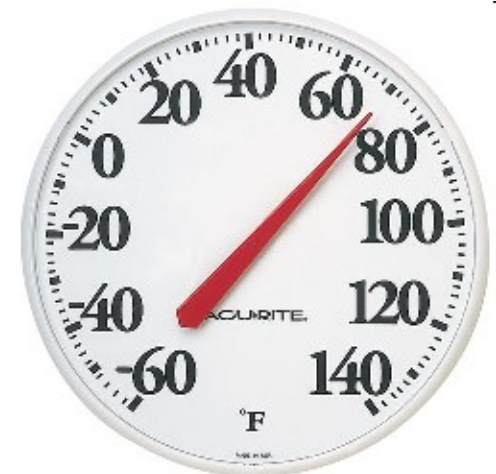


Bits, bytes, binary numbers, and the representation of information

- **computers represent, process, store, copy, and transmit everything as numbers**
 - hence "digital computer"
- **the numbers can represent anything**
 - not just numbers that you might do arithmetic on
- **the meaning depends on context**
 - as well as what the numbers ultimately represent
 - e.g., numbers coming to your computer or phone from your wi-fi connection could be email, movies, music, documents, apps, Skype call, ...

Analog versus Digital

- **analog: "analogous" or "the analog of"**
 - smoothly or continuously varying values
 - volume control, dimmer, faucet, steering wheel
 - value varies smoothly with something else
 - no discrete steps or changes in values
 - small change in one implies small change in another
 - infinite number of possible values
 - the world we perceive is largely analog
- **digital: discrete values**
 - only a finite number of different values
 - a change in something results in sudden change from one discrete value to another
 - digital speedometer, digital watch, push-button radio tuner, ...
 - values are represented as numbers



Digital pictures

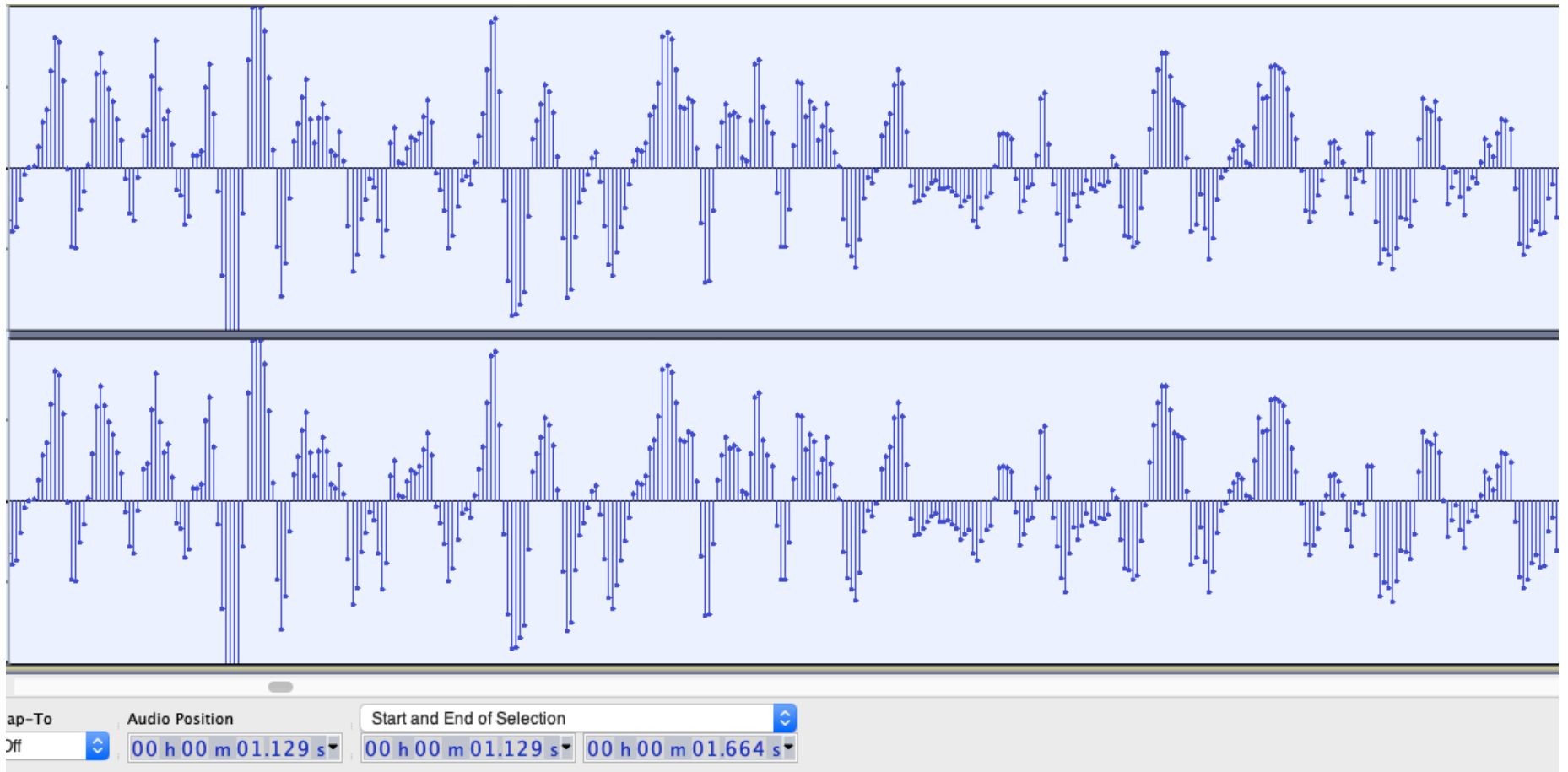
- divide the picture up into a grid of little rectangles (“pixels”)
- assign a different numeric value to each different color value
- the finer the grid and the finer the color distinctions, the more accurate the representation will be



