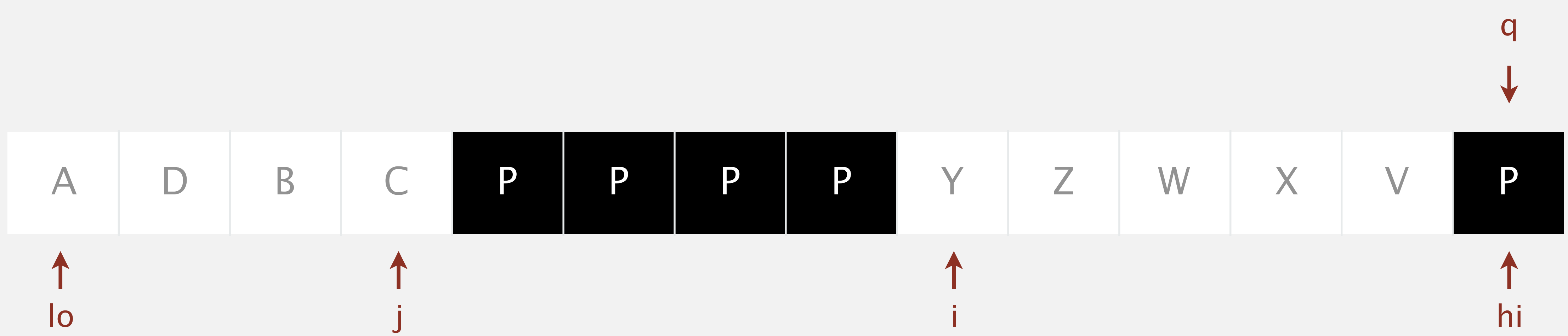

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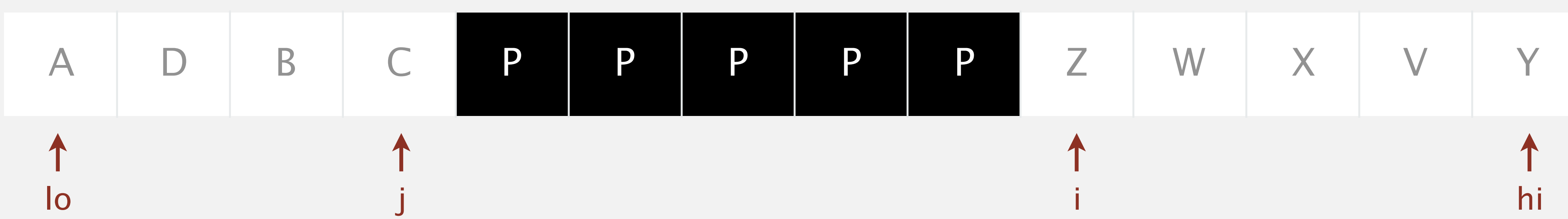


3-way partitioned

https://algs4.cs.princeton.edu

2.3 Partitioning Demos

- *Hoare 2-way partitioning*
- *Dijkstra 3-way partitioning*
- *Bentley–McIlroy 3-way partitioning*
- *dual-pivot partitioning*

Dual-pivot partitioning demo

Initialization.

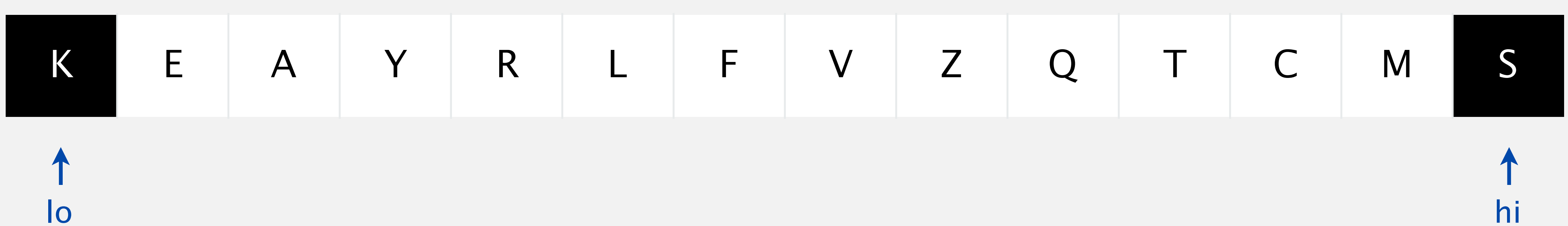
- Choose `a[lo]` and `a[hi]` as partitioning items.
- Exchange if necessary to ensure `a[lo]` ≤ `a[hi]`.

exchange a[lo] and a[hi]

Dual-pivot partitioning demo

Initialization.

