Mutation

COS 326

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Reasoning about Mutable State is Hard

mutable set

```
insert i s1;
f x;
member i s1
```

immutable set

```
let s1 = insert i s0 in
f x;
member i s1
```

Is member i s1 == true? …

- When s1 is mutable, one must look at f to determine if it modifies s1.
- Worse, one must often solve the *aliasing problem*.
- Worse, in a concurrent setting, one must look at *every other function* that *any other thread may be executing* to see if it modifies s1.

Thus far…

We have considered the (almost) purely functional subset of OCaml.

- We've had a few side effects: printing & raising exceptions.

Two reasons for this emphasis:

- *Reasoning about functional code is easier*.
  - Both formal reasoning
    - equationally, using the substitution model
    - and informal reasoning
  - Data structures are *persistent*.
    - They don't change – we build new ones and let the garbage collector reclaim the unused old ones.
  - *Hence, any invariant you prove true stays true*.
    - e.g., 3 is a member of set S.
- *To convince you that you don't need side effects for many things where you previously thought you did*.
  - Programming with *basic immutable data like ints, pairs, lists is easy*.
    - types do a lot of testing for you!
    - do not fear recursion!
  - You can implement *expressive*, *highly reuseable functional* data structures like polymorphic 2-3 trees or dictionaries or stacks or queues or sets or expressions or programming languages with reasonable space and time.

But alas…

Purely functional code is pointless.

- The whole reason we write code is to have some effect on the world.
- For example, the OCaml top-level loop prints out your result.
  - Without that printing (a side effect), how would you know that your functions computed the right thing?

Some algorithms or data structures need mutable state.

- Hash-tables have (essentially) constant-time access and update.
  - The best functional dictionaries have either:
    - logarithmic access & logarithmic update
    - constant access & linear update
    - constant update & linear access
  - Don't forget that we give up something for this:
    - we can't go back and look at previous versions of the dictionary. We *can* do that in a functional setting.
- Robinson's unification algorithm
  - A critical part of the OCaml type-inference engine.
  - Also used in other kinds of program analyses.
- Depth-first search, more ...

However, ~~purely~~ mostly functional code is amazingly productive

OCAML MUTABLE REFERENCES

References

- New type: `t ref`
  - Think of it as a pointer to a *box* that holds a t value.
  - The contents of the box can be read or written.

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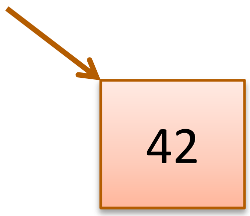
  - Think of it as a pointer to a ***box*** that holds a t value.
  - The contents of the box can be read or written.
- To create a fresh box: `ref 42`
  - allocates a new box, initializes its contents to 42, and returns a pointer:



  - `ref 42 : int ref`

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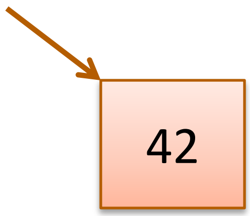

  - `ref 42 : int ref`
- To read the contents: `!r`
  - if `r` points to a box containing 42, then return 42.
  - if `r : t ref` then `!r : t`
- To write the contents: `r := 5`
  - updates the box that r points to so that it contains 5.
  - if `r : t ref` then `r := 5 : unit`

Example

```
let c = ref 0 in

let x = !c in         (* x will be 0 *)

c := 42;

let y = !c in      (* y will be 42.
                      x will still be 0! *)
```

Another Example

```
let c = ref 0 ;;

let next() =
  let v = !c in
  (c := v+1 ; v)
```

Another Example

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let c = ref 0

let next() =
  let v = !c in
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```