Digital sound

- **need to measure intensity/loudness often enough and accurately enough that we can reconstruct it well enough**
- **higher frequency = higher pitch**
- **human ear can hear ~ 20 Hz to 20 KHz**
 - taking samples at twice the highest frequency is good enough (Nyquist)
- **CD audio usually uses**
 - 44,100 samples / second
 - accuracy of 1 in 65,536 ($= 2^{16}$) distinct levels
 - two samples at each time for stereo
 - data rate is $44,100 \times 2 \times 16$ bits/sample
 - $= 1,411,200$ bits/sec $= 176,400$ bytes/sec ~ 10.6 MB/minute
- **MP3 audio compresses by clever encoding and removal of sounds that won't really be heard**
 - data rate is ~ 1 MB/minute

Digital sound sampling (using Audacity)



A review of how decimal numbers work

- **how many digits?**

we use 10 digits for counting: "decimal" numbers are natural for us

other schemes show up in some areas

clocks use 12, 24, 60; calendars use 7, 12

other cultures use other schemes (quatre-vingts)

- **what if we want to count to more than 10?**

0 1 2 3 4 5 6 7 8 9

1 decimal digit represents 1 choice from 10; counts 10 things; 10 distinct values

00 01 02 ... 10 11 12 ... 20 21 22 ... 98 99

2 decimal digits represents 1 choice from 100; 100 distinct values

we usually elide zeros at the front

000 001 ... 099 100 101 ... 998 999

3 decimal digits ...

- **decimal numbers are shorthands for sums of powers of 10**

$$1492 = 1 \times 1000 + 4 \times 100 + 9 \times 10 + 2 \times 1$$

$$= 1 \times 10^3 + 4 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$$

- **counting in "base 10", using powers of 10**

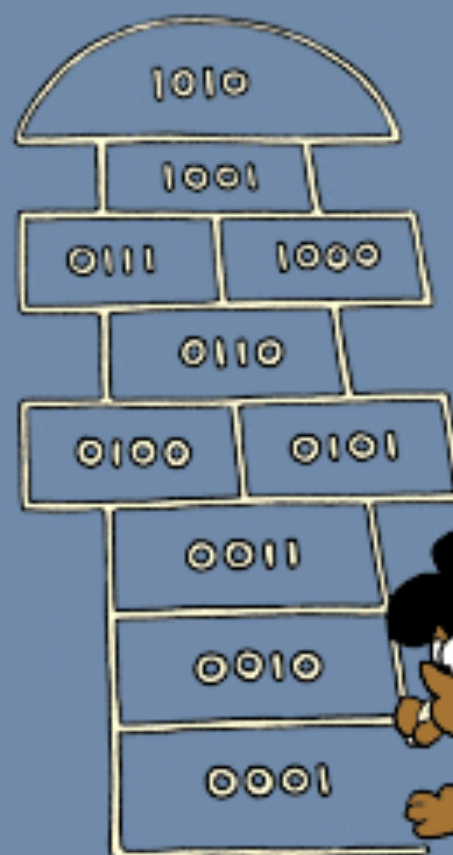
Binary numbers: only use the digits 0 and 1 to represent numbers