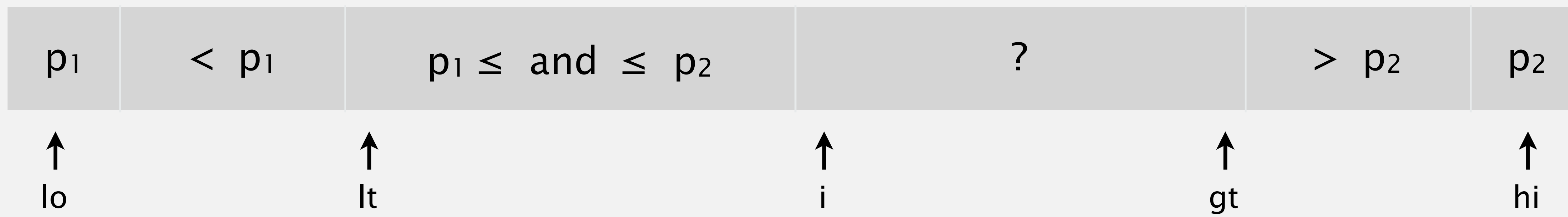
- Choose `a[lo]` and `a[hi]` as partitioning items.
- Exchange if necessary to ensure `a[lo] ≤ a[hi]`.

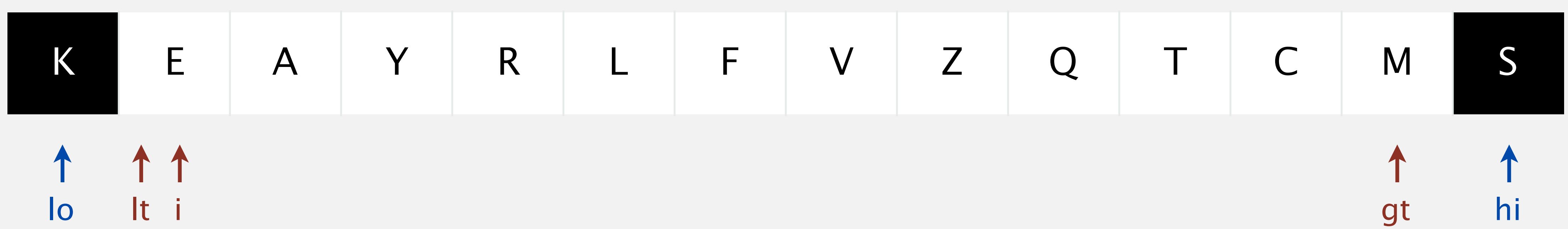


Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If `(a[i] < a[lo])`, exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
- Else if `(a[i] > a[hi])`, exchange `a[i]` with `a[gt]` and decrement `gt`.
- Else, increment `i`.





exchange a[i] and a[lt]; increment lt and i

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

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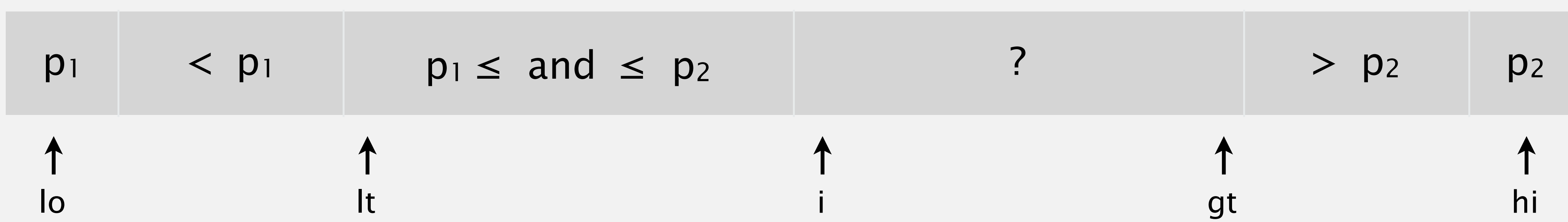