If `e1 : unit`
and `e2 : t` then
`(e1 ; e2) : t`

You can also write it like this:

```
let c = ref 0

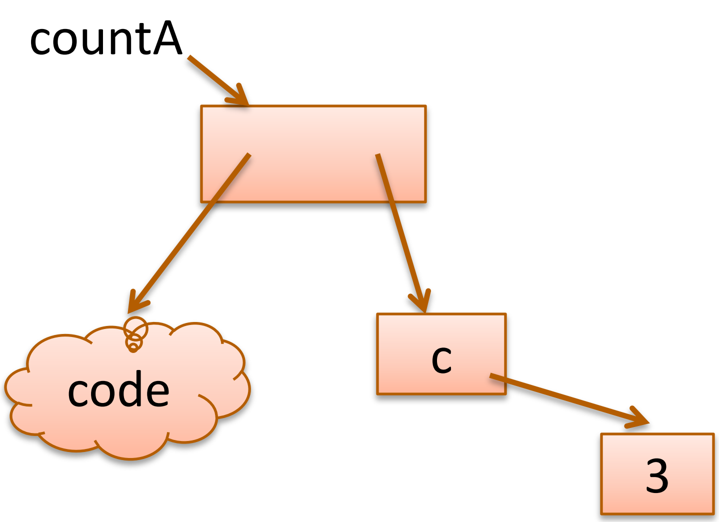
let next() =
  let v = !c in
  let _ = c := v+1 in
  v
```

Another Idiom

Global Mutable Reference

```
let c = ref 0

let next () : int =
  let v = !c in
  (c := v+1 ; v)
```



Mutable Reference Captured in Closure

```
let counter () =
   let c = ref 0 in
  fun () ->
    let v = !c in
    (c := v+1 ; v)

let countA = counter() in
let countB = counter() in
countA() ; (* 0 *)
countA() ; (* 1 *)
countB() ; (* 0 *)
countB() ; (* 1 *)
countA() ; (* 2 *)
```

Imperative loops

```
(* sum of 0 .. n *)

let sum (n:int) =
  let         s = ref 0 in
  let current = ref n in
  while !current > 0 do
    s := !s + !current;
    current := !current - 1
  done;
  !s
```

```
(* print n .. 0 *)
let count_down (n:int) =
  for i = n downto 0 do
    print_int i;
    print_newline()
  done


(* print 0 .. n *)
let count_up (n:int) =
  for i = 0 to n do
    print_int i;
    print_newline()
  done
```

Imperative loops?

```
(* print n .. 0 *)

let count_down (n:int) =
  for i = n downto 0 do
    print_int i;
    print_newline()
  done
```

```
(* for i=n downto 0 do f i *)

let rec for_down
         (n : int)
         (f : int -> unit)
            : unit =
  if n >= 0 then
   (f n; for_down (n-1) f)
  else
   ()


let count_down (n:int) =
  for_down n (fun i ->
    print_int i;
    print_newline()
  )
```

REFS AND MODULES

Types and References

Concrete, first-order type tells you a lot about a data structure:

- int ==> immutable
- int ref ==> mutable
- int * int ==> immutable
- int * (int ref) ==> 1st component immutable, 2^{nd} mutable
- ... etc

What about higher-order types?

- int -> int ==> the function can't be changed
  ==> what happens when we run it?

What about abstract types?

- stack, queue? stack * queue?

Functional Stacks

```
module type STACK =
  sig
    type ‘a stack
    val empty : unit -> ‘a stack
    val push : ‘a -> ‘a stack -> ‘a stack
    val peek : ‘a stack -> ‘a option
     ...
  end
```

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```

A functional interface takes in arguments, analyzes them, and produces new results

Imperative Stacks

```
module type IMP_STACK =
  sig
    type 'a stack
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    val peek : 'a stack -> 'a option
     ...
  end
```

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      ...
  end
```

When you see "unit" as the return type, you know the function is being executed for its side effects. (Like void in C/C++/Java.)

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Unfortunately, we can't always tell from the type that there are side-effects going on. It's a good idea to document them explicitly if the user can perceive them.

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Sometimes, one uses references inside a module but the data structures have functional (persistent) semantics

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```

This is a terrific way to use references in ML. Look for these opportunities

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Imperative Stacks