- just like decimal except there are only two digits: 0 and 1
- everything is based on powers of 2 (1, 2, 4, 8, 16, 32, ...)
 - instead of powers of 10 (1, 10, 100, 1000, ...)
- counting in binary or base 2:
 - 0 1
1 binary digit represents 1 choice from 2; counts 2 things; 2 distinct values
 - 00 01 10 11
2 binary digits represents 1 choice from 4; 4 distinct values
 - 000 001 010 011 100 101 110 111
3 binary digits ...
- binary numbers are shorthands for sums of powers of 2
 - $11011 = 1 \times 16 + 1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1$
 $= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
- counting in "base 2", using powers of 2



FoxTrot

by Bill Amend



SEE? IT'S KINDA
SORTA A LITTLE
LIKE A VIDEO
GAME THIS WAY!



ON SECOND THOUGHT,
LET'S NOT PLAY
HOPSCOTCH.



Binary (base 2) arithmetic

- works like decimal (base 10) arithmetic, but simpler

- addition:

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10$$

- subtraction, multiplication, division are analogous

Converting binary to decimal

from right to left:

if bit is 1 add corresponding power of 2

i.e. 2^0 , 2^1 , 2^2 , 2^3

(rightmost power is zero)

$$\begin{aligned} 1101 &= 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 \\ &= 1 \times 1 + 0 \times 2 + 1 \times 4 + 1 \times 8 \\ &= 13 \end{aligned}$$

Converting decimal to binary

repeat while the number is > 0 :

divide the number by 2

write the remainder (0 or 1)

use the quotient as the number and repeat

the answer is the resulting sequence

in reverse (right to left) order

divide 13 by 2, write "1", number is 6

divide 6 by 2, write "0", number is 3

divide 3 by 2, write "1", number is 1

divide 1 by 2, write "1", number is 0

answer is 1101

What's a bit?

- **a bit represents one 2-way decision or a choice out of two possibilities**
 - yes / no, true / false, on / off, up / down, north / south, ...
- **the abstraction of all of these is represented as 0 or 1**
 - enough to tell which of TWO possibilities has been chosen
 - a single digit with one of two values
 - hence "binary digit"
 - hence bit
- **binary is used in computers because it's easy to make fast, reliable, small devices that have only two states**
 - high voltage/low voltage, current flowing/not flowing (chips)
 - electrical charge present/not present (RAM, flash)
 - magnetized this way or that (disks)
 - light bounces off/doesn't bounce off (cd-rom, dvd)
- **all information in a computer is stored and processed as bits**

Powers of two, powers of ten

1 bit = 2 possibilities

2 bits = 4 possibilities

3 bits = 8 possibilities

...

n bits = 2^n possibilities

$2^{10} = 1,024$ is about 1,000 or 1K or 10^3

$2^{20} = 1,048,576$ is about 1,000,000 or 1M or 10^6

$2^{30} = 1,073,741,824$ is about 1,000,000,000 or 1G or 10^9

the approximation is becoming less good

but it's still good enough for estimation

- **terminology is often imprecise:**

- " 1K " might mean 1000 or 1024 (10^3 or 2^{10})

- " 1M " might mean 1000000 or 1048576 (10^6 or 2^{20})