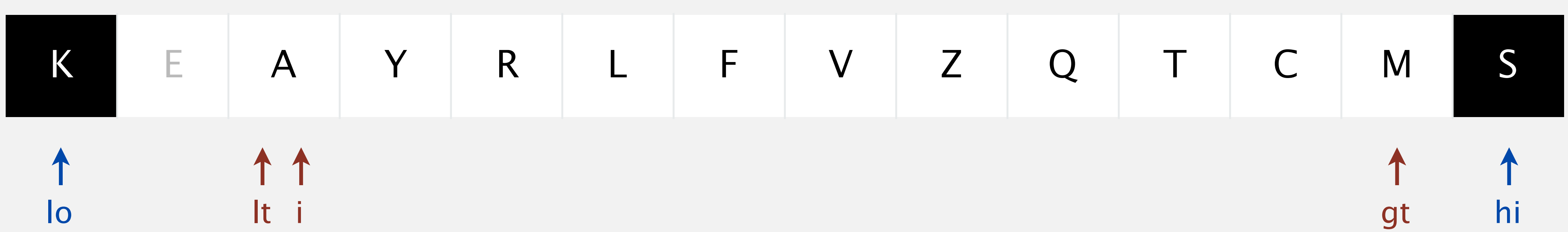

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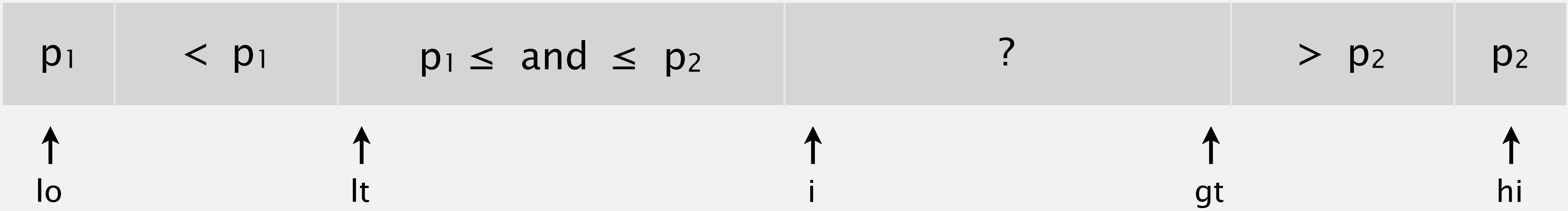
- Else, increment `i`.

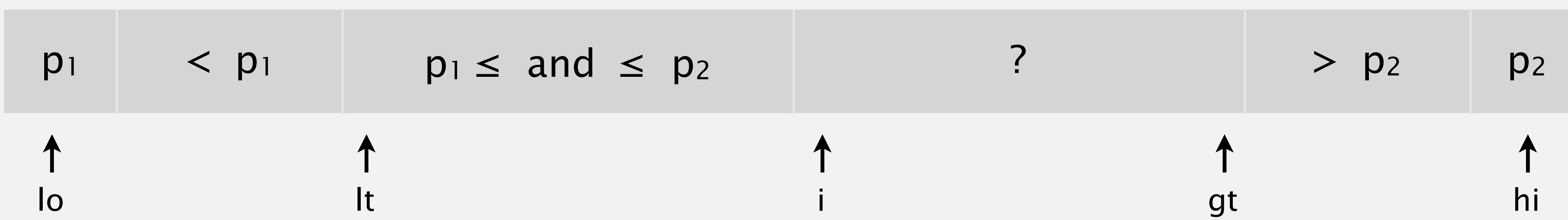


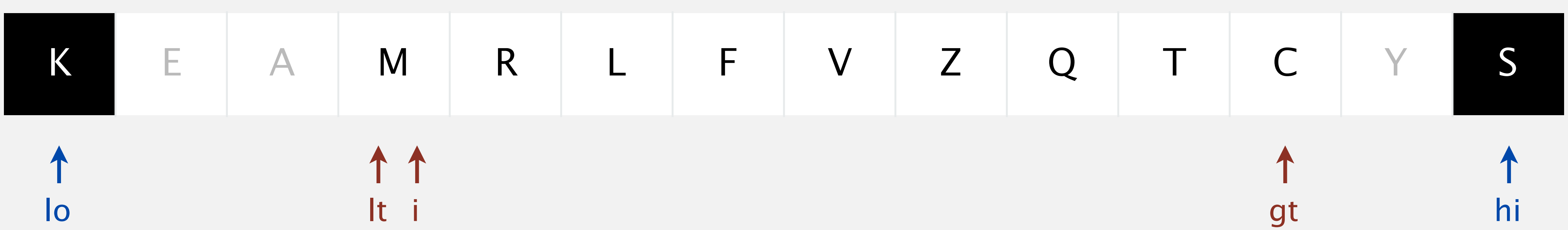
exchange a[i] and a[gt]; decrement gt

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If `(a[i] < a[lo])`, exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
- Else if `(a[i] > a[hi])`, exchange `a[i]` with `a[gt]` and decrement `gt`.
- Else, increment `i`.





increment i

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If `(a[i] < a[lo])`, exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
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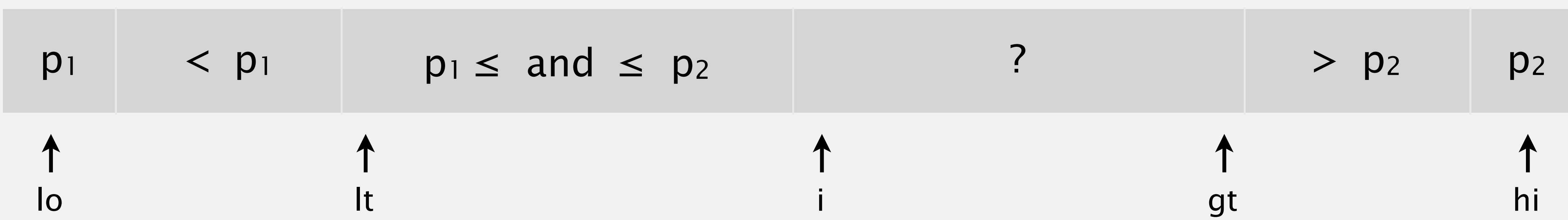