```
module ImpStack : IMP_STACK =
  struct
    type ‘a stack = (‘a list) ref

    let empty() : ‘a stack = ref []

    let push(x:’a)(s:’a stack) : unit =
       s := x::(!s)

    let pop(s:’a stack) : ‘a option =
      match !s with
       | [] -> None
       | h::t -> (s := t ; Some h)
  end
```

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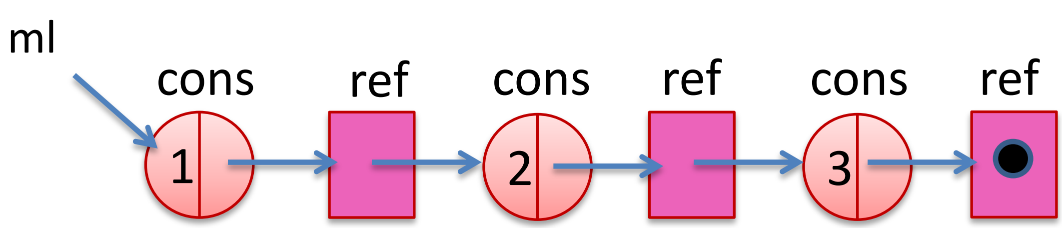
  end
```

Note: We don't have to make *everything* mutable. The list is an immutable data structure stored in a single mutable cell.

Fully Mutable Lists

```
type 'a mlist =
  Nil | Cons of 'a * ('a mlist ref)

let ml = Cons(1, ref (Cons(2, ref
    (Cons(3, ref Nil)))))
```



Fraught with Peril

```
type 'a mlist =
  Nil | Cons of 'a * ('a mlist ref)

let rec mlength(m:'a mlist) : int =
  match m with
  | Nil -> 0
  | Cons(h,t) -> 1 + length(!t)

let r = ref Nil ;;
let m = Cons(3,r) ;;
r := m ;;
mlength m ;;
```

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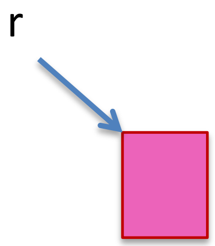

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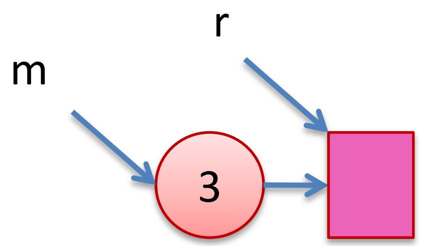

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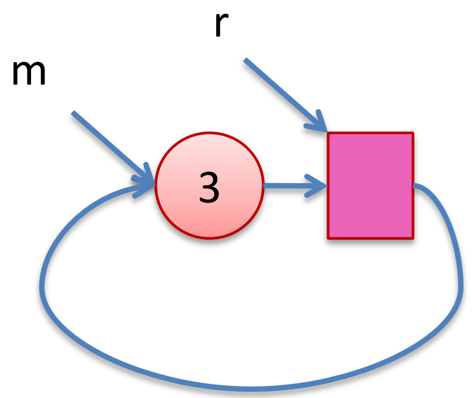

Add mutability judiciously

Two types:

```
type 'a very_mutable_list =
  Nil
| Cons of 'a * ('a very_mutable_list ref)
```

```
type 'a less_mutable_list = 'a list ref
```

The first makes cyclic lists possible, the second doesn't

- the second preemptively avoids certain kinds of errors.
- often called a *correct-by-construction design*

Is it possible to avoid all state?

Yes! (in single-threaded programs)

- Pass in old values to functions; return new values from functions ... but this isn't necessarily the most efficient thing to do

Consider the difference between our functional stacks and our imperative ones:

- `fnl_push : 'a -> 'a stack -> 'a stack`
- `imp_push : 'a -> 'a stack -> unit`

In general, we could pass a dictionary into and out of every function.

- That dictionary would map "addresses" to "values"
  - it would record the value of every reference
- But then accessing or updating a reference takes O(lg n) time.
- ... (wonder how bad the constant factors would be, too) ...