FoxTrot

by Bill Amend



Bytes

- **"byte" = a group of 8 bits treated as a unit**
 - on modern machines, the fundamental unit of processing and memory addressing
 - can encode any of $2^8 = 256$ different values, e.g., numbers 0 .. 255 or a single letter like A or digit like 7 or punctuation like \$
ASCII character set defines values for letters, digits, punctuation, etc.
- **group 2 bytes together to hold larger entities**
 - two bytes (16 bits) holds $2^{16} = 65,536$ values
 - a bigger integer, a character in a larger character set
Unicode character set defines values for almost all characters anywhere
- **group 4 bytes together to hold even larger entities**
 - four bytes (32 bits) holds $2^{32} = 4,294,967,296$ values
 - an even bigger integer, a number with a fractional part (floating point), a memory address
 - current machines use 64-bit integers and addresses (8 bytes)
 $2^{64} = 18,446,744,073,709,551,616$
- **no fractional bytes: the number of bytes is always an integer**

Interpretation of bits and bytes depends on context

- **meaning of a group of bits depends on how they are interpreted**
- **1 byte could be**
 - 1 bit in use, 7 wasted bits (e.g., M/F in a database)
 - 8 bits representing a number between 0 and 255
 - an alphabetic character like W or + or 7
 - part of a character in another alphabet or writing system (2+ bytes)
 - part of a larger number (2 or 4 or 8 bytes, usually)
 - part of a picture or sound
 - part of an instruction for a computer to execute
 - instructions are just bits, stored in the same memory as data
 - different kinds of computers use different bit patterns for their instructions
 - laptop, cellphone, game machine, etc., all potentially different
 - part of the location or address of something in memory
 - ...
- **one program's instructions are another program's data**
 - when you download a new program from the net, it's data
 - when you run it, it's instructions

Hexadecimal notation

- **binary numbers are bulky**
- **hexadecimal notation is a shorthand**
- **it combines 4 bits into a single digit, written in base 16**
 - a more compact representation of the same information
- **hex uses the symbols A B C D E F for the digits 10 .. 15**

0 1 2 3 4 5 6 7 8 9 A B C D E F

0	0000	1	0001	2	0010	3	0011
4	0100	5	0101	6	0110	7	0111
8	1000	9	1001	A	1010	B	1011
C	1100	D	1101	E	1110	F	1111

ASCII, using hexadecimal numbers

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	NUL	SOH	STX	ETX	EOT	ENQ	ACK	BEL	BS	HT	LF	VT	FF	CR	SO	SI
1	DLE	DC1	DC2	DC3	DC4	NAK	SYN	ETB	CAN	EM	SUB	ESC	FS	GS	RS	US
2	SPC	!	"	#	\$	%	&	'	()	*	+	,	-	.	/
3	0	1	2	3	4	5	6	7	8	9	:	;	<	=	>	?
4	@	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
5	P	Q	R	S	T	U	V	W	X	Y	Z	[\]	^	_
6	`	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
7	p	q	r	s	t	u	v	w	x	y	z	{		}	~	DEL

Color

- TV & computer screens use Red-Green-Blue (RGB) model



- each color is a combination of red, green, blue components
 - $R+G$ = yellow, $R+B$ = magenta, $B+G$ = cyan, $R+G+B$ = white
- for computers, color of a pixel is usually specified by three numbers giving amount of each color, on a scale of 0 to 255
- this is often expressed in hexadecimal so the three components can be specified separately (in effect, as bit patterns)
 - 000000 is black, FFFFFFFF is white
- printers, etc., use cyan-magenta-yellow[-black] (CMY[K])