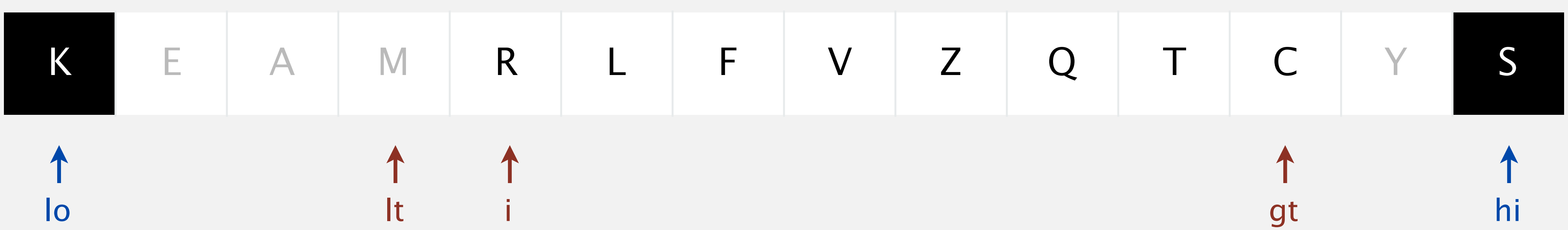

increment i

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

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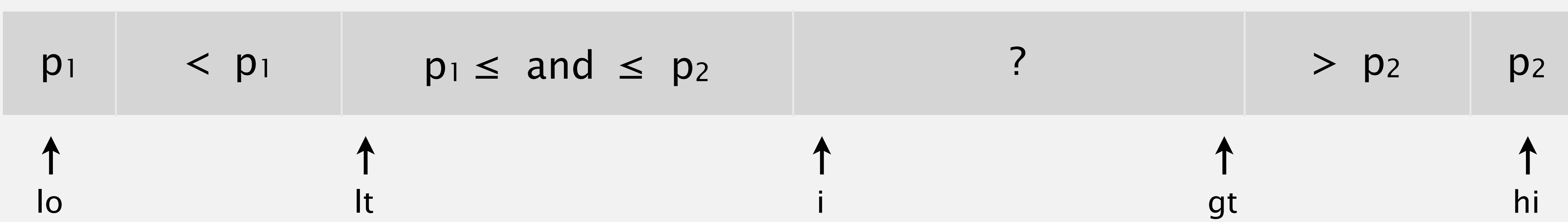

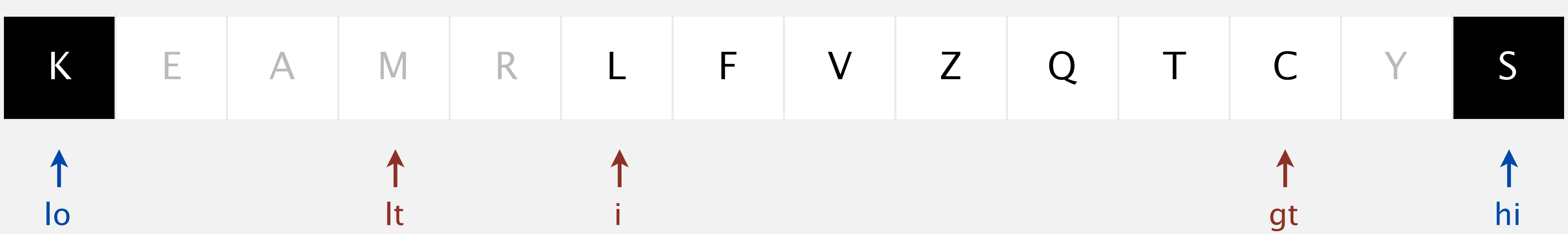

increment i

Dual-pivot partitioning demo

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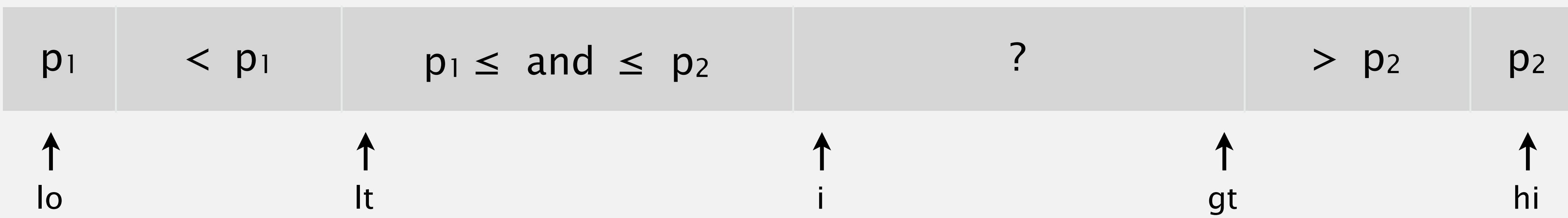
- Else, increment `i`.