MUTABLE RECORDS AND ARRAYS

Records with Mutable Fields

OCaml records with mutable fields:

```
type 'a queue1 =
  {front : 'a list ref;
   back  : 'a list ref }

type 'a queue2 =
  {mutable front : 'a list;
   mutable back : 'a list}

let q1 = {front = [1]; back = [2]} in
let q2 = {front = [1]; back = [2]} in

let x = q2.front @ q2.back in

q2.front <- [3]
```

In fact: `type 'a ref = {mutable contents : 'a}`

Mutable Arrays

For arrays, we have:

`A.(i)`

- to read the ith element of the array `A`

`A.(i) <- 42`

- to write the ith element of the array `A`

`Array.make : int -> 'a -> 'a array`

- `Array.make 42 'x'` creates an array of length 42 with all elements initialized to the character `'x'`.

See the reference manual for more operations.

www.caml.inria.fr/pub/docs/manual-ocaml/libref/Array.html

Factoring!

```
let factor n =
  let s = int_of_float (sqrt (float_of_int n)) in
  let rec f i =
    if i<=s then
      if n mod i = 0 then
        Some i
      else
        f (i+1)
    else
      None
   in f 2
```

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```
factor 77 = Some 7
```

```
factor 97 = None
```

Caveats

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```

Caveat 1:
Many applications of prime numbers are for many-bit (500-bit, 2000-bit) numbers; OCaml ints are 31-bit or 63-bit, so you'd want a version of this for the bignums

Caveat 2:
This primitive factoring algorithm, already obsolete 2000 years ago, is not what you'd really use. Modern algorithms based on fancy number theory are much faster.

Caveat 3:
Even the fancy number-theory algs take superpolynomial time (as function of the number of bits in n)

Memoized factoring

```
let table = Hashtbl.create 1000

let memofactor n =
  try Hashtbl.find table n
  with Not_found ->
       let p = factor n
       in Hashtbl.add table n p; p
```

```
memofactor 77 = Some 7
```

```
memofactor 97 = None
```

Encapsulating the side effects

```
struct
  let table = Hashtbl.create 1000

  let memofactor n =
    try Hashtbl.find table n
    with Not_found ->
         let p = factor n
         in Hashtbl.add table n p; p

  let factor n = memofactor n
end
```

```
sig
  val factor : int -> int
end
```

The table is hidden inside the function closure.

There's no way for the client to access it, or know it's there.

We can pretend memofactor is a pure function.

OCaml Objects

```
class point =
  object
    val mutable x = 0
    method get_x = x
    method move d = x <- x + d
  end;;
```

```
let p = new point in
let x = p#get in

p#move 4;

x + p#get   (* 0 + 4 *)
```

http://caml.inria.fr/pub/docs/manual-ocaml-4.00/manual005.html

Xavier Leroy (OCaml inventor):

- No one ever uses objects in OCaml!
- Adding objects to OCaml was one of the best decisions I ever made!

SUMMARY

Summary: How/when to use state?

- A complicated question!
- In general, I try to write the functional version first.
  - e.g., prototype
  - don't have to worry about sharing and updates
  - don't have to worry about race conditions
  - reasoning is easy (the substitution model is valid!)
- Sometimes you find you can't afford it for efficiency reasons.
  - example: routing tables need to be fast in a switch
  - constant time lookup, update (hash-table)
- When I do use state, I try to *encapsulate* it behind an interface.
  - try to reduce the number of error conditions a client can see
    - correct-by-construction design
  - module implementer must think explicitly about sharing and invariants
  - write these down, write assertions to test them
  - if encapsulated in a module, these tests can be localized
  - *most of your code should still be functional*

Summary

Mutable data structures can lead to *efficiency improvements*.

- e.g., Hash tables, memoization, depth-first search

But they are *much* harder to get right, so don't jump the gun

- *updating in one place may have an effect on other places.*
- *writing and enforcing invariants becomes more important*.
  - e.g., assertions we used in the queue example
  - why more important? because the types do less ...
- *cycles in data (other than functions) can't happen until* we introduce refs.
  - must write operations much more carefully to avoid looping
  - more cases to deal with and the compiler doesn’t help you!
- we haven’t even gotten to the multi-threaded part.

So use refs when you must, but try hard to avoid it.