A very important idea

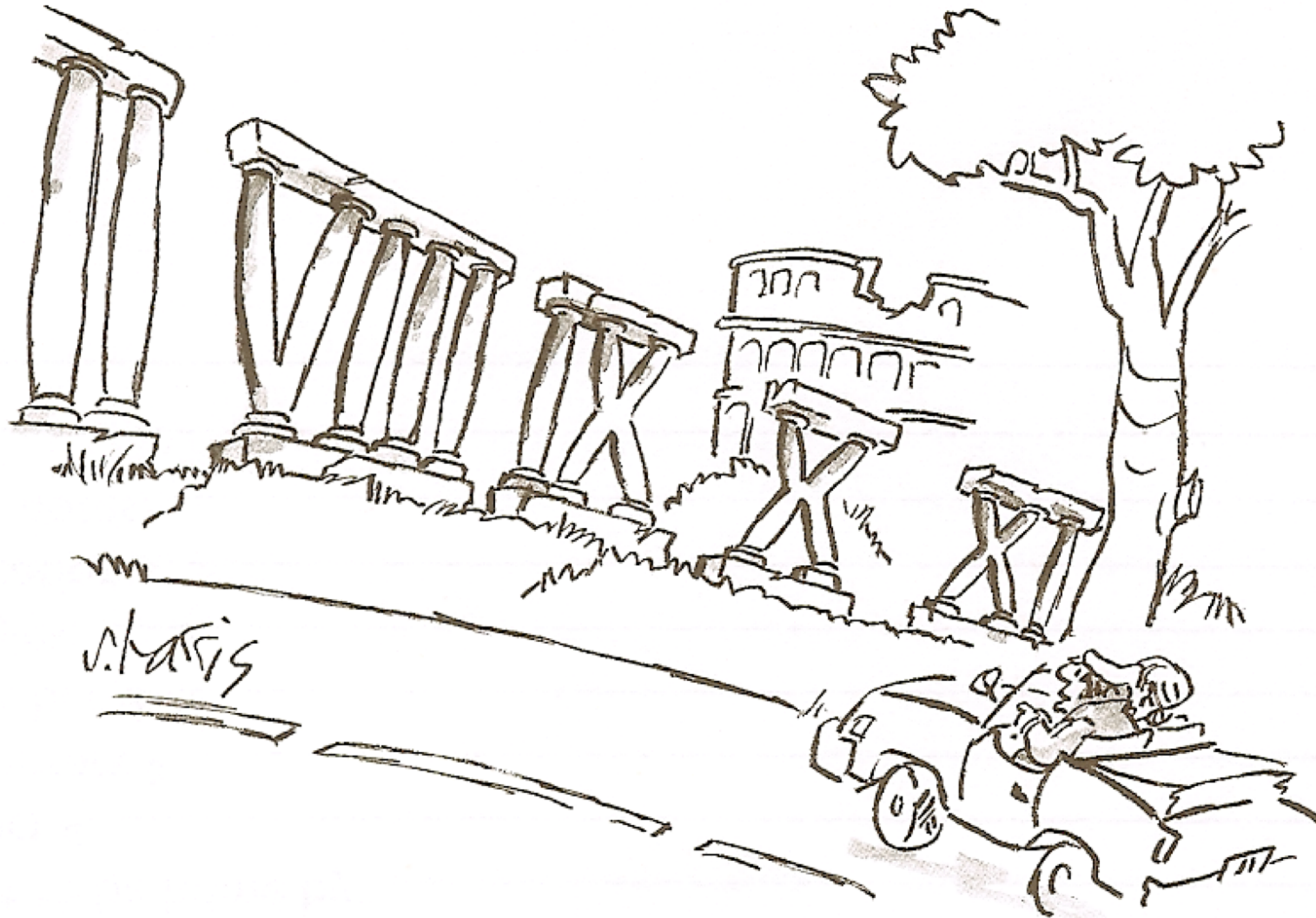
- **number of items and number of digits are tightly related:**
 - one determines the other
 - maximum number of different items = base^{number of digits}
 - e.g., 9-digit SSN: $10^9 = 1$ billion possible numbers
 - e.g., to represent up to 100 “characters”: 2 digits is enough
 - but for 1000 characters, we need 3 digits
 - the same for bits: 9 bits can represent up to $2^9 = 512$ items
- **interpretation depends on context**
 - without knowing that, we can only guess what numbers mean

Things to remember

- **digital devices represent everything as numbers**
 - discrete values, not continuous or infinitely precise
- **all modern digital devices use binary numbers (base 2)**
 - instead of decimal (base 10)
- **it's all bits at the bottom**
 - a bit is a "binary digit", that is, a number that is either 0 or 1
 - computers ultimately represent and process everything as bits
- **groups of bits represent larger things**
 - numbers, letters, words, names, pictures, sounds, instructions, ...
 - the interpretation of a group of bits depends on their context
 - the representation is arbitrary; standards (often) define what it is
- **the number of digits used in the representation determines how many different things can be represented**
 - number of values = base ^{number of digits}
 - e.g., 10^2 , 2^{10}



THE NEW YORKER



"If you think this is annoying, just wait. In a few miles, they switch to binary."