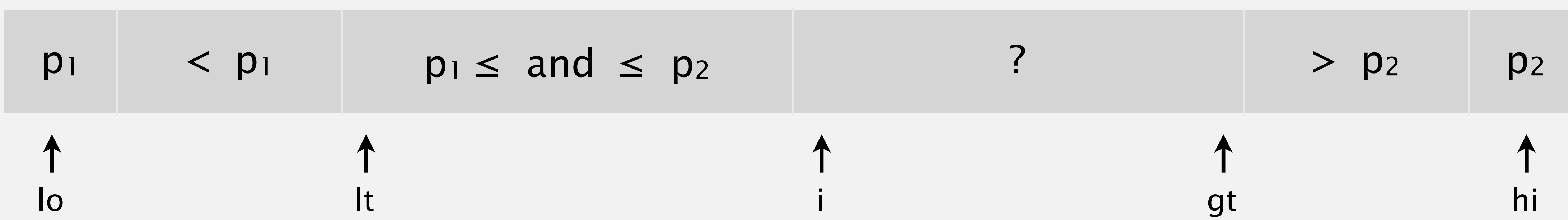


exchange a[i] and a[lt]; increment lt and i

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If `(a[i] < a[lo])`, exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
- Else if `(a[i] > a[hi])`, exchange `a[i]` with `a[gt]` and decrement `gt`.
- Else, increment `i`.

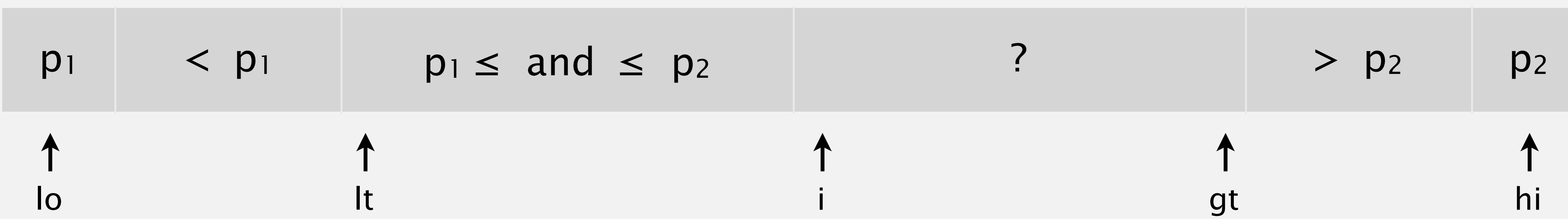


exchange a[i] and a[gt]; decrement gt

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If `(a[i] < a[lo])`, exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
- Else if `(a[i] > a[hi])`, exchange `a[i]` with `a[gt]` and decrement `gt`.
- Else, increment `i`.

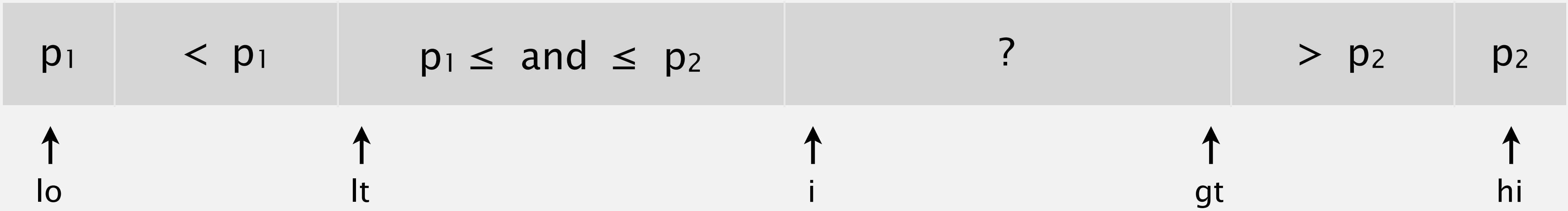


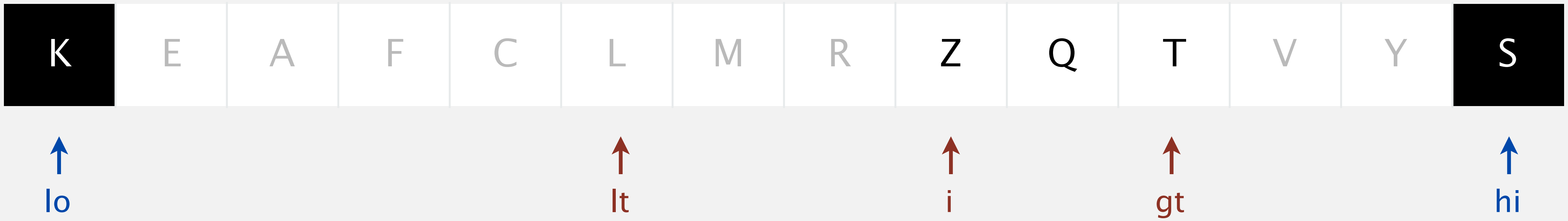
exchange a[i] and a[lt]; increment lt and i

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If (`a[i] < a[lo]`), exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
- Else if (`a[i] > a[hi]`), exchange `a[i]` with `a[gt]` and decrement `gt`.
- Else, increment `i`.





exchange a[i] and a[gt]; decrement gt

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If `(a[i] < a[lo])`, exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
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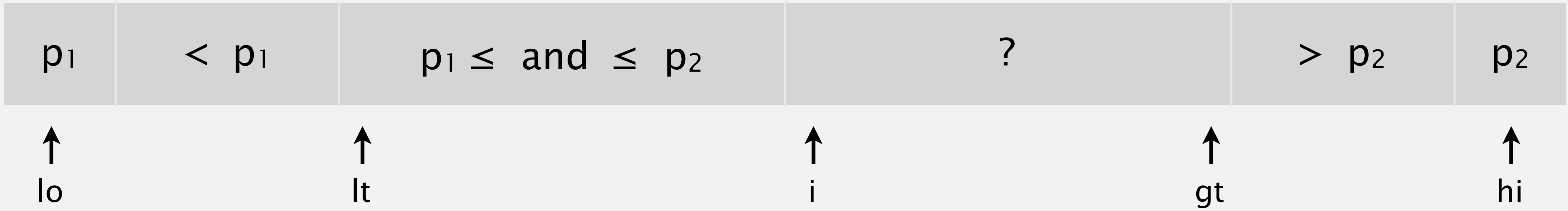
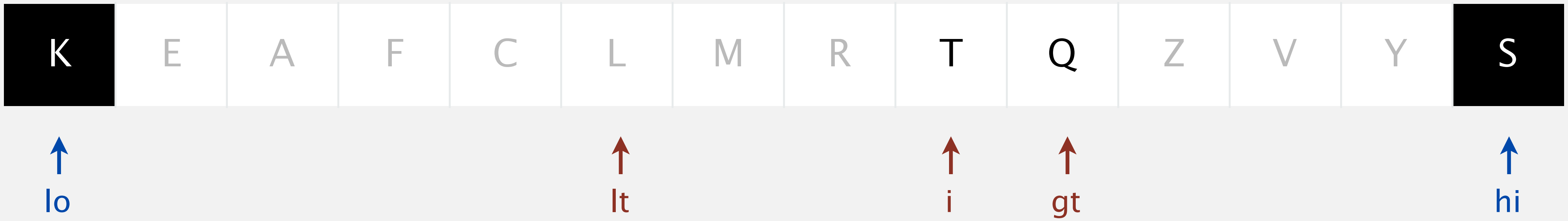
- Else, increment `i`.





exchange a[i] and a[gt]; decrement gt

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If (`a[i] < a[lo]`), exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
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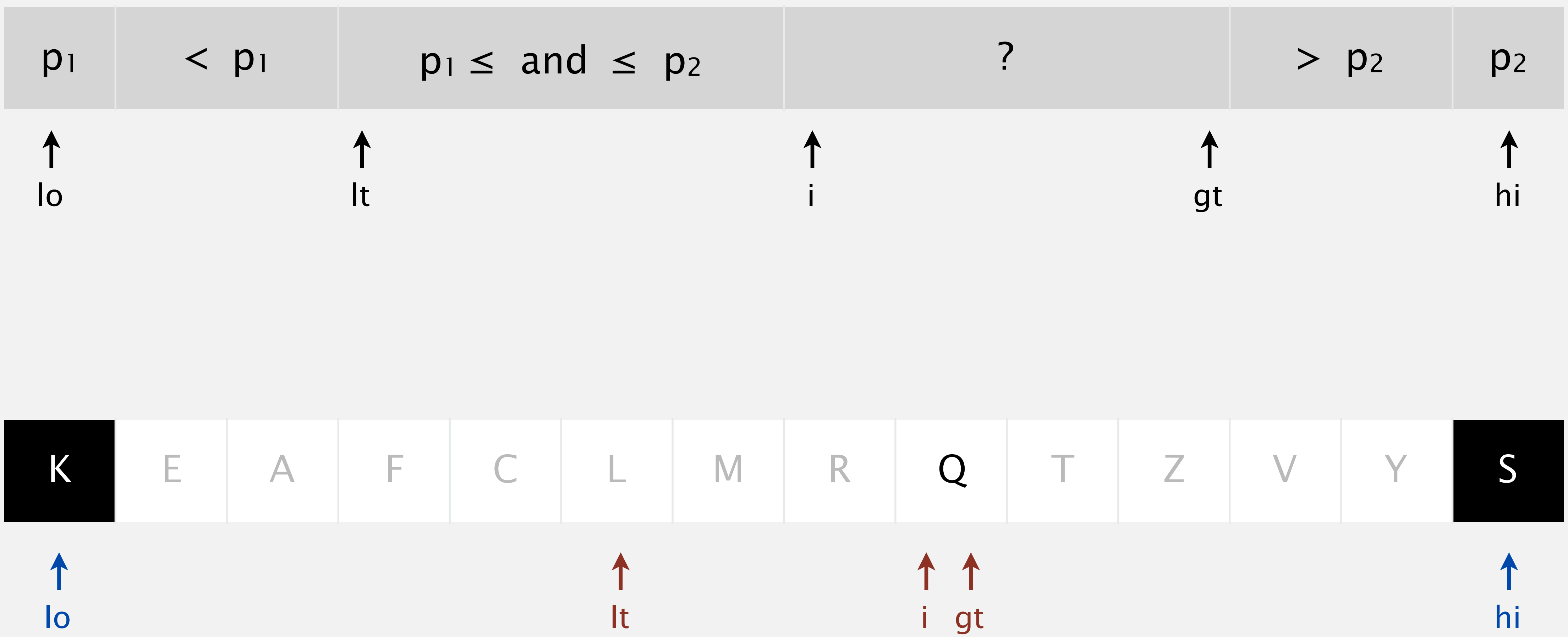

increment i

Dual-pivot partitioning demo

Main loop. Repeat until `i` and `gt` pointers cross.

- If `(a[i] < a[lo])`, exchange `a[i]` with `a[lt]` and increment `lt` and `i`.
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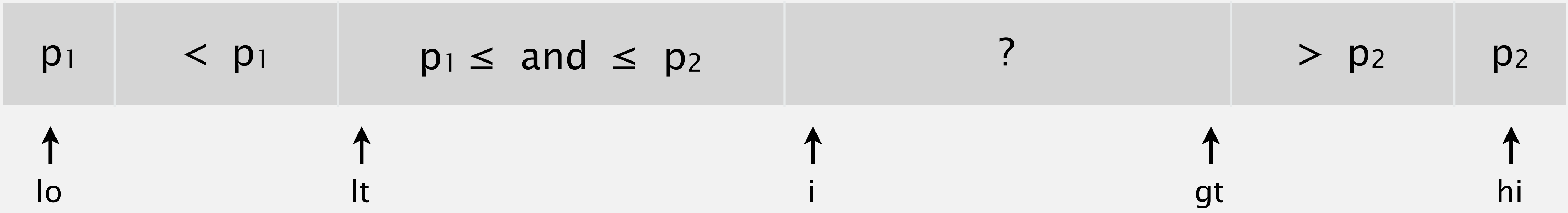
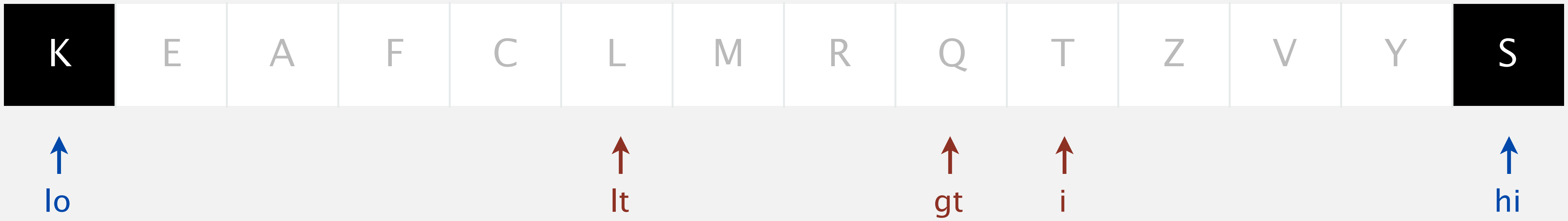


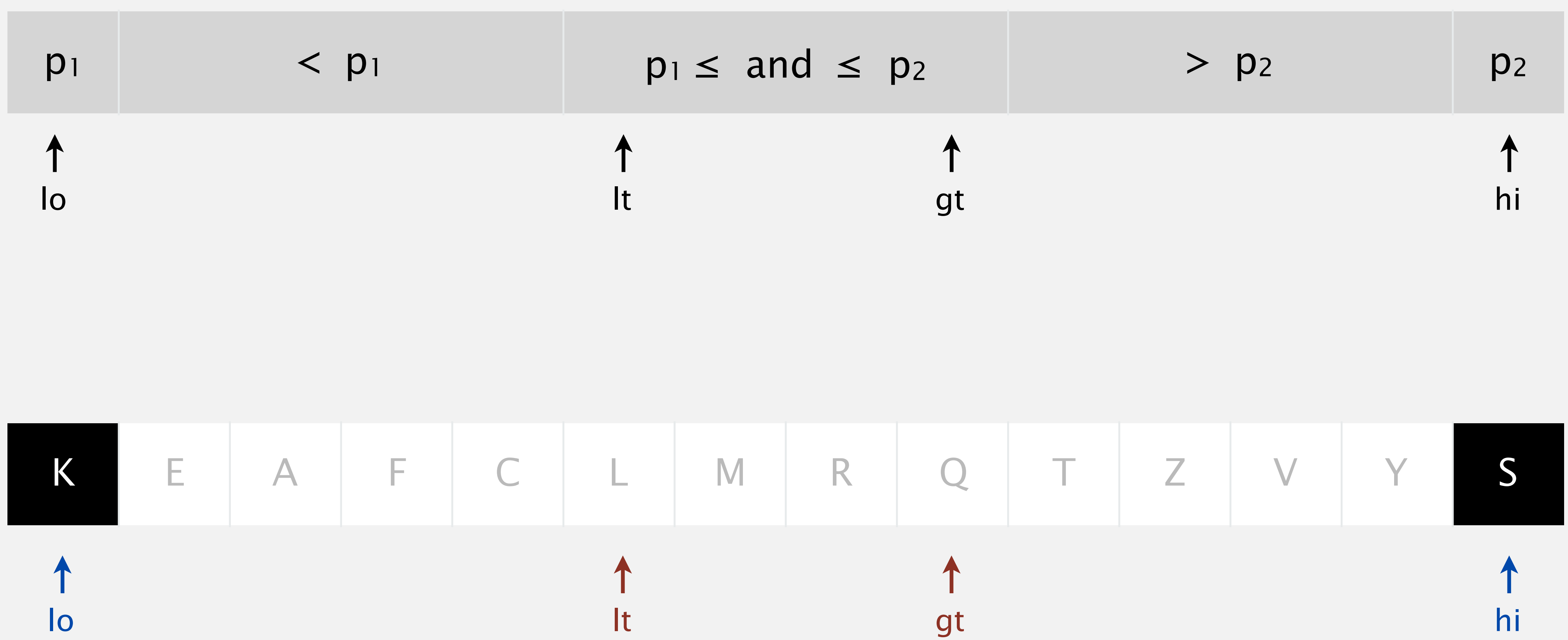


stop when pointers cross

Dual-pivot partitioning demo

Finalize.

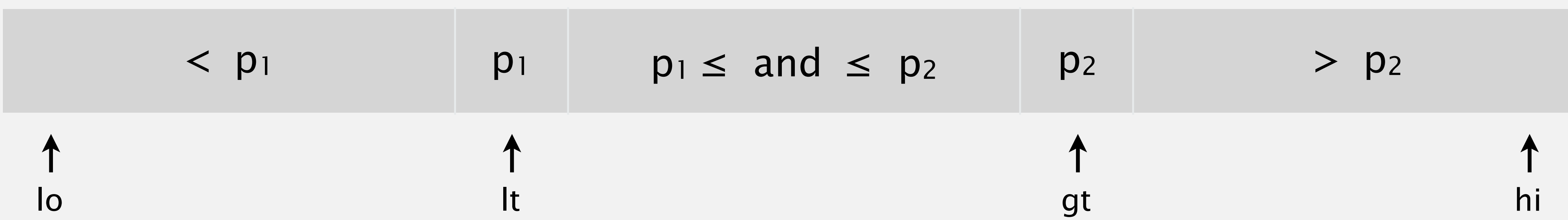
- Exchange `a[lo]` with `a[--lt]`.
- Exchange `a[hi]` with `a[++gt]`.

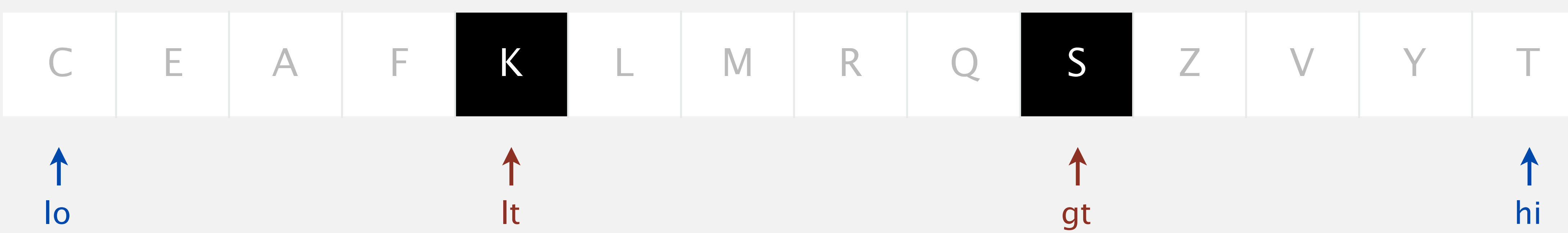


Dual-pivot partitioning demo

Finalize.

- Exchange `a[lo]` with `a[--lt]`.
- Exchange `a[hi]` with `a[++gt]`.





3-way partitioned